



## Optical characterization of the Advanced Virgo gravitational wave detector for the O4 observing run

This is the peer reviewed version of the following article:

*Original:*

Acernese, F., Agarwal, D., Aiello, L., Ain, A., Albanesi, S., Allene, C., et al. (2025). Optical characterization of the Advanced Virgo gravitational wave detector for the O4 observing run. APPLIED OPTICS, 64(17), 4710-4726 [10.1364/AO.555312].

*Availability:*

This version is available <http://hdl.handle.net/11365/1307474> since 2026-01-19T23:00:56Z

*Published:*

DOI: <http://doi.org/10.1364/AO.555312>

*Terms of use:*

Open Access

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. Works made available under a Creative Commons license can be used according to the terms and conditions of said license.

For all terms of use and more information see the publisher's website.

(Article begins on next page)

1 **Optical characterization of the Advanced Virgo**  
2 **gravitational wave detector for the O4**  
3 **observation run**

4 THE VIRGO COLLABORATION: F. ACERNESE, D. AGARWAL, L.  
5 AIELLO, A. AIN, S. ALBANESI, C. ALLÉNÉ, A. ALLOCCA, A. AMATO,  
6 C. AMRA, M. ANDIA, M. ANDRÉS-CARCASONA, T. ANDRADE, T.  
7 ANDRIĆ, S. ANSOLDI, S. ANTIER, E. Z. APPAVURAVTHER, F.  
8 ARMATO, N. ARNAUD, M. ASSIDUO, S. ASSIS DE SOUZA MELO, P.  
9 ASTONE, F. ATTADIO, F. AUBIN, G. AVALLONE, S. BABAK, F.  
10 BADARACCO, S. BAGNASCO, E. BAGUI, T. BAKA, G. BALLARDIN, B.  
11 BANERJEE, P. BARNEO, F. BARONE, M. BARSUGLIA, D. BARTA, A.  
12 BASTI, M. BAWAJ, M. BAZZAN, F. BEIRNAERT, M. BEJGER, D.  
13 BELARDINELLI, L. BELLIZZI, I. BENTARA, S. BERNUZZI, D.  
14 BERSANETTI, A. BERTOLINI, G. BEVILACQUA, U. BHARDWAJ, V.  
15 BIANCALANA, A. BIANCHI, A. BINETTI, S. BINI, M. BITOSSO, M.-A.  
16 BIZOUARD, F. BOBBA, G. BOILEAU, M. BOLDRINI, A. BOLLIAND, L.  
17 D. BONAVENTA, R. BONDARESCU, F. BONDU, R. BONNAND, V.  
18 BOSCHI, V. BOUDART, A. BOUDON, A. BOZZI, C. BRADASCHIA, M.  
19 BRANCHESI, T. BRIANT, A. BRILLET, M. L. BROZZETTI, G. BRUNO, F.  
20 BUCCI, O. BULASHENKO, T. BULIK, H. J. BULTEN, R. BUSCICCHIO,  
21 D. BUSKULIC, C. BUY, G. CABRAS, R. CABRITA, G. CAGNOLI, E.  
22 CALLONI, M. CANEPA, G. CANEVA SANTORO, E. CAPOCASA, G.  
23 CAPURRI, G. CARAPELLA, F. CARBOGNANI, M. CARPINELLI, G.  
24 CARULLO, J. CASANUEVA DIAZ, C. CASENTINI, S. CAUDILL, R.  
25 CAVALIERI, G. CELLA, P. CERDÁ-DURÁN, E. CESARINI, W. CHAIBI,  
26 E. CHASSANDE-MOTTIN, S. CHATY, C. CHECCHIA, P. CHessa, F.  
27 CHIADINI, G. CHIARINI, R. CHERICI, A. CHINCARINI, M. L.  
28 CHIOFALO, A. CHIUMMO, N. CHRISTENSEN, G. CIANI, M. CIEŚLAR,  
29 P. CIECIELAG, M. CIFALDI, R. CIOLFI, S. CLESSE, E. COCCIA, E.  
30 CODAZZO, P.-F. COHADON, S. COLACE, A. COLOMBO, M. COLPI, L.  
31 CONTI, I. CORDERO-CARRIÓN, S. COREZZI, S. CORTESE, A.  
32 COUINEAUX, J.-P. COULON, J.-F. COUPECHOUX, J. R. CUDELL, E.  
33 CUOCO, M. CUSINATO, B. D'ANGELO, S. D'ANTONIO, L. D'ONOFRIO,  
34 D. D'URSO, G. DÁLYA, P. DABADIE, S. DALL'OSSO, T. DAL CANTON,  
35 S. DAL PRA, S. DANILISHIN, V. DATTILO, A. DAUMAS, N. DAVARI, M.  
36 DAVIER, P. DAVIS, J. DEGALLAIX, S. DELÉGLISE, D. DELL'AQUILA, F.  
37 DELLA VALLE, W. DEL POZZO, G. DEMASI, A. DEPASSE, J. DE  
38 BOLLE, M. DE LAURENTIS, F. DE LILLO, F. DE MARCO, F. DE  
39 MATTEIS, R. DE PIETRI, R. DE ROSA, C. DE ROSSI, R. DE SIMONE,  
40 D. DIKSHA, J. DING, M. DI CESARE, L. DI FIORE, M. DI GIOVANNI, T.  
41 DI GIROLAMO, A. DI MICHELE, S. DI PACE, I. DI PALMA, F. DI  
42 RENZO, T. DOONEY, O. DOROSH, M. DRAGO, J.-G. DUCOIN, U.

43 DUPLETSKA, H. DUVAL, M. EBERSOLD, H. EINSLE, L. ERRICO, M.  
44 ESPOSITO, F. FABRIZI, F. FAEDI, V. FAFONE, G. FAVARO, M. FAYS, R.  
45 . FELICETTI, E. FENYVESI, T. FERNANDES, S. FERRAIUOLO, I.  
46 FERRANTE, F. FIDECARO, P. FIGURA, A. FIORI, I. FIORI, R.  
47 FITTIPALDI, V. FIUMARA, R. FLAMINIO, J. A. FONT, K.  
48 FRANCESCHETTI, N. FRANCHINI, S. FRASCA, F. FRASCONI, A.  
49 FRATTALE MASCIOLI, A. FREISE, O. FREITAS, G. G. FRONZÉ, B.  
50 GADRE, S. GALAUDAGE, R. GAMBA, B. GARAVENTA, J.  
51 GARCÍA-BELLIDO, J. GARGIULO, F. GARUFI, C. GASBARRA, F.  
52 GAUTIER, G. GEMME, A. GENNAI, V. GENNARI, ARCHISMAN GHOSH,  
53 P. GIRI, S. GKAITATZIS, F. GLOTIN, S. GOMEZ LOPEZ, B.  
54 GONCHAROV, M. GOSSELIN, R. GOUATY, A. GRADO, M. GRANATA, V.  
55 GRANATA, G. GRECO, A. C. GREEN, G. GRIGNANI, C. GRIMAUD, D.  
56 GUERRA, D. GUETTA, G. M. GUIDI, F. GULMINELLI, Y. GUO, P.  
57 GUPTA, N. GUTIERREZ, M. HANEY, S. HARIKUMAR, K. HARIS, T.  
58 HARMARK, J. HARMS, B. HASKELL, A. HEIDMANN, H. HEITMANN, P.  
59 HELLO, G. HEMMING, S. HILD, S. HOANG, D. HOFMAN, N. A.  
60 HOLLAND, L. HONET, C. A. HRISHIKESH, W.-F. HSU, V. HUI, L.  
61 IAMPIERI, G. A. IANDOLO, M. IANNI, A. IERARDI, A. IESS, G. IORIO,  
62 P. IOSIF, C. JACQUET, P.-E. JACQUET, J. JANQUART, K. JANSSENS,  
63 S. JARABA, P. JARANOWSKI, V. JUSTE, I. KHAN, W. KIENDREBEOGO,  
64 G. KOEKOEK, S. KOLEY, A. KOUSHIK, A. KRÓLAK, S. L.  
65 KRANZHOFF, J. KUBISZ, P. KUIJER, D. LAGHI, M. LALLEMAN, A.  
66 LARTAUD-VOLLARD, C. LAZARTE, C. LAZZARO, A. LA RANA, P.  
67 LEACI, A. LEMAÎTRE, M. LENTI, M. LEONARDI, M. LEQUIME, N.  
68 LEROY, N. LETENDRE, M. LETHUILLIER, K. LEYDE, M. LE JEAN, T.  
69 G. F. LI, J.-P. LOCQUET, L. LONDON, A. LONGO, D. LOPEZ, M.  
70 LOPEZ PORTILLA, M. LORENZINI, V. LORIETTE, G. LOSURDO, L.  
71 LUCCHESI, D. LUMACA, A. MACQUET, S. MAENAUT, R. MAGGIORE,  
72 M. MAGNOZZI, E. MAJORANA, N. MAN, V. MANGANO, M.  
73 MANTOVANI, M. MAPELLI, D. MARÍN PINA, F. MARCHESONI, C.  
74 MARINELLI, F. MARION, S. MARSAT, F. MARTELLI, M. MARTINEZ, V.  
75 MARTINEZ, A. MARTINI, L. MASSARO, A. MASSEROT, M.  
76 MASTRODICASA, S. MASTROGIOVANNI, T. MATCOVICH, Q. MEIJER, S.  
77 MELLAERTS, L. MERENI, M. MERZOUGUI, B. MESTICHELLI, A. MIANI,  
78 C. MICHEL, A. L. MILLER, E. MILOTTI, V. MILOTTI, Y. MINENKOV, LL.  
79 M. MIR, L. MIRASOLA, M. MIRAVET-TENÉS, C.-A. MIRITESCU, A. L.  
80 MITCHELL, L. MOBILIA, C. MONDAL, M. MONTANI, G. MORRAS, A.  
81 MOSCATELLO, B. MOURS, C. M. MOW-LOWRY, F. MUCIACCIA,  
82 SUVODIP MUKHERJEE, D. NABARI, A. NAGAR, D.  
83 NANADOUMGAR-LACROZE, V. NAPOLANO, I. NARDECCHIA, H.  
84 NAROLA, L. NATICCHIONI, G. NIERADKA, S. NISSANKE, E. NITOGLIA,  
85 F. NOCERA, J. NOVAK, J. F. NUÑO SILES, M. OERTEL, A.

86 OFFERMANS, G. OGANESYAN, R. OLIVERI, M. ORSELLI, A. OUZRIAT,  
87 C. PÉRIGOIS, L. PAIELLA, M. A. PALAIA, P. P. PALMA, C. PALOMBA,  
88 P. PALUD, R. PANAI, P. T. H. PANG, F. PANNARALE, F. PAOLETTI, A.  
89 PAOLONE, L. PAPALINI, G. PAPIGKIOTIS, A. PAQUIS, A. PARISI, D.  
90 PASCUCCI, A. PASQUALETTI, R. PASSAQUIETI, D. PASSUELLO, B.  
91 PATRICELLI, R. PEGNA, A. PEREGO, G. PERNA, A. PERRECA, J.  
92 PERRET, S. PERRIÈS, J. W. PERRY, D. PESIOS, C. PETRILLO, K. S.  
93 PHUKON, M. PIARULLI, L. PICCARI, M. PICHOT, M. PIENDIBENE, F.  
94 PIERGIOVANNI, L. PIERINI, G. PIERRA, V. PIERRO, M. PIETRZAK, M.  
95 PILLAS, F. PILO, L. PINARD, I. M. PINTO, M. PINTO, A. PLACIDI, E.  
96 PLACIDI, W. PLASTINO, R. POGGIANI, E. POLINI, E. PORCELLI, E. K.  
97 PORTER, R. POULTON, M. PRACCHIA, T. PRADIER, G. PRINCIPE, M.  
98 PRINCIPE, G. A. PRODI, P. PROSPERI, P. PROSPPOSITO, A. PUECHER,  
99 M. PUNTURO, P. PUPPO, G. QUÉMÈNER, G. RAAIJMAKERS, I.  
100 RAINHO, A. RAMOS-BUADES, P. RAPAGNANI, M. RAZZANO, M.  
101 RECAMAN PAYO, T. REGIMBAU, L. REI, A. RENZINI, P. RETTEGNO, B.  
102 REVENU, A. S. REZAEI, F. RICCI, M. RICCI, A. RICCIARDONE, S.  
103 RINALDI, F. ROBINET, A. ROCCHI, L. ROLLAND, R. ROMANO, A.  
104 ROMERO, S. RONCHINI, L. ROSA, D. ROSIŃSKA, D. ROZZA, P.  
105 RUGGI, E. RUIZ MORALES, P. SAFFARIEH, T. SAINRAT, S. SAJITH  
106 MENON, O. S. SALAFIA, L. SALCONI, F. SALEMI, M. SALLÉ, S.  
107 SALVADOR, N. SANCHIS-GUAL, F. SANTOLIBUENDO, F. SARANDREA, A.  
108 SASLI, P. SASSI, B. SASSOLAS, S. SAYAH, V. SCACCO, S. SCHMIDT,  
109 K. SCHOUTEDEN, M. SCIALPI, M. SEGLAR-ARROYO, D. SENTENAC,  
110 J. W. SEO, V. SEQUINO, M. SERRA, G. SERVIGNAT, A. SEVRIN, N. S.  
111 SHCHEBLANOV, L. SILENZI, A. SINGHA, V. SORDINI, F. SORRENTINO,  
112 F. SPADA, V. SPAGNUOLO, M. SPERA, P. SPINICELLI, D. A. STEER, J.  
113 STEINLECHNER, S. STEINLECHNER, N. STERGIIOULAS, P. STEVENS,  
114 F. STOLZI, G. STRATTA, M. SUCHENEK, S. SUDHAGAR, J. SURESH,  
115 B. L. SWINKELS, A. SYX, M. J. SZCZEPAŃCZYK, P. SZEWCZYK, M.  
116 TACCA, N. TAMANINI, E. N. TAPIA SAN MARTÍN, C. TARANTO, A.  
117 THEODOROPOULOS, J. TISSINO, P. TIWARI, A. TORRES-FORNÉ, M.  
118 TOSCANI, I. TOSTA E MELO, E. TOURNEFIER, M. TRAD NERY, A.  
119 TRAPANANTI, F. TRAVASSO, M. C. TRINGALI, G. TROIAN, A.  
120 TROVATO, L. TROZZO, K. TURBANG, M. TURCONI, C. TURSKI, H.  
121 UBACH, M. VACATELLO, M. VALENTINI, S. VALLERO, N. VAN BAKEL,  
122 M. VAN BEUZEKOM, M. VAN DAEL, J. F. J. VAN DEN BRAND, C. VAN  
123 DEN BROECK, M. VAN DER SLUYS, A. VAN DE WALLE, J. VAN  
124 DONGEN, H. VAN HAEVERMAET, J. V. VAN HEIJNINGEN, P. VAN HOVE,  
125 Z. VAN RANST, N. VAN REMORTEL, M. VARDARO, G. VEDOVATO, S.  
126 VENIKOUDIS, P. VERDIER, M. VEREECKEN, D. VERKINDT, P. VERMA,  
127 F. VETRANO, A. VEUTRO, A. VICERÉ, J.-Y. VINET, S. VIRET, A.  
128 VIRTUOSO, H. VOCCA, A. WAJID, M. WAS, M. WILS, J. WOehler, N.

129 YADAV, N. YADAV, A. ZADROŻNY, T. ZELENOWA, J.-P. ZENDRI, M.  
130 ZEOLI, M. ZERRAD, Y. ZHAO

131 *Virgo Collaboration*  
132 \*yuefan.guo@maastrichtuniversity.nl

133 **Abstract:** The Advanced Virgo Plus detector, an upgrade of the Advanced Virgo Detector, is a  
134 Dual-Recycled Fabry-Perot Michelson Interferometer characterized by 3 km long arm cavities.  
135 The main upgrades in view of the fourth Observing Run (O4) were the implementation of the  
136 Signal Recycling Cavity and the installation of the Frequency Dependent Squeezing system.  
137 Another upgrade was the increasing of the laser power at the input of the detector, which could  
138 lead to more severe thermal aberrations impacting the achievement of the interferometer optimal  
139 working point. Therefore, also the fine tuning of the Thermal Compensation System, optimized  
140 with respect to the one implemented for the O3 run, was also challenging. In order to achieve  
141 the best performance of such a sophisticated optical system, having a clear knowledge of all its  
142 optical parameters is crucial. The optical characterization of the detector in different working  
143 conditions could help in understanding its behavior and optimizing the global control system.  
144 Moreover, the characterization in different thermal conditions, i.e. different values of the input  
145 laser power or different configuration of the Thermal Compensation System, could provide a  
146 significant guidance to the optimization of the thermal tuning. In this paper, we will describe  
147 all the methodologies adopted for the optical characterization activities performed in Advanced  
148 Virgo Plus, presenting the experimental results for all the relevant parameters obtained in view of  
149 the O4 run.

150 © 2025 Optica Publishing Group

## 151 1. Introduction

152 In 2020, the Advanced LIGO [1] and Advanced Virgo [2] gravitational-wave (GW) detectors  
153 officially concluded the third scientific Observing Run (O3) and started an upgrade phase aimed  
154 to improve their sensitivity. The upgraded configuration of the Advanced Virgo detector is  
155 called Advanced Virgo Plus (AdV+) and the project was conceived to be implemented into two  
156 different phases. The phase I, planned to be accomplished for the fourth scientific Observing  
157 Run (O4) [3], aimed at reducing the impact of the quantum noise on the sensitivity. This target  
158 was planned to be reached thanks to the increase of the input power, the implementation of the  
159 Signal Recycling Cavity (SRC) and the installation of the Frequency Dependent Squeezing (FDS)  
160 system. The phase II target was initially planned to reduce the impact of the coating thermal noise  
161 on the sensitivity. It consisted in the modification of the optical geometry of the arm cavities,  
162 increasing the size of the optical laser beam impinging on the end test masses, that implies the  
163 installation of larger mirrors (in conjunction with a coating with potentially lower mechanical  
164 loss) [4]. The target is to conclude the phase II installation before the fifth scientific Observing  
165 Run (O5) [5]. However, due to issues discovered during the phase I commissioning, the plan  
166 for phase II has shifted toward increasing the robustness of the recycling cavities. Since Initial  
167 Virgo, the recycling cavities of Virgo are marginally stable with no focusing element in those  
168 cavities, similar to flat-flat cavities whose g-factor are close to 1, at the limit of the stability. Such  
169 configuration, limiting the number of mirrors, fits well in the infrastructure with constraint space.  
170 However, it is extremely sensitive to optical aberrations. So during the years 2023-2024, it was  
171 decided to install stable recycling cavities in Virgo before O5, so the current goal of phase II is  
172 now a large infrastructure change in the central part of the interferometer rather than installing  
173 large end test masses.

174

175 During the commissioning, to optimize the working point of the instrument, fundamental  
176 information should be deduced by performing measurements of its optical performance. A  
177 comprehensive optical characterization of a gravitational-wave interferometer includes the  
178 analysis of the Fabry-Pérot arm cavity, power recycling cavity, and dark fringe. The methods and  
179 results of these characterization will be discussed in Sections 3 and 4, respectively, following  
180 a detailed overview of the Advanced Virgo Plus detector for the O4 run in Section 2. To  
181 fully understand the performance of the interferometer, key parameters such as the length, the  
182 mode-matching, the finesse and the round-trip loss (RTL) of the Fabry-Pérot arm cavities, and  
183 the estimation of the Radius of Curvature (RoC) of the various test masses were measured  
184 through characterization of the arm cavities; the carrier and sidebands recycling gain, the arm  
185 circulating power and the power recycling cavity lengths were deduced from the characterization  
186 of the power recycling cavity and the contrast defect of the interferometer is calculated while the  
187 interferometer is under dark fringe condition. In this paper we describe the techniques used to  
188 perform the measurements of the various optical properties of Advanced Virgo and we present  
189 the obtained results during the commissioning phase in view of O4.

## 190 2. Advanced Virgo configuration for O4

### 191 2.1. Dual-recycled Fabry-Perot Michelson Interferometer

192 A sketch of the optical configuration of AdV+ phase I is shown in Figure 1: it is a Dual-Recycled,  
193 Fabry-Pérot, Michelson Interferometer (DRFPMI), working with a laser beam at a wavelength of  
194 1064 nm.

195 In this configuration, seven main optics are installed: a beam-splitter (BS), a power recycling  
196 mirror (PRM), a signal recycling mirror (SRM), an input mirror and an end mirror (WI and  
197 WE, respectively) for the arm West arm cavity (WA) and an input mirror and an end mirror  
198 (NI and NE, respectively) for the North arm cavity (NA). WA and NA are Fabry-Pérot (FP)  
199 cavities used to increase the effective arm length of the Michelson interferometer. The working  
200 point of the interferometer is tuned to have a destructive interference for the beams reflected  
201 by the arm cavities, the so-called dark fringe (DF) configuration. Therefore, most of the light  
202 is reflected back towards the laser source and lost. However, by installing a semi-reflective  
203 mirror (PRM) between the BS and the main laser, one can recycle this light and inject it back  
204 into the interferometer [6]. The PRM and the two input mirrors form a power recycling cavity  
205 (PRC), that allows to increase the effective power circulating inside the detector. The passage  
206 of a gravitational wave causes a differential change in the arm length, inducing a phase-shift of  
207 the recombined laser beam at the output port. Measuring this out-of-phase beam leaks at the  
208 output port leads to the detection of GWs. The installation of a semi-reflective mirror (SRM)  
209 between the BS and the output port of the detector allows to shape and optimize the detector  
210 frequency bandwidth for specific GW sources. The SRM and the input mirrors form a signal  
211 recycling cavity (SRC). Therefore, considering all the main mirrors, there are five longitudinal  
212 degrees of freedom that must be controlled:

- 213 •  $\mathbf{DARM} = \frac{L_N - L_W}{2}$ : Differential **ARM** length of the two Fabry-Pérot cavities in the arms.
- 214 •  $\mathbf{CARM} = \frac{L_N + L_W}{2}$ : Common **ARM** length, average of the lengths of the Fabry-Pérot  
215 cavities.
- 216 •  $\mathbf{MICH} = l_N - l_W$ : Difference between the lengths from the BS and, respectively, the NI  
217 and the WI mirrors, forming the **MICH**elson interferometer.
- 218 •  $\mathbf{PRCL} = l_{PR} + \frac{l_N + l_W}{2}$ : **Power Recycling Cavity Length**.
- 219 •  $\mathbf{SRCL} = l_{SR} + \frac{l_N + l_W}{2}$ : **Signal Recycling Cavity Length**.

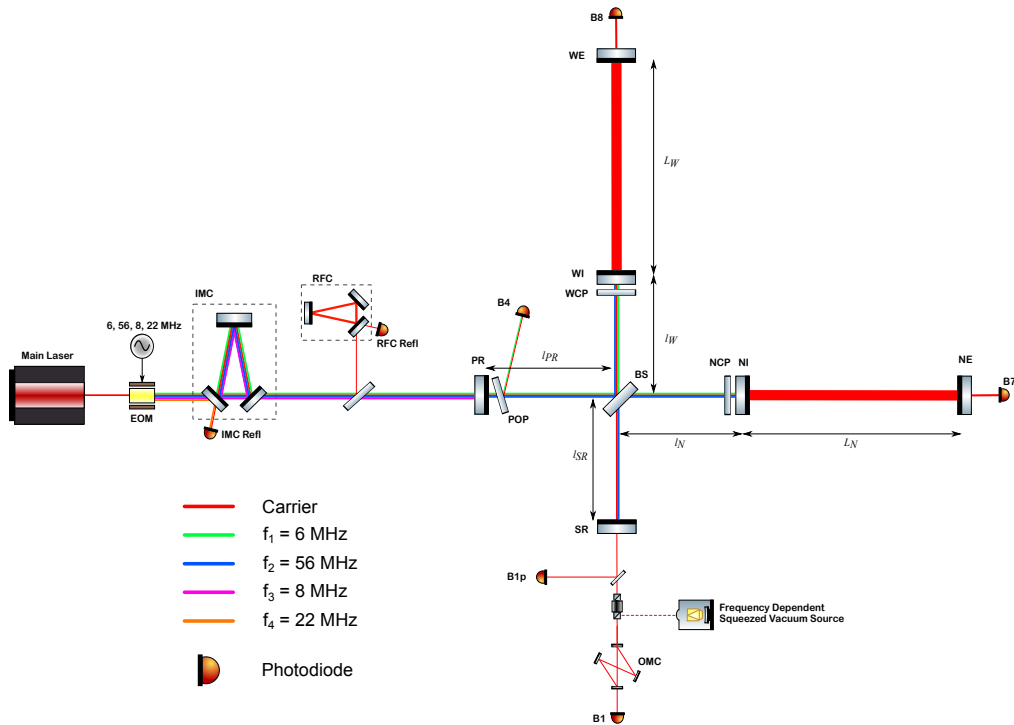


Fig. 1. Advanced Virgo Plus optical configuration for the O4 observing run. The laser beam travels towards the interferometer. It passes through the IMC cavity and the PR mirror, then the laser beam is split at the BS and directed towards the North and West arms. Then, the beams are reflected by the end mirrors of each arm and directed back to the BS, where the beams are recombined and split again towards the input port and the output port passing through the SR mirror. The carrier is shown in red. The different sidebands  $f_1$ ,  $f_2$ ,  $f_3$ ,  $f_4$  are indicated in green, blue, pink and orange respectively.  $L_N$ ,  $L_W$ ,  $l_{PR}$  and  $l_{SR}$  represents the length of the North arm cavity, West arm cavity, power recycling cavity and signal recycling cavity.

220 The two recycling cavities and the BS form the so-called central interferometer, including the  
 221 MICH, PRCL and SRCL degrees of freedom.  
 222 Other systems and relevant devices are installed in Advanced Virgo and shown in Figure 1: among  
 223 them, a high-power stabilized laser, few electro-optic modulators (EOMs), some photodiodes  
 224 (PDs) and auxiliary optical systems and cavities, such as the Input Mode Cleaner cavity (IMC).  
 225 The main laser is pre-stabilized and phase modulated at different frequencies,  $f_1 = 6$  MHz,  
 226  $f_2 = 56$  MHz,  $f_3 = 8$  MHz and  $f_4 = 22$  MHz, in order to provide the Pound-Drever-Hall  
 227 (PDH) [7] control signals for all the degrees of freedom of the whole experiment. The 6 MHz  
 228 sidebands are resonant inside the power recycling cavity, and anti-resonant inside the arm and  
 229 signal recycling cavities, the 56 MHz sidebands are resonant inside both the power and signal  
 230 recycling cavities, but anti-resonant in the arm cavities, the 8 MHz sidebands are anti-resonant  
 231 and, therefore, fully reflected by the interferometer and the 22 MHz sidebands, reflected by the  
 232 IMC, are used to control the IMC length. The input mode cleaner cavity filters out higher-order  
 233 modes, in order to have only the fundamental Gaussian mode entering into the interferometer.  
 234 Beam jitter and high-frequency noises are both suppressed as well. A reference cavity (RFC), in  
 235 combination with the IMC, is used to further stabilize the frequency of the main laser. A pick-off  
 236 of the beam circulating in the power recycling cavity is extracted just after the PRM through a

237 Pick-Off-Plate (POP) and acquired by a photodiode, named B4. The beams transmitted by the  
 238 two arm cavities are collected by photodiodes and cameras installed on auxiliary benches. They  
 239 are called B7 and B8, respectively for the NA and the WA. Two anti-reflective compensation  
 240 plates (NCP and WCP, respectively for the NI and WI) are located in front of each input test  
 241 mass, to be shined with the light of CO<sub>2</sub> auxiliary lasers used for the compensation of the thermal  
 242 aberrations induced by the high circulating power. The beams reflected back from the arms are  
 243 recombined on the beam splitter. The interference signal passes through the SR mirror and the  
 244 Output Mode Cleaner cavity (OMC) before arriving on the output detectors, named B1. A  
 245 pick-off of the output beam is extracted before entering the OMC cavity and is acquired by the  
 246 B1p sensors.  
 247 The optical properties of the main test masses, as measured at the Laboratoire des Matériaux  
 248 Avancés (LMA) [8] are shown in Table 1.

Table 1. Optical parameters of the main test masses individually measured at LMA before the installation in the detector. T (HR) is the power transmissivity of the Highly Reflective (HR) side of the mirror. R (AR) is the power reflectivity of the Anti-Reflective (AR) side of the mirror. RoC (HR) is the radius of curvature of the mirror on the HR side.

Parameters	NI	NE	WI	WE	PRM	SRM
T (HR)	1.377%	4.4 ppm	1.375%	4.3 ppm	4.835%	39.6%
R (AR)	32 ppm	133 ppm	58 ppm	155 ppm	160 ppm	151 ppm
RoC (HR)	1425 m	1695 m	1425 m	1696 m	1477 m	1443 m

249 Starting from the third Observing Run, the laser input power has been increased from about  
 250 10 W to 26 W, to reduce the impact of the shot noise in the high-frequency band. Higher  
 251 circulating power inside the detector also leads to more power absorbed in the substrate and  
 252 coating of the mirrors, inducing different optical aberrations. The thermal lensing increases the  
 253 optical path length in the substrates of the mirrors and the thermo-elastic deformation induces a  
 254 detuning of the Radius of Curvature (RoC) of the mirrors. These effects could shift the resonances  
 255 of the recycling cavity higher-order modes (HOMs), creating a degeneracy with the fundamental  
 256 mode and, consequentially, inducing an offset in the control signals, directly degrading the  
 257 performance of the detector. A Thermal Compensation System (TCS) [9] was implemented to  
 258 correct the optical aberrations and find the optimal optical working pointing of the detector.  
 259 The TCS consists of sensors and actuators. In particular, the sensors are the Hartmann wavefront  
 260 sensors [10] used to directly measure the wavefront aberrations, and the phase cameras, which  
 261 are installed near the B4 and B1p photodiodes and are used to study the carrier and the sideband  
 262 beams circulating into the recycling cavities. The actuators are the CO<sub>2</sub> projectors and the ring  
 263 heaters [11], used to induce on purpose a gradient of temperature in the optics to compensate  
 264 for the aberrations. The ring heaters, which are installed around the mirrors of the Fabry-Pérot  
 265 cavities, PRM and SRM, could reduce the radius of curvature of each mirror up to 100 m. The  
 266 CO<sub>2</sub> lasers are installed on two auxiliary benches close to the input mirrors. Their beams are  
 267 projected on the CPs and not directly on the mirrors, due to their residual intensity fluctuations  
 268 that could inject displacement noise into the interferometer, degrading the sensitivity. The  
 269 wavelength of the CO<sub>2</sub> lasers is about 10.6 μm, which is chosen since it is totally absorbed by the  
 270 fused silica substrate of the CPs. The CO<sub>2</sub> laser could generate different projection patterns: a  
 271 central heating (CH) beam whose shape is Gaussian, and a double ring produced by a double  
 272 axicon system (DAS). The rings produced by the DAS are used to compensate for the thermal  
 273 lensing effect inside the input mirrors. On the other hand, when the interferometer is not locked,

274 with a small amount of light inside, the beams produced by the CH are used to mimic the  
275 thermal lensing effect introduced by the main laser beam to keep a constant thermal state. The  
276 different TCS actuators are employed to tune specific parameters of the interferometer. The  
277 NE and WE ring heaters can adjust the g-factor of the arm cavities and help to move away the  
278 resonance of the fundamental mode to the resonances of the 8th and 9th higher-order modes,  
279 reducing cavity optical losses. The differential adjustment of these same ring heaters optimizes  
280 the interferometer's contrast defect by making the two arms more symmetric. Additionally,  
281 tuning the CH and DAS improves the recycling gains and corrects the imbalance between the  
282 lower and upper sidebands by reducing the thermal aberration in the recycling cavities. For the  
283 PRM and SRM, other actuators, the Central Heating Radius of Curvature Correction (CHRoCC)  
284 systems were implemented [12]. These actuators consist of black-body emitters projecting a  
285 circular pattern either on the POP installed close to the PRM or directly on the SRM, which can  
286 reduce the radius of curvature of the mirrors. In the final configuration for O4, the CHRoCC  
287 actuation is applied on the PRM, while the ring heater is switched on the SRM. Both actuators  
288 were tuned in their optimal working points.

289 One of the most important upgrade of the AdV+ phase I is the implementation of the SRC. As  
290 mentioned above, the installation of the SRM introduces a new degree of freedom (SRCL) that  
291 should be controlled. Therefore, a new control strategy has been implemented. This requires  
292 the installation of an Auxiliary Laser System (ALS) [13]. The ALS is composed of three  
293 phase-locked laser beams at 532 nm, generated by doubling the frequency of a pick-off beam of  
294 the Virgo main laser [14]: one is implemented near the Virgo main laser and two at the end of  
295 the WA and NA, respectively. The two latter beams at 532 nm are then injected with orthogonal  
296 polarizations from the end mirrors of the Fabry-Pérot arm cavities. Interfering them with the  
297 third beam allows to build the error signals to control DARM and CARM. It is used to arbitrary  
298 control the resonance condition of the two arm cavities, while acquiring the control of the central  
299 interferometer. During this process, the arm cavities are brought out of resonance for the infrared  
300 beam adding an offset on the CARM control loop. By changing this offset, we are able to perform  
301 relatively slow scans of a few Free Spectral Range (FSR) of the Fabry-Pérot cavities for the  
302 infrared beam.

## 303 *2.2. Different working conditions*

### 304 2.2.1. Different input power

305 In order to find the optimal working point of the detector with respect to the noise for the  
306 Observing Run O4, the interferometer has been tested and characterized using different input  
307 powers. In particular, input powers of  $P = 11$  W,  $P = 17$  W,  $P = 22$  W,  $P = 31$  W have been  
308 injected into the interferometer. The final input power used for the Observing Run O4 is 17 W.  
309 Different input powers imply different circulating powers in the detector and, therefore, different  
310 tunings of the TCS actuators.

### 311 2.2.2. Different TCS tuning

312 To minimise the noise coupling and the contrast defect, while optimizing the working point of the  
313 interferometer, in terms of both stability and performance, different tunings of the TCS actuators  
314 have been tested. More specifically, the EM heating ring are used to fine tuned the geometry of  
315 the arm cavities to prevent the resonances of the carrier fundamental mode at the same time as  
316 some higher order modes (modes of order 8 and 9 are worrisome). A small differential tuning of  
317 the rings can also help to improve the symmetry of the cavities and so reduce the power on the  
318 dark fringe. The DAS and CH actions are for the recycling cavities, where they compensate the  
319 thermal effects (dominated by the aberrations in the IMs) and helps to maximise the recycling  
320 gains and to reduce sidebands imbalance.

321 Moreover, also measurements have been done with these different tunings to better characterize  
322 the detector. For example, when the ring heaters on the end test masses were switched on with  
323 the same set points as during O3, we observed a clear change in the spacing among the resonance  
324 modes in the arm cavities with respect to the condition with the ring heaters off. Moreover, this  
325 change was different between the NA and WA. Furthermore, asymmetrical changes in the optical  
326 properties of the two arms have been observed also switching on the CH and DAS actuators.

### 327 2.2.3. Different SRM alignment

328 During commissioning, it was discovered that misaligning the signal recycling mirror could  
329 improve the contrast defects of the interferometer. By construction, a marginally stable SRC  
330 tuned to enlarge the bandwidth of the detector, brings all the higher order modes on resonance  
331 within the cavity. So to reduce the gain of the higher order modes, the signal recycled mirror is on  
332 purpose misaligned at the price of higher loss for the fundamental mode. Measurements reported  
333 in this paper include both ITF configurations with SRM aligned and with SRM misaligned

### 334 2.2.4. Hot and cold cavity

335 When the interferometer is fully and stably controlled in its final working point, high circulating  
336 power inside the Fabry-Pérot cavities leads to a temperature increase in the mirrors due to the  
337 absorption in their coatings and substrates. This non-uniform increase of temperature induces  
338 wavefront distortions, due to thermorefractive and thermoelastic effects. When the interferometer  
339 is stably controlled with the maximum recycling gain, the arm cavities are in a state referred  
340 to as ‘hot cavities’. On the other hand, at the beginning of the lock acquisition procedure,  
341 the amount of power circulating in the arm cavities is low since the power recycling mirror is  
342 misaligned. This state is referred to the ‘cold cavities’ configuration. In order to perform a  
343 complete characterization of the detector, optical properties are measured in both configurations.  
344 Ideally, measurements done as soon as the interferometer loses the control could represent the  
345 ‘hot cavities’ state. However, after the unlock happens, at least 3 to 4 minutes are needed to recover  
346 the status of the detector that allows to perform any measurement, inducing a not negligible  
347 decrease in the magnitude of the thermal effects.

## 348 3. Optical parameter characterization methods

349 The optical characterization of the AdV+ detector was carried out during the whole commissioning  
350 period towards the Observing Run O4. We give an overview of all the methods used to characterize  
351 the optical properties of the different cavities of the detector. In particular, we focus on the  
352 characterization of two main parts of the interferometer: the Fabry-Pérot arm cavities and the  
353 recycling cavities.

### 354 3.1. Arm cavities

355 For the characterization of the arm cavities, we present two different groups of measurements:  
356 Free Spectral Range scans and round trip loss measurements. The FSR scans can be performed  
357 in two different ways, the free swing of one of the cavity mirrors or the slow scan of the frequency  
358 of the auxiliary laser beam. The method used to measure the round trip loss is a comparison  
359 between the power reflected by the cavity when it is locked and unlocked.

#### 360 3.1.1. Free Spectral Range scan

361 From the scans of Free Spectral Range, different parameters can be extracted, such as: the length  
362 difference between two Fabry-Pérot cavities, the mode matching of the input beam into one  
363 cavity, the Radius of Curvature (RoC) of the arm cavity mirrors from the cavity g factor and the  
364 finesse of the cavity. In Figure 2, a typical FSR of the Virgo arm cavities is shown and it is used

365 to demonstrate the methods of estimating the optical parameters. Performing a scan of an FSR,  
 366 we can excite all the resonant modes of the cavity, in particular, the TEM00 and the HOMs of the  
 carrier and sideband beams, as highlighted in Figure 2. From Figure 2, we could get the height

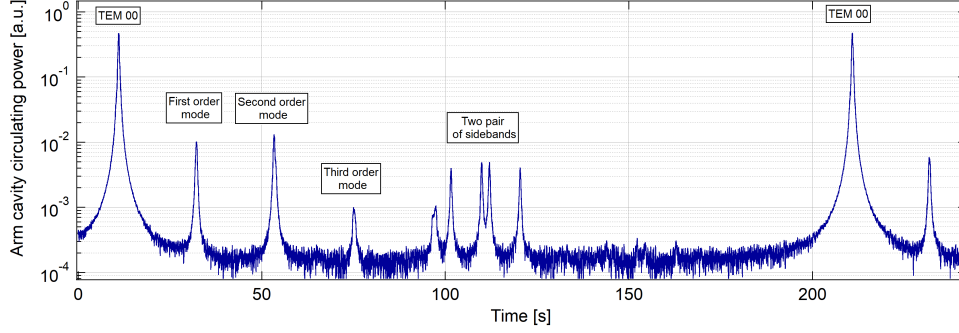


Fig. 2. FSR scan of one Fabry-Pérot arm cavity of the AdV+ detector with the fundamental and high-order modes of the carrier and the fundamental modes of the sidebands labeled.

367 of each peak and the distance between peaks. In order to estimate precise values of peak heights  
 368 and distances, we use the Gaussian approximation of the Airy function to fit each peak:  
 369

$$I(x) = \frac{a}{1 + \mathcal{F} \cdot \sin^2 \frac{\pi(x-x_0)}{f_{\text{FSR}}}} + o \quad (1)$$

370 where  $I(x)$  is the intensity of the peak as a function of frequency, due to the constant speed of  
 371 the scan, the  $x$  axis can be easily converted from time to frequency,  $\mathcal{F}$  is the finesse,  $a$  is the  
 372 amplitude of the peak,  $x_0$  is the expectation value of the distribution and  $o$  is an offset.  $f_{\text{FSR}} = \frac{c}{2L}$ ,  
 373 where  $c$  is the speed of light and  $L$  is arm cavity length, is the Free Spectral Range, i.e. the  
 374 distance between two consecutive peaks of the TEM00 mode. All the peaks are normalized to the  
 375 height of the TEM00 mode and all the distances between peaks are normalized to the FSR.

376 Before the implementation of the Auxiliary Laser System, the FSR scan was acquired from the  
 377 free swing of the end mirror of the cavity. In order to induce a larger swing, the mirror is usually  
 378 excited by the actuators, while the dampers, used to damp the mirror motion, are switched off.  
 379 Defining  $\tau_{\text{sto}}$  the storage (or decay) time of a cavity, which quantifies how long a photon can  
 380 stay inside the cavity before it is absorbed or transmitted, when the time needed for the cavity  
 381 to cross the resonance is shorter than  $\tau_{\text{sto}}$ , the peaks of the modes are distorted by the so-called  
 382 ringing effect [15]. An example of a distorted resonance peak with the ringing is shown in Figure  
 383 3. In this condition, the light does not have enough time to completely fill the cavity and the  
 384 beat between the incoming field and the evolving field stored in the cavity creates an oscillating  
 385 pattern, which is called the ringing. As a consequence, a precise measurement of finesse cannot  
 386 be done using a simple Airy function. However, it's still possible to estimate the storage time by  
 387 modelling the fields affected by the ringing as [16]:

$$E(t) = D(t)\Theta(t) \quad (2)$$

388 where:

$$D(t) = \left(\frac{i\pi}{\alpha}\right)^{\frac{1}{2}} t_1 A e^{-\frac{i\beta^2}{\alpha}}$$

$$\Theta(t) = \left(\frac{\alpha}{i\pi}\right)^{\frac{1}{2}} \sum_{n=0}^{\infty} e^{i\alpha(n-\beta\alpha)^2} \quad (3)$$

389 and:

$$\alpha = \frac{2\pi v}{\lambda f_{\text{FSR}}} \quad (4)$$

$$\beta = \frac{2\pi}{\lambda} vt - i \frac{L}{c\tau_{\text{sto}}}$$

390 where  $\lambda$  is the wavelength of the laser beam,  $v$  is the speed of the mirror, and  $n$  is the number of  
 391 round trips. Then the Finesse can be estimated using the storage time:

$$\mathcal{F} = \pi \cdot f_{\text{FSR}} \cdot \tau_{\text{sto}} \quad (5)$$

392 In Figure 3, the red dots present the fit of the ringing using the above mentioned equations.

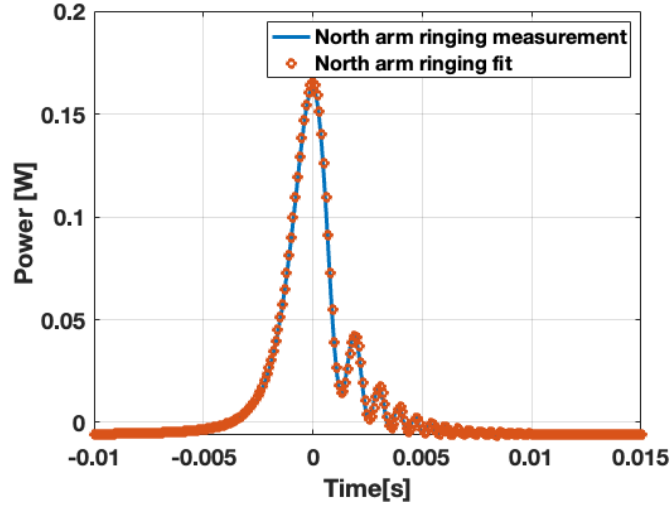


Fig. 3. Resonance peak of one of the AdV+ arm cavities distorted by the ringing effect.

393 The ALS implemented during the upgrade period after the Observing Run O3 provides a more  
 394 controllable way to perform scans of the FSR. The ALS allows to lock the arm cavities using a  
 395 beams at 532 nm, for which the finesse is lower than the one for the infrared beam. The first step  
 396 to perform the scan is to move the control of the arm cavities from the infrared to the green beam.  
 397 Using the infrared beam, the arm cavities are controlled and kept at the same length (DARM = 0)  
 398 by adjusting the mirror positions. Two Acousto-Optic Modulators (AOMs) are installed on the  
 399 two green beam paths before entering the cavities to control the green frequency. Acting on these  
 400 AOMs, the green frequency is shifted in order to have the green beam resonant inside the cavities.  
 401 Beating the green beam generated in the central area with the two transmitted by the cavities,  
 402 two new signals are produced and combined to form the green CARM and DARM error signals,  
 403 used to control the cavities instead of the infrared ones. After the lock using the 532 nm beam is  
 404 acquired, the frequency of the beam is varied at a constant speed in order to scan several FSRs of  
 405 the infrared beam. To perform slow scans, a gradual change on the offset of the CARM loop  
 406 is applied, acting on the input mirror positions. The cavity is kept on resonance for the green  
 407 beam acting on the AOMs. In this way, we can get a more precise measure of the height and the  
 408 position of the peaks of the fundamental mode, higher-order modes and sidebands.

#### 409 **Cavity length**

410 The Fabry-Pérot arm cavity length can be estimated through the sideband positions in the  
 411 FSR scan. From Figure 4, the sidebands  $f_2 = 56$  MHz of the WA and NA have a clear shift  
 412 between them. It is a zoom of the full free spectral range around the sidebands when the TEM00

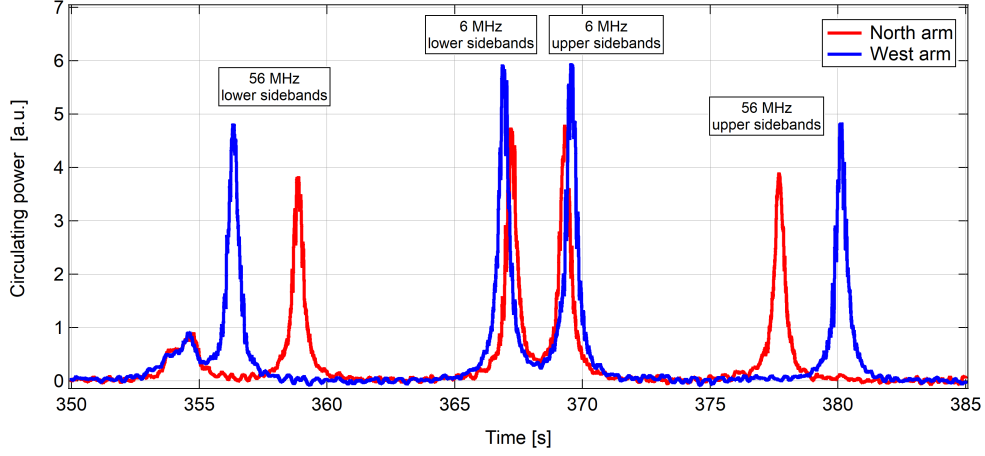


Fig. 4. FSR scan of the North (red) and West (blue) arms of the AdV+ detector zoomed on the 56 MHz and 6 MHz sidebands.

413 peaks of the carrier are superposed. The scan was repeated multiple times and the results were  
 414 consistent. This shift in the resonance of the sidebands is due to a small difference in the arm  
 415 length between the two cavities. Since the cavity length is also crucial for the calculation of other  
 416 optical parameters, we estimated their values from the following equation:

$$f_{\text{SB}} = N \times f_{\text{FSR}} + d_{\text{SB}} \times f_{\text{FSR}} \quad (6)$$

417 where  $f_{\text{SB}}$  is the frequency of the 56 MHz sidebands,  $N$  is an integer number and  $d_{\text{SB}}$  is the  
 418 normalized distance between the 56 MHz sideband peak and the TEM00 carrier peak. **Mode-**  
 419 **Matching**

420 Comparing the amplitude between the TEM00 and the higher-order modes in the FSR scan, we  
 421 can extract information for characterizing the arm cavities. Especially, the misalignment and  
 422 mismatching of the beam can be estimated, since both of them couple the fundamental mode  
 423 into higher-order modes. The presence of higher-order modes indicates imperfections in the  
 424 optical system which lead to a decrease in the circulating power and offsets in the error signals,  
 425 worsening the performance of the interferometer. Therefore, the mismatching and misalignment  
 426 are crucial information for the control of the interferometer. While improving the mode-matching  
 427 of the beam into the cavity is related to the fine tuning of the input telescope, reducing the residual  
 428 misalignment needs to consider the alignment control strategy, that is beyond the work of optical  
 429 characterization. However, from the scan, information to quantify the misalignment could be  
 430 extracted. It is given by the comparison between the peak height of the first order Hermite Gauss  
 431 mode and the fundamental mode. In this paper, our work was mainly focus on the mismatch  
 432 estimation.

433 Using the polar coordinates, the amplitude of the fundamental eigenmode of one cavity can be  
 434 written as [17]:

$$V_0(r) = \sqrt{\frac{2}{\pi}} \frac{1}{\omega_0} \exp\left(-\frac{r^2}{\omega_0^2}\right) \quad (7)$$

435 where  $\omega_0$  is the beam waist size. The field at the input of a cavity can be defined as  $\Psi(r) = AV_0(r)$ ,  
 436 where  $A$  is the amplitude coefficient. The higher-order modes can be written as the generalized  
 437 Laguerre polynomials  $L_l^p$  weighted by a Gaussian function, where  $p$  and  $l$  are the radial and  
 438 angular mode numbers. The mode order is defined as  $k = 2p + l$ . Since the angular dependence is

439 negligible for our purpose, we set  $l = 0$ . The amplitude of the first order radial Laguerre-Gaussian  
 440 (LG) mode, where  $p = 1$  and  $l = 0$ , can be written as:

$$V_{10}(r) = \sqrt{\frac{2}{\pi}} \frac{1}{\omega_0} \left( 1 - \frac{2r^2}{\omega_0^2} \right) \exp\left(-\frac{r^2}{\omega_0^2}\right) \quad (8)$$

441 Considering a mismatch caused by a small difference  $\epsilon$  between the waist size of the input beam  
 442 and the one of the cavity, we can define a new beam waist size  $\omega'_0$  as:

$$\omega'_0 = \omega_0(1 + \epsilon) \quad (9)$$

443 Thus, the input field  $\Psi(r)$  becomes:

$$\Psi(r) = A \sqrt{\frac{2}{\pi}} \frac{1}{\omega_0} (1 + \epsilon) \exp\left[-\frac{r^2}{\omega_0^2} (1 + \epsilon)^2\right] \quad (10)$$

444 Since  $\epsilon \ll \omega_0$ , the exponential can be expanded and considered only up to the first order of  $\epsilon$ ,  
 445 resulting in:

$$\Psi(r) \simeq A [V_0(r) + \epsilon V_{10}(r)] \quad (11)$$

446 which means that a small deviation of the beam waist size will couple the power from the  
 447 fundamental mode to the second order LG<sub>10</sub> mode. A similar result could be obtained considering  
 448 the mismatch caused by a small difference in the position of the waist. Therefore, the level of  
 449 mismatching is estimated from the comparison between the peak height of the second order  
 450 mode and the one of the fundamental mode.

#### 451 **Radius of Curvature**

452 The Radius of Curvature of the arm cavity mirrors is another parameter that has been regularly  
 453 measured during the commissioning period, when the TCS actuators were constantly tuned to  
 454 find the optimal working point of the interferometer. A precise measurement of the RoCs could  
 455 also be used in the tuning of the simulations for a better interpretation of the detector behaviour.  
 456 An estimation of the RoCs can be done starting from the resonance frequency of the different  
 457 peaks excited performing the scan of the cavity.

458 The different spatial modes have different resonant frequencies inside the cavity due to their  
 459 different round trip Gouy phase. The frequency difference between the TEM<sub>00</sub> mode and the  
 460 order  $k$  mode is given by:

$$\delta\nu = k\nu_0 \quad (12)$$

461 where:

$$\nu_0 = \frac{c}{2L} \frac{1}{\pi} \arccos \sqrt{g_{\text{IM}} g_{\text{EM}}} \quad (13)$$

462 where  $L$  is the cavity length,  $g_{\text{XX}}$ , the g-factor of the input and end mirror as defined by  
 463  $g_{\text{XX}} = (1 - L/\text{RoC}_{\text{XX}})$  with  $\text{RoC}_{\text{IM}}$  and  $\text{RoC}_{\text{EM}}$  are the Radius of Curvature of the input mirror  
 464 and end mirror of the cavity, respectively. To be noted the g-factor of the arm cavity is given by  
 465 the product  $g_{\text{IM}} g_{\text{EM}}$  and is around 0.87 in the case of the Virgo arm cavities with the nominal  
 466 RoC.

467 The distance between different modes normalized to the FSR is referred as the mode spacing.  
 468 For example, the mode spacing between the zero and the first order modes can be written as:

$$\Delta\nu_1 = \frac{1}{\pi} \arccos \sqrt{g_{\text{IM}} g_{\text{EM}}} \quad (14)$$

469 From Figure 2, using Equation 1, we could get the precise position of each peak and calculate  
 470 the distance  $\delta\nu$  between different orders. In order to calculate the Radius of Curvature of both

471 mirrors, we use the arm length  $L = 2999.8$  m and the RoC values listed in Table 1 as a starting  
 472 point, assuming the measurement error of both mirrors is the same. To be noted, as we are  
 473 measuring the g-factor of the arm cavity and not directly the one of each individual mirror, we  
 474 can not determine unambiguously the RoC of the input and end mirrors.

#### 475 **Finesse**

476 The finesse of a Fabry-Pérot cavity can be defined as the ratio between the FSR and the Full  
 477 Width Half Maximum (FWHM) of the TEM00 peak:

$$\mathcal{F} = \frac{f_{\text{FSR}}}{f_{\text{FWHM}}} \quad (15)$$

478 The finesse can be estimated using both the slow scans of the FSR and the fast swing of the cavity.  
 479 Using the slow FSR scan performed with the ALS, both the  $f_{\text{FSR}}$  and the  $f_{\text{FWHM}}$  can be precisely  
 480 measured and fit with a simple Airy function. We can directly use Equation 15 to calculate the  
 481 finesse.

482 Using the ringing effect caused by the fast swing of the cavity mirror, the storage time of the cavity  
 483 can be estimated from Equation 2, then finesse can be calculated using Equation 5. Statistically,  
 484 higher speed induces more ringings, giving a more precise estimation on the finesse.

485 The finesse of the Fabry-Pérot cavity is determined by the reflectivity of the input and end  
 486 mirrors, mathematically defined as:

$$\mathcal{F} = \frac{\pi \sqrt{r_{\text{in}} r_{\text{end}}}}{1 - r_{\text{in}} r_{\text{end}}} \quad (16)$$

487 where  $r_{\text{in}}$  and  $r_{\text{end}}$  are the amplitude reflectivity of the HR side of the input and end mirrors.  
 488 In the Advanced Virgo Plus detector, manufacturing reasons have led to an asymmetry in the  
 489 finesse between the two arms, significantly affecting the detector's Binary Neutron Star (BNS)  
 490 range [18]. Therefore, it is crucial to correct this asymmetry. The input mirrors of the Advanced  
 491 Virgo Plus interferometer have parallel surfaces, creating an etalon within the substrate. The  
 492 optical path length of this etalon causes a modification on the transmissivity on the input mirror,  
 493 consequentially changing the Finesse of the Fabry-Pérot cavity. The optical length inside the  
 494 input mirror substrates can be controlled by adjusting temperature of the mirror itself, since it  
 495 depends on the thermal expansion coefficient of the mirror material.

#### 496 3.1.2. Reflected power comparison of lock/unlock states of the cavity

497 Among the multiple ways to define the round trip loss, if we consider the energy conservation  
 498 principle, the loss is [19]:

$$\Lambda_{rt} = \frac{P_{\text{in}} - P_{\text{ref}} - P_{\text{trans}}}{P_{\text{circ}}} \quad (17)$$

499 where  $P_{\text{in}}$ ,  $P_{\text{ref}}$ ,  $P_{\text{trans}}$ ,  $P_{\text{circ}}$  are, respectively, the input, reflected, transmitted and circulating  
 500 power of the Fabry-Pérot cavity. Loss could be caused by different reasons, such as absorption in  
 501 substrates and coatings or clipping, which could be due to a beam size larger than the mirror  
 502 numerical aperture, or scattering from mirror surface defects. Measuring the RTL is needed to  
 503 confirm that the quality of the optics is within the requirements. Moreover, it is also crucial to  
 504 optimize the control of the interferometer, since loss could lead to smaller control signals and  
 505 recycling gain. Furthermore, the performance of the squeezing can be largely affected by the loss  
 506 level, higher loss means higher squeezing degradation and, as a consequence, less quantum noise  
 507 reduction.

508 As mentioned in [20], we can assume the loss as an increase of the end mirror transmission.  
 509 Therefore, we can estimate it from the reflectivity of the Fabry-Pérot cavity. In the case of a  
 510 perfect cavity, without any loss and the end mirror perfectly reflective, all the power must be

511 reflected back from the cavity. Therefore, in this case, with the cavity locked or unlocked, the  
512 reflectivity (in power) of the cavity is always 1. As soon as small optical losses are introduced,  
513 the reflected power decreases linearly when the cavity is locked. So to estimate the RTL, we  
514 make a relative measurement looking at the reflected power ( $P_{\text{ref}}$ ) with the same photodiode  
515 when the cavity is unlocked and locked. When the cavity is unlocked, the reflected power is  
516 equal to the incident power ( $P_{\text{ref}} = P_{\text{in}}$ ), which is assumed to be also the reflected power when  
517 there is no loss in the locked cavity and the end mirror is perfectly reflective. Then, we lock the  
518 cavity and the reflected power decreases, the drop in power is directly related to the RTL. In this  
519 condition, we define  $P_{\text{ref}} = P_{\text{lock}}$ . Since it is a relative measurement, we do not need to know the  
520 optical losses between the cavity and the photodiode used for measurement, that is the strong  
521 advantage of this technique.

522 We first estimate the end reflectivity as:

$$r_{\text{end}} = \sqrt{1 - T_{\text{end}} - \Lambda_{\text{rt}}} \quad (18)$$

$$\simeq \sqrt{1 - \Lambda_{\text{rt}}}$$

523 where  $r_{\text{end}}$  and  $T_{\text{end}}$  are the amplitude reflectivity and the power transmissivity of the end mirror.  
524 Then we could express the cavity reflectivity when it is locked as:

$$R_{\text{cav}} = \left[ \frac{r_{\text{in}} - r_{\text{end}}}{1 - r_{\text{in}} r_{\text{end}}} \right]^2 \quad (19)$$

$$\simeq \left[ \frac{r_{\text{in}} - \sqrt{1 - \Lambda_{\text{rt}}}}{1 - r_{\text{in}} \sqrt{1 - \Lambda_{\text{rt}}}} \right]^2$$

525 Considering a locked cavity, we can write  $P_{\text{lock}} = P_{\text{in}} \cdot R_{\text{cav}}$  and, combining with Equation 19,  
526 we can express the round trip loss as:

$$\Lambda_{\text{rt}} = \frac{T_{\text{in}}}{2} \frac{1 - \frac{P_{\text{lock}}}{P_{\text{in}}}}{1 + \frac{P_{\text{lock}}}{P_{\text{in}}}} \quad (20)$$

527  $P_{\text{lock}}$  and  $P_{\text{in}}$  can be acquired from both B4 and B1p PDs for the estimation of the round trip  
528 loss. This technique has been successfully used in the past to measure round trip losses in single  
529 Fabry-Pérot cavities [21], and more recently, also to characterize RTL in Virgo filter cavity,  
530 as a function of the beam position on the mirrors [22]. However, in a complex system as a  
531 gravitational wave detector, spurious beams also reach the PDs leading to unreliable results. An  
532 alternative way to estimate the round trip loss is through the recycling gain, as mentioned in the  
533 Section 3.2.1.

### 534 3.2. Recycling cavities

535 In addition to the arm cavities, we are also interested in the optical parameters of the recycling  
536 cavities, including the power recycling cavity length, the recycling gain of the carrier and  
537 sidebands, the arm circulating power and the contrast defect.

#### 538 3.2.1. Carrier recycling gain

539 The optical gain of the PRC is called the recycling gain and is defined as follow when the  
540 interferometer is controlled in the dark fringe working point:

$$G_{\text{PR}} = \left( \frac{t_{\text{PR}}}{1 - r_{\text{PR}} \cdot r_{\text{FP}}} \right)^2 \quad (21)$$

541 where  $r_{PR}$  and  $t_{PR}$  are the amplitude reflectivity and transmissivity of the PR mirror and  $r_{FP}$  is  
 542 the amplitude reflectivity of the Fabry-Pérot arm cavity, which can be expressed as:

$$r_{FP} = 1 - G_{FP} * \Lambda_{rt} \quad (22)$$

543 where  $G_{FP} \approx 286$  is the gain of the Fabry-Pérot cavity of AdV+ and  $\Lambda_{rt}$  is the round trip loss  
 544 defined in Equation 20. The recycling gain of the PRC can be estimated using the DC power  
 545 measured by the B7 or B8 photodiodes in transmission of two arm cavities in two different  
 546 interferometer configurations. The first one is when only the Fabry-Pérot arm cavities are locked,  
 547 and the second one is when the interferometer is fully locked. For example, if we consider the B7  
 548 photodiode, in the first working condition, the power measured can be written as:

$$P_{B7}^{FP} = P_{in} * T_{PR} * T_{BS} * G_{FP} * T_{NEM} \quad (23)$$

549 where  $P_{in}$  is the input power of the interferometer,  $T_{PR}$  is the power transmissivity of the PRM,  
 550  $T_{BS}$  is the power transmissivity of the BS,  $G_{FP}$  is the gain of the Fabry-Pérot arm cavity, and  
 551  $T_{NEM}$  is the power transmissivity of the North end mirror. On the other hand, the power measured  
 552 by the B7 photodiode when the interferometer is fully locked can be expressed as:

$$P_{B7}^{DRFPMI} = P_{in} * G_{PR} * T_{BS} * G_{FP} * T_{NEM} \quad (24)$$

553 Combining Equations 23 and 24, the recycling gain is:

$$G_{PR} = T_{PR} * \frac{P_{B7}^{DRFPMI}}{P_{B7}^{FP}} \quad (25)$$

554 where the value of  $T_{PR}$  is given in Table 1.

555 Using the measured carrier recycling gain, together with Equation 22 and 21, the round trip loss  
 556 of the arm cavities can be estimated.

557 From the recycling gain, another parameter which could also be estimated is the circulating  
 558 power inside the arm cavities, defined as:

$$P_{circ} = \frac{1}{2} * P_{in} * G_{PR} * G_{FP} \quad (26)$$

### 559 3.2.2. Sideband recycling gain

560 To measure the gain of the sidebands, it is essential to obtain a power measurement independent of  
 561 the carrier field. This is achieved using the phase camera, which can separately and simultaneously  
 562 measure the powers of the carrier and sidebands. In order to compute the sideband gain, the  
 563 measurement must be normalized against a simpler reference configuration, where the power can  
 564 be easily estimated. This reference is provided by what we call "single bounce configuration"  
 565 described in the next section.

566 To avoid multiple bounces in the recycling cavities, the single bounce configuration is often set  
 567 during the phase of commissioning. The single bounce condition for one arm cavity is created by  
 568 misaligning its end mirror, while the input and end mirrors of the other arm cavity are completely  
 569 misaligned. Power and signal recycling mirrors are also misaligned. In this state, the B1p and  
 570 B4 sensors only measure the reflection from one input mirror. This setup accounts for losses  
 571 introduced by all auxiliary optics from the input mirror to the sensors, and serves as a baseline  
 572 for characterizing the full interferometer.

573 The phase cameras once calibrated are a unique tool for estimating various optical parameters  
 574 of the interferometer, such as the sideband gain and contrast defect (CD). The intensity of  
 575 the different fields captured by the phase camera results from the beating signal between the

576 interferometer beam (comprising the carrier, 6 MHz, and 56 MHz sidebands) and a reference  
 577 beam, which is a pickoff of the Virgo main laser offset in frequency. This beating signal is then  
 578 demodulated at different frequencies to retrieve information about the individual components.  
 579 Comparing the sideband power between the single bounce and the fully locked configurations,  
 580 the sideband gain can be estimated. The sideband reference power measured by the B4 phase  
 581 camera in the single bounce configuration can be written as:

$$P_{B4}^{\text{ref}} = P_{\text{in}} * T_{\text{PR}} * T_{\text{BS}} * R_{\text{in}} * T_{\text{BS}} \quad (27)$$

582 where  $T_{\text{BS}}$  is the power transmissivity of the beam splitter, and  $R_{\text{in}}$  is the power reflectivity of the  
 583 input mirror HR surface. When the interferometer is fully locked, the sideband power measured  
 584 by B4 phase camera can be expressed as:

$$P_{B4}^{\text{DRFPMI}} = P_{\text{in}} * G_{\text{SB}} \quad (28)$$

585 where  $G_{\text{SB}}$  is the sideband gain. Combining Equations 27 and 28, the recycling gain for the  
 586 sidebands is:

$$G_{\text{SB}} = T_{\text{PR}} * T_{\text{BS}}^2 * R_{\text{in}} * \frac{P_{B4}^{\text{DRFPMI}}}{P_{B4}^{\text{ref}}} \quad (29)$$

587 The theoretical values of the gain of the sidebands can be calculated as the carrier one using  
 588 Equation 21, with the  $r_{\text{FP}} = 1$  as the sidebands are not resonant in the arms. The formula is  
 589 correct for the 6 MHz sidebands, which are almost on the dark fringe and not impacted by the  
 590 presence of SRM. For the 56 MHz sidebands, due to the Michelson length asymmetry explained  
 591 in Section 3.5 and since they are also resonant inside the SRC, although it is possible to derive an  
 592 analytical model, the calculation of  $r_{\text{FP}}^{56\text{ MHz}}$  is more complex and the values of the gain can be  
 593 easier estimated through simulations.

### 594 3.3. Contrast defect

595 The contrast defect of an interferometer is used to characterize the amount of excess light reaching  
 596 the dark port. It can be expressed as the ratio between the power of the excess light and the  
 597 circulating power inside the interferometer. Mathematically, it can be written as:

$$\text{CD} = 2 * \frac{P_{\text{min}}}{P_{\text{max}}} \quad (30)$$

598 where  $P_{\text{min}}$  and  $P_{\text{max}}$  are the minimum and maximum powers measured at the output of the  
 599 interferometer, and correspond to the power measured by the B1p and B4 sensors. Using the  
 600 power from the B1p and B4 phase cameras when the interferometer is in single bounce and in  
 601 the fully locked states, we can deduct the contrast defect expression of the interferometer before  
 602 the OMC as:

$$\text{CD} = 2 * T_{\text{SR}} * \frac{\frac{P_{B1p}^{\text{DRFPMI}}}{P_{B1p}^{\text{ref}}}}{\frac{P_{B4}^{\text{DRFPMI}}}{P_{B4}^{\text{ref}}}} \quad (31)$$

603 where  $T_{\text{SR}}$  is the power transmissivity of the signal recycling mirror.

### 604 3.4. Modulation depth of the sidebands

605 The modulation depth of the 56 MHz and 6 MHz sidebands is estimated by the relative power  
 606 between the sidebands and the carrier, defined as:

$$m = \sqrt{\frac{2 * P_{\text{SB}}}{P_{\text{car}}}} \quad (32)$$

607 where  $P_{\text{SB}}$  and  $P_{\text{car}}$  are the total powers of the sidebands and of the carrier. These powers can be  
 608 estimated through data collected using two different approaches: the slow FSR scans and the  
 609 phase camera in the single bounce configuration:

- 610 • From the slow FSR scans, as the one shown in Figure 2, the peak height of the 56 MHz  
 611 and 6 MHz sidebands can be estimated using Equation 1.
- 612 • The power of the sidebands can be acquired independently from the phase camera used in  
 613 the single bounce configuration. The result obtained acquiring the modulation depth from  
 614 the phase camera data can be used to confirm the estimation made analyzing the data from  
 615 the scan.

#### 616 3.4.1. Power recycling cavity length

617 During the interferometer steady-state operation, the carrier and the sidebands at 6 MHz and at  
 618 56 MHz have to resonate in the PRC. While the carrier resonance is guranteed by changing the  
 619 microscopic length of the PRC, using a Pound-Drever-Hall (PDH) locking error signal obtained  
 620 by beating the carrier with the anti-resonant sidebands at 8 MHz, the resonance condition of the  
 621 sidebands depends on the macroscopic length of the cavity. The sideband resonance condition  
 622 can be written as:

$$2 \frac{2\pi f}{c} l_{\text{PR}} + \pi = 2N\pi \quad (33)$$

623 where the additional  $\pi$  picked up after a round trip is due to the anti-resonant sidebands in the  
 624 arms, and  $N$  is an integer. The relation between the cavity length and the resonance frequency  $f$   
 625 can be defined as:

$$f = (2N - 1) \frac{c}{4l_{\text{PR}}} \quad (34)$$

626 If the macroscopic cavity length is such that the sidebands are resonant in the PRC, the optimal  
 627 demodulation phase of the PDH signal will have a difference of  $180^\circ$  between the carrier and the  
 628 sidebands. A change in the PRC length moves the sidebands away from the resonance resulting  
 629 in a shift of the optimal demodulation phase difference away from  $180^\circ$ . The PRC length change  
 630  $\Delta l_{\text{PR}}$  can be measured by checking how much the optimal PDH demodulation phase shift deviates  
 631 from the expected  $180^\circ$  when optimizing the demodulation frequency to lock the PRC purely on  
 632 the resonance of the carrier or of the sidebands. The relation between the phase shift  $\delta\phi_{\text{PDH}}$  and  
 633 the length mistuning depends on the PRC finesse  $\mathcal{F}_{\text{PR}}$  as:

$$\Delta l_{\text{PR}} = \frac{c}{4f\pi} \frac{2\mathcal{F}_{\text{PR}}}{\pi} \frac{\delta\phi_{\text{PDH}}}{180} \quad (35)$$

634 The measurement has been performed using the 6MHz RF signal on B2 photodiode, with only  
 635 the central interferometer locked.

#### 636 3.5. Michelson arm length (Schnupp) asymmetry

637 A macroscopic asymmetry in the lengths of the arms of the short Michelson interferometer,  
 638 known as Schnupp asymmetry, is required to transmit part of the sidebands toward the dark  
 639 port where they are use for generating control error signals. The beam reflected by the two  
 640 arm cavities travel different distances resulting in a relative phase shift, which depends on the  
 641 Schnupp asymmetry  $\delta l_{\text{MICH}}$  and on the modulation frequency  $f_i$  ( $i$  indicating which modulation  
 642 frequency) as:  $\delta\phi = 2\delta l_{\text{MICH}} \frac{2\pi f_i}{c}$  for one round trip. This can be measured by locking one arm at  
 643 a time, and recording the optimal demodulation phase for each cavity. From the difference in the  
 644 optimal demodulation phase of the two arms we can directly compute the Schnupp asimmetry as:

$$\delta l_{\text{MICH}} = \delta\phi \frac{c}{4\pi f_i} \quad (36)$$

645 The measurement has been performed using the 6MHz RF signal on B4 photodiode.

#### 646 4. Arm cavity optical parameter measurements

647 All the measurements were performed using the arm cavity slow FSR scan method during the  
648 commissioning activities needed to find the best working point of the interferometer in view  
649 of O4. Therefore, they have been performed with different input powers and TCS tunings, as  
650 described in section 2.2. The main results presented in the following sections have been obtained  
651 with the final configuration of the interferometer chosen for O4, meaning that the input power is  
652 17 W, and they are compared with the values obtained in different working conditions. The speed  
653 of the scan is about 1 kHz/s, while the cavity FSR is about 50 kHz. Each group of measurements  
654 consists of four full FSRs, obtained by scanning the frequency with a ramp. They have been  
655 repeated for some minutes to give enough statistic for more accurate parameter estimations.

##### 656 4.1. Arm cavity lengths

657 From the measurement of the FSR scan as illustrated by the figure 2, the value  $d_{SB}$  (position  
658 of the SBs in the FSR, see Equation 1) can be derived using the 56 MHz sidebands. We found  
659  $d_{SB}$  equals to  $0.4536 \pm 0.0002$  and  $0.4416 \pm 0.0002$ , respectively for the NA and WA. The  
660 corresponding arm lengths are  $2999.8203 \pm 0.0004$  m and  $2999.7884 \pm 0.0005$  m, deducted from  
661  $f_{FSR}$ . The difference between arm lengths  $L_N - L_W$  is  $0.0319 \pm 0.0006$  m. It was also confirmed  
662 in the simulation made with both OSCAR [23] and Finesse [24] that the NA is about 3 cm long  
663 than the WA.  
664

##### 665 4.2. Finesse

666 The design finesse value of the arm cavities can be calculated using Equation 16. The nominal  
667 value for the Advanced Virgo Plus cavities is about 450. Considering the etalon effect [25], the  
668 finesse of the North arm could change between 448 and 458, while the one of the West arm varies  
669 between 447 and 461.

670 From the slow scan, the finesse was estimated through Equation 15. Using the data of 6 scans,  
671 the values obtained for the estimations of the finesse of the cavities are  $459.51 \pm 6.19$  for NA and  
672  $454.73 \pm 8.14$  for WA.

673 The finesse has been estimated also using the ringing effect, with Equation 5. Figure 5 shows the  
674 histograms of the finesse estimated using the ringing. The results are  $452 \pm 4$  for NA and  $448 \pm 7$   
675 for WA, slightly different compared with the ones obtained using the slow scan. This could be  
due to different etalon conditions of the input mirrors between the two finesse measurements.

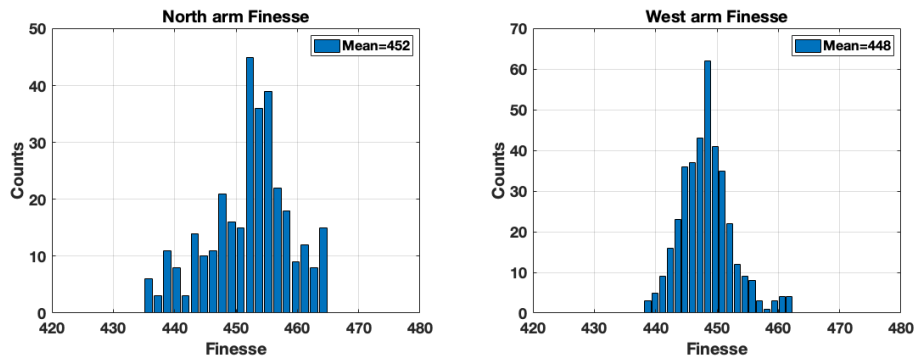


Fig. 5. Histograms of the estimated Finesse using the ringing effect.

677 **4.3. Mode-matching**

678 The values of the mode-matching of the beam into the Fabry-Pérot cavities are estimated  
 679 through the relative peak height between the second-order mode and the fundamental mode,  
 680 as described in section 3.1. For the cold cavities, the obtained results are  $97.19 \pm 0.15 \%$  for  
 681 NA and  $98.93 \pm 0.06 \%$  for WA, respectively. For the hot cavities, the obtained values are  
 682  $97.45 \pm 0.04 \%$  and  $99.12 \pm 0.01 \%$ . The difference between the two arms could be caused by  
 683 the slightly different RoCs of the mirrors, the different RoCs of the CPs and the independent  
 684 tuning of the TCS actuators.

685 **4.4. Radius of curvature**

686 The radii of curvature of the cavity mirrors are estimated through Equation 14, using slow FSR  
 687 scan and then measuring the distance between the second order modes and the fundamental one.  
 688 Once normalised by the FSR, this value is directly related to the g-factor of the cavity. We choose  
 689 to use the position of the second order modes in the FSR as it appears to be the most robust mode,  
 690 insensitive to misalignment.

691 The same technique can also be used to estimate the g-factor of the arm cavities when the full  
 692 power is circulating inside the interferometer. For that, we lock the interferometer for at least one  
 693 hour and then on purpose unlock the arms and start the slow scan. For the final O4 configuration  
 694 with 17 W of input power, the evolution of the g-factor for both cavities from hot to cold is shown  
 695 in figure 6.

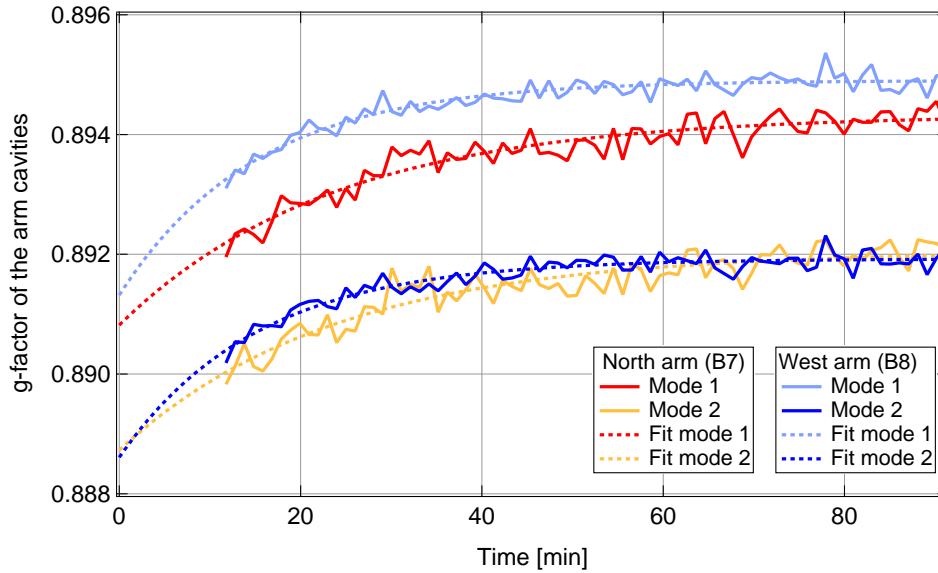


Fig. 6. Evolution of the g-factor of the arm cavities from hot to cold. This measurement is derived from the position of the second order modes inside the FSR. The position is fitted as 2 peaks per cavity to take into account the presence of astigmatism, usually visible on all the modes. So in total 4 curves are presented and fitted with a decay function (simple exponential).

696 For O4, the ring heaters around the end mirrors are switched on permanently to optimise  
 697 the contrast defect, by comparing the g-factor of the cavity with the ring heater off and on, it  
 698 is possible to estimate the change of RoC due to the action of the ring heater. Using the cold  
 699 g-factor from the figure 6, with a ring heater power of 7.1 W for the North arm and 8.2 W for

700 the West one, it was found that the measured RoC changes are respectively about  $-24.6 \pm 0.9$  m  
 701 and  $-27.3 \pm 0.6$  m for the end mirrors. To be noted that before the start O4, the ring heater were  
 702 further tuned by 0.1 W but no FSR scan was done after.

703 The radii curvature of the mirrors in the hot cavity configuration were also estimated from  
 704 Figure 6 for both cavities. The time  $t = 0$  in the abscissa represents the moment of the unlock  
 705 in the interferometer. Fitting the data with a decay function ( $f(t) = a \times \exp(-bt) + c$ ) with the  
 706 coefficients  $a$ ,  $b$  and  $c$ , the g-factor in the hot cavity configuration can be estimated by  $f(t = 0)$ .  
 707 The three coefficients represent:  $a$ , the change in the g-factor between the hot and the cold cavity  
 708 configurations.  $b$ , the inverse of the time constant and,  $c$ , g-factor in the cold state. In Figure 6,  
 709 the fits are shown in red and blue dashed lines for the North and West arms. 4 fits are done in  
 710 total, 2 per arms one for each second order modes but the same time constant is used for a given  
 711 arm.

Table 2. Fitted parameters from the evolution of the cavity g-factor with data from figure 6.

Mode	$a \times 1000$	$b \times 10000$	$c$
NA Mode 1	$-3.5 \pm 0.2$	$-7.0 \pm 0.6$	$0.8943 \pm 0.0001$
NA Mode 2	$-3.3 \pm 0.2$	$-7.0 \pm 0.6$	$0.8920 \pm 0.0001$
WA Mode 1	$-3.6 \pm 0.3$	$-10.9 \pm 0.6$	$0.8949 \pm 0.0001$
WA Mode 2	$-3.3 \pm 0.2$	$-10.9 \pm 0.6$	$0.8919 \pm 0.0001$

712 From the parameter  $a$  in table 2, it is obvious that the thermal effect is quite similar for both  
 713 arms. Assuming the change in RoC distributes equally on the input and end mirrors (which is not  
 714 rigorously true due to the difference in beam size), we can conclude the RoC change per mirror is  
 715 about  $1.5 \pm 0.1$  m for the North and West arms.

716 An overlapping double resonance peak was observed in some first higher-order modes during  
 717 the scan, suggesting astigmatism, leading to different resonance conditions in 2 different directions.  
 718 That is the reason why in the analysis of the cavities g-factor, 2 peaks are fitted for the second  
 719 order mode. From the difference in the  $c$  parameters for the same arm, one can express the  
 720 astigmatism as the difference of RoC in 2 different directions. Based on the value from the table  
 721 2, we found the RoC difference for North arm to be  $1.8 \pm 0.2$  m and  $2.4 \pm 0.2$  m for the West arm  
 722 if we assume the astigmatism to be induced by one single mirror of the cavity.

723 To further investigate the level of astigmatism in each arm, angular misalignments were applied  
 724 separately (and sequentially) to each degree of freedom of the cavity mirror while scanning the  
 725 cavity. As the result we can excite only one first order mode for example the TEM10 as shown in  
 726 figure 7. The resulting spacing for the first-order mode resonances TEM10 and TEM01 were  
 727 computed. From this result, the difference of RoC between the 2 perpendicular directions is  
 728 found to be  $1.3 \pm 0.7$  m and  $2.5 \pm 0.6$  m, respectively for the NA and WA arms. It should be  
 729 noted that the astigmatism is not necessary along the direction of misalignment of the mirrors  
 730 and so the analysis could be more complex but the values are compatible with the analysis with  
 731 the second order modes.

#### 732 4.5. Round trip losses

733 Using Equation 20, the round trip loss of a Fabry-Pérot cavity can be calculated comparing the  
 734 power acquired by the photodiodes in reflection, while the cavity is locked and unlocked. In  
 735 the case of AdV+, the usual photodiodes used to collect the data are the B4 or the B1p ones.  
 736 The data is acquired for one cavity at a time, with cavity locking/unlocking periods of 30 s. The

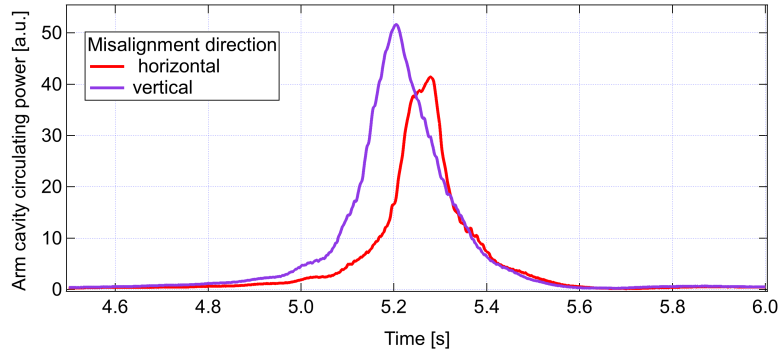


Fig. 7. Zoom on the resonances of the first order mode during a FSR scan with the arm cavity misaligned in the horizontal and vertical directions. The shift in the resonances of the mode TEM 10 and 01 could be explained by the presence astigmatism in the arm cavity.

737 measurements are repeated few times to obtain some statistic. However, as mentioned in Section  
 738 3.1.2, the presence of different spurious beams when the cavity is locked and unlocked led to  
 739 not reliable estimations of the round trip loss. In Figure 8, examples of such measurements for  
 740 cold cavities are shown using two B4 photodiodes. The estimated RTL is  $100 \pm 10$  ppm for both  
 741 the NA and the WA. The cold arm cavity RTL was measured to be 55 ppm [26]. Compared to  
 742 simulations based on the characterization of individual mirrors, including low-frequency surface  
 743 aberrations and scatterometer data, an additional loss of around 10 ppm per mirror was observed  
 744 inside the interferometer. In the next section, the results obtained for the estimation of the RTL  
 starting from the power recycling gain will be presented.

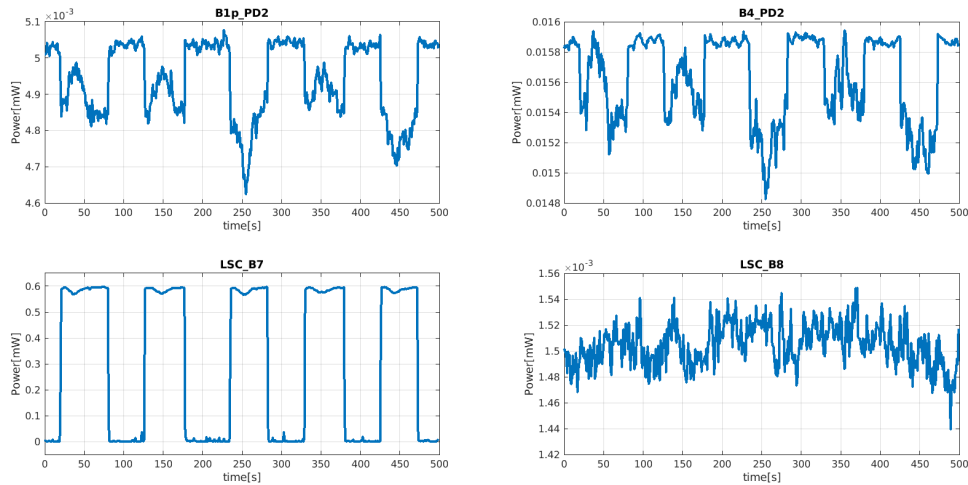


Fig. 8. The two top plots show an example of the RTL measurement of the North arm, with 17 W of input power, using two different photodiodes: one is installed at the dark port along the B1p path, and the other one is on the B4 path, a pick-off extracted in the PRC. The two bottom two plots show the transmissions of the NA and WA, respectively acquired by the B7 and B8 photodiodes. The two different phases of the measurement correspond to lock and unlock of the NA cavity, while the WA cavity is completely misaligned.

745 **5. Recycling cavity optical parameter measurements**

746 *5.1. Power recycling cavity length*

747 Based on the estimation from the analytical model and the simulations, the optimal power  
748 recycling cavity length  $l_{PR}$  should be around 12.073 m. Applying Equation 35, with the 56 MHz  
749 sidebands, a demodulation phase shift of 0.4 rad was measured, which corresponds to about  
750  $3.3 \pm 0.5$  mm of length mistuning from the optimal  $l_{PR}$  without a definite sign.

751 *5.2. Michelson length (Schnupp) asymmetry*

752 By locking the cavities formed by PR and NI or PR and WI, one at a time, and optimizing the  
753 demodulation phases with a precision of  $0.1^\circ$ , the asymmetry in the Michelson arm length, i.e.  
754 the Schnupp asymmetry, was measured to be  $22.9 \pm 0.4$  cm.

755 *5.3. Carrier gain*

756 The recycling gain of the carrier has been estimated through the Equation 25 at different input  
757 powers of the interferometer during the whole commissioning period. For each input power, the  
758 power transmitted by NA has been measured by the B7 photodiode in two different states of  
759 the interferometer: the first one is when only the arm cavities are locked and the second one is  
760 when the full interferometer is locked. The results are shown in the left plot of Figure 9. We  
761 can observe that increasing the input power, the recycling gain is decreasing due to partially  
762 uncompensated thermal aberration. The same trend has been observed also in LIGO [27]. The  
763 right plot of Figure 9 shows the circulating power in the arms as a function of the input power.  
764 One thing to be noted is that the North end mirror of the AdV+ detector was replaced in June  
765 2023. The blue and red dots in the figure represent the measurements done before and after the  
766 mirror replacement. A slight increase in the recycling gain was measured after changing the  
767 North end mirror. From the recycling gain, we also estimated the round trip loss of the arm  
768 cavities listed in Table 4, assuming all the gain reduction are induced only by the round trip loss.

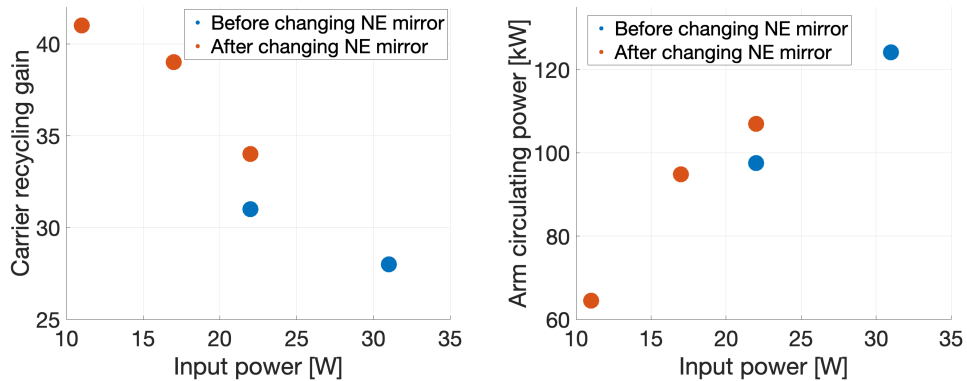


Fig. 9. Left plot: Measured recycling gain of the carrier for different input powers of the interferometer. Right plot: Arm circulating power inside the interferometer for different input powers.

769 **5.4. Sidebands gain**

770 The recycling gain of the sidebands has been calculated through Equation 29 with different input  
771 powers. Assuming the transmissivity of the beam splitter  $T_{BS} = 0.5$ , each sideband power has  
772 been acquired by the B4 phase camera in two interferometer states: the first one is the single  
773 bounce configuration, to have a reference power, while the second one is with the interferometer  
774 fully locked. For the second state, the measurements were taken both right after the lock (cold  
775 state) and one hour after the lock (hot/steady state). In Table 3, the values estimated for the  
776 gains of the 56 MHz and 6 MHz sidebands for the cold and hot cavity configurations are listed,  
777 compared to the theoretical ones. These values are the averages between the values of the upper  
778 and the lower sidebands. The upper and lower sidebands were well balanced for the 6 MHz,  
779 for which the difference was less than 1.6%. On the other hand, for 56 MHz, the difference  
780 was around 11%. For the final O4 configuration, the recycling gain of the 6 MHz sideband  
781 was  $24 \pm 0.7$ , while the one of the the 56 MHz sideband was  $17 \pm 0.9$  for for the hot cavity  
782 configuration.

Table 3. Comparison of the recycling gain for the carrier, 6 MHz sidebands and 56 MHz sidebands among the theoretical values, cold (right after the lock of the full interferometer) and hot (1 h after the lock of the full interferometer) states for 17 W of input power. The theoretical carrier and sidebands recycling gain value is given for a perfect interferometer with the unique defect of 60 ppm of round trip loss in the arm cavities

	$G_{\text{car}}$	$G_{6\text{MHz}}$	$G_{56\text{MHz}}$
Theoretical	42	72	53
Cold state	42	48	26
Hot state	39	24	17

783 **5.5. Arm circulating power estimation**

784 Using the recycling gain of the carrier estimated in Section 5.3 and Equation 26, we calculated  
785 the circulating powers in the arm cavities for different interferometer input powers. The results  
786 are shown on the right plot of Figure 9. For the final O4 configuration, the circulating power is  
787  $96 \pm 1$  kW.

788 **5.6. Contrast defect**

789 The contrast defects for different interferometer input powers have been estimated through  
790 Equation 31. The power of the carrier measured by the B1p and B4 phase cameras in two  
791 different interferometer states are used: the first state is the single bounce configuration, while the  
792 second state refers to the fully locked interferometer with the signal recycling mirror misaligned  
793 by approximately  $2 \mu\text{rad}$  in the vertical direction, resulting in a 75% reduction in the contrast  
794 defect. For the final O4 configuration, the optimized contrast defect of the interferometer is  
795  $(2.86 \pm 0.24) \times 10^{-4}$  before the OMC, and  $6 \times 10^{-8}$  after the OMC. Although not the focus of  
796 the article, optical simulations of the full interferometer were conducted, including wavefront  
797 distortions from the cold mirrors (assuming perfect TCS compensation of the thermal effects).  
798 The results showed that the measured contrast defect after the SRM was twice higher than the  
799 estimate provided by the simulations. More accurate simulations would require to include all the  
800 wavefront distortions including the effects of the optical absorption and the TCS corrections.

801 **5.7. Summary**

802 In Table 4, we summarize all the parameters mentioned in Section 5 that have been measured with  
 803 different interferometer input powers during the commissioning period. Except the measurements  
 804 of 31 W of input power, all the other measurements were performed after the replacement of  
 805 the North end mirror. Moreover, the contrast defects for all the input power except 31 W were  
 measured with misaligned SRM.

Table 4. Summary of all the mentioned parameters measured with different input powers during the O4 commissioning period. Except for 31 W of input power, all the results are given with SR misaligned.

$P_{in}$ (W)	$G_{PR}$	$G_{SB}$ 6 MHz	$G_{SB}$ 56 MHz	$P_{circ}$ (kW)	CD	RTL (ppm)
11	41	29	20	65	$4.2 \times 10^{-4}$	61
17	39	24	17	96	$2.9 \times 10^{-4}$	68
22	34	22	17	108	$7.3 \times 10^{-4}$	85
31	28	27	18	125	$1.8 \times 10^{-3}$	112

806

807 **6. Modulation depth measurements**

808 The modulation depth of the sidebands was estimated through two different measurements, as  
 809 mentioned in Section 3.4. The obtained results are summarized in Table 5 for the 56 MHz and  
 810 6 MHz upper and lower sidebands. For the phase camera method, data were acquired using both  
 811 the B1p and B4 sensors. The estimated values of the modulation depth obtained using the two  
 methods listed in the table are in a good agreement.

Table 5. Modulation depth of the 56 MHz and 6 MHz upper and lower sidebands estimated analysing the data obtained both through the FSR scan, and from the B1p and the B4 phase cameras used in the single bounce configuration. The results from the different measurements are in good agreement.

	FSR scan	B1p phase camera	B4 phase camera
56 MHz upper sideband	0.189	0.193	0.197
56 MHz lower sideband	0.183	0.193	0.197
6 MHz upper sideband	0.201	0.206	0.210
6 MHz lower sideband	0.202	0.206	0.210

812

813 **7. Conclusion**

814 In this paper, we describe the methods adopted for the characterization of the optical parameters  
 815 of the AdV+ detector during the commissioning phase in view of the O4 Observing Run. In  
 816 particular, for the characterization of the Fabry-Pérot arm cavities, the implementation of an  
 817 Auxiliary Laser System provided the possibility of performing slow scans of the free spectral  
 818 range. Using this method we could estimate the arm cavity length, the mode-matching and  
 819 alignment between the input beam and the arm cavities, the radius of curvature of the cavity  
 820 mirrors and the cavity finesse. All the measurements needed for the characterization could be

821 performed with different configurations of the interferometer and various tunings of the Thermal  
822 Compensation System. Also the estimation of some parameters of the recycling cavities was  
823 performed in different interferometer configurations. Among these parameters we could estimate  
824 the recycling gain for both the carrier and the sidebands, the power circulating in the arm cavities  
825 and the contrast defect. In particular, in this papers the optical parameters measured in the final  
826 Advanced Virgo Plus configuration optimized for O4 have been presented. These measurements  
827 triggered a simulation work for a more deeper understanding of the detector, which is out of the  
828 scope of this paper and will be detailed in another article.

## 829 Acknowledgments

830 The authors gratefully acknowledge the Italian Istituto Nazionale di Fisica Nucleare (INFN), the  
831 French Centre National de la Recherche Scientifique (CNRS) and the Netherlands Organization  
832 for Scientific Research (NWO), for the construction and operation of the Virgo detector and the  
833 creation and support of the EGO consortium. The authors also gratefully acknowledge research  
834 support from these agencies as well as by the Spanish Agencia Estatal de Investigación, the  
835 Consellera d'Innovació, Universitats, Ciència i Societat Digital de la Generalitat Valenciana  
836 and the CERCA Programme Generalitat de Catalunya, Spain, the National Science Centre of  
837 Poland and the European Union – European Regional Development Fund; Foundation for Polish  
838 Science (FNP), the Polish Minister of Science, the Hungarian Scientific Research Fund (OTKA),  
839 the French Lyon Institute of Origins (LIO), the Belgian Fonds de la Recherche Scientifique  
840 (FRS-FNRS), Actions de Recherche Concertées (ARC) and Fonds Wetenschappelijk Onderzoek  
841 – Vlaanderen (FWO), Belgium, the Aristotle University of Thessaloniki (AUTH), the European  
842 Commission. The authors gratefully acknowledge the support of the NSF, STFC, INFN, CNRS  
843 and Nikhef for provision of computational resources. The authors would like to extend special  
844 thanks to the control room operators, whose efforts made this work possible: Andrea Magazzu,  
845 Beatrice Montanari, Fabio Gherardini, Francesco Berni, Gianmatteo Sposito, Lorenzo Irace and  
846 Nicola Menzione.

## 847 References

- 848 1. B. P. Abbott *et al.*, “LIGO: the laser interferometer gravitational-wave observatory,” *Reports on Prog. Phys.* **72**,  
849 076901 (2009).
- 850 2. F. Acernese *et al.*, “Advanced Virgo: a second-generation interferometric gravitational wave detector,” *Class.*  
851 *Quantum Gravity* **32**, 024001 (2015).
- 852 3. T. V. Collaboration, “Advanced virgo plus phase i design report,” [https://tds.virgo-gw.eu/?content=](https://tds.virgo-gw.eu/?content=3&r=20962)  
853 [3&r=20962](https://tds.virgo-gw.eu/?content=3&r=20962).
- 854 4. T. V. Collaboration, “Advanced virgo plus phase ii design report,” [https://tds.virgo-gw.eu/?content=](https://tds.virgo-gw.eu/?content=3&r=20962)  
855 [3&r=20962](https://tds.virgo-gw.eu/?content=3&r=20962).
- 856 5. B. P. Abbott, R. Abbott, T. D. Abbott, *et al.*, “Prospects for observing and localizing gravitational-wave transients  
857 with advanced ligo, advanced virgo and kagra,” *Living Rev. Relativ.* **23** (2020).
- 858 6. M. Ando, K. Arai, K. Kawabe, and K. Tsubono, “Demonstration of power recycling on a fabry-perot-type prototype  
859 gravitational wave detector,” *Phys. Lett. A* **248**, 145–150 (1998).
- 860 7. E. D. Black, “An introduction to Pound–Drever–Hall laser frequency stabilization,” *Am. J. Phys.* **69**, 79–87 (2001).
- 861 8. <http://lma.in2p3.fr/Lmagb.htm>.
- 862 9. L. Aiello, E. Cesarini, V. Fafone, *et al.*, “Thermal compensation system in advanced and third generation gravitational  
863 wave interferometric detectors,” *J. Physics: Conf. Ser.* **1226**, 012019 (2019).
- 864 10. A. F. Brooks, T.-L. Kelly, P. J. Veitch, and J. Munch, “Ultra-sensitive wavefront measurement using a hartmann  
865 sensor,” *Opt. Express* **15**, 10370–10375 (2007).
- 866 11. I. Nardecchia, Y. Minenkov, M. Lorenzini, *et al.*, “Optimized radius of curvature tuning for the virgo core optics,”  
867 *Class. Quantum Gravity* **40**, 055004 (2023).
- 868 12. T. Accadia, F. Acernese, M. Agathos, *et al.*, “Central heating radius of curvature correction (chrocc) for use in large  
869 scale gravitational wave interferometers,” *Class. Quantum Gravity* **30**, 055017 (2013).
- 870 13. I. Khan, E. Genin, V. Fafone, *et al.*, “Auxiliary lasers for advanced virgo gravitational wave detector using single  
871 pass second harmonic generation in periodically poled lithium niobate crystal,” *J. Physics: Conf. Ser.* **1548**, 012025  
872 (2020).

- 873 14. C. De Rossi, J. Brooks, J. Casanueva Diaz, *et al.*, “Development of a frequency tunable green laser source for  
874 advanced virgo+ gravitational waves detector,” *Galaxies* **8** (2020).
- 875 15. J. Poirson, F. Bretenaker, M. Vallet, and A. L. Floch, “Analytical and experimental study of ringing effects in a  
876 fabry–perot cavity. application to the measurement of high finesse,” *J. Opt. Soc. Am. B* **14**, 2811–2817 (1997).
- 877 16. M. Rakhmanov, “Dynamics of laser interferometric gravitational wave detectors,” Phd thesis, California Institute of  
878 Technology (2000).
- 879 17. D. Z. Anderson, “Alignment of resonant optical cavities,” *Appl. optics* **23**, 2944–2949 (1984).
- 880 18. A. Chiummo and G. Vajente, “Itf asymmetries and coupling of laser technical noises,” [https://tds.virgo-gw.  
881 eu/?content=3&r=9206](https://tds.virgo-gw.eu/?content=3&r=9206).
- 882 19. R. B. Jerome Degallaix, Massimo Galimberti and Q. Benoit, “Defining the arm cavity loss for advanced virgo,” Virgo  
883 internal note:VIR-706A-10.
- 884 20. M. L. F. Bondu and J.-Y. Vinet, “From mirror maps to tem00 model: evaluation of effective losses,” [https:  
885 //tds.virgo-gw.eu/?content=3&r=1940](https://tds.virgo-gw.eu/?content=3&r=1940).
- 886 21. E. Capocasa, Y. Guo, M. Eisenmann, *et al.*, “Measurement of optical losses in a high-finesse 300 m filter cavity for  
887 broadband quantum noise reduction in gravitational-wave detectors,” *Phys. Rev. D* **98**, 022010 (2018).
- 888 22. Y. Zhao, M. Vardaro, E. Capocasa, *et al.*, “Optical losses as a function of beam position on the mirrors in a 285-m  
889 suspended fabry-perot cavity,” *Phys. Rev. Appl.* **22**, 054040 (2024).
- 890 23. <https://nl.mathworks.com/matlabcentral/fileexchange/20607-oscar>.
- 891 24. <https://finesse.ifosim.org>.
- 892 25. J. Brooks, M. Mantovani, A. Allocca, *et al.*, “Temperature control for an intra-mirror etalon in interferometric  
893 gravitational wave detector fabry–perot cavities,” *Galaxies* **8**, 80 (2020).
- 894 26. J. Degallaix, C. Michel, B. Sassolas, *et al.*, “Large and extremely low loss: the unique challenges of gravitational  
895 wave mirrors,” *J. Opt. Soc. Am. A* **36**, C85–C94 (2019).
- 896 27. A. F. Brooks, G. Vajente, H. Yamamoto, *et al.*, “Point absorbers in advanced ligo.” *Appl. optics* **60** **13**, 4047–4063  
897 (2021).