Preface to the third volume

Mathematical fuzzy logic (MFL) is a subdiscipline of mathematical logic. Comprising the mathematical study of a certain family of formal logical systems whose algebraic semantics involve some notion of *truth degree*. The main motivations for MFL, its historical origins, and its later evolution were briefly explained in the preface of the first volume of this handbook series.

Fuzzy logics can be seen as a particular kind of many-valued logical system where the intended semantics is typically based on algebras of linearly ordered truth values. As a subdiscipline of mathematical logic, MFL has acquired the typical core agenda of this field and is studied by many mathematically-minded researchers regardless of its original motivations, as witnessed by the plethora of works on a diversity of topics that have been accumulating over the years.

This handbook series aims to be an up-to-date, systematic presentation of the best developed areas of MFL. The first two volumes were released in 2011, also in *Studies in Logic, Mathematical Logic and Foundations* (vols. 37 and 38), College Publications. The first volume starts with a gentle introduction to MFL assuming only some basic knowledge of classical logic. The second chapter presents and develops a general and uniform framework for MFL based on the notions and methods of abstract algebraic logic. The third chapter is a presentation of the deeply developed proof theory of fuzzy logics. The fourth chapter presents the standard algebraic semantics for fuzzy logics based on classes of semilinear residuated lattices. The fifth chapter closes the first volume with the study of Hájek's BL logic and its algebraic counterpart.

The second volume of the series starts with the sixth chapter and is devoted to another widely studied fuzzy logic, Łukasiewicz logic Ł, and MV-algebras as its algebraic semantics. The seventh chapter deals with a third distinguished fuzzy logic, Gödel–Dummett logic, and its variants. The eighth chapter studies fuzzy logics in expanded languages providing greater expressive power. The ninth chapter collects results on functional representations of fuzzy logics and their free algebras. The last two chapters are dedicated to complexity issues: Chapter X studies the computational complexity of propositional fuzzy logics, while Chapter XI is devoted to the arithmetical hierarchy of first-order predicate fuzzy logics.

Intense research in MFL has continued in recent years, bringing to a higher level of maturity some of the topics that were omitted from the previous volumes of the handbook. This third volume collects chapters on seven such areas. It starts with Chapter XII, a systematic study of the most prominent part of the algebraic semantics for fuzzy logics: integral residuated chains. The thirteenth chapter presents a different type of semantics for some prominent fuzzy logics obtained by extending Hintikka's and Giles' semantic games. In a similar fashion, the fourteenth chapter is devoted to another game-theoretic interpretation of t-norm-based fuzzy logics, namely, that given by Ulam–Rényi games arising from the theory of error correcting codes. The fifteenth chapter surveys a series of works on fuzzy logics with evaluated syntax in which intermediate truth-values are incorporated as syntactical devices that accompany each formula as a lower bound for its truth-degree. Chapter XVI surveys another area that has recently been intensively developed: fuzzy description logics, understood as tractable fragments of first-order fuzzy logics amenable to knowledge representation. The seventeenth chapter focuses on another application of MFL by presenting the theory of MV-algebras endowed with functions, called *states*, that allow one to model finitely additive probability measures. The volume is concluded by Chapter XVIII offering a philosophical discussion of the role of MFL in the study of vagueness.

It should be emphasized that this is not a handbook written by a single team of authors, but a collection of chapters prepared by distinguished experts in each area. Nevertheless, the editors have encouraged a reasonable level of homogeneity between the chapters, as regards their structure and notation. The series is conceived as a unit with consecutive page and chapters enumeration. The majority of chapters in this volume contain a purely theoretical (mathematical) presentation, making it mostly a book on mathematical logic, focusing on the study of a particular family of many-valued non-classical logics. However, as briefly outlined before, some chapters feature connections to other mathematical fields, to applications in computer science and, in the case of the last chapter, to developments in analytical philosophy. The intended audience of the book is quite wide, comprising at least the following groups of readers: (1) students of logic looking for a systematic presentation of MFL where they can study the discipline from scratch, (2) experts on MFL that may use it as a reference book for consultation, (3) readers interested in fuzzy set theory and its applications looking for the logical foundations of (some parts) of the area, and (4) readers interested in philosophical and linguistic issues related to reasoning in presence of vagueness, looking for a mathematical apparatus that can be applied to some aspects of those issues.

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Dedication This volume is dedicated to two great researchers who passed away during the preparation of the book: Siegfried Gottwald and Franco Montagna. Siegfried was one of the pioneers in the area of MFL, which he deeply influenced with his monograph *A Treatise on Many-Valued Logics*, and remained a constant contributor to the field and a close collaborator of this handbook series. Franco joined the MFL community later, but he soon became one of the field's major figures, as witnessed by the fact that he is the only author who contributed to each of the three volumes of this series.

Petr Cintula, Christian G. Fermüller, and Carles Noguera
Editors

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LIST OF AUTHORS

FERNANDO BOBILLO
Department of Computer Science and Systems Engineering
University of Zaragoza
C. Maria de Luna 1
50018 Zaragoza, Spain
Email: fbobillo@unizar.es

MARCO CERAMI
Department of Computer Science
Palacký University Olomouc
17. listopadu 2
771 46 Olomouc, Czech Republic
Email: marco.cerami@upol.cz

FERDINANDO CICALESE
Department of Computer Science
University of Verona
Strada Le Grazie 15
37134 Verona, Italy
Email: ferdinando.cicalese@univr.it

FRANCESC ESTEVA
Artificial Intelligence Research Institute (IIIA)
Spanish National Research Council (CSIC)
Campus de la Universitat Autònoma de Barcelona s/n
08193 Bellaterra, Catalonia, Spain
Email: esteva@iiia.csic.es

CHRISTIAN G. FERMÜLLER
Theory and Logic Group 185.2
Vienna University of Technology
Favoritenstraße 9-11
1040 Vienna, Austria
Email: chrisf@logic.at

TOMMASO FLAMINIO
Dipartimento di Scienze Teoriche e Applicate
Universitá dell'Insubria
Via Mazzini 5
21100 Varese, Italy
Email: tommaso.flaminio@uninsubria.it

ÀNGEL GARCÍA-CERDAÑA
Artificial Intelligence Research Institute (IIIA)
Spanish National Research Council (CSIC)
Campus de la Universitat Autònoma de Barcelona s/n
08193 Bellaterra, Catalonia, Spain
and
Information and Communication Technologies Department
Universitat Pompeu Fabra
Tànger 122-140
08018 Barcelona, Catalonia, Spain
Email: angel@iiia.csic.es

TOMÁŠ KROUPA
Institute of Information Theory and Automation
Czech Academy of Sciences
Pod Vodárenskou věží 4
182 08 Prague, Czech Republic
Email: kroupa@utia.cas.cz

FRANCO MONTAGNA
Department of Information Engineering and Mathematics
University of Siena
Via Roma 56
53100 Siena, Italy

VILÉM NOVÁK
Institute for Research and Applications of Fuzzy Modeling
University of Ostrava & NSC IT4Innovations
30. dubna 22
701 03 Ostrava 1, Czech Republic
Email: Vilem.Novak@osu.cz

RAFAEL PEÑALOZA
KRDB Research Centre
Free University of Bozen-Bolzano
Piazza Domenicani 3
I-39100 Bozen-Bolzano BZ, Italy
Email: rafael.penaloza@unibz.it

NICHOLAS J.J. SMITH
Department of Philosophy
Main Quadrangle A14
University of Sydney
NSW 2006, Australia
Email: njjsmith@sydney.edu.au

UMBERTO STRACCIA
Istituto di Scienze e Tecnologie dell'Informazione (ISTI)
Consiglio Nazionalle delle Ricerche (CNR)
Via G. Moruzzi 1
56124 Pisa, Italy
Email: straccia@isti.cnr.it

THOMAS VETTERLEIN
Department of Knowledge-Based Mathematical Systems
Johannes Kepler University Linz
Altenberger Straße 69
4040 Linz, Austria
Email: Thomas.Vetterlein@jku.at

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