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Abstract

In a recent article, we extended Goodwin's (1967) model to study the interaction between distributive cycles and international trade for economies in which growth is balance-of-payments constrained (BoPC). Building on that set up, we investigate the implications of allowing exports to be a function of the capital stock. Using the existence part of the Hopf bifurcation theorem, we show that the resulting 3-dimensional system admits a limit cycle solution. We rely on numerical simulations to verify if fluctuations are persistent and bounded. Applying panel cointegration techniques, we also provide empirical evidence for a sample of 19 OECD countries between 1950-2014 that gives support to the formulation adopted for the exports function. Our main contribution lies in providing a simple base-line model to study distributive dynamics in open economies in line with recent developments in the BoPC growth literature.

Keywords: Growth cycle, Path dependence, Thirlwall's law, Distributive cycles, Hopf bifurcation, Cointegration.

JEL: E12; E32; O40

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1 Introduction

The relationship between growth and income distribution has for a long time been a central issue in theories of social conflict. In this respect, Goodwin's (1967) distributive growth cycle system has in recent decades established itself as a powerful "model for doing macrodynamics". Despite its simplicity, the elegance and flexibility of this model has over time caught the attention of a great number of researchers.

In the last fifty years, a large number of scholars have generalised its formulation in all possible directions. Recently, in Dávila-Fernández and Sordi (2018, hereafter DF&S), we studied the interaction between distributive cycles and international trade for economies in which growth is balance-of-payments constrained (BoPC), i.e. follows Thirlwall's (1979) law. In doing this, our contribution is related to other recent efforts devoted to the analysis of how deviations from long-run paths are generated and corrected (e.g. Soukiazis et al 2012; 2014; Garcimartin et al, 2016). The implication is that, although the BoPC growth approach is focussed on the long-run, it also has profound implications for short-run dynamics.

Formally, Goodwin's (1967) model consists of two simultaneous non-linear dynamic equations, one for the employment rate and one for the wage share, and assumes full capacity utilisation. This assumption was relaxed by DF&S in two steps. First, the basic motion of the system was modified to include the rate of capacity utilisation as an endogenous variable while the balance-of-payments was supposed to be in permanent equilibrium. We showed that a Hopf-Bifurcation analysis established the possibility of persistent and bounded cyclical fluctuations for the resulting 3-dimensional non-linear dynamic system. Then, a 4-dimensional system was developed in which deviations from the external constraint were investigated introducing an independent investment function.

In the BoPC growth model literature, world demand constitutes the main limitation to which individual country growth adjusts. In fact, the growth rate of exports is taken as a function of the growth rate of foreign demand mediated by its elasticity. However, as discussed not long ago by Ros (2013, p. 239-245), there is no demand for a country's exports, say a demand for Brazilian or Italian cars. What there is, rather, is a demand for cars in a market in which Brazilian and Italian car producers compete. Therefore, domestic conditions are particularly important to describe export behavior and should be explicitly modelled.

Fagerberg (1988) seems to have been the first one formally to do it based on the concept of technological competitiveness. Kaldorian and Schumpeterian insights have also been put together in a recent contribution by Romero and McCombie (2016a). Using the KLEMS database for a sample of seven European countries, they estimated an elasticity of exports with respect to the capital stock between 0.27 and 0.57. Razmi (2016) formalised a similar idea making exports a function of foreign demand and the capacity to export. In this way, capital accumulation that results in an expansion of the exportable sector can allow for an increase in imports without an exploding current account deficit.

Taking as starting points DF&S and the aforementioned contributions, in this paper we study the implications for the former of allowing exports to respond to capital accumulation. Using the existence part of the Hopf bifurcation theorem, we show that the resulting 3-dimensional system admits a limit cycle solution. Since the existence part of the theorem leaves us in the dark regarding the nature of the cycle (see Gandolfo, 2009, p. 479-484), we rely on numerical simulations to investigate if fluctuations are persistent and bounded. Our exercise confirms that we are dealing with a supercritical Hopf bifurcation and there is convergence to a limit cycle.

The view that an economic system may be permanently influenced by the extent to which

it has changed in the past seems highly intuitive. In economics this is commonly referred to as "path dependent behaviour" and is associated with the idea that outcomes are historically contingent. The dynamic system we obtain is capable of providing different representations of this. On the one hand, we are able to identify the coexistence of two limit cycles so that different initial conditions lead to cycles of different amplitude. On the other hand, once we take into account the possibility of periodic autonomous investment motivated by innovation waves *à la* Schumpeter, we are able to provide an explanation of the irregularity of economic time-series.

Using panel cointegration techniques, we also provide empirical evidence for a sample of 19 OECD countries between 1950-2014 that gives support to the formulation adopted for the exports function. Our estimations indicate that an increase of 100 US dollars in the capital stock is related to an increase of exports between 3.5 and 4.1 dollars. Such a result emphasises the importance of explicitly incorporating capital when modelling the BoPC.

Our main contribution lies in providing a simple base-line model to study distributive dynamics in open economies in line with recent developments in the BoPC growth literature. The paper is organised as follows. In the next section we present our modification of DF&S. Section 3 puts together the dynamic system and provides the local stability and Hopf bifurcation analysis. In section 4 we present a numerical simulation exercise showing the existence of persistent oscillations. Some final considerations follow.

2 The basic structure of the model

In his growth cycle paper, Goodwin (1967) aimed at building a system capable of generating cycles in the growth rate of output rooted in the functioning of the labour market and the dynamics of distributive conflict. The model was originally conceived for a closed economy without government. DF&S's extension introduces international trade to take into account Thirlwall's law. In what follows, we will revisit their derivation of the dynamic system of the model. This is done with the purpose of highlighting the novelties introduced in the present exercise. We divide our exposition into four blocks of equations: (i) aggregate demand conditions; (ii) supply conditions; (iii) distributive conditions, and (iv) behavioural relations.

2.1 Aggregate demand conditions

In an open economy without government, the expenditure identity is given by:

$$Y = C + I + X - M$$

where C stands for consumption, I is investment, X corresponds to exports, and M stands for imports. It immediately follows that the external constraint is the goods market equilibrium condition:

$$S - I = X - M \tag{1}$$

with savings, S, equal to total output minus consumption. Equilibrium in the balance-of-payments implies equilibrium between investment and savings decisions.

Taking logarithms and time derivatives we obtain a dynamic version of the external constraint:[1]

$$\theta \frac{\dot{M}}{M} + (1-\theta)\frac{\dot{S}}{S} = \Omega \frac{\dot{X}}{X} + (1-\Omega)\frac{\dot{I}}{I} \tag{2}$$

[1] For any variable x, $\dot{x}$ indicates its time derivative (dx/dt).

where $\theta = M/(M+S)$, $\Omega = X/(X+I) \in [0;1]$.

Even though behavioural relations constitute a single block of equations, for expositional purposes it is useful to introduce the following traditional function for imports:

$$M = M(Y), \; M_y > 0 \tag{3}$$

Since we are abstracting from any price consideration, the real exchange rate is held constant and equal to one. For simplicity, it is also assumed that all trade consists in the exchange of final goods.

Taking logarithms and time derivatives of equation (3), and substituting in the dynamic external constraint given by (2), we obtain:

$$\frac{\dot{Y}}{Y} = \frac{1}{\pi}\left[\frac{(1-\Omega)\frac{\dot{I}}{I} - (1-\theta)\frac{\dot{S}}{S}}{\theta} + \left(\frac{\Omega}{\theta}\right)\frac{\dot{X}}{X}\right] \tag{4}$$

where $\pi = (dM/dY)(Y/M)$ is the income elasticity of imports which, for simplicity, is assumed to be constant. The expression above separates the growth rate of output into two components. On the one hand, there is disequilibrium between investment and savings. That is, a higher growth rate of investment relative to savings implies a higher growth rate of output. On the other hand, we have the growth rate of exports that, as we will show, is a "quasi-autonomous" component of aggregate demand.

DF&S evaluated empirically Thirlwall's law for a sample of 16 OECD countries between 1950 and 2014 using data from the Penn World Table.[2] They found that actual and estimated growth rates are indeed very close, thus, supporting the hypothesis that, for those economies, growth follows the BoPC. Still, notice that by virtue of (1), we have $\Omega/\theta = X/M$. If it is true that in the long run the BoP is in equilibrium, $\Omega/\theta \approx 1$ is a fair approximation. Thus, we can rewrite equation (4) as:

$$\frac{\dot{Y}}{Y} = \frac{1}{\pi}\left[\frac{(1-\theta)}{\theta}\left(\frac{\dot{I}}{I} - \frac{\dot{S}}{S}\right) + \frac{\dot{X}}{X}\right] \tag{5}$$

One could argue that, even though $\Omega/\theta \approx 1$, the dynamics behind this ratio are not to be ignored when studying the growth cycle and, therefore, we cannot just fix it equal to one. Our answer to that observation is that at a first stage of analysis such approximation is quite reasonable. This is because, to a great extent, variations in Ω/θ are determined by the income elasticities of exports and imports which are structural parameters of the economy. Following the BoPC literature, the ratio between those elasticities is considered to capture the level of diversification and technological complexity of the economy's productive structure.[3] As such, we can treat them as non-cyclical.

From the expression above, once again, we have a disaggregation of output's growth rate into (i) disequilibrium between investment and savings, and (ii) quasi-autonomous aggregate demand growth. Notice that as long as $\dot{I}/I = \dot{S}/S$, it follows that:

$$\frac{\dot{Y}}{Y} = \frac{\dot{X}/X}{\pi} \tag{6}$$

[2] Several scholars (e.g. Bagnai, 2010; Gouvea and Lima, 2010; 2013; Lanzafame, 2014; Bagnai et al, 2016) have tested over the years the validity of Thirlwall's law for different countries and using different methodologies. For a survey see Thirlwall (2011).

[3] Romero and McCombie (2016b) and Martins Neto and Porcile (2017) provide empirical evidence of such an interpretation. See Dávila-Fernández et al (2018) for a theoretical perspective on how foreign trade elasticities might change over time.

i.e. Thirlwall's law.

Equation (5) corresponds to a simple extension of the main insight of the BoPC approach to the short-term. The growth rate of aggregate demand is constrained by the capacity of the economy to sustain balance-of-payments imbalances. The strength of the expression comes from the fact that (1) is straightforward manipulation of an accounting identity.

2.2 Supply conditions

Consider the following production function:

$$Y = \min\{Ku; qNe\} \tag{7}$$

where K corresponds to the capital stock, u stands for effective capacity utilisation, q is labour productivity, N is total labour force, and e is the employment rate. Effective capacity utilisation is given by $(Y/Y^*)(Y^*/K)$, where Y^* is production at full capacity. That is, u corresponds to the output-capital ratio. Moreover, the employment rate is given by L/N, where L is the level of employment.

The Leontief dynamic efficiency condition states that:[4]

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} + \frac{\dot{u}}{u} = \frac{\dot{q}}{q} + \frac{\dot{N}}{N} + \frac{\dot{e}}{e} \tag{8}$$

For an exogenous labour force growth rate, n, it follows that:

$$\frac{\dot{u}}{u} = \frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} \tag{9}$$

$$\frac{\dot{e}}{e} = \frac{\dot{Y}}{Y} - \frac{\dot{q}}{q} - n \tag{10}$$

Variations in capacity utilisation are given by the difference between output's growth rate and capital accumulation. If output is growing faster (slower) than the increase in productive capacity, the rate of utilisation increases (decreases). On the other hand, employment rates fundamentally depend on the difference between the growth rate of output and labour productivity. That is, if output grows faster (slower) than the increase in the productivity of workers, employment will expand (reduce).

2.3 Distributive conditions

In an economy with two factors of production and no government, the income identity is:

$$Y = wL + rK$$

where w and r are, respectively, real wages and the rate of return on capital.

Hence, the wage share is defined as the share of wages over total output, i.e. $\varpi = wL/Y = 1 - rK/Y = w/q$. Therefore, we have that variations in functional distribution are given by:

$$\frac{\dot{\varpi}}{\varpi} = \frac{\dot{w}}{w} - \frac{\dot{q}}{q} \tag{11}$$

If real wages are growing faster (slower) than labour productivity the share of wages in income is going to increase (decrease).

[4] Notice that the Leontief production function is in a sense an accounting identity because $Y = K\,(Y/Y^*)\,(Y^*/K) = (Y/L)\,N\,(L/N)$.

2.4 Behavioural equations

In order to close the model, we need to introduce a set of behavioural equations that together with the accounting identities provide the basic structure of our economy. Most of them were previously discussed by DF&S and therefore will be shortly described here.

2.4.1 Real wage Phillips curve

We begin presenting a generic real wage Phillips curve of the type:

$$\frac{\dot{w}}{w} = f\left(e, \frac{\dot{e}}{e}\right), \; f_e > 0, \; f_{\dot{e}/e} > 0, \; f\left(e, 0\right) \neq 0 \tag{12}$$

indicating that the bargaining power of workers increases as employment expands. Furthermore, not only does the rate of employment matter but also the variation rate. As pointed out by Sordi (2001), this seems to be the case considered by Phillips himself. High employment is associated with greater bargaining power of workers to the extent that, for example, there is an increase in their fall back position given that it is easier to find a new job. On the other hand, $\dot{e}/e$ captures the intensity of this process. Empirical evidence relating employment and the growth of real wages can be found in Grasselli and Maheshwari (2018).

2.4.2 Labour productivity

Labour productivity gains are supposed to be a function of effective capacity utilisation:

$$\frac{\dot{q}}{q} = G(u), \; G_u > 0 \tag{13}$$

In a recent survey on endogenous technical change in alternative theories of growth and distribution, Tavani and Zamparelli (2017) distinguished three different assumptions on the determinants of labour productivity gains: (i) capital accumulation, (ii) income distribution, and (iii) labour market tightness. In an ongoing research project, we are studying the implications of adopting any (or a combination) of them for the main results of our model. Here, however, we maintain the original specification under the following motivation.

Kaldor developed different ways to endogenise technological change (Kaldor, 1957; 1961; 1966). For instance, in his technical progress function, he anticipated some of the basic insights behind Arrow's learning-by-doing model. We are particularly interested in capturing learning-by-doing associated with the presence of economies of scale in the use of capital. The basic idea is that, to a great extent, technical progress is capital embodied. Nevertheless, machines must be operating for productivity gains to be effectively incorporated.

Notice that since $u = (Y/Y^*)\,(Y^*/K)$, there are two possible channels for effective capacity utilisation to influence labour productivity. The first one is related to the level of capacity utilisation, Y/Y^*. High rates of idle capacity leave little room for learning-by-doing because the potential productivity growth embodied in machinery cannot be incorporated by labour. The second one is through an increase in the productivity of machines, Y^*/K. That is, the adoption of modern production techniques comes with spillover effects on workers' productivity.

In either case, for the purposes of our exercise, labour productivity should be understood as an increase in the capacity of workers to produce more instead of the adoption of labour saving techniques. The difference is subtle but important. In the first case, for a given amount of labour, the firm is able to produce more. In the second case, for a given amount of output, the firm uses fewer workers.

The way in which we understand the three determinants in Tavani and Zamparelli's analysis is that they are related to this second situation. An increase in capital accumulation, for instance, would be associated with a reduction in the cost of capital and a relative increase in the cost of labour. Analogously, an increase in the wage share corresponds to a direct increase of production costs given that real wages are higher relative to labour productivity. Finally, labour market tightness indicates that labour is not available to firms even if they want to hire more workers. In all those cases the alternative is to adopt or develop production techniques that save labour.

On the other hand, our argument states that, for the aforementioned reasons, as long as machines are being used, a given set of workers will produce more.[5] Moreover, $G(\cdot)$ also captures the idea that labour productivity is to some extent pro-cyclical, as shown by Baily et al (2001) and Basu and Fernald (2001). Whether measured as labour productivity or total factor productivity, it seems to rise in booms and fall in recessions, being considered in several macroeconomic manuals as an essential feature of the business cycle (see, for example, Romer, 2012, p. 193).

We rely on non-parametric locally weighted (lowess) and kernel-weighted local polynomial (lpoly) regressions further empirically to motivate our modelling choice. These methods combine much of the simplicity of linear least squares with the flexibility of nonlinear models. Fig. 1 depicts both curves for a sample of 19 OECD countries between 1950-2014. Details on data sources are provided in the Econometric Appendix at the end of the paper. Overall, there seems to be a positive relationship between utilisation and the growth rate of labour productivity, especially for $u > 0.3$.

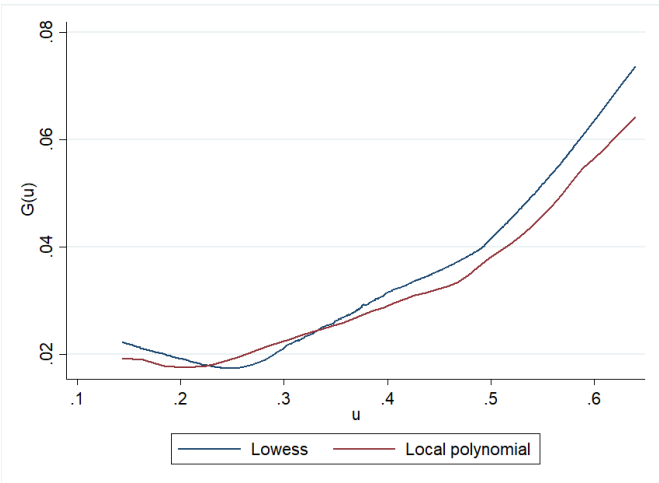


Figure 1: $G(u)$ and effective capacity utilisation, 19 OECD countries, 1950-2014.

2.4.3 Investment

The determination of investment is central to Keynesian theories of effective demand. We adopt the following general specification for the investment-capital stock ratio:

$$\frac{I}{K} = h(r), \;\; h_r > 0 \tag{14}$$

[5]Hein and Tarassow (2010), and Rezai (2012) are examples of contributions suggesting that the two formulations might not be incompatible. One should notice that Sasaki et al (2013) uses $G(u)$ but maintains the inter-class conflict interpretation making reference to Okun's law.

which for simplificity is assumed to be linear.

Capital accumulation is positively related to the profit-rate to the extent that firms respond to profitability opportunities. Still, notice that $\varpi = wL/Y = 1 - rK/Y$, which implies $r = (1 - \varpi)u$. Hence, our behavioural equation can be rewritten as:

$$\frac{I}{K} = H(\varpi, u), \;\; H_\varpi < 0, \;\; H_u > 0 \tag{15}$$

The intuition of the expression above is similar to the one discussed by Bhaduri and Marglin (1990) and is recurrent in the literature. The accelerator component of investment has been for a long time well documented both in the theoretical and empirical literature. Furthermore, there is also some empirical support for the hypothesis that investment depends on the functional income distribution (e.g. Stockhammer et al, 2009; Onaran and Galanis, 2014; 2016).

2.4.4 Savings

Suppose that all savings come from profits and that firms retain a share s of them. Hence:

$$\frac{S}{K} = s(1 - \varpi)u \tag{16}$$

Increases in capacity utilisation and reductions in the wage share increase the savings-capital ratio. The hypothesis of savings coming only from profits is only a simplifying assumption and different authors have investigated the implications of allowing workers to save (though for closed economies, e.g. Sordi, 2001). It is well known that propensities to save out of capital are greater than out of wages, while they also increase as we move from the bottom to the top quintile of wage earners (see, for example, Carvalho and Rezai, 2016).

2.4.5 Exports

Finally, exports are modelled as a function of the capital stock and the level of output of the rest of the world, Z, that is

$$X = X(K, Z), \;\; X_k, \;\; X_z > 0 \tag{17}$$

Recall that we are abstracting from any price considerations. Equation (17) corresponds to a simplification of Razmi's (2016) formulation and indicates that exports depend not only on foreign demand but also on the export capacity of the economy. Secular changes in capital stock are accompanied by advances in infrastructure, for example, that will lead to an increase in the export supply at any given level of export prices, a reasoning similar to the one provided by Goldstein and Khan (1985). One could also think in terms of the tradable industrial sector being generally more capital-intensive, especially in developing countries, which further explains $X_k > 0$. Exports are a "quasi-autonomous" component of aggregate demand because, on the one hand, they depend on Z which is exogenous to the domestic economy, but, on the other hand, also depend on capital that in turn hinges on domestic accumulation.

The idea that exports are conditional upon the technological competitiveness of a country, which in turn is a function of some measure of productive capacity, either capital or investment, goes back at least to Fagerberg (1988). Fagerberg's modification of Thirlwall's model was recently rescued by Romero and McCombie (2016a) who explicitly made exports a function

of the capital stock and estimated the respective elasticity using the System-GMM estimator and data for seven European countries over the period 1984-2006.

Given the importance of this specification for our model and for the next generation of models in the BoPC literature, in this paper we provide some empirical evidence of our own giving support to equation (17). We rely on panel cointegration techniques for a sample of 19 OECD countries between 1950-2014, which allow us better to explore the time dimension of data. Cointegrating equations are estimated using Dynamic Ordinary Least Squares, DOLS. This estimator deals with the problem of second-order asymptotic bias arising from serial correlation and endogeneity. Details of the estimation procedure and the innovative aspects of our exercise are presented in the Econometric Appendix at the end of the paper.

Overall, we find that there is a long-run relationship between exports, foreign demand, and capital, i.e. series are cointegrated. Table 1 brings our main results. Their robustness can be appreciated under different specifications of the econometric model. An increase of 100 US dollars in foreign demand is associated with an increase between 0.13-0.28 dollars in exports. On the other hand, an increase of 100 US dollars in the capital stock is related to an increase in exports by 3.5-4.1 dollars. Such results emphasise the importance of explicitly incorporating capital when modelling the BoPC.

Table 1: DOLS estimations $X(K,Z)$

Dependent variable	Exports			
Model	I	II	III	IV
Capital Stock	0.037566***	0.038262***	0.040994***	0.035234***
Foreign GDP	0.002878***	0.002086***	0.001706***	0.001330***
Panel method	Pooled	Weighted	Pooled	Weighted
Time dummies	No	No	Yes	Yes
T-dimension	64	64	64	64
Cross-sections	19	19	19	19
Obs.	1198	1198	1190	1190
Adj. R2	0.954754	0.953296	0.982371	0.981860
Adj. sample	1951-2014	1951-2014	1951-2014	1951-2014

*, **, *** stand by 10%, 5%, and 1% of significance.

3 The dynamic system

Substituting equation (13) in (10) gives us:

$$\frac{\dot{e}}{e} = \frac{\dot{Y}}{Y} - G(u) - n \tag{18}$$

A constant rate of employment is the result of labour force and productivity growth rates matching output's growth rate. A higher rate of capacity utilisation reduces employment through its effect on the productivity of workers.

Making use of equations (11), (12), and (13), distributive dynamics become:

$$\frac{\dot{\varpi}}{\varpi} = f\left(e, \frac{\dot{e}}{e}\right) - G(u) \tag{19}$$

Functional income distribution can only be stable if real wages grow at the same pace as productivity gains. Moreover, employment and effective capacity utilisation have opposite effects on the wage share. An increase in the employment rate increases worker's bargaining power allowing a rise in wages which in turn has a positive impact on the wage share. On the other hand, an increase in the rate of capacity utilisation increases labour productivity through our learning-by-doing mechanism reducing the share of wages in income.

Capital accumulation is given by the investment function in (14). Therefore, substituting (14) into (9), we obtain the equation for the dynamics of capacity utilisation:

$$\frac{\dot{u}}{u} = \frac{\dot{Y}}{Y} - h(r) \tag{20}$$

The effect of the rate of growth of output on effective capacity utilisation is straightforward. Higher demand increases capacity utilisation. Nevertheless, an increase in capacity utilisation or a reduction in the wage share decreases u. This is because both have a positive impact on capital accumulation through investment profitability, given that $r = (1 - \varpi)u$.

At this point of the analysis, we ask the reader to recall equation (5) in which we showed that output's growth rate is a function of disequilibrium between the growth rate of investment and savings. From the last two behavioural relations, specified in (14) and (16), we have that:

$$\frac{\dot{I}}{I} = \frac{\dot{K}}{K} + \left[h_r \frac{r}{h(r)}\right] \frac{\dot{r}}{r} \tag{21}$$

$$\frac{\dot{S}}{S} = \frac{\dot{K}}{K} - \left(\frac{\dot{\varpi}}{1-\varpi}\right) + \frac{\dot{u}}{u} \tag{22}$$

where by virtue of the linear specification, $h_r r/h(r) = 1$.

Furthermore, the growth rate of exports can be easily obtained from (17) as:

$$\frac{\dot{X}}{X} = \psi_1 \frac{\dot{K}}{K} + \psi_2 \frac{\dot{Z}}{Z} \tag{23}$$

where $\psi_1 = (dX/dK)(K/X)$ and $\psi_2 = (dX/dZ)(Z/X)$ are the scale elasticities of supply (with respect to the capacity to export) and demand (with respect to foreign income), respectively. For simplicity, we assume both are constant so that, from Euler's homogeneity theorem, it follows that $X(\cdot)$ is an homogeneous function of degree $\psi_1 + \psi_2$.

Substituting equations (21)-(23) in (5), and applying the definition of the profit rate, we obtain:

$$\frac{\dot{Y}}{Y} = \phi_1 \frac{\dot{K}}{K} + \phi_2 \frac{\dot{Z}}{Z} \tag{24}$$

with $\phi_i = \psi_i/\pi : i = \{1, 2\}$.

Then, inserting (24) into equations (18) and (20) we obtain the dynamics of employment and the wage share. Finally, equation (19) closes our dynamic system, now given by:

$$\begin{aligned} \dot{e} &= \left[\phi_1 h(r) + \phi_2 \frac{\dot{Z}}{Z} - G(u) - n\right] e \\ \dot{\varpi} &= \left[f\left(e, \frac{\dot{e}}{e}\right) - G(u)\right] \varpi \\ \dot{u} &= \left[\phi_1 h(r) + \phi_2 \frac{\dot{Z}}{Z} - H(\varpi, u)\right] u \end{aligned} \tag{25}$$

which, making use of (15), is equivalent to:

$$\begin{aligned}
\dot{e} &= \left[\phi_1 H(\varpi, u) + \phi_2 \frac{\dot{Z}}{Z} - G(u) - n\right] e = j_1(e, \varpi, u) \\
\dot{\varpi} &= \left[F\left(e, \varpi, u\right) - G(u)\right] \varpi = j_2\left(e, \varpi, u\right) \\
\dot{u} &= \left[\phi_1 H(\varpi, u) + \phi_2 \frac{\dot{Z}}{Z} - H(\varpi, u)\right] u = j_3\left(e, \varpi, u\right)
\end{aligned} \tag{26}$$

where $f\left(e, \frac{\dot{e}}{e}\right) = f\left[e, \phi_1 H(\varpi, u) + \phi_2 \frac{\dot{Z}}{Z} - G(u) - n\right] = F\left(e, \varpi, u\right)$.

3.1 Equilibrium points, local stability analysis and Hopf bifurcation

In steady state $\dot{e}/e = \dot{\varpi}/\varpi = \dot{u}/u = 0$. This gives us the following equilibrium conditions:

$$\phi_1 H(\varpi, u) + \phi_2 \frac{\dot{Z}}{Z} = G(u) + n \tag{27}$$

$$F\left(e, \varpi, u\right) = G(u) \tag{28}$$

$$\phi_1 H(\varpi, u) + \phi_2 \frac{\dot{Z}}{Z} = H\left(\varpi, u\right) \tag{29}$$

which are basically the same as in DF&S.

Equation (27) shows that in equilibrium the sum of labour productivity and labour force growth rates must be equal to the BoPC growth rate. There is a simultaneous adjustment between the external constraint and the so called "natural rate of growth", this last one being endogenous, pro-cyclical and to some extent determined by the external constraint, as several empirical studies have shown to be the case (León-Ledesma and Thirlwall, 2002; Libânio, 2009; León-Ledesma and Lanzafame, 2010; Lanzafame, 2014). The equilibrium condition (28) simply states that real wages and labour productivity must grow at the same rate in order for the wage share to be constant. Finally, condition (29) implies that the rate of growth of output must equal the rate of growth of the capital stock so as not to generate permanently increasing or decreasing idle capacity.

In our model, output's growth rate follows Thirwall's law which in turn was shown to depend on foreign demand and the economy's capacity to export. However, if the sensitiveness of exports to the capital stock is greater than one, it is easy to see that this would lead to explosive growth rates. Therefore, we need to state the following crucial assumption:

Assumption The sensitivity of exports to the capital stock is such that

$$\phi_1 < 1$$

For $\phi_1 > 1$, capacity to export would expand above the expansion of capital itself, infinitely relaxing the external constraint. Such outcome is of little use to us, justifying our assumption. Furthermore, our empirical exercise also gives some support to it. The elasticity of exports with respect to the capital stock is given by $\psi_1 = X_k\left(K/X\right)$. Using $X_k = 0.04$, we can, accordingly, compute the corresponding elasticity for every country and year. In table 2 we report our estimates for the last year of our sample. With the exception of the United States, ψ_1 was found to be significantly lower than one. Still, notice that $\phi_1 = \psi_1/\pi$. DF&S estimated

an average π of 1.5 that is not very different from the values usually found in the literature. Hence, we are quite comfortable making $\phi_1 < 1$.

Table 2: Estimated elasticity of exports with respect to capital, 2014

Country	ψ_1	Country	ψ_1
Australia	0.467456112	Austria	0.33905967
Belgium	0.155409216	Canada	0.377168144
Denmark	0.293482157	Finland	0.414601799
France	0.652635572	Germany	0.266694675
Ireland	0.250067503	Italy	0.761596803
Netherlands	0.181095316	New Zeland	0.331200668
Norway	0.238695187	Portugal	0.900050394
Spain	0.794515391	Sweden	0.329647717
Switzerland	0.185633622	United Kingdom	0.699536417
United States	1.003928617		

*, **, *** stand by 10%, 5%, and 1% of significance.

Given the equilibrium conditions (27)-(29), we can state and prove the following Proposition regarding the existence and uniqueness of an internal equilibrium.

Proposition 1 *The dynamic system (26) has a unique internal equilibrium point* (e^*, ϖ^*, u^*) *that satisfies:*

$$F(e^*, \varpi^*, u^*) = f\left(e^*, 0\right) = y_{bp} - n \tag{30}$$

$$H\left(\varpi^*, G^{-1}\left(y_{bp} - n\right)\right) = h\left((1-\varpi^*)G^{-1}\left(y_{bp} - n\right)\right) = y_{bp} \tag{31}$$

$$u^* = G^{-1}\left(y_{bp} - n\right) \tag{32}$$

where the equilibrium BoPC growth rate is defined and given by:

$$y_{bp} = \left(\frac{\phi_2}{1-\phi_1}\right)\frac{\dot{Z}}{Z}$$

Proof. See Mathematical Appendix B.1. ■

There are two possible ways to foster long-run growth: first, and quite obviously, by means of higher foreign demand; secondly, through a process of structural change that can be divided into (i) a reorientation of capital accumulation towards exports, increasing ϕ_1; and (ii) an increment of product diversification and exports complexity, so as to increase ϕ_2. In both cases, a higher BoPC growth rate delivers a higher rate of employment and effective capacity utilisation. As pointed out by DF&S, such correspondence between e and u resembles Okun's rule.

Inversely, a higher growth rate of the labour force is associated with lower employment and effective capacity. As far as capacity utilisation is concerned, this is because, for a given y_{bp}, higher n requires a reduction in the growth rate of labour productivity in order to satisfy (27). Such reduction is achieved due to a reduction in u. Furthermore, since real wages are determined in the labour market, e must also be reduced in order to keep income distribution stable. After that, the wage share will adjust in order to ensure that capital accumulation is such as to maintain the equilibrium rate of effective capacity utilisation.

The economic intuition works as follows. The external constraint determines the long-run growth trend, y_{bp}. Firms adjust their effective capacity so as to meet that trend. If u is set below u^*, they are not able to match demand growth. On the other hand, if they make $u > u^*$, then $\dot{Y}/Y > y_{bp}$ and there is an increase in employment rates that leads to an increase in real wages and the wage share compromising profitability. A strong learning-by-doing effect means that small increases in capacity utilisation by firms are enough to meet the long-run trend. Therefore, a strong (weak) G_u implies a lower (higher) equilibrium of effective capacity.

Analogously, highly combative workers are able to obtain strong real wage increases. This means that, for a given long-run growth trend of demand determined by y_{bp}, small increases of employment by firms are potentially harmful to investment profitability because of their impact on the wage share. In this way, a strong f_e is related to a lower equilibrium employment rate. Still, this is not to say that strong labour unions lead to lower e. The slope of function f captures how combative workers are and not necessary unionisation levels.

Hence, the external constraint first determines the level of capacity utilisation through an adjustment of labour productivity growth rates. Once firms decide about u, they adjust employment rates taking into account the extension of the distributive conflict and how combative workers are. Finally, for a given rate of growth of output and effective utilisation, income distribution delivers the profit rate that equalises capital accumulation and y_{bp}. In other words, the sequence of the equilibrium values determination follows the order described below:

$$u^* \Rightarrow e^* \Rightarrow \varpi^*$$

Notice that ϖ^* is a function of h_r so that a high sensitiveness of investment to profitability implies a greater wage share. In this case, a lower ϖ^* is unfeasible because firms would respond by increasing capital accumulation which in turn would lead to higher employment, increasing the bargaining power of workers and finally bringing up the wage share.

With regard to the unique internal equilibrium point, we can now state and prove the following Proposition regarding its local stability.

Proposition 2 *If the sensitivity of capital accumulation to profitability is such that:*

$$h_r - \frac{G_u e^* f_e (1-\phi_1)}{\{[(1+f_{\hat{e}})\phi_1 - 1]\varpi^*(h_r) - (\phi_1 - 1)\}[G_u(1+f_{\hat{e}})(1-\phi_1)u^* + \phi_1 e^* f_e]} > 0$$

the internal equilibrium (e^*, ϖ^*, u^*) *of the dynamic system (26) is locally asymptotically stable.*

Proof. See Mathematical Appendix B.2. ■

However, for certain values of h_r, it may happen that the inequality is not satisfied. Thus, the dynamic behaviour of the model may drastically change, from the qualitative point of view, as the sensitivity of investment to r varies, with all the other parameters remaining constant. Using h_r as a bifurcation parameter, our purpose is now to apply the existence part of the Hopf Bifurcation Theorem (HBT) for 3D systems (see Gandolfo, 2009) to show that persistent cyclical behaviour of the variables can emerge as h_r changes.

Proposition 3 *For values of* h_r *in the neighbourhood of the critical value* $h_{r\ HB}$ *such that:*

$$h_{r\ HB} - \frac{G_u e^* f_e (1-\phi_1)}{\{[(1+f_{\hat{e}})\phi_1 - 1]\varpi^*(h_{r\ HB}) - (\phi_1 - 1)\}[G_u(1+f_{\hat{e}})(1-\phi_1)u^* + \phi_1 e^* f_e]} = 0$$

and for which the real negative root of the characteristic equation, λ_1, altogether with the coefficient of the imaginary part of the non-real pair of complex roots, ω, satisfies:

$$Q\lambda_1 + R \neq \omega^2 P$$

where

$$\begin{aligned} P &= \left[f_{\hat{e}}\phi_1 + (\phi_1 - 1)\right] u^* \left[\varpi^*(h_r) + \varpi^*_{h_r} h_r\right] - (\phi_1 - 1)\, u^* \\ Q &= \left[G_u (1 + f_{\hat{e}})(1 - \phi_1)\, u^* + \phi_1 e^* f_e\right] u^* \left[\varpi^*(h_r) + \varpi^*_{h_r} h_r\right] \\ R &= G_u e^* f_e (1 - \phi_1)\, u^{*2} \left[\varpi^*(h_r) + \varpi^*_{h_r} h_r\right] \end{aligned}$$

the dynamic system (26) has a family of periodic solutions.

Proof. See Mathematical Appendix B.3. ■

Finding an economic interpretation for the second part of Proposition 3 is not an easy task. At first, it corresponds only to a mathematical requirement, also easily satisfied as we show in the next section. Still, the first part of the Proposition brings at least one interesting insight. The sensitivity of investment to the profit rate is crucial for the qualitative properties of the system and emphasises the role of profitability behind the dynamics of capitalist economies.

These results are in line with Goodwin's (1967) aim of generating persistent endogenous cycles, though in this case the key parameter is related to the sensitivity of investment to profitability. This is an important change in comparison to DF&S that used as bifurcation the response of real wages to the employment rate. Our choice here was motivated, first, by the desire to show the flexibility of the model in generating fluctuations. Secondly, given the importance of capital accumulation to export dynamics, we wanted our bifurcation parameter to reflect this component.

4 Numerical simulations

We proceed by presenting a numerical simulation exercise to illustrate and provide an economic interpretation of the limit cycle, whose existence was proved in the last section. As discussed by Gandolfo (2009, p. 479-484), the existence part of the Hopf bifurcation theorem leaves us in the dark regarding the nature of the cycle. One possibility is that orbits spiral toward a stable limit cycle. This is called a supercritical Hopf bifurcation. Another possibility is that an unstable cycle exists, also referred to as a subcritical Hopf bifurcation. In what follows, we show that in our case the bifurcation is supercritical.[6]

To this end, we must first of all choose functional forms for the three behavioural equations of the model, namely, $f(\cdot)$, $G(\cdot)$, and $h(\cdot)$. We specify these functions as follows:

$$f\left(e, \frac{\dot{e}}{e}\right) = \beta(e - \bar{e}) + \delta\dot{e}/e \tag{33}$$

$$G(u) = \alpha u \tag{34}$$

$$h(r) = \zeta + \gamma r \tag{35}$$

[6]By means of the first Lyapunov coefficient, we could formally determine the direction of the limit cycle bifurcation and, consequently, if it is super- or sub-critical. However, given that it is difficult to provide an economic interpretation of the required conditions, we directly rely on numerical simulations. For a rigorous reference on the topic, see Kuznetsov (1998, p. 151-186).

where $\bar{e}$ is the rate of employment above which workers are able to obtain real wage increases, β captures the sensitiveness of $\dot{w}/w$ to the employment rate, δ stands for the response of real wages to $\dot{e}/e$, α represents our learning-by-doing mechanism, while γ corresponds to the sensitiveness of investment to the profit rate. Finally, notice that for $\varpi = 1$ or $u = 0$, the profit rate is equal to zero so that capital accumulation is given by $\dot{K}/K = \zeta$. Thus, ζ stands as some sort of capital depreciation. It is unreasonable to suppose that firms continue to invest even when, for example, effective utilisation equals zero.

An important remark follows. All functional forms we have chosen are linear, so that the dynamics obtained are not due to *ad-hoc* non-linearities. The system is intrinsically non-linear as a result of the interaction between its basic structure, given by equations (9)-(11), and the adopted behavioural rules.

Recalling Proposition 1, the equilibrium values become:

$$e^* = \bar{e} + \frac{\left(\frac{\phi_2}{1-\phi_1}\right)\frac{\dot{Z}}{Z} - n}{\beta}$$

$$\varpi^* = 1 + \frac{\alpha}{\gamma}\left[\frac{\zeta - \left(\frac{\phi_2}{1-\phi_1}\right)\frac{\dot{Z}}{Z}}{\left(\frac{\phi_2}{1-\phi_1}\right)\frac{\dot{Z}}{Z} - n}\right]$$

$$u^* = \frac{1}{\alpha}\left[\left(\frac{\phi_2}{1-\phi_1}\right)\frac{\dot{Z}}{Z} - n\right]$$

In order to choose plausible parameter values, we have considered the evidence given in a number of empirical studies to provide outcomes with economic meaning.

$$\phi_1 = 0.148605,\ \phi_2 = 0.851395,\ \frac{\dot{Z}}{Z} = 0.03,\ n = 0.01,$$
$$\beta = 0.4,\ \delta = 0.05,\ \bar{e} = 0.85,\ \alpha = 0.05,\ \zeta = -0.07$$

Taking $h_r = \gamma$ as the bifurcation parameter, we have that $h_{rHB} \approx 0.4\bar{9}$. Consequently, in our simulation, we follow a three step procedure. First, we take a value for h_r that is greater than h_{rHB} and show that there is convergence to the equilibrium solution. Fig. 2 displays the solution path for $\gamma = 0.6$ and initial conditions (e_0, ϖ_0, u_0) equal to $(0.92, 0.53, 0.43)$, showing convergence to the fixed point $(e^*, \varpi^*, u^*) = (0.9, 0.58\bar{3}, 0.4)$.

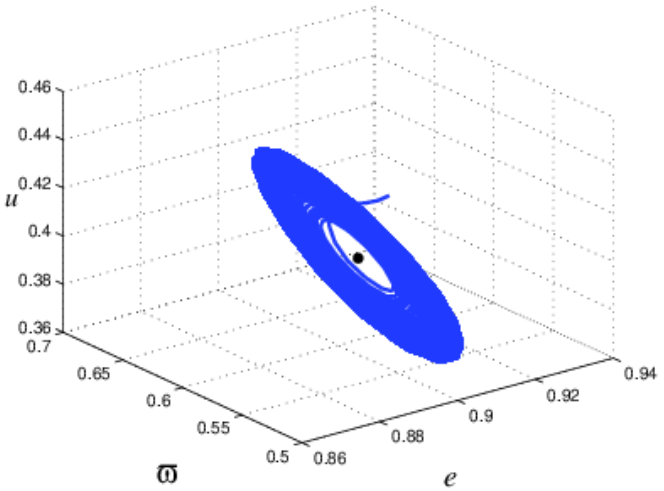
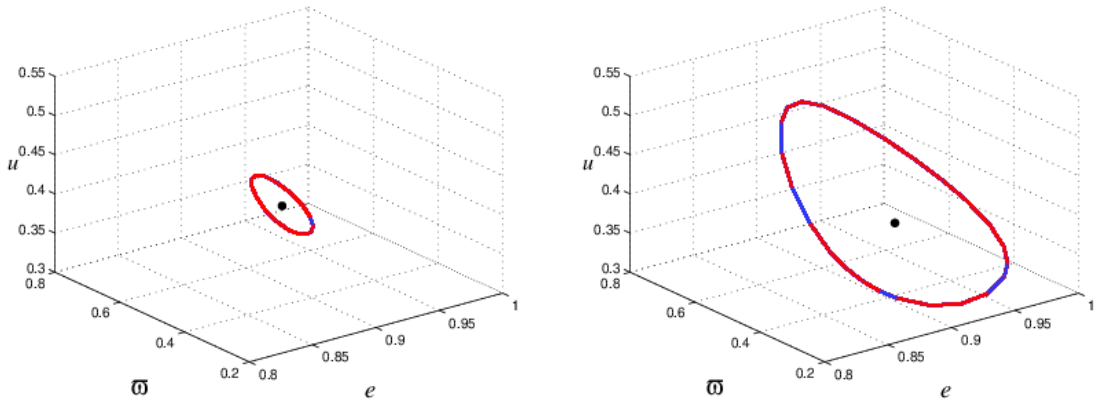


Figure 2: Convergence to the equilibrium solution for $_{\gamma=0.6}$

We proceed by considering the case in which there is a stable limit cycle. This confirms that we are dealing with a supercritical Hopf bifurcation. Fig. 3(a) depicts the case with $\gamma = 0.48\bar{9}$ and shows the asymptotic behaviour of two different trajectories with initial conditions $(e_0, \varpi_0, u_0) = (0.92, 0.53, 0.43)$ and $(e_0, \varpi_0, u_0) = (0.9, 0.45, 0.4)$, both converging to a limit cycle around the equilibrium point of coordinates $(e^*, \varpi^*, u^*) = (0.9, 0.48979, 0.4)$. The robustness of our findings is verified by further reducing the value of γ. As shown in Fig. 3(b), taking $\gamma = 0.3\bar{9}$ and maintaining the previous initial conditions, the trajectories now converge to a limit cycle around $(e^*, \varpi^*, u^*) = (0.9, 0.37499, 0.4)$. As expected, a reduction in the response of investment to changes in profitability leads to a reduction of the equilibrium wage share. Still, the most important insight is the enlargement in the amplitude of the fluctuations: a reduction in the sensitiveness of investment to the profit rate actually increases the volatility of the economy.



(a) (b)

Figure 3: Robustness of the limit cycle for (a) γ=0.48$\bar{9}$ and (b) γ=0.3$\bar{9}$

A possible explanation for this feature is the following. A gradual reduction of γ leads to a reduction in the equilibrium values of the wage share while equilibrium capacity utilisation does not change. This means that there is a decrease in $H_\varpi = h_r u^*$ while $H_u = h_r(1 - \varpi^*)$ remains relatively stable. In terms of our numerical example, the ratio H_u/H_ϖ increases from 1.04 to 1.28 and finally 1.56 as γ is reduced from 0.6 to 0.489 and 0.39. Such an increase in the relative importance of capacity utilisation for capital accumulation, the so called accelerator effect, is responsible for a higher volatility of the system.

Taking a closer look at the figures above, we can attempt to sketch a description of the dynamic interactions among the three variables along any given cycle. An expansion in employment leads to an increase in the bargaining power of workers increasing real wages above labour productivity. This results in a higher wage share which implies a reduction in the profitability of investment which decreases capital accumulation. As investment goes down, there is a reduction in output's growth rate through a reduction in the growth rate of exports that reduces employment rates. Moreover, given $\phi_1 < 1$, a reduction in capital accumulation increases effective capacity utilisation which in turn increases the growth rate of labour productivity through our learning-by-doing mechanism. This further decreases the rate of employment. A reduction in e reduces the wage share which in turn makes possible a recovery of investment profitability. A recovery in capital accumulation follows, leading to higher employment and lower capacity utilisation that ultimately are also reflected in higher employment

rates. At this point the cycle restarts.

$$e \uparrow \Rightarrow \varpi \uparrow \Rightarrow \frac{\dot{K}}{K} \downarrow \Rightarrow \begin{cases} e \downarrow \\ u \uparrow \Rightarrow \frac{\dot{q}}{q} \uparrow \Rightarrow e \downarrow \end{cases}$$

$$e \downarrow \Rightarrow \varpi \downarrow \Rightarrow \frac{\dot{K}}{K} \uparrow \Rightarrow \begin{cases} e \uparrow \\ u \downarrow \Rightarrow \frac{\dot{q}}{q} \downarrow \Rightarrow e \uparrow \end{cases}$$

The view that an economic system may be influenced permanently by the extent to which it has changed in the past seems highly intuitive. In economics, this is commonly referred to as path dependency and is associated with the idea that outcomes are historically contingent. At this point of the analysis we are unable to generate trajectories that are sensitive to initial conditions. However, there is some degree of path dependency due to the coexistence of attractors. In other words, different initial conditions can potentially lead to cycles around the same equilibrium values but with different amplitude. Fig. 4 displays this property showing the asymptotic behaviour of two trajectories with initial conditions $(e_0, \varpi_0, u_0) = (0.92, 0.53, 0.43)$ – in blue – and $(e_0, \varpi_0, u_0) = (0.84, 0.59, 0.35)$ – in red – when $\gamma = 0.48\bar{9}$. Comparing the two cycles, we can see how fluctuations in the employment rate, income distribution, and effective utilisation are more volatile in the first case.

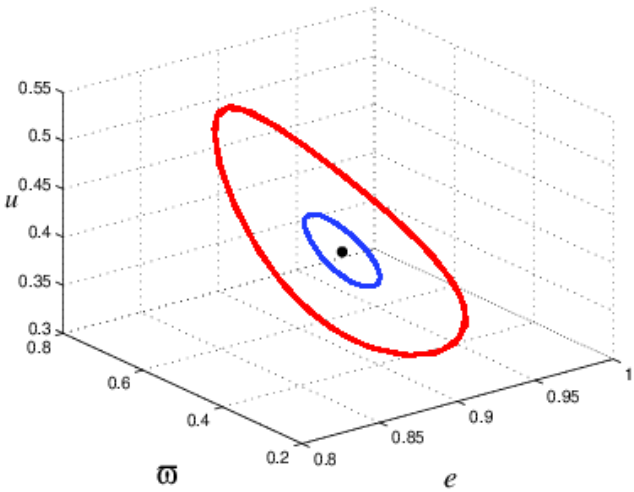


Figure 4: Coexistence of attractors for $\gamma = 0.48\bar{9}$

One should emphasise that such result was obtained even though all behavioural rules chosen were linear. Thus, there is the opportunity to explore more complex dynamics through an increase in the non-linearity of the model. One way to do this consists in adding a non-linear forcing term in the capital accumulation function. –Following Goodwin's (1951) insight that Schumpeterian innovations requiring investment occur periodically, we redefine capital accumulation, $h(\cdot)$, as:

$$h(r) = \zeta + \gamma r + \tau_1 \cos(\tau_2 t)$$

where τ_1 and τ_2 are parameters.

In this way, we obtain a scenario in which a non-linear system with a "natural" oscillation frequency interacts with an external "force" resulting in a chaotic attractor as shown in Fig. 5. The interplay between two or more independent frequencies characterising the dynamics of

the system is a well-known route to more complex behaviour.

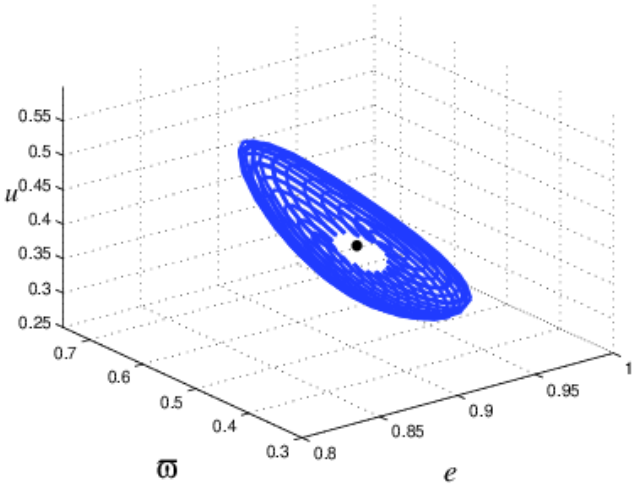


Figure 5: Chaotic attractor when τ_1=0.03 and τ_2=0.2

Fig. 6 depicts the solution paths for the two initial conditions $(e_0, \varpi_0, u_0) = (0.92, 0.53, 0.43)$ – continuous line – and $(e_0, \varpi_0, u_0) = (0.92, 0.53, 0.431)$ – dashed line. It provides a fair representation of the statement "history matters". A prediction of the future values of the variables would be possible only if the initial conditions could be measured with infinite precision. Very small differences in the initial conditions give rise to widely different trajectories.

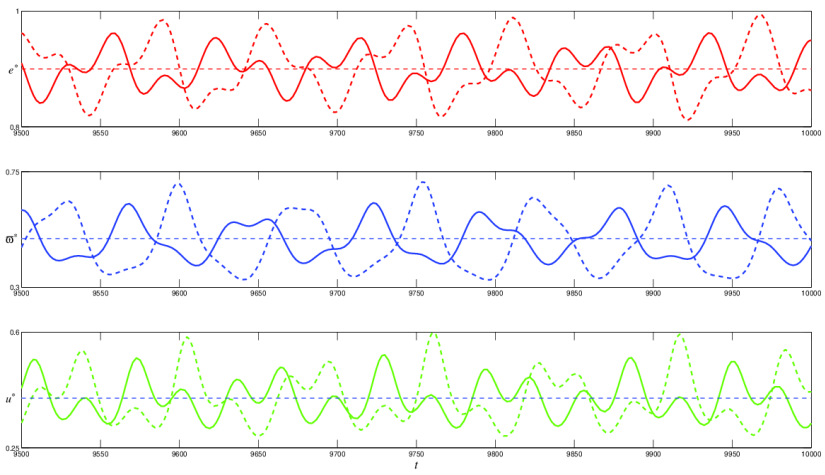


Figure 6: Sensitivity to initial conditions in the chaotic attractor when τ_1=0.03 and τ_2=0.2

5 Final considerations

In the past forty years, Goodwin's distributive cycle model and Thirlwall's law have consolidated themselves among alternative theories of growth and distribution as essential macroeconomic features of capitalist economies. In DF&S, we extended the former to an open economy set up in a way that incorporates the external constraint on growth. We did it by relying

on a learning-by-doing mechanism that allowed us to endogenise the growth rate of labour productivity.

It has been argued, however, that the BoPC growth model should take into account not only foreign demand but also domestic capacity to export. The idea that exports depend on productive capacity, either capital or investment, goes back at least to Fagerberg (1988) but has been recently rescued and further discussed by authors such as Romero and McCombie (2016a) or Razmi (2016). Given the importance of this modification for our model and future contributions in the BoPC literature, we investigated in this paper the implications of allowing exports to respond to capital accumulation.

Using panel cointegration techniques, we also provided empirical evidence for a sample of 19 OECD countries between 1950-2014 that gives support to the formulation adopted for the exports function. Exports, capital stock and foreign income were found to be cointegrated. Our DOLS estimations indicate that an increase of 100 US dollars in the capital stock is related to an increase of exports between 3.5 and 4.1 dollars. Such results emphasize the importance of explicitly incorporating capital when modelling the BoPC.

Furthermore, the model developed here is a three dimensional dynamic system that includes the employment rate, the wage share, and the rate of effective capacity utilisation. We showed that without having to impose any special condition on the values of the parameters, a Hopf-Bifurcation analysis establishes the possibility of persistent and bounded cyclical paths providing insights to enable better understanding of the nature of real-world fluctuations.

Our numerical simulations confirm that the emerging limit cycles are stable and robust to variations in the bifurcation parameter. Persistent bounded fluctuations emerge as we reduce the sensitiveness of investment to profitability. Further reductions in the bifurcation parameter increase the amplitude of the generated cycles, reflecting the importance of capital accumulation and profitability to the dynamics of capitalist economies.

Finally, the model is capable of providing different representations of the idea that "history matters". On the one hand, we were able to identify the coexistence of attractors so that different initial conditions lead to cycles of different amplitude. On the other hand, once we take into account the possibility of periodic innovations *à la* Schumpeter, we are able to provide an endogenous explanation of the irregularity of economic time-series.

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A Econometric Appendix

It is quite common now to have panels in which both T – the number of time series observations – and N – the number of cross sections – are quite large. In most applications of this type, the parameters of interest are the long-run effects and the speed of adjustment to the long-run. However, for a larger T, Pesaran and Smith (1995) among others have shown that the traditional procedures for estimation of pooled models, such as fixed effects (FE), instrumental variables (IV), and generalised method of moments (GMM), can produce inconsistent, and potentially very misleading estimates. These methods require the assumption of homogeneity of slope parameters that is often inappropriate when T is large. Moreover, for large T panels, non-stationarity may also be a concern.

On the other hand, the use of cointegration techniques to test for the presence of long-run relationships among integrated variables has enjoyed growing popularity in the empirical literature. Different techniques have been developed to address these issues. One in particular is very useful for the purposes of this study because it provides consistent estimates in a dynamic panel context, namely, the Dynamic Ordinary Least Square (DOLS) model. Extensions of DOLS to a panel setting were developed by Kao and Chiang (2001), Pedroni (2001), and

Mark and Sul (2003). In the presence of a cointegrating relationship, those estimators control for serial correlation and endogeneity.

We use data from the Penn World Table 9.0 (PWT), which contains standardized macro series for a large number of economies from the 1950s onwards. Our sample consists of 19 OECD countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, New Zeland, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdon, and United States) between 1950 and 2014.

Output is measured as real Gross Domestic Product (GDP) at current PPPs (in millions of 2011 US dollars). Capital stock is also measured at current PPPs (in millions of 2011 US dollars). Exports and imports are obtained by multiplying the respective shares in output by total output. Foreign GDP was calculated subtracting the country's GDP from the world's GDP, this last one being equal to the sum of every country's output available in the PWT. Finally, we also assess the robustness of our exercise using a second measure of productive capacity, namely, the output-capital ratio, which is easily obtained by dividing domestic output by the capital stock. Labor productivity was obtained as the ratio between GDP and number of persons engaged in production (in million). The respective growth rate was computed as $(q_{t+1} - q_t)/q_t$.

Ascertaining the order of integration of the variables under analysis is an essential precondition to establish whether the use of panel cointegration tests is warranted. In this respect, we performed the Im, Pesaran and Shin test, the ADF and Phillips Perron (PP) tests that assume individual unit root processes. Results are reported in table A1.

Table A1: Panel Unit Root tests (levels)

	Exports		Capital stock	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	1.0000	1.0000	1.0000	1.0000
ADF	1.0000	0.9997	1.0000	0.7744
PP	1.0000	1.0000	1.0000	1.0000
	Foreign GDP		Output-capital ratio	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	1.0000	1.0000	0.9962	0.3507
ADF	1.0000	1.0000	0.9940	0.6443
PP	0.0000	1.0000	0.9949	0.8333

Automatic lag selection based on SIC. Newey-West automatic Bandwidth selection.

As expected, series are found to be strongly non-stationary in levels. In all tests we reject the null hypothesis of stationarity. Hence, we proceed by performing the same set of tests now for series in first differences. Results are reported in table A2.

Table A2: Panel Unit Root tests (1st differences)

	Exports		Capital stock	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	0.0000	0.0000	0.0697	0.0000
ADF	0.0000	0.0000	0.0000	0.0000
PP	0.0000	0.0000	0.0000	0.0000
	Foreign GDP		Output-capital ratio	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	0.0000	0.0000	0.0000	0.0000
ADF	0.0000	0.0000	0.0000	0.0000
PP	0.0000	0.0000	0.0000	0.0000

Automatic lag selection based on SIC. Newey-West automatic Bandwidth selection.

Our estimates indicate that series become stationary once we take first differences. We verify the robustness of our exercise using the output-capital ratio as a second measure of productive capacity. Using u instead of K changes significantly the mechanisms discussed in this paper. Still, from an empirical point of view, it allows us to address the robustness of stating that exports depend on foreign demand and domestic capacity to respond to it.

Once we have determined that series are integrated of order one, $I(1)$, we can continue looking for cointegration. We make use of Pedroni (1999; 2004) tests, based on the Engle-Granger two-step cointegration test, as reported in table A3. We performed the tests using Dickey-Fuller corrected variances, but the results would not have changed had we not done so. First, we investigate the presence of cointegration for exports, capital stock and foreign income.

Table A3: Pedroni cointegration test (X, K, Z)

	Trend		Trend&Intercept	
	Prob.		Prob.	
Statistic	Pooled	Weighted	Pooled	Weighted
Panel v	0.0000	0.0000	0.0000	0.0000
Panel rho	0.0000	0.0003	0.0000	0.0001
Panel PP	0.0000	0.0044	0.0000	0.0001
Panel ADF	0.0000	0.0041	0.0000	0.0006
Group rho	0.0053		0.0000	
Group PP	0.0176		0.0000	
Group ADF	0.0060		0.0004	

Automatic lag selection based on SIC. Newey-West automatic Bandwidth selection.

The null of no-cointegration is clearly rejected at the 1% level of significance for all statistics. Still, we repeat the exercise substituting the capital stock by the output-capital ratio. Results do not change much, as we can see in table A4. The only exception is the Group ADF statistic (within dimension) which is not significant even at the 10% level.

Table A4: Pedroni cointegration test (X, u, Z)

	Trend		Trend&Intercept	
	Prob.		Prob.	
Statistic	Pooled	Weighted	Pooled	Weighted
Panel v	0.0000	0.0000	0.0000	0.0000
Panel rho	0.0003	0.0006	0.0031	0.0019
Panel PP	0.0010	0.0106	0.0027	0.0031
Panel ADF	0.0048	0.0509	0.0001	0.0125
Group rho	0.0023		0.0010	
Group PP	0.0353		0.0020	
Group ADF	0.3246		0.0147	

Automatic lag selection based on SIC. Newey-West automatic Bandwidth selection.

As a final step, we can now estimate the respective DOLS model to check the sign and magnitudes involved. Optimal number of lags was chosen using the Akaike informational criteria (AIC). AIC was preferred over the SIC in this case because it assigns a higher number of lags and thus avoids serial correlation problems.

We have already reported in table 1 our main estimates. Here, we limit ourselves to providing outcomes when capital is substituted by the output-capital ratio as measure of capacity to export. Table A5 brings our estimated coefficients. It is important to mention that u has a maximum of 0.639, a minimum of 0.143, and a mean of 0.35. Hence, in terms of the interpretation of results, an increase of 0.1 units of this ratio is related approximately to an increase between 27 000 - 53 000 US dollars of exports. Furthermore, higher foreign GDP is again positively and significantly related to exports. An increase of 100 dollars of Z leads to an increase between 0.5 - 0.7 dollars of exports.

Table A5: DOLS estimations $X(u,Z)$

Dependent variable	Exports			
Model	V	VI	VII	VIII
Capital Stock	529553.1***	313153.2***	528462.8***	270340.4***
Foreign GDP	0.006850***	0.005658***	0.006475***	0.004957***
Panel method	Pooled	Weighted	Pooled	Weighted
Time dummies	No	No	Yes	Yes
T-dimension	64	64	64	64
Cross-sections	19	19	19	19
Obs.	1204	1204	1201	1201
Adj. R2	0.892437	0.889752	0.970485	0.969738
Adj. sample	1951-2014	1951-2014	1951-2014	1951-2014

*, **, *** stand by 10%, 5%, and 1% of significance.

To assess a valid inference and not spurious regressions, residuals of all six regressions were checked for serial correlation. If residuals are correlated the estimated coefficients will be biased and inconsistent. In table A6, we report unit root tests on residuals of the first four sets of DOLS estimations. They are found to be stationary, thus, we conclude that our estimates are consistent and the cointegrating regressions are not spurious.

Table A6: Panel Unit Root tests, DOLS residuals of $x_{(K,Z)}$

	I		II	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	0.0000	0.0000	0.0000	0.0000
ADF	0.0000	0.0000	0.0000	0.0000
PP	0.0000	0.0001	0.0000	0.0006
	III		IV	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	0.0000	0.0000	0.0000	0.0000
ADF	0.0000	0.0000	0.0000	0.0000
PP	0.0000	0.0000	0.0000	0.0000

Automatic lag selection based on SIC. Newey-West automatic Bandwidth selection.

Finally, in table A7, we present unit root tests on residuals of the second set of DOLS estimations. We reject the null hypothesis of non-stationarity in all cases. Hence, errors are stationary, confirming the validy of our analysis.

Table A7: Panel Unit Root tests, DOLS residuals of $x_{(u,Z)}$

	V		VI	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	0.0000	0.0000	0.0000	0.0000
ADF	0.0000	0.0000	0.0000	0.0000
PP	0.0000	0.0000	0.0000	0.0006
	VII		VIII	
	Intercept	Trend&Intercept	Intercept	Trend&Intercept
Method	Prob.	Prob.	Prob.	Prob.
Im, Pesaran and Shin	0.0000	0.0000	0.0000	0.0012
ADF	0.0000	0.0000	0.0000	0.0000
PP	0.0000	0.0000	0.0000	0.0000

Automatic lag selection based on SIC. Newey-West automatic Bandwidth selection.

B Mathematical appendix

B.1 Proof of Proposition 1

To prove Proposition 1 we proceed in four steps. First, recall that in steady-state the rate of growth of output is equal to capital accumulation so as to keep effective capacity utilisation constant. From equation (29), it immediately follows that $H(\varpi^*, u^*) = \left(\frac{\phi_2}{1-\phi_1}\right)\frac{\dot{Z}}{Z} = y_{bp}$.

Continuing, from equations (27) and (29) we have $y_{bp} = G(u^*) + n$, where $G : \Re \to \Re$ is a function monotonically increasing in u. The inverse of $G(\cdot)$ is also monotonically increasing so that $u^* = G^{-1}(y_{bp} - n)$ is the unique equilibrium value of effective capacity utilisation.

Making use of equation (28) and the equilibrium expression for effective utilisation, it follows that $F(e^*, \varpi^*, u^*) = f(e^*, 0) = y_{bp} - n$. Recall that $F : \Re \to \Re$ is monotonically increasing in e. Therefore, its inverse is also an increasing function and there is a unique equilibrium value of the rate of employment, e^*, for which $f(e^*, 0) = y_{bp} - n$ is satisfied.

The equilibrium wage share is determined as the value of the wage share that brings effective capacity utilisation and the balance-of-payments to equilibrium. Our investment function $H : \Re \to \Re$ is monotonically increasing in u and decreasing in ϖ. Making use of the equilibrium value of capacity utilisation and the rate of growth of output, we have that there is a ϖ^* for which $H\left[\varpi^*, G^{-1}\left(y_{bp} - n\right)\right] = h\left[(1 - \varpi^*)G^{-1}\left(y_{bp} - n\right)\right] = y_{bp}$. It follows that the unique equilibrium for the wage share is determined and defined by ϖ^* which satisfies that condition. Finally, in order to obtain values with economic meaning we have to impose $0 < e^* < 1$, $0 < \varpi^* < 1$, and $0 < u^* < 1$.

B.2 Local stability analysis for the 3D dynamic system and proof of Proposition 2

In this Appendix we first derive the characteristic equation of the dynamic system (26) and prove Proposition 2. To do this, we linearise the dynamic system around the internal equilibrium point so as to obtain:

$$\begin{bmatrix} \dot{e} \\ \dot{\varpi} \\ \dot{u} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ 0 & J_{32} & J_{33} \end{bmatrix}}_{J^*} \begin{bmatrix} e - e^* \\ \varpi - \varpi^* \\ u - u^* \end{bmatrix}$$

where the elements of the Jacobian matrix J^* are given by:

$$J_{11} = \left.\frac{\partial j_1\left(e, \varpi, u\right)}{\partial e}\right|_{(e^*, \varpi^*, u^*)} = 0$$

$$J_{12} = \left.\frac{\partial j_1\left(e, \varpi, u\right)}{\partial \varpi}\right|_{(e^*, \varpi^*, u^*)} = -\phi_1 h_r u^* e^* < 0$$

$$J_{13} = \left.\frac{\partial j_1\left(e, \varpi, u\right)}{\partial u}\right|_{(e^*, \varpi^*, u^*)} = \left[\phi_1 h_r (1 - \varpi^*) - G_u\right] e^* \gtreqless 0$$

$$J_{21} = \left.\frac{\partial j_2\left(e, \varpi, u\right)}{\partial e}\right|_{(e^*, \varpi^*, u^*)} = f_e \varpi^* > 0$$

$$J_{22} = \left.\frac{\partial j_2\left(e, \varpi, u\right)}{\partial \varpi}\right|_{(e^*, \varpi^*, u^*)} = -f_{\hat{e}} \phi_1 h_r u^* \varpi^* < 0$$

$$J_{23} = \left.\frac{\partial j_2\left(e, \varpi, u\right)}{\partial u}\right|_{(e^*, \varpi^*, u^*)} = \left\{f_{\hat{e}}\left[\phi_1 h_r (1 - \varpi^*) - G_u\right] - G_u\right\} \varpi^* \gtreqless 0$$

$$J_{31} = \left.\frac{\partial j_3\left(e, \varpi, u\right)}{\partial e}\right|_{(e^*, \varpi^*, u^*)} = 0$$

$$J_{32} = \left.\frac{\partial j_3\left(e, \varpi, u\right)}{\partial \varpi}\right|_{(e^*, \varpi^*, u^*)} = \left(1 - \phi_1\right) h_r u^{*2} > 0$$

$$J_{33} = \left.\frac{\partial j_3\left(e, \varpi, u\right)}{\partial u}\right|_{(e^*, \varpi^*, u^*)} = \left(\phi_1 - 1\right) h_r (1 - \varpi^*) u^* < 0$$

so that the characteristic equation can be written as

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$$

where the coefficients are given by:

$$\begin{aligned} b_1 &= -\operatorname{tr} J^* = -(J_{22} + J_{33}) \\ &= f_{\hat{e}}\phi_1 h_r u^* \varpi^* - (\phi_1 - 1)\, h_r(1 - \varpi^*)u^* > 0 \end{aligned} \tag{36}$$

$$\begin{aligned} b_2 &= \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & J_{13} \\ 0 & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & J_{12} \\ J_{21} & J_{22} \end{vmatrix} = J_{22}J_{33} - J_{23}J_{32} - J_{12}J_{21} \\ &= -f_{\hat{e}}\phi_1 h_r u^* \varpi^* (\phi_1 - 1)\, h_r(1 - \varpi^*)u^* \\ &- \{f_{\hat{e}}\,[\phi_1 h_r(1 - \varpi^*) - G_u] - G_u\}\, \varpi^* (1 - \phi_1)\, h_r u^{*2} + \phi_1 h_r u^* e^* f_e \varpi^* \\ &= G_u (1 + f_{\hat{e}})\, \varpi^* (1 - \phi_1)\, h_r u^{*2} + \phi_1 h_r u^* e^* f_e \varpi^* > 0 \end{aligned} \tag{37}$$

$$\begin{aligned} b_3 &= -\det J = -J_{13}J_{21}J_{32} + J_{12}J_{21}J_{33} \\ &= -[\phi_1 h_r(1 - \varpi^*) - G_u]\, e^* f_e \varpi^* (1 - \phi_1)\, h_r u^{*2} - \phi_1 h_r u^* e^* f_e \varpi^* (\phi_1 - 1)\, h_r(1 - \varpi^*)u^* \\ &= G_u e^* f_e \varpi^* (1 - \phi_1)\, h_r u^{*2} > 0 \end{aligned} \tag{38}$$

The necessary and sufficient condition for the local stability of (e^*, ϖ^*, u^*) is that all roots of the characteristic equation have negative real parts, which, from Routh–Hurwitz criteria, requires:

$$b_1 > 0,\ b_2 > 0,\ b_3 > 0 \text{ and } b_1 b_2 - b_3 > 0.$$

Hence, the crucial condition for local stability becomes the last one. Through direct computation we find that:

$$\begin{aligned} b_1 b_2 - b_3 &= -(J_{22} + J_{33})\,(J_{22}J_{33} - J_{23}J_{32} - J_{12}J_{21}) + J_{13}J_{21}J_{32} - J_{12}J_{21}J_{33} \\ &= [f_{\hat{e}}\phi_1 h_r u^* \varpi^* - (\phi_1 - 1)\, h_r(1 - \varpi^*)u^*]\, \left[G_u (1 + f_{\hat{e}})\, \varpi^* (1 - \phi_1)\, h_r u^{*2}\right. \\ &\left. + \phi_1 h_r u^* e^* f_e \varpi^*\right] - G_u e^* f_e \varpi^* (1 - \phi_1)\, h_r u^{*2} \\ &= [f_{\hat{e}}\phi_1 u^* \varpi^* - (\phi_1 - 1)\,(1 - \varpi^*)u^*]\, \left[G_u (1 + f_{\hat{e}})\, \varpi^* (1 - \phi_1)\, u^{*2} + \phi_1 u^* e^* f_e \varpi^*\right] h_r^2 \\ &- G_u e^* f_e \varpi^* (1 - \phi_1)\, u^{*2} h_r \\ &= (A h_r - B)\, h_r \end{aligned} \tag{39}$$

with

$$\begin{aligned} A &= [f_{\hat{e}}\phi_1 u^* \varpi^* - (\phi_1 - 1)\,(1 - \varpi^*)u^*]\, \left[G_u (1 + f_{\hat{e}})\, \varpi^* (1 - \phi_1)\, u^{*2} + \phi_1 u^* e^* f_e \varpi^*\right] > 0 \\ B &= G_u e^* f_e \varpi^* (1 - \phi_1)\, u^{*2} > 0 \end{aligned}$$

that is satisfied when:

$$h_r > \frac{B}{A}$$

Notice, however, that the equilibrium wage share is a function of the sensitiveness of capital accumulation to the profit rate, i.e. $\varpi^*(h_r)$. Therefore, if the sensitivity of capital accumulation to profitability is such that:

$$h_r - \frac{G_u e^* f_e (1 - \phi_1)}{\{[(1 + f_{\hat{e}})\, \phi_1 - 1]\, \varpi^*(h_r) - (\phi_1 - 1)\}\, [G_u (1 + f_{\hat{e}})\, (1 - \phi_1)\, u^* + \phi_1 e^* f_e]} > 0$$

the internal equilibrium (e^*, ϖ^*, u^*) of the dynamic system (26) is locally asymptotically stable.

B.3 Proof of Proposition 3

To prove Proposition 3 using the (existence part of) the Hopf Bifurcation Theorem and using h_r as bifurcation parameter, we must: (HB1) show that the characteristic equation possesses a pair of complex conjugate eigenvalues $\theta(h_r) \pm i\omega(h_r)$ that become purely imaginary at the critical value $h_{r\ HB}$ of the parameter, i.e. $\theta(h_{r\ HB}) = 0$ with $\omega(h_{r\ HB}) \neq 0$, while no other eigenvalues with zero real part exists at $h_{r\ HB}$; and (HB2) check that the derivative of the real part of the complex eigenvalues with respect to the bifurcation parameter is different from zero at the critical value.

(HB1) Given that the conditions $b_1 > 0$, $b_2 > 0$ and b_3 are all fulfilled, in order that the characteristic equation has one negative real root and a pair of complex roots with zero real part we must have:

$$b_1 b_2 - b_3 = 0$$

a condition which, given the expression for $b_1 b_2 - b_3$ derived in (39), is satisfied for

$$h_{r\ HB} - \frac{G_u e^* f_e (1-\phi_1)}{\{[(1+f_{\hat{e}})\phi_1 - 1]\varpi^*(h_{r\ HB}) - (\phi_1 - 1)\}[G_u(1+f_{\hat{e}})(1-\phi_1)u^* + \phi_1 e^* f_e]} = 0$$

(HB2) By using the so-called sensitivity analysis, it is then possible to show that the second requirement of the Hopf Bifurcation Theorem is also met. Substituting the elements of the Jacobian matrix into the respective coefficients of the characteristic equation:

$$\begin{aligned}
b_1 &= f_{\hat{e}}\phi_1 h_r u^* \varpi^*(h_r) - (\phi_1 - 1) h_r [1 - \varpi^*(h_r)] u^* \\
&= [f_{\hat{e}}\phi_1 + (\phi_1 - 1)] u^* h_r \varpi^*(h_r) - (\phi_1 - 1) u^* h_r \\
b_2 &= G_u (1+f_{\hat{e}}) \varpi^*(h_r)(1-\phi_1) h_r u^{*2} + \phi_1 h_r u^* e^* f_e \varpi^*(h_r) \\
&= [G_u(1+f_{\hat{e}})(1-\phi_1)u^* + \phi_1 e^* f_e] u^* h_r \varpi^*(h_r) \\
b_3 &= G_u e^* f_e \varpi^*(h_r)(1-\phi_1) h_r u^{*2} \\
&= G_u e^* f_e (1-\phi_1) u^{*2} h_r \varpi^*(h_r)
\end{aligned}$$

so that

$$\begin{aligned}
\frac{\partial b_1}{\partial h_r} &= [f_{\hat{e}}\phi_1 + (\phi_1 - 1)] u^* \left[\varpi^*(h_r) + \varpi^*_{h_r} h_r\right] - (\phi_1 - 1) u^* > 0 \\
\frac{\partial b_2}{\partial h_r} &= [G_u(1+f_{\hat{e}})(1-\phi_1)u^* + \phi_1 e^* f_e] u^* \left[\varpi^*(h_r) + \varpi^*_{h_r} h_r\right] > 0 \\
\frac{\partial b_3}{\partial h_r} &= G_u e^* f_e (1-\phi_1) u^{*2} \left[\varpi^*(h_r) + \varpi^*_{h_r} h_r\right] > 0
\end{aligned}$$

When $h_{r\ HB} - \frac{G_u e^* f_e (1-\phi_1)}{\{[(1+f_{\hat{e}})\phi_1 - 1]\varpi^*(h_{r\ HB}) - (\phi_1 - 1)\}[G_u(1+f_{\hat{e}})(1-\phi_1)u^* + \phi_1 e^* f_e]} = 0$, apart from $b_1 > 0$, $b_2 > 0$ and $b_3 > 0$ one also has $b_1 b_2 - b_3 = 0$. In this case, one root of the characteristic equation is real negative (λ_1), whereas the other two are a pair of complex roots with zero real part ($\lambda_{2,3} = \theta \pm i\omega$, with $\theta = 0$). We thus have:

$$\begin{aligned}
b_1 &= -(\lambda_1 + \lambda_2 + \lambda_3) \\
&= -(\lambda_1 + 2\theta) \\
b_2 &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \\
&= 2\lambda_1\theta + \theta^2 + \omega^2 \\
b_3 &= -\lambda_1\lambda_2\lambda_3 \\
&= -\lambda_1\left(\theta^2 + \omega^2\right)
\end{aligned}$$

such that:

$$\begin{aligned}
\frac{\partial b_1}{\partial h_r} &= -\frac{\partial \lambda_1}{\partial h_r} - 2\frac{\partial \theta}{\partial h_r} = P > 0 \\
\frac{\partial b_2}{\partial h_r} &= 2\theta\frac{\partial \lambda_1}{\partial h_r} + 2\left(\lambda_1 + \theta\right)\frac{\partial \theta}{\partial h_r} + 2\omega\frac{\partial \omega}{\partial h_r} = Q > 0 \\
\frac{\partial b_3}{\partial h_r} &= -\left(\theta^2 + \omega^2\right)\frac{\partial \lambda_1}{\partial h_r} - 2\lambda_1\theta\frac{\partial \theta}{\partial h_r} - 2\lambda_1\omega\frac{\partial \omega}{\partial h_r} = R > 0
\end{aligned}$$

where

$$\begin{aligned}
P &= \left[f_{\hat{e}}\phi_1 + \left(\phi_1 - 1\right)\right]u^*\left[\varpi^*\left(h_r\right) + \varpi^*_{h_r}h_r\right] - \left(\phi_1 - 1\right)u^* \\
Q &= \left[G_u\left(1 + f_{\hat{e}}\right)\left(1 - \phi_1\right)u^* + \phi_1 e^* f_e\right]u^*\left[\varpi^*\left(h_r\right) + \varpi^*_{h_r}h_r\right] \\
R &= G_u e^* f_e\left(1 - \phi_1\right)u^{*2}\left[\varpi^*\left(h_r\right) + \varpi^*_{h_r}h_r\right]
\end{aligned}$$

For $\theta = 0$, the system to be solved becomes:

$$\begin{aligned}
-\frac{\partial \lambda_1}{\partial h_r} - 2\frac{\partial \theta}{\partial h_r} &= P \\
2\lambda_1\frac{\partial \theta}{\partial h_r} + 2\omega\frac{\partial \omega}{\partial h_r} &= Q \\
-\omega^2\frac{\partial \lambda_1}{\partial h_r} - 2\lambda_1\omega\frac{\partial \omega}{\partial h_r} &= R
\end{aligned}$$

or

$$\begin{bmatrix} -1 & -2 & 0 \\ 0 & 2\lambda_1 & 2\omega \\ -\omega^2 & 0 & -2\lambda_1\omega \end{bmatrix}\begin{bmatrix} \frac{\partial \lambda_1}{\partial h_r} \\ \frac{\partial \theta}{\partial h_r} \\ \frac{\partial \omega}{\partial h_r} \end{bmatrix} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

Thus:

$$\begin{aligned}
\left.\frac{\partial \theta}{\partial h_r}\right|_{\partial h_r = \partial h_r\ HB} &= \frac{\begin{vmatrix} -1 & P & 0 \\ 0 & Q & 2\omega \\ -\omega^2 & R & -2\lambda_1\omega \end{vmatrix}}{\begin{vmatrix} -1 & -2 & 0 \\ 0 & 2\lambda_1 & 2\omega \\ -\omega^2 & 0 & -2\lambda_1\omega \end{vmatrix}} \\
&= \frac{\omega(Q\lambda_1 - \omega^2 P + R)}{\left(\lambda_1^2 + \omega^2\right)}
\end{aligned}$$

and $\left.\frac{\partial \theta}{\partial h_r}\right|_{\partial h_r = \partial h_r\ HB} \neq 0$ as long as $Q\lambda_1 + R \neq \omega^2 P$.