Loss aversion, habit formation and the term structures of equity and interest rates

(Article begins on next page)

Loss Aversion, habit formation and the term structures of equity and interest rates

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Abstract

I propose a consumption-based asset pricing model that jointly explains the high equity premium, the counter-cyclical behaviour of stock returns, the upward sloping term structure of interest rates and the downward sloping term structure of equity. The driving forces behind these results are loss-aversion and time-varying habits. The high premium is the reward for holding assets that deliver low returns when consumption descends below habits. The term structure of interests rates is upward sloping because long-term bonds are more sensitive to fluctuations of discount rates. The term structure of equity is downward sloping because long-horizon equity gives higher chances to beat consumption habits than short-horizon equity.

Keywords: Loss-aversion, Habit Formation, Yield curve, Dividend strips, General Equilibrium.

JEL Classification Numbers: D51; D91; E20; G12.

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1 Introduction

In the past two decades, habit formation has been used to explain many empirical regularities of asset returns. In their seminal work, Campbell and Cochrane (1999) propose an economy with external habit formation that explains the equity premium puzzle and the high and counter-cyclical volatility of stock returns. More recently, Buraschi and Jiltsov (2007) and Wachter (2006) explore the implications of habit formation for the term structure of interest rates. Dai (2003) builds a production economy with stochastic internal habits that explains the main properties of bond and stock returns. Santos and Veronesi (2010) and Lettau and Wachter (2007) analyse the effect of habit formation on the cross-section of stock returns.

The literature on habit formation is based on the "additive" property of consumption, that is, individual consumption never falls below the standard of living. In this paper, I deviate from the assumption of addictive habits and consider an asset pricing model with heterogeneous agents where consumption falls below the standard of living when economic conditions deteriorate. I believe this is consistent with the low level of consumption experienced during the recent economic crises. The model explains level and cyclicality of stock returns as well as the the term structures of equity and interest rates. The main point of departure from the previous literature on habit formation is the consumption utility. More precisely, I assume that agents are equipped with the gain-loss utility of Kahneman and Tversky (1979, 1991, 1992) and evaluate consumption relative to a time-varying reference level. Moreover, agents are heterogeneous in the reference level of consumption.

Loss-aversion in consumption has already proved useful to explain the high equity premium. According to Yogo (2008), when agents care about fluctuations of consumption around a certain reference point, stocks are risky because they deliver low returns in recession when consumption approaches or falls below the reference level. Thus, the equity premium is the required compensation to hold assets that are positively correlated with consumption losses. The main contribution of this paper is to propose a general equilibrium model that jointly explains the stock returns and the term structure of equity and interest rates. Empirically, the yield curve is upward sloping while the term structure

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of equity is downward sloping. Lettau and Wachter (2011) point out a difficulty for general equilibrium models that aim at explaining the two term structures simultaneously: the upward sloping term structure of interest rates suggests that investors require a premium for holding high duration assets; This mechanism also implies an upward sloping term structure of equity, contrary to the empirical evidence[1]. For instance, Wachter (2006) shows that a model with external habit formation, in the spirit of Campbell and Cochrane (1999), generates the observed upward sloping term structure of interest rates. On the other hand, van Binsbergen et al. (2012) show that the external habit model of Campbell and Cochrane (1999) generates a term structure of equity that slopes upward. Lettau and Wachter (2011) resolve this tension between the two term structures by exogenously specifying a risk-free rate that is negatively correlated with fundamentals and a price of risk process which is not correlated with fundamentals. However, as suggested by the authors themselves, this approach leaves a question open: can the two term structures be endogenized within a single micro-founded equilibrium model?

A model with loss-aversion and heterogeneity in the reference level of consumption offers a positive answer to the previous question. Intuitively, the model in this paper combines habit formation with an utility function that allows consumption to fall below the reference level. At the same time, loss-aversion implies that agents suffer huge utility losses when consumption descends below the reference level. As pointed out by Lettau and Wachter (2007) and van Binsbergen et al. (2012), in a consumption-based model with habit formation, long-maturity assets are more sensitive to fluctuations of discount rates and, thus, command higher premium than short term assets. This mechanism makes both the term structures of equity and interest rates upward sloping. Instead, when agents care about losses in consumption, they are willing to pay more for holding long-horizon equity that, having higher expected pay-off, gives lower chances of consumption losses than short-horizon equity. This insurance property makes the term structure of equity downward sloping. In contrast, the insurance property of long-horizon equity is of no

[1]The argument in Lettau and Wachter (2011) is actually more general than this: they study the cross-section of stock returns and build an equilibrium models that explains the returns of value and growth stocks. However the mechanism that generates the upward sloping term structure of equity is the same as the one I describe here.

value in standard models with habit formation because consumption never falls below the standard of living.

In summary, from the point of view of a loss-averse investor, expected returns are generated by two opposite effects: the discount rate and the insurance effect. Pure-discount bonds, having a constant pay-off, are subject to the first effect only and, as a result, the term structure of interest rates slopes upward. Stocks are subject to both effects but, thanks to loss-aversion, the insurance effect dominates and the term structure of equity slopes downward.

Within a representative agent framework, Hung and Wang (2010) show that loss-aversion in consumption reproduces the observed upward sloping term structure of interest rates. My work differs from theirs for two reasons: first I consider an economy where agents differ with respect to the reference level of consumption while Hung and Wang (2010) work in the representative agent framework. Second, I study the implication of loss-aversion for the term structure of equity while they only look at the implications for the term structure of interest rates. However, they are able to provide a remarkably good match of the most important observed moments of interest rates, while I am more interested in the qualitative implications of loss-aversion for the joint slope of the term structures of equity and interest rates.

Finally, this paper offers a technical contribution and shows that a continuum distribution of reference levels is essential for the existence of the equilibrium which, on the contrary, does not obtain in a representative agent model or with a discrete number of agents. Loss-aversion over consumption may be at odds with the existence of the equilibrium because the optimal consumption of loss-averse agents is discontinuous: as consumption price falls below a given threshold, the optimal consumption jumps above the reference level. With a discrete number of agents, the demand curve of the consumption good jumps when the consumption of loss-averse agents rises above (or falls below) the reference point, and the equilibrium fails to hold. Differently, when agents are distributed over a continuum of reference levels, their contribution to the aggregate demand of consumption is infinitesimal. As a result, the aggregate demand of consumption is continuous even if the single-agent's demand is not, thus, allowing for the existence of the equilibrium. A similar result is obtained by De Giorgi et al. (2010) who assume that agents have (cumulative) prospect theory preferences over final wealth and evaluate outcomes according to subjective decision weights rather than using the true probabilities.

The rest of the paper is organized as follow: Section 2 introduces the model and the primitives of the economy. Section 3 characterizes the competitive equilibrium. Section 4 presents the results of the quantitative analysis and Section 5 concludes. Technical details and proofs can be found in Appendix A.

2 The Model

I consider an infinite-horizon, pure-exchange economy where a single consumption good serves as numeraire and investors trade continuously on a complete financial market to share risk. The uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ on which I define a two-dimensional Brownian motion $B_t = [B_{1,t}, B_{2,t}]$ with instantaneous correlation $\langle dB_{1,t}, B_{2,t} \rangle = \rho_{1,2} dt$.

The economy features two non-standard elements. First, I assume that agents are equipped with the gain-loss utility of Kahneman and Tversky (1979, 1991, 1992). More precisely, agents in the economy i) care about fluctuations of consumption around a time-varying reference level, ii) are more sensitive to losses than gains and iii) show risk-seeking behaviour in the domain of losses. I assume that the reference point of consumption is represented by an index of the social standard of living defined as the geometric average of past realizations of the aggregate consumption. Second, the economy is populated with a continuum of investors who differ from each other with respect to the sensitivity to the social standard of living.

Preferences Agents in this economy maximize expected utility of the form

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} U(c_t, Z_t, b) dt\right] \tag{1}$$

where

$$U(c_t, Z_t, b) = \begin{cases} -B\frac{(n(b)Z_t - c_t))^{1-\gamma}}{1-\gamma}, & \text{if } c_t < n(b)Z_t, \\ \frac{(c_t - n(b)Z_t)^{1-\gamma}}{1-\gamma}, & \text{if } c_t \geq n(b)Z_t, \end{cases} \tag{2}$$

$\gamma < 1$, c_t is the consumption rate at time t and Z_t is an exogenous state variable which is interpreted as the standard of living in the economy. $n(b)$ is a density function that represents the agent-specific sensitivity to the standard of living. In addition, I assume that optimal consumption cannot descend below the agent-specific subsistence level $n(b)\underline{Z}_t$. Utility function 2 captures the main features of prospect theory: i) it is steeper for losses than for gains (i.e. agents are loss-averse); ii) it is convex in the domain of losses and concave in the domain of gains, i.e. (agents are risk-seeking when consumption falls below the reference level).

There is a continuum of preference types, defined over $b \in [0, \infty)$. Given the distribution of agent-type, the aggregate reference level of consumption and the aggregate subsistence level in the economy are determined as $\int_0^\infty n(b)Z_t dt = Z_t$ and $\int_0^\infty n(b)\underline{Z}_t dt = \underline{Z}_t$, respectively. Note that, since agents' consumption is not allowed to fall below the subsistence level, a restriction on the aggregate endowment process is required to ensure market clearing (see Detemple and Zapatero (1991) and Chapman (1998) for a detailed discussion of similar problems in pure-exchange economies with habit formation). The next paragraph describes in detail the aggregate endowment process consistent with market clearing.

Aggregate endowment and habits Let e_t be the aggregate endowment of the economy at time t. An endowment process which is consistent with the existence of the equilibrium is

$$\frac{dY_t}{Y_t} = \mu_y dt + \phi_y dB_{1,t} \tag{3}$$

where $Y_t \equiv e_t - \underline{Z}_t$, μ_y and ϕ_y are positive constants and $\mu_y > \phi_y^2/2$. In other words, the difference between the aggregate endowment and the subsistence level is defined as a log-normal random variable. In this way, it is always possible to consume the subsistence level and the agents demand the entire endowment at all dates. Conversely, if the endowment is not enough to finance the consumption of the subsistence level at some dates, the market-clearing mechanism may fail at those dates and the equilibrium does not exist.

It is convenient to define the variable $\Delta Z_t \equiv Z_t - \underline{Z}_t$. I assume that the habit process Z_t is stochastic and satisfies:

$$d \lg \Delta Z_t = \lambda \left(\omega_t + \mu_z\right) dt + \phi_z dB_{2,t} \tag{4}$$

where $\omega_t \equiv \lg Y_t - \lg \Delta Z_t$ measures the relative difference between consumption and habits and describes that state of the economy: High (low) values of ω represents good (bad) states of the economy[2]. In line with the habit formation literature, I call ω relative consumption. An application of the Ito's lemma reveals that ω follows the mean-reverting dynamics

$$d\omega_t = -\lambda \left(\omega_t - \bar{\omega}\right) dt + \phi_y dB_{1,t} - \phi_z dB_{2,t} \tag{5}$$

with long-run moments given by

$$\bar{\omega} \equiv \lim_{t\to\infty} \mathbb{E}_0[\omega_t] = \frac{\mu_y - 0.5\phi_y^2}{\lambda} - \mu_z, \quad \sigma_\omega \equiv \left[\lim_{t\to\infty} Var_0[\omega_t]\right]^{1/2} = \sqrt{\frac{\phi_y^2 + \phi_z^2 - 2\phi_y\phi_z\rho_{1,2}}{2\lambda}}. \tag{6}$$

The dynamics of habits differs from previous works on this topic, such as Constantinides (1990) or Chan and Kogan (2001) in two key aspects. First, this model departs from the usual assumption of (instantaneously) deterministic habits by assuming that agents' habits are hit by unexpected shocks. Economically, habits shocks can be interpreted as demand shocks or shocks in the process of acquiring new habits from other people. Dai (2002), shows that a model with stochastic habit formation accounts for the risk-free rate puzzle, the equity premium puzzle and the expectation puzzle, simultaneously[3].

[2]Note that, by specifying the process for the log-difference between Z_t and $\underline{Z}_t$, I ensure that $Z_t - \underline{Z}_t > 0$ at all dates and, as a result, utility function 2 is well defined.

[3]Dai (2002) imposes the additional condition that habit volatility vanishes at zero surplus consumption. Otherwise, the consumption process may descend below the habit level with positive probability. This condition is not required in my model since individual consumption is allowed to fall below habits.

Second, models with deterministic habits usually assume that the stock of habits increases whenever the relative consumption is positive, and decreases otherwise. Differently, I introduce an additional drift component, μ_z, in the habit process. When μ_z is positive, habits increase faster over time, thus, reflecting agents' optimism on the future state of the economy. When μ_z is negative, agents are reluctant to increase the stock of habits, even when the relative consumption is positive, reflecting pessimistic views on the future state of the economy. From an economic point of view, the case $\mu_z < 0$ seems to make more sense than $\mu_z > 0$. This is so because I expect the habit process to be non-increasing, on average, at zero surplus-consumption ratio. In fact, when the aggregate consumption equals the habit level, an additional increase of ΔZ_t would imply an increase of the probability that future consumption drops below habits, thus, causing an increase in the probability of future losses and a drop in expected utility. Anyway, I discuss the implication of μ_z for stock returns in Section 4 below.

Capital markets Agents can invest in a risk-less bond in zero net supply and a risky asset, the stock, in net supply of 1. The risky stock is a claim to the (net) aggregate endowment and its price, S_t, evolves as

$$\frac{dS_t + Y_t dt}{S_t} = \mu_t dt + \sigma_{1,t} dB_{1,t} + \sigma_{2,t} dB_{2,t} \tag{7}$$

The price of the bond is denoted by S^0 and evolves as

$$\frac{dS_t^0}{S_t^0} = r_t dt \tag{8}$$

where r_t is the instantaneous risk-less interest rate. The asset prices coefficients, μ, $\sigma_{1,t}$ $\sigma_{2,t}$, and r are to be determined endogenously in equilibrium.

These two investment opportunities are not enough to render the market complete. Dai (2002) shows that, in a model with stochastic habits, one way to complete the market is to introduce a console bond, in zero net supply, into the investment opportunity set.

An alternative way is to allow the agents to trade options, in zero net supply [4], written on the sources of risk B_2. The options can be interpreted as a consumption insurance which allows the agents to protect themselves against the risk that consumption drops below the reference level. Anyway, since the characterization of the agents' portfolio is beyond the scope of this paper, I assume, without loss of generality, that markets are complete without further restriction on the investment opportunity set.

Market completeness implies the existence of a unique pricing kernel, or state-price density m_t, whose dynamics can be written as

$$\frac{dm_t}{m_t} = -r_t dt - \theta_c dB_{1,t} - \theta_z dB_{2,t} \tag{9}$$

where θ_c and θ_z are the prices of risk and represent the expected return on a claim with unit exposure to consumption and habit risk, respectively. The price of risk processes, θ_c and θ_z are to be determined in equilibrium.

3 The competitive equilibrium

I solve the model following the standard steps in the literature: First, I obtain the optimal consumption sharing rule by solving the social planner problem; then, I determine the state-price density by imposing the market clearing condition for the consumption good and, finally, I use the state-price density to determine the equilibrium price of traded assets.

The social planner distributes the aggregate endowment among agents in a such a way that the consumption allocation is Pareto optimal. Following Chan and Kogan (2001) I assume that $g(b)$ is the exogenous social weights function. Given the distribution of social weights, the social planner solves

$$\max_{c(b)} \mathbb{E} \int_0^\infty e^{-\rho t} \left[\int_0^\infty g(b) U(c_t, Z_t, b) db \right] ds \tag{10}$$

[4] The volatility of the option prices has to be specified in such a way that the variance/covariance matrix of financial assets has full rank.

subject to the resource constraint

$$\int_0^\infty c_t(b)db \leq Y_t + \underline{Z}_t,$$

$$c_t(b) \geq n(b)\underline{Z}_t \quad \forall t, b \tag{11}$$

where U is the utility function of loss-averse agents defined in Eq. 2. In the following, I describe the solution to the social planner problem and the most important equilibrium conditions. A complete list of all equilibrium quantities and their derivation is deferred to Appendix A. The optimal sharing rule is presented in the next Proposition.

Proposition 1 *The optimal consumption sharing rule is given by*

$$c^*(b) = \begin{cases} g(b)^{\frac{1}{\gamma}} k_t^{-\frac{1}{\gamma}} + n(b)Z_t, & \text{if } k_t \leq \overline{k}(b) \\ n(b)\underline{Z}_t, & \text{if } k_t > \overline{k}(b). \end{cases} \tag{12}$$

where $k_t \equiv e^{\rho t} m_t$ *and* $\overline{k}(b)$ *is implicitly defined by*

$$g(b)^{\frac{1}{\gamma}} \overline{k}^{1-\frac{1}{\gamma}} \frac{\gamma}{1-\gamma} - \overline{k} n(b)\Delta Z + g(b) B \frac{(n(b)\Delta Z)^{1-\gamma}}{1-\gamma} = 0. \tag{13}$$

Proposition 1 says that the optimal consumption profile of prospect theory is discontinuous: when the state-price density falls below the threshold $\overline{k}$, the optimal consumption jumps above the reference level nZ_t; when the state-price density exceeds the threshold $\overline{k}$, the optimal consumption falls to the subsistence level $n\underline{Z}$. In other worlds, in the states of the world where consumption is excessively expensive (i.e. when $k_t > \overline{k}$), the risk-seeking behaviour prevails and induces loss-averse agents to decrease consumption at the subsistence level and invest as much as possible in the stock market in order to maximize the probability of beating the reference level in the next future. We notice that the threshold $k_t > \overline{k}$ is agent-specific. This implies that the economy is characterized by the coexistence of agents whose consumption exceeds the reference level (risk-averse agents), and agents who consume at the subsistence level (risk-seeking agents). This interaction has strong implications for the equilibrium properties of asset returns that are analysed in this paper.

We obtain more insights on the optimal sharing rule by adding more structure to the problem. If we further assume that $g(b) = n(b)$ and $\frac{\partial g}{\partial b} < 0$ we can rewrite the optimal sharing rule as follow:

Proposition 2 *Assume that $g(b) = n(b)$ and $\frac{\partial g}{\partial b} < 0$. Then, the optimal sharing rule is given by*

$$c^*(b) = \begin{cases} g(b)^{\frac{1}{\gamma}} k_t^{-\frac{1}{\gamma}} + n(b) Z_t, & \text{if } b \leq \bar{b} \\ n(b) \underline{Z}_t, & \text{if } b > \bar{b}. \end{cases} \tag{14}$$

where $\bar{b}$ solves

$$g(x)^{\frac{1}{\gamma}} \left(\frac{S(x)}{G(x)} \right)^{1-\gamma} \frac{\gamma}{1-\gamma} - \left(\frac{S(x)}{G(x)} \right)^{-\gamma} n(x) e^{-\omega} + g(x) B \frac{n(x)^{1-\gamma}}{1-\gamma} e^{-(1-\gamma)\omega} = 0, \tag{15}$$

$S = 1 - N(x) e^{-\omega}$ *and* $G(x) \equiv \int_0^x g(u)^{\frac{1}{\gamma}} du$.

The assumptions on g and n are made to ensure that $\bar{k}(b)$ is monotonically decreasing in b. This implies that the agents' consumption profile is determined by $\bar{b}$ only: agents with high standard of living ($b < \bar{b}$) are those whose consumption exceeds the reference level; agents with low standard of living ($b > \bar{b}$) are more likely to consume at the subsistence level. Moreover, $\bar{b}$ is increasing in ω. This implies that, as the economic conditions improve, the fraction of agents whose consumption exceeds the reference level increases. The assumption $g(b) = n(b)$ implies that agents with high standard of living also hold larger fractions of initial wealth[5]. In other words, agents who start the economy with a larger fraction of wealth, have higher standard of living and are more likely to preserve their standard of living when economic conditions deteriorate[6].

I conclude this section with a remark on the importance of heterogeneity. Assume that the economy is populated with a representative agent characterized by reference

[5]In fact, given the one to one mapping between social weights and the Lagrange multiplier of the single-agent optimization problem in the the decentralized economy, higher social weights imply larger endowment of initial wealth.

[6]The model can be solved even in absence of these two assumption. However these assumptions greatly simplify computations and, in addition, generate economic implications, I believe, in line with empirical observations

level Z. According with Proposition 1, for any level of the pricing kernel below $\overline{k}$, the optimal consumption is given by the subsistence level $\underline{Z}$ and, as a result, it does not exist a value of the pricing kernel such that demand and supply of consumption are equal. Suppose now that the economy is populated with two loss-averse agents, say 1 and 2, with corresponding state-price density threshold $\overline{k}_1$ and $\overline{k}_2$, social weights g_1 and g_2, and sensitivities n_1 and n_2. Without loss of generality, I assume $\overline{k}_1 < \overline{k}_2$. There are 3 relevant cases to be considered: $k < \overline{k}_1 < \overline{k}_2$, $\overline{k}_1 < k < \overline{k}_2$ and $\overline{k}_1 < \overline{k}_2 < k$. The case $\overline{k}_1 < \overline{k}_2 < k$ is analogous to the case of a representative agent consuming at the subsistence level where there is no solution to the market clearing equation.

When $\overline{k}_1 < k < \overline{k}_2$ only consumption of agent 2 exceeds the reference level and the market clearing condition writes as

$$\begin{aligned} & Zn_2 + k^{-\frac{1}{\gamma}} g_2^{\frac{1}{\gamma}} + n_1 \underline{Z} \\ & = Zn_2 + k^{-\frac{1}{\gamma}} g_2^{\frac{1}{\gamma}} + (1 - n_2)\underline{Z} \\ & = Y + \underline{Z} \end{aligned} \tag{16}$$

with solution $k = \left(\frac{Y - n_2 \Delta Z}{g_2^{\frac{1}{\gamma}}} \right)^{-\gamma}$. The assumption $\overline{k}_1 < k < \overline{k}_2$ implies that Eq. 16 has a solution only when

$$\overline{Y}_2 < Y < \overline{Y}_1 \tag{17}$$

where:

$$\begin{aligned} \overline{Y}_1 & \equiv n_2 \Delta Z + \overline{k}_1^{-\frac{1}{\gamma}} g_2^{\frac{1}{\gamma}} \\ \overline{Y}_2 & \equiv n_2 \Delta Z + \overline{k}_2^{-\frac{1}{\gamma}} g_2^{\frac{1}{\gamma}}. \end{aligned}$$

Since $\overline{k}_1 < \overline{k}_2$ we conclude that $\overline{Y}_1 > \overline{Y}_2$. Repeating the same reasoning for the other cases we can characterize k as follow

$$k = \begin{cases} \left(\frac{y+-n_2\Delta Z}{g_2^{\frac{1}{\gamma}}}\right)^{-\gamma}, & \text{if } \overline{Y}_2 \leq Y \leq \overline{Y}_1 \\ \left(\frac{Y-\Delta z}{\sum_{i=1}^{2} g_i^{\frac{1}{\gamma}}}\right)^{-\gamma}, & \text{if } Y > \overline{Y}_3 \end{cases} \tag{18}$$

where:

$$\overline{Y}_3 \equiv \Delta Z + \overline{k}_1^{-\frac{1}{\gamma}} \sum_{i=1}^{2} g_i^{\frac{1}{\gamma}} \tag{19}$$

Since $\overline{Y}_3 > \overline{Y}_1$, the state-price density is not defined when $\overline{Y}_1 < Y < \overline{Y}_3$. The problem here is the discontinuous adjustment of the aggregate demand of consumption when the consumption of agent 1 jumps above the reference level.

Therefore, in a pure-exchange economy with loss-averse agents, heterogeneity in the attitude toward losses not only is consistent with empirical and experimental evidence on loss-aversion (Dimmock and Kouwenberg (2010) and Gill and Prowse (2012)), but also necessary for the existence of the equilibrium [7].

4 Quantitative Analysis

In this section I quantify the implications of loss-aversion and time varying habits for the moments of asset returns. In particular I am interested in the usual conditional and unconditional moments of stock returns, such as the risk-free rate, the equity premium and the returns volatility. In addition, I examine the effect of loss-aversion on the term structures of equity and interest rates. The derivation of these quantities is deferred to Appendix A.

[7]More precisely, the problem is the discontinuous consumption demand *in conjunction* with the exogenous endowment process. The exogenous consumption process does not adapt to jumps in the demand of consumption. This suggests that a production economy, where the supply of consumption is *endogenously* determined by a representative firm's choices, should allow the equilibrium to exist even in absence of heterogeneity across agents. This idea is left for future research.

4.1 Calibration

The parameters are chosen in order to illustrate the economic link between loss-aversion, time-varying habit and asset returns and, at the same time, producing asset pricing quantities not very different from their empirical counterpart.

The drift and volatility of the consumption process are $\mu_y = 1.7\%$ and $\phi_y = 4.05$ %, which belong to the typical range of values in the literature. In particular, these parameters are very similar to those in Chan and Kogan (2001). Concerning the preference parameters, $B = 2.25$ is the usual loss-aversion parameter and corresponds to the estimates of Kahneman and Tversky (1992). s-shaped utility requires $\gamma < 1$ which limits the ability of the model to generate high equity premium. Thus, I choose $\gamma = .98$ in order to obtain a sizeable equity premium. In the habit formation literature, the persistence parameter λ typically ranges in between .01 and .05. In a recent working paper, Lynch and Randall (2011) show that these values are quite low and, arguably, inconsistent with the micro evidence of habit formation. In order to account for this new evidence, I choose $\lambda = 0.15$

Concerning heterogeneity, I assume that the cross sectional distributions of agents/types and social weights are described by two exponential functions with parameters l_1 and l_2, respectively. To satisfy the restriction of Proposition 2, l_1 and l_2 have to be equal, thus, I have to calibrate one parameter only, say $l \equiv l_1 = l_2$. l represents the importance of loss-averse agents in the economy and is crucial for the shape of the price of risk. Empirically, the price of risk is counter-cyclical, rising during recession and decreasing in good times. In consumption based asset pricing models, this property is generally captured by a counter-cyclical aggregate risk aversion. In my model there are two countervailing forces that determine the dynamics of the aggregate risk aversion: after a negative economic shock, the consumption of a group of agents approaches the reference level, thus, increasing the aggregate risk aversion in the economy. On the other hand, after a negative shock, the consumption of some agents drops at the subsistence level and those agents become risk-seeking, thus, decreasing the aggregate risk aversion in the economy. Therefore, in order to generate the counter-cyclical pattern of risk aversion, I have to assign low weight to agents who are more likely to consume at the subsistence level, that is, those with high sensitivity b. Note also that, if the weight attached to loss averse agents is excessively low, asset prices would be similar to those in Campbell and Cochrane (1999) where, by construction, consumption never falls below the reference level and the term structure of equity is upward sloping. For this reason, I select $l = 0.5$, so that the distribution of agents/type and social weights are both centred at $1/l = 2$.

Finally, keeping constant the previous parameters, I construct four economies based on different specifications of drift and volatility of the habit process. In the first economy, which I label as *Model 1*, I consider the standard locally deterministic habit process of Chan and Kogan (2001) where $\mu_z = \phi_z = 0$. In *Model 2* and *Model 3* I keep $\phi_z = 0$ but take into account the role of pessimisms and optimism on the future state of the economy. μ_z is not directly observable and can be chosen to obtain realistic moments of asset returns. In *Model 2* I set $\mu_z = -0.11$ to avoid excessive risk-taking behaviour which would result in low equity premium, low return volatility and excessive risk-free rate volatility[8]. In *Model 3*, I choose $\mu_z = 0.11$ for the sake of symmetry. We will see that *Model 3* produces counter-factual properties of asset returns and therefore I introduce stochastic habits in *Model 2* only. This leads to *Model 4* where I allow for stochastic habit formation and choose $\phi_z = 0.02$. The low value of habit volatility is in line with Dai (2002) who shows that low habit volatility is consistent with the main observed properties of stock returns.

4.2 Moments of asset returns

The moments of asset return are computed using the expressions derived in Section A. To compute conditional expectations I simulate 100 paths, each of them containing 10000 realizations (at monthly frequency) of the state variable ω, for any given initial condition ω_0. Unconditional returns statistics are computed by integrating conditional moments over the interval $[\overline{\omega} - 5\sigma_\omega, \overline{\omega} + 5\sigma_\omega]$.

[8]More precisely, given this choice of μ_z, $P(\omega < 0) = 0.13\%$. The low probability of $\omega < 0$ limits the effect of risk-seeking agents on asset prices

4.2.1 Unconditional moments

The unconditional moments of asset returns are reported in Table 2. The implied equity premium is quite large, increases with the habit drift μ_z and ranges from 2.51% to 11.92% across the different calibrations. The link between μ_z and equity premium is intuitive: An increase in μ_z decreases $\overline{\omega}$, for the same level of σ_ω. This makes consumption more likely to fall below the reference level and, in turn, increases the required compensation for holding the risky asset. The same economic mechanism explains the positive relation between stock returns volatility and μ_z. Furthermore, it is important to note that the implied equity premium and stock returns volatility are remarkably high numbers, especially if compared with the corresponding unconditional risk aversion of the economy. For instance, *Model 2* produces an equity premium of 4.40% and a stock market volatility of 21.70% in conjunction with unconditional risk aversion of 4.69. For comparison, a model with power utility and risk aversion equal to 4.69 would have produced an equity premium of 0.75% and a stock market volatility of 4%.

In models with locally deterministic habits (i.e. $\sigma_z = 0$) and with $\mu_z \leq 0$, the risk free rate and its volatility are low and similar to the values observed in the data. This stands in contrast with standard models of habit formation which tend to produce excessively volatile risk-free rates. Differently, when $\mu_z > 0$ and/or $\sigma_z > 0$ consumption fluctuates "too much" around the reference level and the effect of loss-aversion increases: loss-averse agents borrow more than risk-averse agents to finance their aggressive trading strategy and this increases both the risk free rate and its volatility.

The model also reproduces the level of the price-dividend ratio but not its volatility which is in general lower than that observed in the data. Stochastic habits generate higher price-dividend volatility but at the cost of increasing the risk-free rate volatility at about 5% which is higher than what observed in the data.

Taking all this information together we conclude that a combination of loss-aversion, locally deterministic habits and pessimistic expectations about the future economic conditions is reasonably consistent with the basic stock market properties. Therefore, in the sections that follow I focus on *Model 2* and its implications for the conditional moments of stock returns and the term structures of equity and interest rates.

4.2.2 Conditional moments

In this section I explore the relation between relative consumption and conditional moments of stock returns which are crucial to understand the statistical properties of stock returns previously derived. In Figure 1 we observe that the price-dividend ratio is an increasing function of the relative consumption. In recessions, consumption is low relative to habits, the aggregate risk aversion increases and prices are low relative to dividend. Figure 2 shows the most relevant conditional moments: the risk-free rate,

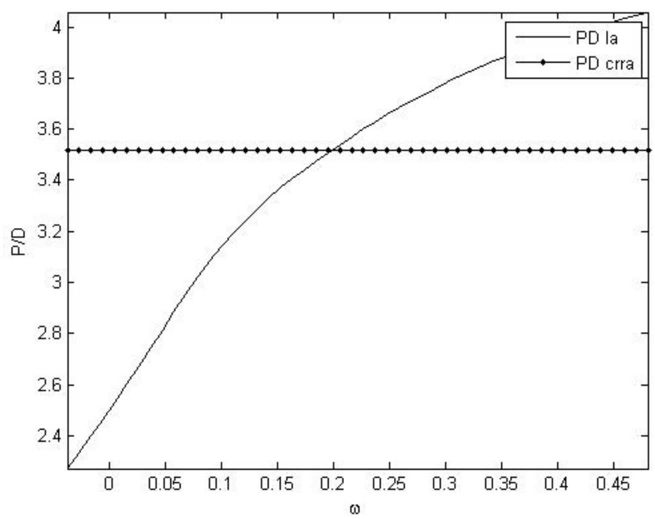

Figure 1: Log price-dividend ratio as a function of ω. Parameters are from Table 1.

the equity premium, the stock return volatility and the Sharpe ratio. The risk-free rate is counter-cyclical and, compared to the power utility case, is higher in recession and lower in boom. Economically, this is explained by the dynamics of precautionary saving motives: in boom, marginal utility is low and agents are motivated to save to be insured against possible recessions tomorrow, thus, decreasing the risk-free rate; in recession, marginal utility is low and agents are motivated to borrow to allow their consumption to catch up with habits, thus, increasing the risk-free rate [9]. The second panel (clockwise) of Figure 2 shows that the implied equity premium is counter-cyclical over the relevant range

[9]Moreover, in recession, the proportion of loss-averse agents increases. Loss-averse agents are motivated to borrow in order to finance an "aggressive" trading strategy. This reinforces the precautionary saving effect and drives further up the risk-free rate.

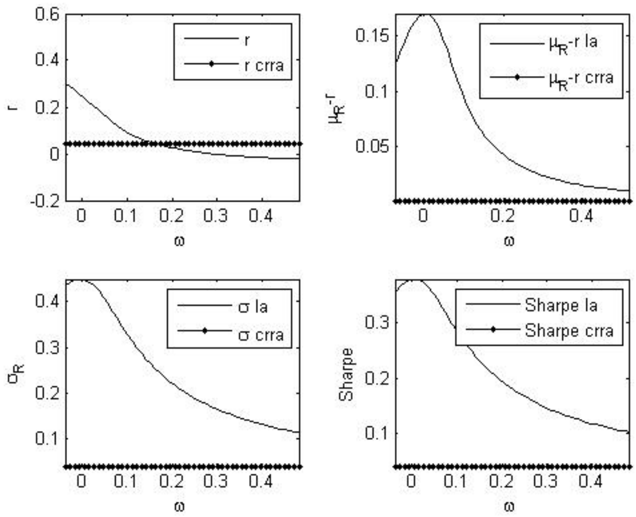

Figure 2: Conditional moments of asset returns as a function of ω. Parameters are from Table 1.

of relative-consumption. This result is intuitive given the pro-cyclical behaviour of the price-dividend ratio and the counter-cyclical behaviour of the risk-free rate. This suggests that the equity premium in the model is the reward that agents require for the risk of low returns in recession when consumption approaches or falls below the reference level. From the third panel of Figure 2 we observe that the stock return volatility increases as consumption falls toward the habits. Thus the stock returns volatility is counter-cyclical as empirically observed. Finally, the fourth panel of Figure 2 shows that the conditional Sharpe ratio is decreasing in the relative consumption.

These results suggest that the model replicates the observed counter-cyclical fluctuations of asset returns. It is important to emphasize that the proportion of loss-averse agents is crucial for this result. Counter-cyclical fluctuation in asset returns obtain because the aggregate risk aversion in the economy is itself counter-cyclical. However, in models with loss-averse agents the counter-cyclical patter of the aggregate risk aversion is not straightforward: when relative consumption decreases, a fraction of agents becomes risk-seeking, thus, decreasing the aggregate risk aversion. As a consequence, an high fraction of loss-averse agents would make the aggregate risk aversion, and thus expected returns, pro-cyclical in contrast to the empirical evidence[10]. On the other hand, decreasing t he

[10] A more detailed analysis of the effect of loss-aversion on asset prices can be found in Curatola (2012)

fraction of loss-averse agents to zero would give a model similar to that of Campbell and Cochrane (1999), with adverse consequences for the term structure of equity as I argue in the next section.

4.2.3 Implications for the term structures of interest rates and equity

In this section I study the model's implications for the unconditional returns of dividend strips and bond yield[11]. The dividend strip is defined as an asset that pays the aggregate endowment Y at a given maturity τ. The pure-discount bond is an asset that pays the fixed pay-off 1 at a given maturity τ. Details for the calculations of prices and returns of dividend strip and pure-discount bonds are given in Appendix A. The first panel of Figure 3 shows that the yield curve is up-ward sloping. This is a discount factor effect: long-term bond are more sensitive to fluctuations of discount rates and thus command an higher return. The economic mechanism behind this result is illustrated by Wachter (2006) and can be summarized with the following example: a positive consumption shock increases relative consumption and, in turn, decreases the aggregate risk aversion; The lower risk aversion implies that agents discount future pay-off at a lower rates which increases prices. Therefore, bond prices are positively correlated to consumption shocks and earn a positive risk premium; Since this effect is more pronounced for long-term bonds, the term structure of interest rates is up-ward sloping. The previous result leads to the following question: does the discount factor effect also implies that the term structure of equity slopes upward? On the one hand, long-horizon equity is subject to the discount factor effect, in a similar fashion to long-term bonds. This effect implies that, ceteris paribus, the term structure of equity will rotate upward. On the other hand, while pure-discount bonds have a fixed pay-off at maturity, the pay-off of dividend strips is random, and increases on average over time. In other words, long-horizon equity has higher ex-

[11]Admittedly, a full characterization of the empirical properties of the term structure of interest rates requires to model nominal and real bonds and, thus, to specify a process for inflation. However, in this paper, I do not aim at matching returns and volatility of (nominal and real) bonds, nor at explaining the expectations puzzle. Instead, my objective is to explore the economic link between loss-aversion, the distribution of temporal risk and, in turn, the slope of the term structure of interest rates. For this goal, a simple characterization of the real term structure (which is provided by my equilibrium model) is enough.

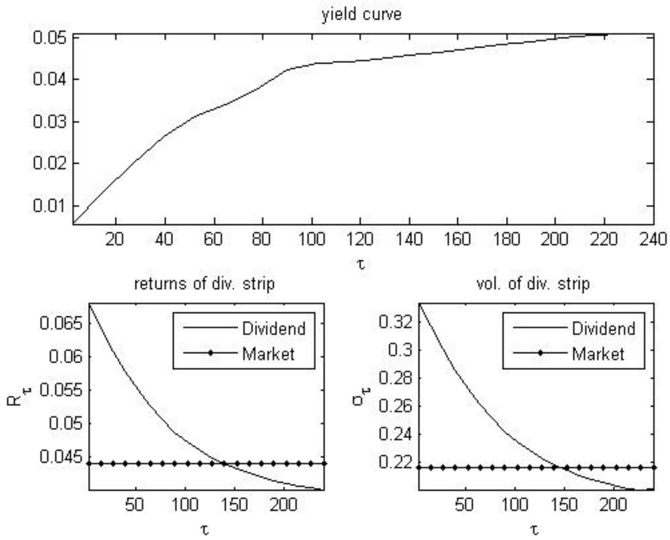

Figure 3: Term structures. The yield curve, the returns of dividend strips and the volatility of dividend strips are plotted against maturity τ expressed in months. Parameters are from Table 1.

pected pay-off than short-horizon equity. From the perspective of a loss-averse agent, the higher expected pay-off makes long-horizon equity less risky than short-horizon equity because it implies a lower probability of future consumption losses. This effect rotates the term structure of equity downward. The final slope of the term structure depends on the sum of these two opposite forces. The last two panel of Figure 3 show that the second effect is more important for loss-averse agents and the term structure of equity slopes downward, consistently with the empirical evidence. Note also, that in standard habit formation model, without loss-aversion, the insurance effect of long-horizon equity has no value, and the term structure of equity slopes upward, as showed by van Binsbergen et al. (2012).

van Binsbergen et al. (2012) also show that returns on short-horizon equity are higher than aggregate market returns. As evident in Figure 3, this finding can be replicated in a model with loss-aversion and time-varying habits. The market asset, which is a long-lived asset, incorporates the cash flow risk for all maturities; As a result, the return of the market reflects the average of equity returns at all horizons, and lies in between the returns of short/median horizon equity and the returns of long-horizon equity. This is not only consistent with the empirical results of van Binsbergen et al. (2012), but also confirms that, in this model, asset returns are mainly determined by fluctuations of cash-flows.

These results, taken all together, suggest that a combination of loss-aversion, time-varying habits and preference heterogeneity can simultaneously account for the behaviour of the aggregate stock market, the upward-sloping yield curve and the downward sloping term structure of equity returns. The explanation proposed in this paper is purely preference based in the sense that the previous results are explained by the fact that loss-averse agents are willing to pay more for assets that give higher chances to beat their reference of consumption. Recently, Collin-Dufresne et al. (2013) show that the term structure of equity can be explained if dividend dynamics are derived endogenously under the assumption that firms employ a capital structure policy that generates stationary leverage ratios. Their approach is more "structural" than mine, in the sense that the downward slopes of equity returns comes mainly from the dividend dynamics than from preferences. Thus, in a sense, these two explanations can be considered complementary to each other.

5 Conclusion

This paper proposes a framework to understand the behaviour stock returns and the joint term structure of equity and interest rates. The economic building blocks of the model are loss-aversion and heterogeneous habit formation. An interesting extension for future research would be to add multiple assets in the economy and study the implications of loss-aversion in consumption for the cross-section of stock returns. Lettau and Wachter (2007) show that models where 1) value stocks vary more with fluctuations in cash flow while growth stocks vary more with fluctuations in discount rates and 2) agents care more about fluctuations in cash flows, account for the observed value premium. In models with loss-aversion, short-horizon equity is indeed more sensitive to fluctuations in cash-flows and agents care more about cash-flow because they fear consumption losses. Therefore, loss-aversion in consumption has the potential to explain the value premium puzzle. However, loss-aversion with multiple assets poses an additional challenge: are agents loss-averse over single assets or over portfolios of assets? As shown by Barberis and Huang (2001) this is not irrelevant for asset prices. I leave this challenge for future research.

6 Appendix A: Model solution and proof

6.1 Optimal consumption and equilibrium

To find the optimal consumption I follow Berkelaar et al. (2004). The time index is removed for simplicity. Since the s-shaped utility is not quasi concave, the first order conditions only describe local maxima: if $c < nZ$ the utility function is convex and the Weirestrass theorem implies that the maximum must lie on the boundaries, $n\underline{Z}$ or nZ; If $c \geq nZ_t$ the utility function is concave and the optimal consumption can be obtained by applying the Kuhn-Tucker theorem:

$$e^{-\rho t} g \left(c - nZ\right)^{-\gamma} = m - \lambda$$
$$\lambda(c - n\underline{Z}), \quad \lambda \geq 0 \tag{20}$$

where λ is the Lagrange multiplier associated to the susbsistence-level constraint. Solving the system 20 we are left with the following three candidates for a global maximum, $c_1 = g^{\frac{1}{\gamma}} k^{-\frac{1}{\gamma}} + nZ$, $c_2 = nZ$ and $c_3 = n\underline{Z}$, where $k = e^{\rho t} m$. Let the function $L : \mathbb{R} \to \mathbb{R}$ be the Legendre-Fenchel transform (or convex conjugate) of the agent's maximization problem:

$$L(k, c) = \max_{c \geq 0} \{g(b)U(c) - kc\}. \tag{21}$$

We first note that

$$L(k, c_1) > L(k, c_2) \Longleftrightarrow \tag{22}$$
$$g^{\frac{1}{\gamma}} k^{1-\frac{1}{\gamma}} \frac{\gamma}{1-\gamma} - nZ \geq -nZ \Longleftrightarrow \tag{23}$$
$$g^{\frac{1}{\gamma}} k^{1-\frac{1}{\gamma}} \frac{\gamma}{1-\gamma} \geq 0 \Longleftrightarrow \tag{24}$$
$$\gamma < 1 \tag{25}$$

which implies that, for $\gamma < 1$, c_1 is always preferred to c_2. Second,

$$L(k, c_2) > L(k, c_3) \iff \tag{26}$$

$$-knZ \geq -gB\frac{(n\Delta Z)^{1-\gamma}}{1-\gamma} - kn\underline{Z} \iff \tag{27}$$

$$k < k^* \equiv gB\frac{(n\Delta Z)^{-\gamma}}{1-\gamma} \tag{28}$$

The above inequality implies that, for $k < k^*$, c_1 is the optimal solution to the agent's maximization problem. Instead, when $k \geq k^*$ the optimal consumption is either c_1 or c_3:

$$L(k, c_1) > L(k, c_3) \iff \tag{29}$$

$$g^{\frac{1}{\gamma}}k^{1-\frac{1}{\gamma}}\frac{\gamma}{1-\gamma} - knZ \geq -gB\frac{(n\Delta Z)^{1-\gamma}}{1-\gamma} - kn\underline{Z} \iff \tag{30}$$

$$g^{\frac{1}{\gamma}}k^{1-\frac{1}{\gamma}}\frac{\gamma}{1-\gamma} - kn\Delta Z + gB\frac{(n\Delta Z)^{1-\gamma}}{1-\gamma} \equiv f(k, b, \Delta Z) \geq 0 \tag{31}$$

which leads to Eq. 13 in Proposition 1. Let now c^* be the consumption that maximizes $L(k, c)$ and $\tilde{c}$ an alternative solution to the social planner problem, that satisfies the resource constraint either with equality or inequality. Thus we can write

$$\begin{aligned}
&\mathbb{E}\int_0^\infty e^{-\rho t}\left[\int_0^\infty g(b)U(c_t^*, Z_t, b)db\right]ds - \mathbb{E}\int_0^\infty e^{-\rho t}\left[\int_0^\infty g(b)U(\tilde{c}_t, Z_t, b)db\right]ds \\
&= \mathbb{E}\int_0^\infty \left[e^{-\rho t}\int_0^\infty g(b)\left(U(c_t^*, Z_t, b) - U(\tilde{c}_t, Z_t, b)\right)db\right]ds + k(y_t + \underline{Z}) - k(y_t + \underline{Z}) \\
&\geq \mathbb{E}\int_0^\infty \left[e^{-\rho t}\int_0^\infty g(b)\left(U(c_t^*, Z_t, b) - U(\tilde{c}_t, Z_t, b)\right)db\right]ds + k\int_0^\infty c_t^* db - k\int_0^\infty \tilde{c}_t db \\
&= \mathbb{E}\int_0^\infty e^{-\rho t}\left[\int_0^\infty (g(b)U(c_t^*, Z_t, b) - kc_t^*) - (g(b)U(\tilde{c}_t, Z_t, b) - k\tilde{c}_t)\right]dbds \geq 0
\end{aligned}$$

where, the first inequality follows because the resource constraint holds with equality for c^* and with inequality for $\tilde{c}$; the last inequality follows because c^* maximizes $L(k, c)$. This shows that c^* solves the original social planner problem. Simple algebra reveals that

$$f(k^*, b, \Delta Z) = g^{\frac{1}{\gamma}} (k^*)^{1-\frac{1}{\gamma}} \frac{\gamma}{1-\gamma} > 0, \tag{32}$$

$$\lim_{k \to \infty} f(k, b, \Delta Z) = -\infty, \tag{33}$$

$$\frac{\partial f}{\partial k} = -g^{\frac{1}{\gamma}} k^{-\frac{1}{\gamma}-1} - n\Delta Z < 0 \tag{34}$$

which implies that Eq. $f(k, b, \Delta Z) = 0$ admits a unique solution $\overline{k}$ in the interval (k^*, ∞), for any b. We conclude that c_1 is optimal for any $k \leq \overline{k}$, and c_3 is optimal for any $k > \overline{k}$. This concludes the proof of Proposition 1.

We are now ready to prove Proposition 2. An application of the implicit function theorem on $f(k, b, \Delta Z)$ reveals that

$$\frac{\partial \overline{k}}{\partial b} = -\frac{\partial f / \partial b}{\partial f / \partial k} = \frac{g' \left(\frac{g^{\frac{1}{\gamma}-1} k^{1-\frac{1}{\gamma}}}{1-\gamma} + B \frac{(n\Delta Z)^{1-\gamma}}{1-\gamma} \right) + n' \left(gBn^{-\gamma} \Delta Z^{1-\gamma} - k\Delta Z \right)}{g^{\frac{1}{\gamma}} k^{-\frac{1}{\gamma}} + n\Delta Z} \tag{35}$$

where $g' \equiv \frac{\partial g(b)}{\partial b}$ and $n' \equiv \frac{\partial n(b)}{\partial b}$. Under the additional assumptions $g = n$ the above derivative can be written as

$$\begin{aligned} \frac{\partial \overline{k}}{\partial b} &= \frac{\frac{g'}{g} \left(g^{\frac{1}{\gamma}-1} k^{1-\frac{1}{\gamma}} + f(\overline{k}, b, \Delta Z) + g^2 B \left(n\Delta Z \right)^{1-\gamma} \right)}{g^{\frac{1}{\gamma}} k^{-\frac{1}{\gamma}} + n\Delta Z} \\ &= \frac{\frac{g'}{g} \left(g^{\frac{1}{\gamma}-1} k^{1-\frac{1}{\gamma}} + g^2 B \left(n\Delta Z \right)^{1-\gamma} \right)}{g^{\frac{1}{\gamma}} k^{-\frac{1}{\gamma}} + n\Delta Z} < 0 \end{aligned} \tag{36}$$

where the last inequality follows because $g' < 0$. The above inequality implies that it exist $\overline{b} \in [0, \infty)$ such that the optimal consumption plan can be written as

$$c^* = \begin{cases} c_1, & \text{if } b \leq \overline{b} \\ c_3, & \text{if } b > \overline{b}. \end{cases} \tag{37}$$

As a result, the market clearing condition for consumption writes as

$$\begin{aligned}&\int_0^{\bar{b}} \left(g(b)^{\frac{1}{\gamma}} k^{-\frac{1}{\gamma}} + n(b)Z\right) db + \int_{\bar{b}}^{\infty} n(b)\underline{Z} db \\ =&G(\bar{b})k^{-\frac{1}{\gamma}} + N(\bar{b})Z + \left(1 - N(\bar{b})\right) \underline{Z} = Y + \underline{Z}\end{aligned} \tag{38}$$

with solution $k = \left(\frac{Y - N(\bar{b})\Delta Z}{G(\bar{b})}\right)^{-\gamma}$ where $G(\bar{b}) \equiv \int_0^{\bar{b}} g^{\frac{1}{\gamma}} db$ and $N(\bar{b}) \equiv \int_0^{\bar{b}} n db$. Plugging the equilibrium value of k into the equation $f(k, b, \Delta Z) = 0$ we obtain $\bar{b}$ as the unique solution to the following equation:

$$h(b, \omega) = g^{\frac{1}{\gamma}} \left(\frac{S}{G}\right)^{1-\gamma} \frac{\gamma}{1-\gamma} - \left(\frac{S}{G}\right)^{-\gamma} n e^{-\omega} + gB \frac{n^{1-\gamma}}{1-\gamma} e^{-(1-\gamma)\omega} = 0 \tag{39}$$

where $S = 1 - Ne^{-\omega}$. Uniqueness of $\bar{b}$ follows from the property of h. First we note that

$$\frac{\partial h}{\partial b} = \frac{\partial f}{\partial k}\frac{\partial k}{\partial b} + \frac{\partial f}{\partial b}; \tag{40}$$

A close inspection to the equilibrium value of k reveals that $\frac{\partial k}{\partial b} > 0$; from Eq. 34 and 36 we have that $\frac{\partial f}{\partial k} < 0$ and $\frac{\partial f}{\partial b} < 0$, respectively. Thus, $\frac{\partial h}{\partial b} < 0$ which means that h is monotonically decreasing in b. Second, by taking limits of h we obtain $\lim_{b\to 0} h = \infty$. The right limit of h is a bit more tricky: we impose the additional restriction $\bar{b} \in \mathbb{R}$. This requires $S > 0$ and therefore the upper bound of b is given by $N(e^{-\omega})^{-1}$ and $\lim_{b\to N(e^{-\omega})^{-1}} h = -\infty$. Thus, in the relevant domain of b, h changes sign, is monotonically decreasing in b and therefore admits a unique $\bar{b}$ such that $h(\bar{b}, \omega) = 0$. We conclude this section by noting that, from Eq. 39, $\bar{b}$ is a function of ω only and, thus, is stationary.

6.2 Asset prices

First, I derive the risk free rate r and the price of risk process θ. Applying Ito's lemma on the equilibrium state price density m we obtain:

$$\frac{dm}{m} = -\rho dt + \frac{m_Y}{m} Y \mu_Y dt - \frac{m_\omega}{m} \lambda(\omega - \bar{\omega}) dt + \left(\frac{m_Y}{m} Y + \frac{m_\omega}{m}\right) \phi_y dB_1 - \frac{m_\omega}{m} \phi_z dB_2 + \frac{m_{\omega,Y}}{m} \rho_{1,2} \phi_z \phi_y dt + \frac{1}{2} \frac{m_{YY}}{m} Y^2 \phi_y^2 dt + \frac{1}{2} \frac{m_{\omega\omega}}{m} \left(\phi_y^2 + \phi_z^2 - 2\rho_{1,2}\phi_z\phi_y\right) dt \tag{41}$$

Comparing the coefficient with Eq. 9 we obtain

$$r_t = \rho - \frac{m_Y}{m} Y \mu_Y dt - \frac{1}{2} \frac{m_{YY}}{m} Y^2 \phi_y^2 + \frac{m_\omega}{m} \lambda(\omega - \bar{\omega}) + \frac{1}{2} \frac{m_{\omega\omega}}{m} \left(\phi_y^2 + \phi_z^2 - 2\rho_{1,2}\phi_z\phi_y\right) + \frac{m_{\omega,Y}}{m} \rho_{1,2} \phi_z \phi_y \tag{42}$$

and

$$\theta_c = -\left(\frac{m_Y}{m} Y + \frac{m_\omega}{m}\right) \phi_y \tag{43}$$

$$\theta_z = \frac{m_\omega}{m} \phi_z \tag{44}$$

where the derivatives of m can be computed in closed form as follow:

$$\frac{m_Y}{m} Y = -\gamma \tag{45}$$

$$\frac{m_{YY}}{m} Y^2 = \gamma(\gamma + 1) \tag{46}$$

$$\frac{m_\omega}{m} = -\gamma A(\omega) \tag{47}$$

$$\frac{m_{\omega\omega}}{m} = \gamma(\gamma + 1) \left(\frac{G}{S}\right)^2 A(\omega)^2 - \gamma \frac{G}{S} \frac{\partial A(\omega)}{\partial \omega} \tag{48}$$

$$\frac{m_{\omega,Y}}{m} Y = \gamma^2 \frac{G}{S} A(\omega) \tag{49}$$

where $A(\omega) \equiv \frac{e^{-\omega}(N - nb_\omega) - Sg^{1/\gamma} b_\omega}{SG}$. We conclude that r and θ are stationary function of ω only.

The price of the risky asset can be determined using the state-price density m and the endowment process Y:

$$\begin{aligned} S_t &= \mathbb{E}\int_t^\infty \frac{m_s}{m_t} Y_s ds \\ &= Y_t \left(\frac{S(\omega_t)}{G(\omega_t)}\right)^\gamma \mathbb{E}\int_t^\infty e^{-\rho s}\left(\frac{S(\omega_s)}{G(\omega_s)}\right)^{-\gamma} e^{(1-\gamma)\left(\mu_y - .5\phi_y^2\right)(s-t)+(1-\gamma)\phi_y B_1} ds \\ &= Y_t \Theta(\omega_t) \end{aligned} \tag{50}$$

which implies that the price-dividend ratio is a stationary function of ω only. Applying Ito's lemma on Eq 50 we obtain

$$\frac{dS_t}{S_t} = [...]dt + \gamma\sigma_C\left(1 + \frac{G}{S}A - \xi(\omega)\right)dB_1 + \gamma\sigma_Z\left(\frac{G}{S}A + \xi(\omega)\right)dB_2$$

from which we deduce that the stock return volatility is given by

$$\sigma_R = \gamma\sqrt{\left(\phi_y^2\left(1 + \frac{G}{S}A - \xi(\omega)\right)^2 + \phi_y^2\left(\frac{G}{S}A + \xi(\omega)\right)\right)^2} \tag{51}$$

where

$$\xi(\omega) \equiv -\gamma\frac{\mathbb{E}_t\int_t^\infty e^{-(\rho+\lambda)(s-t)}\left(\frac{S(\omega_s)}{G(\omega_s)}\right)^{-\gamma-1} A e^{(1-\gamma)\left(\mu_y - .5\phi_y^2\right)(s-t)+(1-\gamma)\phi_y B_1} ds}{\mathbb{E}_t\int_t^\infty e^{-\rho s}\left(\frac{S(\omega_s)}{G(\omega_s)}\right)^{-\gamma} e^{(1-\gamma)\left(\mu_y - .5\phi_y^2\right)(s-t)+(1-\gamma)\phi_y B_1} ds}. \tag{52}$$

The equity premium is computed using the price of risk process:

$$\mu_R - r = \gamma\left(\theta_c\phi_y\left(1 + \frac{G}{S}A - \xi(\omega)\right) + \theta_z\phi_z\left(\frac{G}{S}A + \xi(\omega)\right)\right). \tag{53}$$

The price of an asset that pays the dividend Y at maturity τ is defined as follow

$$S_\tau = \mathbb{E}_t\left(\frac{m_\tau}{m_t}Y_\tau\right). \tag{54}$$

An application of Ito's lemma on Eq. 54 reveals that the returns volatility of the short term asset is given by Eq. 51 where the function ξ is replaced by

$$\xi_1(\omega) \equiv -\gamma \frac{\mathbb{E}_t e^{-(\rho+\lambda)(\tau-t)} \left(\frac{S(\omega_\tau)}{G(\omega_\tau)}\right)^{-\gamma-1} A e^{(1-\gamma)\left(\mu_y - .5\phi_y^2\right)(\tau-t)+(1-\gamma)\phi_y B_1} ds}{\mathbb{E}_t e^{-\rho\tau} \left(\frac{S(\omega_\tau)}{G(\omega_\tau)}\right)^{-\gamma} e^{(1-\gamma)\left(\mu_y - .5\phi_y^2\right)(\tau-t)+(1-\gamma)\phi_y B_1}}. \tag{55}$$

Finally, the price of a pure-discount bond for maturity τ is given by

$$P_\tau = \mathbb{E}_t \left(\frac{m_\tau}{m_t}\right) \tag{56}$$

and the corresponding term structure of interest rates is computed as

$$r_{t,\tau} = -\frac{\log P_\tau}{\tau}. \tag{57}$$

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7 Appendix B: Tables

Symbol	Value	Description
γ	.98	Curvature of utility function
B	2.25	Degree of loss aversion
ρ	.03	Subjective discount factor
μ_y	.0172	Consumption drift
ϕ_y	.0405	Consumption volatility
μ_z	.11, -.11	Drift of the habit process
ϕ_z	.02	Volatility of the habit process
λ	.15	Persistence of the habit process
$l_1 = l_2$	.5	Importance of loss-averse agents
ρ_{12}	-.8	Correlation consumption/habits

Table 1: Model parameters: preferences, consumption and habit process.

	Data	$\mu_z = 0,\ \phi_z = 0$	$\mu_z = -.11,\ \phi_z = 0$	$\mu_z = .11,\ \phi_z = 0$	$\mu_z = -.11,\ \phi_z = .02$
$\%LA$	–	2.72	1.35	6.44	1.54
RA	–	6.87	4.69	8.10	4.88
$\overline{\omega}$	–	0.10	0.22	−0.01	0.22
σ_ω (%)	–	7.39	7.39	7.39	10.55
r (%)	2.9	0.47	2.64	−0.79	0.53
σ_r (%)	3.00	5.67	3.47	9.03	4.53
$\mu_R - r$ (%)	4.89	9.42	4.40	12.49	4.46
σ_R (%)	17.92	32.02	21.70	37.42	23.60
$\log P/Y$	3.38	3.55	3.55	3.63	3.61
$\sigma_{\log P/Y}$ (%)	45.00	34.11	22.85	45.86	24.42

Table 2: Model Calibration. Unconditional return moments for various habit processes. Empirical data are from Papanikolaou (2011) and cover the 1962-2008 period. Statistics of the price-dividend ratio are from Constantinides and Ghosh (2011). Consumption parameters from Table 1.