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WRITINGS ON COMMONS, COMMON-POOL RESOURCES, PUBLIC GOODS, AND COOPERATION.

DOCTORAL THESIS

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To my parents, Tere and Martin with affection.
To my "Ma" Lupita Montoya, without you, I would not be me.

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Summary

WRitings *on Commons, Common-Pool Resources (CPRs), Public Goods (PGs), and Cooperation* has a twofold aim: firstly to provide an overview of the different concepts related to Commons, CPRs and PGs in order to help us to clarify their particularities and commonalities, and secondly, to offer some explanations of the phenomenon of cooperation in settings framed by the individuals' actions over the appropriation of CPRs and/or the contribution to PGs, which, in turn, leads to the emergence of conflicting interests in terms of the reasons and benefits involved agents might have in virtue of pursuing a certain behavior —i.e., individually competitive or an individually cooperative behavior. That is, what from the collective point of view is known as a social dilemma.

The work is then composed of four chapters that, apart from the introductory chapter, have in common *commons* (in its various conceptions) as the object of study within different frameworks. Then, although connected so that the reader can proceed through them in order, the chapters are self-contained. Chapter 1 introduces the guiding thread of the thesis: justifications, arguments, and results. Chapter 2 states the different notions of commons. We notice that it can be studied from different angles across social

disciplines. Herein we offer different interpretations and concomitant concepts currently in the literature. That way, we aspire to bring forth, once and for all, an understanding of commons that enables us to differentiate it from other related terms as well as to pinpoint the situations in which they can be overlapped, nested and/or not mutually exclusive.

Chapter 3 and Chapter 4 deal with different approaches of the study of cooperation and formation of groups, both drawing on a simple, but powerful enough, CPRs standard model. In turn, Chapter 3 is composed of two main parts. The first one studies how the presence of a cooperative group in a community of appropriators affects both cooperative and non-cooperative members and how it can drive them to become cooperative or not. In the second part, we apply recent methods of cooperative game theory so as to study the formation and stability of cooperative coalitions; we transform the CPRs game into a partition function CPRs game. We analyze the coalition pattern and its possible implications for our CPRs setting. This transformation was found wanting as for explaining partial cooperation. In addition, we go further in accounting for observed CPRs real situations, so we apply a coalition formation stage game. In general, full cooperation is theoretically observed in this last part of the chapter.

Chapter 4 also considers the formation of groups but from a moral stance. Here, we expand the baseline model used in Chapter 3. That is, we consider simultaneously both the appropriation and conservation problems that arise from the use and management of CPRs, this latter component having a public good nature. We rely on the concept of Kantian optimization to capture the essence of those cooperative moral agents, so we again examine the inferences of having two types of individuals in a fixed community of appropriators (Kantians and Nashers). Then we move forward and introduce random group formation to study the evolutionary stability of both kinds of populations. All in all, both populations will be stable. Finally, chapter 5 brings forth the general conclusions of this thesis as a whole.

Sommario

ARticoli *su Commons, Risorse Comune (RC), Beni Pubblici (BP), e Cooperazione* ha un duplice obiettivo: in primo luogo, fornire una panoramica dei diversi concetti relativi a Commons, RC e BP al fine di aiutarci a chiarire le loro particolarità e comunanze, e in secondo luogo, offrire alcune spiegazioni del fenomeno della cooperazione in ambienti modellati dalle azioni degli individui sull'appropriazione delle risorse comuni e/o il contributo ai beni pubblici. L'impossessamento delle risorse da parte delle singole persone porta all'emergere di interessi contrastanti in termini di ragioni e benefici a causa del perseguimento di un certo comportamento individuale che può risultare competitivo o cooperativo. Cioè, quello che dal punto di vista collettivo è noto come dilemma sociale.

La tesi si compone di cinque capitoli autoconclusivi i quali, ad esclusione del capitolo introduttivo, descrivono i *beni comuni* (nelle sue varie concezioni) come oggetti di studio all'interno di contesti diversi. Il capitolo 1 introduce il filo conduttore della tesi: motivazioni, argomenti e risultati. Il capitolo 2 discute le diverse nozioni di *commons*, osservandole da diverse angolazioni in varie discipline appartenenti alle Scienze Sociali. Vengono analizzate diverse interpretazioni e concetti concomitanti attualmente

in letteratura fornendo al lettore una comprensione dei beni comuni che ci permetta di differenziarli da altri termini correlati.

Il capitolo 3 e il capitolo 4 utilizzano approcci differenti per lo studio della cooperazione e della formazione dei gruppi, entrambi attingendo da un semplice, ma abbastanza potente, modello standard di RC. Il capitolo 3 è composto da due parti principali. La prima studia come la presenza di un gruppo cooperativo all'interno di una comunità di appropriatori influenzi i suoi membri spingendoli a diventare cooperativi o non cooperativi. Nella seconda parte si studia la formazione e la stabilità delle coalizione cooperative applicando modelli recenti della teoria dei giochi transformando il gioco RC in un gioco RC dipendente da una funzione di partizione. Si applica un modello di coalizione e si analizzano le sue possibili implicazioni nel contesto della RC. Questo modelo in funzione di partiaione si è *dimostrato insufficiente per spiegare la cooperazione parziale*. In aggiunta è stato applicato un gioco iterato di formazione della coalizione considerando delle situazioni reali osservate di RC. In generale, la piena cooperazione è stata osservata teoricamente in questa ultima parte del capitolo.

Il capitolo 4 tratta nuovamente la formazione dei gruppi cooperativi e non nel contesto dei beni comuni espandendo il modello utilizzato nel capitolo 3 con una prospettiva inspirata alla morale Kantiana. In questo modo vengono considerati simultaneamente sia i problemi di appropriazione che di conservazione derivanti dall'uso delle RC, rendendo a questa ultima componente una natura di bene pubblico. Usiamo il concetto di ottimizzazione kantiana per catturare l'essenza di questi agenti morali cooperativi, quindi esaminiamo di nuovo le inferenze di avere due tipi di individui in una comunità fissa di appropriatori (kantiani e nashers). Poi andiamo avanti e introduciamo la formazione di gruppi casuali per studiare la stabilità evolutiva di entrambi i tipi di popolazioni. Tutto sommato, entrambe le popolazioni saranno stabili. Infine, il quinto capitolo 5 presenta le conclusioni generali di questa tesi nel suo insieme.

Contents

CHAPTER 1

Introduction

Common-pool resources (CPRs) and Public Goods (PGs) pose real analytic puzzles to the conventional wisdom of economics since theoretical results continue to be at odds with reality and other empirical inquiries. Typically, the core issue is studied through the looking glass of collective action theory. It is formalized as a social dilemma game; individual optimal behavior begets an individually and socially sub-optimal outcome, which in turn leads towards the widely-held concepts of *free-rider* for PGs and the *tragedy* for CPRs. As a matter of fact, nevertheless, some groups of actors are able to successfully surmount social inefficient results. Study cases and experiments show users behaving differently from the traditional *homoeconomicus*. These observations underpin arguments challenging the assumption that sees economic life constrained within the state-market space. Furthermore, recent developments in the field of commons have led to a renewed interest in them due to their lessons and implications for designing public policies.

As indicated above, the standard formal economic framework for studying the problem of provision of a PG and/or extraction of a CPR is non-cooperative game theory. Under certain conditions, however, cooperation succeeds. Hence, of particular concern is the need for a better understanding of behavior of those decision making agents. In this respect, this dissertation deals with cooperation in the context of CPRs and PGs and the coordination of individuals degree of exploiting and providing them. I present, therefore, writings on how cooperation can be theoretically explained relying on simple baseline models and recent cooperative solution concepts.

In its first part, I offer an overview of the recent progress and seminal work on Commons, CPRs, PGs, and cooperative behavior models putting special stress on the former ones. Then, the work starts off by understanding the differences between commons, CPRs and Public Goods. Here, I make clear-cut definitions and concepts often misconceived in the literature. I note that, in the light of the above, besides the group size factor influencing cooperation, one of the crucial mechanisms that allow groups to get better results in appropriation/extraction settings is communication and information sharing among agents (Dubois et al. (2020)). Therefore, I study how the formation of a cooperative group may be favorable to its members and the conditions under which this might be maintained.

Furthermore, I embed a theory of coalitions formation into a CPRs model that, assuming homogeneity across players, precisely accepts communication and accounts for group size. I draw from the appropriation setting of Ostrom et al. (1994), which is a strategic game that captures the social dilemma involved in CPRs. Thus, I transform this game into a partition function game by applying recent approaches that combine both non-cooperative equilibrium concepts with cooperative game solutions (Chander (2019)). That is, we inquire into the upshot of having the strategic CPR game in an associated cooperative theoretic version. Cooperation is then explained in terms of how much collective payoff a set of appropriators might gain by coalescing. In other words, cooperative behavior here is captured by setting the values coalitions can attain by the union of the appropriators. The results suggest that cooperative game theory succeeds in explaining full cooperation above other intermediate forms of cooperation when agents play the CPRs game. Subsequently, since the emergence of coalitions other than the grand coalition —the coalition comprising all appropriators—was not observed, the hypothesis that this cooperative game model is more accurate for analyzing cooperation in CPR scenarios is sustained to the point where we are interested in complete cooperation rather than other intermediate forms of cooperation.

On the other hand, as noted by Mas-Colell (1989) and later endorsed by Roemer (2019b) —as he takes it as one of the arguments for formulating his theory of cooperation—cooperative game theory does not tell us how the members of the potential coalitions may communicate with each other, but rather it assumes pre-play communication and clear understanding of the options of joint action. Thus, although the implications of this approach allow us to give explanation to a situation under which appropriations would generally prefer being part of the the grand coalition, understanding cooperative behavior in this CPR game solely through the value coalitions can generate is somewhat limited.

On these grounds, I went further into looking at other explanations of cooperation in CPR scenarios. Recent research highlights that internally motivated individual actors are one of the reasons of the emergence of self-governance. Certainly, as the name suggests, internal motivations come from within people. And they can result, inter alia, from the *morals* and values those people have to a greater or lesser extent. Thus, when it comes down to cooperation in the context of CPRs, moral inner motivations matter. Moreover, Ostrom acknowledged this fact when she proposed a second generation of models on rational choice theory.

It follows from the foregoing that it may be the case for appropriators to be driven by doing “what is right;" notwithstanding, “what is right" being determined by what an appropriator observes what other appropriators in general do —what Elster (2017) calls a *quasi-moral norm*[1]. Therefore, should an appropriator follow a *quasi-moral norm* in this sense, [s]he will (not) cooperate provided that [s]he observes that most other appropriators do (not) cooperate as well. That is, appropriators follow a *quasi-moral norm* when reducing their level of extraction even without knowing individual extractions but knowing aggregate extraction. That way they can see if, in general, the majority was "cooperating." In this case, the observation that a larger part of them reduces its levels of extractions acts as a catalyst for the *quasi-moral norm*[2]. Then, this perspective took me to consider that theory, which unlike cooperative game theory, accounts for the formal procedure of making decisions and does provide micro-foundations for cooperation. This is the Kantian optimization solution-concept Roemer (2019a) comes up with.

The motivational foundation is built upon a reasoning reminiscent to Kant's hypothetical imperative. It basically consists of altering the optimization process. Roughly, under a *Kantian equilibrium*, each agent takes that action [s]he would most like to be universalized. This notion differs from the traditional optimization in the sense that just as the Nash equilibrium decentralizes competition, Kantian equilibrium does cooperation. Then, this solution-concept helps us to reconcile the theory of CPRs with the evidence from study cases characterized by the presence of somehow a decentralized cooperation.

Hence, in this part of the thesis, I apply the concept of Kantian optimization in an extended version of the CPRs game. The work is somewhat novel in that, in addition to the use of the Kantian optimization as a way of accounting for moral driven appropriators, I consider two main concerns of CPRs in one setting, viz. the classically

[1]According to him, quasi-moral norms are triggered when the agent can observe (or more generally know) what other people are doing. For a quasi-moral norm to be triggered, the agent need not have individual-level knowledge about what others are doing: aggregate information may be sufficient. Its efficacy depends on the agent seeing (or getting to know about) what other people do.

[2]Another example of a *quasi-moral norm* in this sense —drawn from personal experience—occurred at the beginning of the covid-19 pandemic when the use of face masks in public places was recommended. Some people observed that within a certain area some other people were not wearing masks, so the former were tempted to take them off. Then, if they happened to observe that the majority did not wear the mask, they simply stopped wearing it. This contrasts with the kind of people who followed a moral norm, who would wear the mask no matter what.

studied appropriation problem (congestion externalities), and its conservation part devised as a public good. Next, I set a community playing the above game and consisting of both types of appropriators. They are those who follow a *quasi-moral norm* captured by the Kantian behavior (Kantians) as well as those who follow the traditional strategic Nash behavior (Nashers). In this case, I observe that the mere presence of the former types of agents weakens the *tragedy*, albeit Nash players do better. Then, I introduce random group formation into the picture. Herein, the question I now address is whether the Kantian protocol of optimization acts as a mechanism for the evolution of cooperation. For this propose, I study the conditions under which a certain type of appropriators (Nashers or Kantians) replicates itself and invades its counterpart when they enter into evolutionary competition with each other through a simple dynamical system. The findings prove that a community constituted of Kantians is a stable group just as is a community comprised of Nashers. The prevalence of one or another type group will hinge on the initial conditions of the system. In contrast to a fixed group formation, randomness in the formation of groups allows us to observe the presence of stable cooperative communities.

CHAPTER 2

Commons, Common-Pool Resources, and Public Goods

In this chapter, we discern the akin concepts of Commons, Common-Pool Resources (CPRs) and Public Goods (PGs) as well as provide an overview of recent advances and influential insights in the literature.

2.1 Understanding Commons

2.1.1 Commons

The concept of *commons* lends itself to misunderstanding among different scholars. Terms around it might lead to common misconceptions sometimes used interchangeably throughout the literature. Although some of them can be nested into others, there are distinctions and features in terms of their implications and scope of study that are worth mentioning here. For Bollier and Helfrich (2019), “commons are living social systems through which people address their shared problems in self-organized ways." In this sense, Monbiot (2017) sitcks with the notions of commons proposed by Bollier and Helfrich (2015), commons go beyond physical and intangible things (man-made natural). For these authors, commons are more than sharing or having equal rights over land, water, minerals, knowledge, culture, scientific research or software. Accordingly, commons constitute a set of interdependent elements together with people involved in a process through which they themselves organize, in diverse manners, different-communal[1] actions to deal and overcome a common issue for the ultimate propose of benefiting all involved (respecting other forms of live). Thus, in an abstract sense, resources or goods that can be used, transformed, and shared by people in order to take advantage of are just a part of a whole system. In this connection, although delimited within an institutional framework, Madison et al. (2019b, p. 657) reinforce this idea of commons when they mention that “[c]ommons does not denote the resource, the community, a place, or a thing. Commons is the institutional arrangement of these elements and their coordination via combinations of law and other formal rules; social norms, customs, and informal discipline; and technological and other material constraints.”

Under this perception, even science and knowledge can be conceived as commons. Madison et al. (2019a, p. 76) explain “*knowledge commons* refers to an institutional approach (commons) to governing the production, use, management, and/or preservation of a particular type of resource (knowledge).” See Frischmann et al. (2014), Madison et al. (2019b), Ostrom and Hess (2007), Hess (2012), Joranson (2008), Strandburg et al. (2017), Dekker and Kuchař (2018), Sanfilippo et al. (2019), Pelacho et al. (2021), Frischmann (2021), Sanfilippo et al. (2021), Madison et al. (2019a), and Ramakrishnan et al. (2021) for further understanding about *knowledge commons* and their applica-

[1]It sounds like an oxymoron, but a different-communal action is an action that someone takes for a common propose but that is different from actions taken by others. It could be an individual contribution for the commons. Different-communal actions then can be taken just by some individuals or by all involved at the same time.

tions.

Thus, once we accept *commons* as a broad and abstract concept, we are in a position to understand the terms in Table 2.1 and that Bollier and Helfrich (2019) provide some clarifications.

Common Goods	**Common-Pool Resources (CPRs)**	**Common Property**	**Open Access Resources**	**Common**	**The Common Good**

Table 2.1: *Common concepts often confounded as commons*

2.1.2 Common Goods

This term is used to encompass the kind of goods that are not public goods neither private goods but something in between. In conventional economics, the related classification of goods depends upon the features of excludability and rivality. The former refers to the extent of which good is limited to those who can pay for it. Individuals face a cost of exclusion. In other words, (Ottone and Sacconi (2015)) explain to us, there is a technique or a mix of techniques that prevent each agent to get/use/access the good/resource, so one has to pay a price (bear a cost) to get the good. Thus, when the good is non-excludable the technique has no effect on the exclusion of it, or putting it a price is costly, if not infinite. On the other hand, a good is rivalous if its use or consumption reduces its overall availability for others. Four kind of goods have arisen from this angle, see Table 2.2. A common good will be a good that is non-excludable (as a public good) and rivaluous (as a private good).

	Excludable	**Non-Excludable**
Rivalous	Private Good	Common Goods
Non-Rivalous	Club Goods	Public Goods

Table 2.2: *Conventional classification of Goods in Economics*

Following Ottone and Sacconi (2015), this classification of goods relies upon the absence or presence of these two characteristics underpinned by market principles. That is, the framework is built upon the definition of public goods and private goods. The

Goods, therefore, within this taxonomy are considered as though they are to be exchanged in the market. Goods, however, might possess other characteristics which can be taken for setting other classifications. They might be classified according to other values. They might have intrinsic and/or usage values that not necessarily have to coincide with a market value. Moreover, as suggested by Bollier and Helfrich (2019), the two features for defining a good or resource ultimately come from the human ability to create, alter, or transform them and the uselessness they might have. For instance, fish can be considered a common good because while the fish we catch can not be fished by others, we can not avert others to attempt to catch other fish. It does not mean necessarily that the fish itself is inherently rivalrous and non-excludable. People's actions over the fish are what make it display such characteristics. The classification, however, is useful as it leads to the definition of common pool resources. The reader will learn more about this a little later.

2.1.3 Common and the Common Good

Bollier and Helfrich (2019) talk about the term *common* as a concept found on the literature alluding to water or shared land. On another note, however, they consider a further meaning stemmed from Hardt and Negri (2009) and refers to "the language we create, the society we establish, the modes of sociality that define our relationships." It seems that this idea of common is akin to the concept of commons mentioned before. The difference pointed out by Bollier and Helfrish has to do with the purpose behind the *commons*. A commons, they authors state, could admit all forms of cooperation, including unlawful purposes. Diversely, the common good is a bromide we can find in different economic, philosophical and political dialogues referring to the ultimate goal of a society: the benefit or interests of all[2]. Yet the means to achieve it are still subject of debates.

[2]From Oxford English Dictionary. Oxford, Oxfordshire. UK. Retrieved 8.03.2021

2.2 Common-Pool Resources

The categorization of good presented in Section 2.1.2 was unquestionable in economic theory until the works of Ostrom (2010). She argues that this twofold classification is consistent with a dual view of the organizational forms of society. First, that the market is the optimal institution for the production and exchange of private goods. And second, that the government is seen as the owner of a property organized by a public hierarchy. Then, she goes deeper into this simplistic dual division and proposes additional concepts[3]. First, the introduction of the term "**subtractability** of use" instead of "**rivalry** of consumption." Meaning that by using the resource one agent might subtract others from it use or consumption. Second, to conceptualize subtractability of use and excludability vary from low to high rather than characterizing them as either present or absent. Third, on the basis of these concepts, a new type of good is envisaged, the **common-pool resources**. And fourth, another change proposed by her is to shift from considering a good as a "toll good" instead of "club good." In this sense, following Ostrom (2008), "common-pool resources are seen as sufficiently large that it is difficult, but not impossible, to define recognized users and exclude other users altogether. Further, each person's use of such resources subtracts benefits that others might enjoy." These new taxonomic modifications can be arrayed in Table 1, which for clarity contains some examples.

[3]See Ostrom and Ostrom (1999)

		Subtractability of Use	
		High	**Low**
Difficulty of excluding potential beneficiaries	**High**	Common-pool resources: groundwater basins, lakes, irrigation systems, fisheries, forests.	Public goods: peace and security of a community, national defense, fire protection, weather forecasts.
	Low	Private goods: food, clothing, automobiles.	Toll goods: theaters, private clubs, daycare centers.

Table 2.3: *Taken from Ostrom (2010).*

Other examples of common-pool resources that include both natural and human-made systems are grazing lands, mainframe computers, government and corporate treasuries, and the Internet. And instances of the resource units derived from common-pool resources include water, timber, fodder, computer-processing units, information bits, and budget allocations Ostrom (2002b) Ostrom and Blomquist (1985). Moreover, anthropic *climate change* problems can be studied as those of CPRs. Recent work in the area points towards this direction. A broader definition of governance of global and complex environmental resources is suggested. For instance, the case of atmospheric sinks of greenhouse gases (GHGs). They are somehow CPRs. Accordingly, at some point the use of units of GHG sink services fall in the substractability-excludability conception (Paavola (2019)). Although an agent is not able to exclude potential users, there is the fact that a unit of a sink services [s]he uses is a unit subtracted from the total available units and that others cannot use. The issue then is how we can avert a situation where the capacity of the atmospheric GHGs sink to provide sink services is not surpassed.

2.2.1 CPRs as systems

On the other hand, Merino Pérez (2019a) tells us an explication of CPRs that approaches to the term of commons in Section 2.1.1: the fact that (CPRs) involve a configuration of group of users and their actions, either individual or collective, the resource per se or unities of it, its location, and in some cases the State, make it useful to understand CPRs as systems. A system is a regularly interacting or interdependent group of items forming a unified whole[4]. Thus, according to her, understanding CPRs means to identify boundaries. And this might depend on the subject of study. Boundaries can be either geographic or conceptual [García (2006)]. In this line, Merino Perez suggests to pinpoint three aspects. First, the boundaries of the CPR themselves, second, the parts that make up the system and interact within it, and third, the structure of the resource, which begs the question —how does each element relate to the others? It may be the case that there is a horizontal relationship, a vertical one, and that the intensity level of these relationships is different [Adams (1980)]. Under this perception, we see that there may be the case that CPR and commons overlap each other in some situations. Meaning that a CPR situation might be understood as commons, but commons are not always CPRs.

2.2.1.1 Open-Access Resources and Common-Property Resources

Common-pool resources are further classified into two types: open-access resources and common-property resources, in opposition to private property resources. The latter are such that property rights are held by a community of individuals and may include the government and non-government organizations, and their use can be regulated in a variety of ways by a variety of institutions, Common and Stagl (2005). Following Tietenberg and Lewis (2018), some common pool resources might admit property rights. However, such rights may be costly to enforce, so they are not exercised. In contrast, in open access resources nothing is subject to property rights. Nobody owns anything. Anyone can enter freely to exploit the resource on a *first-come, first-served basis* . And no individual or group has the capacity or the legal power to restrict access. Such a characteristic promotes a *use it or lose i*t situation.

Open-access resources unleash what has become known popularly as the "tragedy

[4] From Merriam-Webster Dictionary. Springfield, MA. USA. Retrieved 8.22.2019

of the commons" —see Hardin (1968) and Lloyd (1833). Thus, unlike to open-access resources, which may be over-exploited, common property resources need not suffer overuse and their allocation can be regulated in ways that avoid the tragedy. The distinction between the tragedy and the problem of commons is stated clearly by Ostrom (2009).

> [T]he problem is that people can overuse, they [the CPRs] can be destroyed, and it is a big challenge to try to figure out how to avoid it. That is a problem, that is real. The tragedy is the way he [Hardin (1968)] expresses it, they cannot, ever, solve it. That is different.—It is inevitable and unconquerable. That is why he called it a tragedy. They were trapped... and the only way out was some external government coming in or diving it up into small chunks and everyone owing their own....

In essence, as Elinor Ostrom tells us, some elements of this differentiation are important to notice. It is not merely a tragedy in the first place. Instead, it is a problem or potential problem that does not necessarily need to be dealt through the creation of private property rights nor through top-down regulations. There are different ways of overcoming it, for instance, bottom-up institutions or hybrid regimes constituted by shareholders, regulatory and market-based instruments. Studying what and how could be the best way of preventing the problem is a concern and a matter of debate. Ostrom and Janssen (2006) highlight that there are cases of both successful and unsuccessful efforts to govern and manage common-pool resources by governments, communal groups, cooperatives, voluntary associations, and private individuals of firms ["Berkes (1989), Bromley et al. (1992), Katar et al. (1994),Singh and Ballabh (1996).] In this context, given the nature of the open accesses resources, the "tragedy" may emerge eventually. And this does not mean that only open accesses resources are endangered by overuse. Every common-pool resource can face deterioration by unsustainable use. Thus, whether man-made or natural ones, common-pool resources demand collective action.

2.2.2 Appropriation and Provision of Common-Pool Resources

In the same line of Plott and Meyer (1975), the process of taking units from any kind of common pool resource is termed appropriation, and the person who withdraws such units is, accordingly, an appropriator. Ostrom et al. (1994) separate the problems appropriators might face into two types, appropriation and provision. In the former, there is an assumed production relationship between yield and level of inputs. Here the problem to be solved is how to allocate equitably that yield, or how to allocate input activities to achieve the said yield. Appropriation problems deal with the allocation of the units of extraction of the resource as a flow. More specifically, the problem has to do with the following aspects: one, the quantity of resource units to be appropriated, or the establishment of the efficient level of input resources necessary for obtaining that flow of units of the resource. Second, timing and location of appropriation as well as the technology for appropriation. On the other hand, the provision problems deal with the creation, maintenance, and the improvement of productive capabilities of the resource as well as avoiding its depletion or destruction. Here the units of use of the resource are seen as stock. Notice that in real world situations a common pool resource may be complex and exhibit problems of appropriation and provision.

2.2.3 The Nature of Common-Pool Resources

In this respect, according to Ostrom et al. (1990b), there are four necessary conditions to produce a common-pool resources dilemma, and more notably, to distinguish it from a simple common-pool situation. To begin with, *resource unit substractability* is strongly linked to the definition of a common-pool resource. This condition tells, as it was already mentioned, that a resource unit extracted, harvested or withdrawn by one individual makes it unavailable for another one. Such extracted unit —the argument goes —is possible since the resource provides a never-ending flow of units over time as long as the degree of appropriateness do not outweigh the degree of replacement or regeneration of it. Also, in cases where the resource is exhaustible, there is not a flow but a stock gradually is depleted. The second condition is the *existence of multiple appropriators*, the resource is withdrawn by more than one person or teams of individuals. Third, *sub-optimal outcomes*, which means that the appropriators' strategies yield sub-optimal outcomes given a configuration of their own attributes, the market conditions, technology, and the physical system. Forth is *constitutional feasible alternatives*. Here the authors touch upon the existence of a set of coordinated strategies that are more efficient than current decisions, and that they are constitutionally feasible given the current institutional and constitutional arrangements. Within this condition, in turn, I find that a sufficient condition for such set of feasible alternatives is the existence of a Pareto-optimal set of coordinates strategies that are individually advantageous to the involved appropriators.

As the reader can infer now, the definition of a common-pool resources together with conditions one and two lead to what is called common-pool resources situations (Ostrom et al. (1990b)). Conditions three and four are necessary for a dilemma. And, in the view of Ostrom, we are not in the presence of a dilemma if we do not have sub-optimal outcomes in an setting characterized by the factors described in condition three. Similarly, there is no a dilemma when the set of available actions of appropriators is not able to produce a better outcome for themselves.

2.2.4 Ownership Regimes and Property-Rights in CPRs

For Wall (2014), on the other hand, commons is a form of property ownership. However, ownership regimens are those that entitle the property rights to the commons or CPRs (Merino Pérez (2014)). Merino Pérez (2019c) highlights four maim types of property regimes. Namely, **public ownership** (the state is the owner of the resource),**open access** of the resource for everyone, **private ownership**, and **collective ownership**, which is like a private collective ownership but instead of a single owner, there is a group of owners. The ownership here is well-defined and concedes rights as well as responsibilities and duties concerning the resource. The so-called *ejido* in

Mexico is a clear instance of collective ownership. Notice that ownership regimen and ownership rights are different. The classification of the goods proposed by Ostrom as such is independent of the property/ownership regimes. Merino Pérez (2019c) explains that three cases may turn out from this independence. First, there exist *goods of public ownership but of common use*. For instance, a road or stretch of road of public ownership may behave as a CPR when congested, since it would constitute a resource difficult to exclude and with high substractabilty. Second, there may exist *goods of public ownership but of private use*, or in other terms, private goods of public ownership. Goods that belong to public institutions but used by a single individual are some examples. Third, there exist as well *private ownership of common use*. For example, forests and parks of private property whose users are foreseen to have just the usufruct right. Further, in terms of CPRs, Schlager and Ostrom (1992) distinguish rights at an operational-level and rights a collective-choice level. Operational-level property rights are *access* and *withdrawal*. Collective-choice property rights include management, exclusion and alienation. And holders of these rights in a CPR might have just one, some or all of them.

2.3 Sustainable Management of the CPRs: Incentives and Conditions

Merino Pérez (2019b) tells that studies of CPR show that users may readily overuse the resource when there is no regulation or institutions that monitor the fulfillment of the rules and punish in case of breaking them. That is to say, it may be easy for involved rational individuals having incentives to free-ride off others. Hence the standard economic answer is providing material incentives to participate in the provision of the resource and/or to curb its use, be it by privatizing the resource or by implementing state regulation. That is, we look for ways to induce cooperation. To some degree this approach works, but it can be limited and can have counterproductive results. The implementation of policies influenced by this conclusion underestimates the capacity of users to overcome such genuine problems by themselves. When it comes to monetary incentives, they may just backfire. As Merino mentions, evidence from the CPR fields, in which people self-organize to achieve a common goal without considering the privatization of the resource nor state regulation, is considerable. Self-organization of the sustainable management of the CPR is straightforward when users do know the characteristics of the resource, how it changes, how it behaves, and/or its regeneration capacity. Also it is readier when they do communicate with each other, set rules-in-use, and devise monitoring mechanisms (Merino Pérez (2019b)). In this context, when it is about policies based on the standard model of incentives that aim to enhance common pool resource management, Ostrom (2005) reviews evidence regarding the relationship between intrinsic motivations and those kind of policies. She argues that such policies often have negative impacts on behaviors based on intrinsic preferences. As she posits, in some situations it is observed that external incentives crowd out[5] behaviors that are based on intrinsic preferences, and then decreasing cooperation. Although she recognizes that they also may “crowd in" such behavour and enhance what could have been achieve without those incentives Ostrom (2005). Hence she highlights the necessity of “designing institutions that enhance cooperation rather than crowding it out." She opts for sorts of “policies that involve both public governance mechanisms and private market and community institutions." More recently, Bowles (2016) goes deeper in the debate of the crowing-out effect and material incentives as he distinguishes their mechanisms of action. Bowels' work strengthens the conclusions of Ostrom. He shows recent evidence regarding material incentives that lead policies which far from solving the problem, makes it worse off. New policies, he suggests, should contemplate possible synergies between incentives and social preferences —reciprocity, fairness, altruism, and inequity aversion.

[5]Crowing-out: the negative impact of extrinsic incentives on intrinsic motivations.

2.4 Social Capital

Social capital, understood as *the networks of relationships among people who live and work in a particular society, enabling that society to function effectively*[6] helps to explain the emergence of cooperation in CPR management. Social capital theory contends that social relationships are resources that can lead to the development and accumulation of human capital (Machalek and Martin (2015)). More precisely, social capital can be thought of as the links, shared values and understandings in society that enable individuals and groups to trust each other and to work together (Brain (2007)). In studies such as Aida (2018) Agrawal (2001), Bowles and Gintis (2002) Hayami (2009) social capital is considered an instrument for successful CPR management. In these lines, groups of users' lack of social capital do not bring about coordination. For these authors, cooperation in this context is explained through the presence -or not- of social capital. For Ostrom and Ahn (2009), trust and norms of reciprocity, networks and forms of civic engagement, and institutions are considered causes of collective action under the social capital perspective.

2.5 Common Variables Involved in Common Pool Resources

From the point of view of experimental psychology, Kopelman et al. (2002) identify nine variables that influence cooperation in common dilemmas, namely, social motives, gender, payoff structure, uncertainty, power and status, group size, communication, causes, and frames. In turn, they categorize such variables into individual differences (stable personal traits such as social motives and gender) and situation factors (the environment). The latter category is further differentiated into task structure (which orderly is composed by the decision structure and the social structure) and the perception of the tasks or perceptual factors (causes and frames). Within the decision structure there are the variables of payoff structure and uncertainty, whereas the social structure category includes the variables power and status, communication, and group size. These

[6] From Oxford English Dictionary. Oxford, UK. Retrieved 9.18.2019

two last variables are particularly interesting. The size of the group, and the ability of people to communicate with one another are fundamental elements highly related to the limitations of the standard game theory.

Ostrom (2015) shows CPRs cases of successful groups avoiding the Nash outcome. One of the crucial conditions she detects, under which coordination succeeds, has to do with the number of individuals involved. Also Ostrom et al. (1992) discuss a series of experiments approaching issues of individual behavior under common-pool situations. They set up experiments so as to gain a general explanation over how communication and punishing mechanisms on the group level influence individual behavior. Once they introduce these elements into the mix, they observe that the outcomes of the experiments generate behavior clearly inconsistent with the predictions of non-cooperative game theory. Moreover, when individuals are allowed to communicate with each other, they achieve significant improvements from group interactions even in the absence of punishing mechanisms.

In this connection, group size and communication under a common-pool resource context have been the object of investigation. In Kopelman et al. (2002) there is an interesting discussion of the experimental commons dilemmas literature regarding these two elements. According to them, two explanations of the effect of communication on cooperation, provided by Dawes et al. (1990), are salient. First, group discussion enhances group identity or solidarity, and second, group discussion elicits commitments to cooperate. On the other hand, the group size issue has been highly a matter of debate. So far, there is no consensus on whether small size groups achieve more cooperative outcomes than the larger ones. The discussion presented in Kopelman et al. (2002) is not conclusive. In this line, Allison et al. (1992) explains that small groups are more motivated to divide resources equally than are members of large groups, whereas Agrawal and Goyal (2001) suggest that there is a curvilinear relationship between group size and successful collective action.

On the other hand, Ostrom and Janssen (2006) highlight nine variables commonly found in empirical studies related to self-governed resources use. Firstly, there is the information about the condition of the resource and expected flow of benefits and costs are available at low cost to the participants; second, appropriators plan to live and work in the same area for a long time; third, they are highly dependent on the resource; fourth, appropriators use collective-choice rules that fall between the extremes of unanimity or control by a few; fifth, the group using the resource is relatively stable; sixth, the size of the group is relatively small; seventh, the group is relatively homogeneous; eighth, participants have developed generalized norms of reciprocity and trust that can be used as initial social capital; and ninth; participants can develop relatively accurate and low-cost monitoring and sanctioning arrangements.

2.6 Origin of Peer Governance

We end this chapter by mentioning three different ways, though not the only ones, in which self-governance can arise. Bollier and Helfrich (2019) pinpoint these three patterns typically observed, to be specific, *spontaneous attraction*, *tradition*, and *conscious design*. The first term is related to cases in which there is a problem tackled by an agent such that it draws attention of others as they can benefit from the solution. The problem-solving approach attracts them in such a way that now they want to contribute to it. Some examples of it are open-collaborative online resources. Secondly, ***Tradition***, as the word suggests, refers to nowadays observed traditional practices of self-governance and cooperation traced back to ancient times. People here share not only a resource but values, norms, views, and customs, which are handed down from generation to generation (Magaloni et al. (2019), Monterroso et al. (2019), Joranson (2008)).Traditional commons teach us how to coexist with some forms of natural commons. Although, sometimes values might change over time due to external influence from modern day values and norms as we see in chapter Chapter 4. Thirdly, ***Conscious Design*** refers to the idea that a designed system by people who initially may or may not be related but that work together in a project promotes the origin of self-governance. That is to say, the fact that people jointly work on something makes it easier for the common goal that values, ideas, and ways of proceeding evolve into a commons through conscious design.

CHAPTER 3

Common-Pool Resources (CPRs), Groups, and Coalitions

In economics Common Pool Resources (CPRs), whose characteristics of low degree of excludability and high degree of substractability derived from the actions and decisions of individuals over them, imply mainly two problems: appropriation and contribution to its conservation or maintenance. In this part we focus on the first one. Theoretically and typically the problem of extraction or appropriation of units of a CPR is studied by using non-cooperative games. In this way, the *tragedy of the commons* is explained by the Nash equilibrium of the induced game. The results of experiments presented in Ostrom (2010) show that some people move away from this equilibrium. And many communities are able to develop their own approaches to manage common-pool resources (Ostrom (2015)). Furthermore, there are situations in which the formation of cooperative groups or coalitions is observed. In this chapter we start off by studying the formation of a cooperative group that may be beneficial to their members and the conditions under which it can be sustained (Section 3.1.2). Then, we go further in our analysis, so we use recent cooperative game theory methods to try to explain more accurately observed formation of groups in common pool resources scenarios. Cooperative game theory assumes that homogeneous agents may communicate freely among themselves before the onset of a formal game, and that any potential coalition has an understanding of the options of joint action (Mas-Colell (1989)), such as group size. Thus, cooperation is explained through the material payoffs those coalitions can gain. Communication, on the other hand, is one of the important factors that promotes cooperation in commons dilemmas —as mentioned in the previous chapter Section 2.5—. Evidence from case studies and experiments tells us that involved individuals achieve cooperative outcomes through communication. Thus, the inbuilt assumptions of cooperative game theory allow us to capture this observation. Ergo, having noticed the applicability of this theory in CPRs problems of explaining cooperation, we study to what extent it better gives account of observed cooperative groups in the context of CPRs, as emphasized in the literature. Through communication, agents are able to coordinate their strategies. They engage in agreements that can be binding or not binding. Then, we set the appropriation problem formally within such a framework (Section 3.2). And by relying on Chander (2019) we transform a CPRs strategic game into a partition function game, we observe hat it admits a non empty γ-core, and then we apply a game of coalition formation called the payoff sharing game. Implications of potential coalitions formation are examined.

3.1 The Appropriation Setting, Groups and Individual Behavior

3.1.1 The Appropriation Setting

Here we draw on the common pool resources appropriation strategic game presented in Falk et al. (2002). It depicts the appropriation setting of Ostrom et al. (1994) and underlies the CPR experiments carried out by Ostrom et al. (1990a).

3.1.1.1 The Standard Common-Pool Resources Strategic Game

There is a community in which each of its n members, possessing an initial endowment e, extracts or appropriates[1] a part of a limited CPR for personal benefits. They decide independently and simultaneously how much they want to take from the CPR. Although the appropriation of the resource yields a revenue for the community that depends on the total level of appropriation, it involves an individual cost $c \in \mathbb{R}$ per appropriation unit irrespective of the decisions of all other community members. Moreover, for low levels of the amount of total appropriation, the revenue from the resource is positive and increases —up to a certain level—as the total amount appropriated does. After that point, when individuals appropriate too much, the outcome is detrimental. Also, each appropriator i retains a share of the total revenue obtained as a community. Then, the allocation rule is that they keep a part of revenue in proportion to their share in the total amount of appropriation, which leads the community to implement a proportional sharing rule. This situation defines a game in strategic form (or in normal form) $\Gamma = (N, \chi, u)$, in which:

- $N = \{1, \ldots, n\}$ is a finite set of players/appropriators.
- χ_i is the strategy set of player/appropriator i, for every player $i \in N$. $\chi = \times \chi_i$ denotes the set of all vectors of strategy profiles. A strategy profile is denoted by $x = (x_i, \ldots, x_n) \in \chi$, where x_i corresponds to the amount of the appropriate resource (units of appropriation).

[1] Depending on the context, we also say harvest, fish, extract, or graze.

- $u_i : \chi \mapsto \mathbb{R}$ is the payoff function of player/appropriator i, so $u = (u_i, \ldots, u_n)$ is the vector of payoff functions.

  – The payoff function of i is given by:

$$u_i(x_i, x_{-i}) = e - cx_i + \left[\frac{x_i}{x(N)}\right] f\left(x\left(N\right)\right) \tag{3.1}$$

  where $x_{-i} = (x_1, ...x_{i-1}, ..., x_n)$, and

  * $x(N) = \sum_{i \in N} x_i$ is the amount of total appropriation.
  * $\dfrac{x_i}{x(N)}$ is the sharing rule: an individual appropriator i gets a fraction of the total revenue according to her or his share in total appropriation.
  * $f\left(x\left(N\right)\right)$ is a strictly concave function that governs the total revenue with $f\left(0\right) = 0$ and $f'\left(0\right) > c$. Accordingly, say that $\hat{x}$ is the level of $x\left(N\right)$ such that $f'\left(\hat{x}\right) = 0$, so for a $\bar{x} > \hat{x}$ we have that $f'(\bar{x}) < 0$.

Equation (4.9) allows us to represent that individual appropriation hinges on the aggregate resource extraction and on appropriator's own level of extraction in accordance with the proportional sharing rule (a share of the sum of individual appropriations). Now, consider a specific form of the revenue function used by Ostrom et al. (1990a) in their experiments, which was based on Gordon (1954) classic model.

$$f\left(x\left(N\right)\right) = ax\left(N\right) - b\left[x\left(N\right)\right]^2 \tag{3.2}$$

As assumed, initially the CPR yields positive returns, so $f'(0) > c$, but if appropriators take too much, the revenue decreases. Then, Equation (3.2) necessarily fulfills that $c < a = f'(0)$, and $f'(\hat{x}) = a - 2b\hat{x} < 0$. Concretely this model captures an environment most closely parallel to that of a CPR with limit-access. Next, plug Equation (3.2) into Equation (4.9), the payoff function of individual i is

$$u_i(x_i, x_{-i}) = e - cx_i + \left[\frac{x_i}{x(N)}\right]\left[ax(N) - b\left[x(N)\right]^2\right] = e + (a - c)\,x_i - x_i b\left[x(N)\right] \tag{3.3}$$

Write $(a - c) = \alpha$, so Equation (3.3) is

$$e + \alpha x_i - x_i b\left[x(N)\right] \tag{3.4}$$

Let us determine the equilibrium behavior of this game as traditionally presented in the literature (Ostrom et al. (1994), Elsner et al. (2015)). Appropriator i seeks to grasp some of the resource provided the marginal return of it is initially positive, so (s)he decides the optimal amount to appropriate given the amounts of the other involved appropriators. This situation leads each appropriator i to solve:

$$\begin{aligned} \underset{x_i}{\text{maximize}} \quad & u_i(x_i, x_{-i}) \\ \text{subject to} \quad & 0 \leqslant x_i \end{aligned} \tag{3.5}$$

Given the assumption imposed on $f\left(x\left(N\right)\right)$, there are no corner solutions, and $x_i = 0$ does not solve Equation (3.5), so we are in presence of an interior solution. Thus, we compute the first order condition for appropriator i with respect x_i given a strategy profile of the rest of appropriators. Moreover, the uniqueness of the maximizer is guaranteed since the payoff function is strictly concave in x_i as shown by the second order condition.

First Order Condition (f.o.c):

$$u_i'\left(x_i, x_{-i}\right) = \alpha - x_i b - \left(\sum x_{i\in N}\right) b = 0, \tag{3.6}$$

Second Order Condition:

$$u_i''\left(x_i, x_{-i}\right) = -2b << 0. \tag{3.7}$$

Since there is a first order condition corresponding to each appropriator, we cope with a system of n first order conditions with n unknowns:

$$\begin{cases} \alpha - x_1 b - \left(\sum x_{i\in N}\right) b = 0 \\ \vdots \\ \alpha - x_i b - \left(\sum x_{i\in N}\right) b = 0 \\ \vdots \\ \alpha - x_n b - \left(\sum x_{i\in N}\right) b = 0 \end{cases}$$

However, given the symmetry of the game in terms of having the same strategies and payoff function for each appropriator, we draw on the result of Nash (1951) that every finite game has a symmetric equilibrium point[2]. Then, assume that the maximizer x_i^* is a symmetric Nash equilibrium resource extraction of every appropriator. Then, the total level of extraction is given by $\sum x_i = nx_i^*$, and the system is reduced to Equation (3.8),

$$\alpha - x_i^* b - b\left[nx_i^*\right] = 0. \tag{3.8}$$

Equation (3.8) yields

$$x_i^* = \frac{\alpha}{b\left(n+1\right)} \tag{3.9}$$

Moreover, as Harsanyi and Selten prove in their theory of equilibrium selection, "the solution of a symmetric game should be symmetric" Van Damme and Weibull (1995). Thus, x_i^* is certainly selected. And the total level of exploitation at this equilibrium will be

$$x^* = \sum_{i\in N} x_i^* = nx_i^* = \frac{n\alpha}{b\left(n+1\right)}. \tag{3.10}$$

Let us move on now to check the socially optimal appropriation level. In this case we look at the overall yield of the CPR, which is given by:

$$U\left(x\left(N\right)\right) := \sum u_{i\in N}\left(x\right) = ne + \alpha\left[x\left(N\right)\right] - b\left[x\left(N\right)\right]^2 \tag{3.11}$$

[2]See Theorem 2 of Nash (1951)

If appropriators want to exploit the resource at socially efficient levels, they should implement any extraction strategy profile $(x_1, \ldots, x_n)$ such that $\sum x_{i \in N} = x(N)$ solves

$$\begin{aligned} \underset{x(N)}{\text{maximize}} \quad & U\left(x(N)\right) \\ \text{subject to} \quad & 0 \leqslant x(N) \end{aligned} \tag{3.12}$$

The first order condition is

$$\alpha - 2bx(N) = 0$$

so the socially efficient level of exploitation is

$$x^{SO}(N) = \frac{\alpha}{2b}. \tag{3.13}$$

Comparing Equation (3.10) and Equation (3.13), they yield different results. Observe that the total level of exploitation at the Nash equilibrium, x^*, is higher than x^{SO} as long as $n > 1$. The former is increasing in n whereas the latter does not depend on the number of appropriators in the community. That is to say, the loss of surplus involved by the Nash equilibrium compared to that one associated to the efficient level of extraction is increasing in the number of appropriators. Also notice that $\lim_{n\to\infty} x^* = 2x^{SO}$. Then, the individuals equilibrium behavior is not socially optimal. And $x^{so}(N)$ gives the maximal yield derived from the extraction of the resource, more than this, the return decreases. The revenue from the CPR reaches a maximum net level when individuals appropriate some, but not all of the resource available, see Figure 3.1. Appropriators, however, act in such a way as to end up being worse off individually than if they acted collectively. Hence the term *tragedy of the commons*[3]. Appropriators could be better off should they find ways and means of cooperating/coordinating to tackle the tragedy. Graphically we can see the tragedy in Figure 3.2 as presented in the literature (Sethi and Somanathan (2005)). On the horizontal line we represent the total level of extrac-

[3] In this chapter we will refer to this notion of the tragedy when we mention *the tragedy* for simplicity, although we have already seen that strictly speaking the tragedy is associated with the use of open access resources, a type of commons whose users in principle do not have to have any other type of relationship among them but the exploitation, use, or access to the resource.

tion, that its $\sum x_i$, and on the vertical line we represent the income and cost derived from the extraction. Then, the green straight line shows that the total gross income the community gets due to the exploitation/extraction increases with the total amount of resource extracted, while the red curve shows the total cost that the appropriators face as a whole is small for small quantities of the extracted resource, but as the total resource extracted increases, the costs increases sharply. Observe that under the social optimum level of resource extraction, the difference between the gross income and the total costs is greater than the difference between the income given by the total extraction at the Nash equilibrium and the total costs. That is the, the income after costs is greater under the social optimum level of extraction. The appropriators, however, will extract at the NE. The literature we can find three courses of action to conquer the tragedy: privatization (arrangements creating property rights), top-down regulations (from the state or a third regulator to the community), and bottom-up institutions (solutions from and to the involved actors). In this chapter we will deal with the theoretical study of the emergence of cooperation —protecting the commons by foregoing high levels of resource appropriation—from an individualistic point of view using group and coalition formation approaches. This is related somehow bottom-up ways to avoid failing into the *tragedy*. Thus, just as it is explained from the individualistic assumption of the agents, the same conception can be used to understand cooperative outcomes.

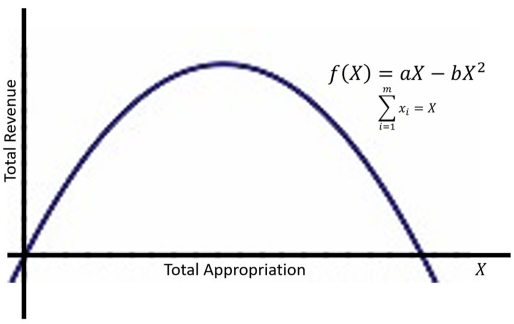

Figure 3.1: *The revenue from the CPR increases with aggregate extraction X up to certain maximum point, after that, it declines.*

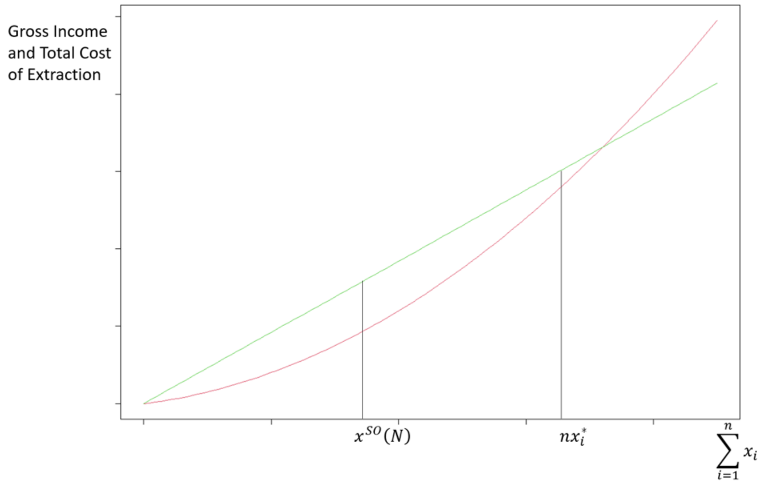

Figure 3.2: *Gross Income and Total Costs of Appropriation*

Further, in the experiments mentioned in Ostrom (2010), the initial endowments were tokens the subject could allocate to the common-pool resource (for their experiment they use eight individuals). The game theoretic outcome involves substantial overuse of a resource while a much better outcome could be reached if the subjects were to reduce their joint allocation, the standard prediction is that subjects would invest according to the Nash equilibrium —8 tokens each for a total of 64 tokens. Subjects could earn considerably more if they reduced their allocation down to a total of 36 tokens in the resource. The results of those experiments suggest people moving away from the theoretical predictions.

In this line, many communities are able to spontaneously develop their own approaches to manage CPRs. See several cases in Ostrom (2015) where people craft arrangements in a fashion different from standard predictions. Now, one way to try to reconcile theory with practice in this subject is to approach the problem through the looking-glass of groups and coalition theory. Formation of groups acting as a single entity might shed light on the coordination of players and overcome individualistic outcomes. Based on the CPR setting introduced above, we will explore the following three scenarios in which we might understand how the formation of one or several cooperative groups can be beneficial to the community with access to a CPR.

3.1.2 Forming a Cooperative Group

3.1.3 Scenario 1

Let us suppose now that there is a group of appropriators S that agrees to cooperate in some way, so they decide to choose a different level of extraction from the CRP. This group commits to the socially optimal level of appropriation:Equation (3.13), so they apply the proportional rule to establish how much of the resource could be appropriated by each member. Then, an individual i in group S will comply with the following level of appropriation:

$$x^c_{i \in S} = \frac{x^{SO}(N)}{n} = \frac{\alpha}{2bn} \tag{3.14}$$

which means that as a group, they will extract the resource at:

$$x^c_S = \frac{s\alpha}{2bn} \tag{3.15}$$

where s stands for the cardinality of S. However, while the in-group cooperative appropriators reduce their amount of extracted/exploited/appropriated resource, the out-group appropriators act individually, so they will choose optimally according to the Nash criteria:

$$\begin{aligned} \underset{x_{j \in N \setminus S}}{\text{maximize}} \quad & se + \alpha x_{j \in N \setminus S} - x_{j \in N \setminus S} b \left[\sum x_{i \in N} \right] \\ \text{subject to} \quad & 0 \leqslant x_{j \in N \setminus S} \end{aligned} \tag{3.16}$$

$$\text{f.o.c: } \alpha - x_{j \in N \setminus S} b - \left(\sum x_{i \in N} \right) b = 0 \tag{3.17}$$

Since we have that each one of the cooperative appropriators in S extracts the resource at the same level, we can write then the total level of appropriation as follows:

$$\sum x_{i \in N} = s x^c_{i \in S} + \sum x_{j \in N \setminus S} \tag{3.18}$$

we plug Equation (3.18) into Equation (3.17):

$$\alpha - x_{j\in N\backslash S}b - b\left[sx^{c}_{i\in S} + \sum x_{j\in N\backslash S}\right] = 0 \tag{3.19}$$

And Equation (3.19) holds for all non-cooperative appropriators in $N\backslash S$, meaning that we have $n - s$ system of equations corresponding to the cooperative appropriators. As the assumption of symmetry in terms of the strategies that each player possesses still holds, we can again assume the selection of a symmetric equilibrium:

$$\sum x_{i\in N} = sx^{c}_{i\in S} + (n-s)\, x^{*}_{j\in N\backslash S} \tag{3.20}$$

where $x^{*}_{j\in N\backslash S}$ denotes the Nash equilibrium level of extraction from the CPR exerted by the non-cooperators, so we can re-write equation Equation (3.19) as follows:

$$\alpha - x^{*}_{j\in N\backslash S}b - b\left[sx^{c}_{i\in S} + (n-s)\, x^{*}_{j\in N\backslash S}\right] = 0 \tag{3.21}$$

so, the best respond of a non-cooperative appropriator will be

$$x^{*}_{j\in N\backslash S} = \frac{\alpha - sbx^{c}_{i\in S}}{b\,(n-s+1)} \tag{3.22}$$

plug $x_{i\in S^{c}} = \frac{\alpha}{2bn}$, we have that

$$x^{*}_{j\in N\backslash S} = \frac{\alpha\,(2n-s)}{2bn\,(n-s+1)} = x^{c}_{i\in S}\left[\frac{2n-s}{n-s+1}\right] \tag{3.23}$$

It is easy to see now that

$$\frac{\alpha\,(2n-s)}{2bn\,(n-s+1)} > \frac{\alpha}{2bn} \tag{3.24}$$

An appropriator who does not belong to S makes his best response given the appropriation quote extraction (s)he expects on the part of each appropriator belonging to S and on the appropriation quote extraction (s)he expects on the part the other non cooperative appropriators. On the other hand, notice that Equation (3.23) becomes $\frac{\alpha}{b(n+1)}$, the Nash equilibrium of the original CPR game when there is no cooperative group $(s = 0)$. Whereas for the case in which $0 < s < n$, the following inequality holds:

$\dfrac{\alpha}{b(n+1)} < \dfrac{\alpha(2n-s)}{2bn(n-s+1)} \iff n > 1 \ \forall n \in \mathbb{Z}^{+}$. Moreover, given that the cooperative group extracts the resource at $x_S^c < (n-s)\, x^*_{j \notin S}$, this implies that they are forsaking some part of the resource, which in turn might be taken by the non-cooperative appropriators. The decision made by the cooperators leads the non-cooperators to an increase in their extraction level with respect the extraction level of the original level: $\Delta_{x^*_{j\in N\setminus S}} = \frac{\alpha(2n-s)}{2bn(n-s+1)} - \frac{\alpha}{b(n+1)} = \left[\dfrac{\alpha(n-1)s}{2n(n+1)(n-s+1)b}\right]$,which represents in increase share of $\dfrac{(n-1)s}{2n(n-s+1)}$ for the non-cooperator. Next, we can check the gains for each type of appropriator. The payoff of a non-cooperative appropriator is given by the next expression:

$$u_{j\in N\setminus S}\left(x^*_{j\in N\setminus S}, x^c_{i\in S}\right) =$$
$$e+\alpha\left[\frac{\alpha(2n-s)}{2bn(n-s+1)}\right]-b\left[\frac{\alpha(2n-s)}{2bn(n-s+1)}\right]\left[s\frac{\alpha}{2bn}+(n-s)\left[\frac{\alpha(2n-s)}{2bn(n-s+1)}\right]\right]$$
$$= e + \left(\frac{\alpha(2n-s)}{2n(n-s+1)\sqrt{b}}\right)^2 \quad (3.25)$$

Naturally, just as expression Equation (3.23) becomes the original level of the extraction when there is no a cooperative group, so does expression Equation (3.25) relative to the total payoff each individual gets in the original situation. On the other hand, the gains of a cooperative appropriator are the next:

$$u_{i\in S}\left(x^c_{i\in S}, x^*_{j\in N\setminus S}\right) =$$
$$e+\alpha\left[\frac{\alpha}{2bn}\right]-\left[\frac{\alpha}{2bn}\right]b\left(s\frac{\alpha}{2bn}+(n-s)\left[\frac{\alpha(2n-s)}{2bn(n-s+1)}\right]\right)$$
$$= e + \left(\frac{\alpha}{2n\sqrt{b}}\right)^2\left(\frac{2n-s}{n-s+1}\right) \quad (3.26)$$

Then, we compare both payoffs Equation (3.25) and Equation (3.26) and notice that

$$u_{j\in N\setminus S}\left(x^*_{j\in N\setminus S}, x^c_{i\in S}\right) = u_{i\in S}\left(x^c_{i\in S}, x^*_{j\in N\setminus S}\right)\left(\frac{2n-s}{n-s+1}\right) \quad (3.27)$$

which means that as long as $\Leftrightarrow n > 1$ and $1 \leqslant s < n$,

$$u_{j\in N\backslash S}\left(x^*_{j\in N\backslash S}, x^c_{i\in S}\right) > u_{i\in S}\left(x^c_{i\in S}, x^*_{j\in N\backslash S}\right) \tag{3.28}$$

Although appropriators belonging to group S set somehow an example for the whole community by extracting the resource at a level such that the social optimum level of appropriation of the CPR would be achieved if individuals who are not in this group were to commit to this level, the non-cooperative appropriators will gain more by acting individually. Thus, it is not in their interest to join group S. Should they do so, their individual payoffs would be undergo a reduction with respect to the payoffs they would achieve on their own.

Now, if non-cooperative appropriators prefer to be outside a cooperative group since they obtain better payoffs, we wonder whether it is in the interest of non-cooperators that a cooperative group be formed as long as they do not belong to it. That is, would non-cooperative appropriators be better off sharing the resource with group S? or do they prefer a situation where each member of the community acts individually? How do group and community size affect the payoffs of non-cooperators so that they prefer one situation or the other? Observe that without the formation of group S, each individual will extract the resource according to the level of extraction given by Equation (3.9), which will yield the following payoff:

$$u_j\left(x^*_j, x^*_{-j}\right) = e + \alpha\left(\frac{\alpha}{b\left(n+1\right)}\right) - \left(\frac{\alpha}{b\left(n+1\right)}\right)\left(\frac{n\alpha}{b\left(n+1\right)}\right)b = e + \frac{\alpha^2}{b\left(n+1\right)^2} \tag{3.29}$$

What we do now is just to contrast this gain to the gain when x^*_i (Equation (3.25)) is extracted. Could we expect one payoff to be always higher than the other? To see this, we set a function of n and s as the difference between both Equation (3.29) and Equation (3.25) payoffs. That is, the difference between the payoff a non-cooperative appropriator gets when there is a cooperative group and the payoff (s)he gets when this groups does not come into play.

$$G(n,s) := u_{j\in N\setminus S}\left(x^*_{j\in N\setminus S}, x^c_{i\in S}\right) - u_j\left(x^*_j, x^*_{-j}\right) =$$
$$\left\{e + \frac{\alpha^2(2n-s)^2}{4bn^2(n-s+1)^2}\right\} - \left\{e + \frac{\alpha^2}{b(n+1)^2}\right\}$$
$$= \left[\left(\frac{2n-s}{2n}\right)^2\left(\frac{\alpha}{n-s+1}\right)^2 - \left(\frac{\alpha}{n+1}\right)^2\right]\frac{1}{b} \quad (3.30)$$

Then, note that $G(n,s) > 0$ increases with $2 \leqslant n$, $0 < s < n$ as $\frac{\partial G(n,s)}{\partial s} = \frac{\alpha^2(n-1)(2n-s)}{2bn^2(n-s+1)^3} > 0$, also notice that as the number of people in the community grows and the size of the cooperative groups approaches it, this difference is strictly positive:

$$\lim_{n\to\infty}\left\{\lim_{s\to(n-1)} G(n,s)\right\} = \lim_{n\to\infty}\left[\frac{\alpha(n+1)}{4n\sqrt{b}}\right]^2 = \left(\frac{\alpha}{4\sqrt{b}}\right)^2 \quad (3.31)$$

Therefore, an appropriator who does not stick with a cooperative level of extraction thrives on the formation of a cooperative group s as long as (s)he does not belong to it as (s)he obtains a better payoff.

3.1.4 Scenario 2

On the other hand, assume now that the individuals interested in cooperating disregard the social optimum exerted in the previous case, but still are willing to form a group. Thus they decide to implement that level of appropriation that maximizes their joint utility taking as given the individual appropriation of the non cooperative individuals. In other words, the players who are not interested in cooperating choose to implement the level dictated by their individual maximization whereas the cooperative players join a group that implements an optimum group extraction as if they were one single player. Under this scenario, the group S chooses optimally:

$$\underset{x_s}{\text{maximize}} \quad se + \alpha x_s - x_s b\left(\sum x_{i\in N}\right)$$
$$\text{subject to} \quad 0 \leqslant x_s$$

$$\text{f.o.c: } \alpha - x_s b - \left(\sum x_{i\in N}\right) b = 0 \quad (3.32)$$

The best response strategy of group S is given by:

$$x_S = \frac{\alpha - \left(\sum x_{i\in N}\right) b}{b} \tag{3.33}$$

Likewise, a non-cooperative appropriator $i \notin S$ will face a similar program as Equation (3.16), with the f.o.c given by:

$$\alpha - x_{i\notin S} b - \left(\sum x_{i\in N}\right) b = 0 \tag{3.34}$$

so the best response strategy is the same as Equation (3.33),

$$x_{i\notin S} = \frac{\alpha - \left(\sum x_{i\in N}\right) b}{b}. \tag{3.35}$$

A non-cooperative appropriator makes his or her best response appropriation strategy given the appropriating quote extraction (s)he expects on the part of the group S acting as a single appropriator as well as given the appropriating quote extraction (s)he expects on the other $n-s-1$ individual members. Again, we assume and selected a symmetric equilibrium. Therefore, the total amount of resource extracted at this equilibrium will be given as follows:

$$\sum x_{i\in N} = x_S + \sum x_{i\notin S} = (n-s+1)\, x^{**} \tag{3.36}$$

where x^{**} is the new Nash equilibrium level of appropriation of the $n-s+1$ "appropriators" in the community. Then Equation (3.33) and Equation (3.35) both reduce to:

$$x^{**} = \frac{\alpha}{b\,(n-s+2)} \tag{3.37}$$

Next, each appropriating member $i \in S$ will get an equal share of Equation (3.37):

$$x^{**}_{i\in S} := \frac{x^{**}}{s} = \frac{\alpha}{b\,(n-s+2)\, s} \tag{3.38}$$

Needless to say, $x^{**}_{i\in S}$ is a smaller amount of resource in relation to the amount of resource appropriated by a non-cooperative individual. Next, note here also that when there is no cooperative group $(s = 1)$, the Nash equilibrium is simply the one corresponding to the symmetric equilibrium of the original game $x^* = \frac{\alpha}{b(n+1)}$. We proceed now to study whether or not the fact that the cooperative appropriators form a group that acts as a single entity makes it more beneficial for out-group appropriators to join S. The first consequence we observe is an increase on the level of extraction of the non-cooperators with respect to the extraction level of the original game, that is $\frac{\alpha}{b(n-s+2)} \geqslant \frac{\alpha}{b(n+1)} \iff 1 \leqslant s < n \; \forall n, s \in \mathbb{Z}^+$. This increase is $\Delta_{x^*_{j \in N \setminus S}} = \frac{\alpha}{b(n-s+2)} - \frac{\alpha}{b(n+1)} = \frac{\alpha}{b}\left[\frac{s-1}{(n+1)(n-s+2)}\right]$, which means an share of $\frac{s-1}{n-s+2}$ for the non-cooperative appropriators. Then, will non-cooperative appropriators be interested in joining to the cooperative group S? The payoff of an individual $i \in S$ is given by:

$$u_{i \in S}\left(x^{**}_{i \in S}, x^{**}_{j \notin S}\right) = \\ e + \alpha\left[\frac{\alpha}{b(n-s+2)s}\right] - \left(\frac{\alpha}{b(n-s+2)s}\right)\left[s\frac{\alpha}{b(n-s+2)s} + (n-s)\frac{\alpha}{(n-s+2)b}\right]b = \\ e + \frac{\alpha^2}{b(n-s+2)^2 s} \quad (3.39)$$

whereas the payoff of an individual $j \notin S$ will be give by

$$u_{j \notin S}\left(x^{**}_{j \notin S}, x^{**}_{i \in S}\right) = \\ e + \alpha\left[\frac{\alpha}{b(n-s+2)}\right] - \left(\frac{\alpha}{b(n-s+2)}\right)\left[s\frac{\alpha}{b(n-s+2)s} + (n-s)\frac{\alpha}{(n-s+2)b}\right]b = \\ e + \frac{\alpha^2}{b(n-s+2)^2} \quad (3.40)$$

Then $u\left(x^{**}_{i \in S}, x^{**}_{j \notin S}\right) \leqslant u_{j \notin S}\left(x^{**}_{j \notin S}, x^{**}_{i \in S}\right)$. Newly, not cooperating yields greater individual benefits than cooperating. Being cooperative is not made up for by the payoff achieved in here. We now move on to check what situation a non-cooperative appropriator would prefer in terms of the benefits (s)he can obtain versus the formation or not of a cooperative group and to what degree the size of this group and the community as a whole would influence this decision. We set a function $H : (n, s) \mapsto \mathbb{R}$ as the difference of the payoff that the non-cooperators obtain in the presence of the cooperative group S (Equation (3.40)) and the payoff they obtain when each of the appropriators in the community acts independently (Equation (3.29)):

$$H(n,s) = \left\{ e + \frac{\alpha^2}{b(n-s+2)^2} \right\} - \left\{ e + \frac{\alpha^2}{b(n+1)^2} \right\} = \frac{\alpha^2 (s-1)(2n-s+3)}{b(n+1)^2 (n+2-s)^2} \tag{3.41}$$

This difference will always be positive as the number of people in the community grows and the size of the cooperative group approaches this number: $\lim_{n\to\infty} \left\{ \lim_{s\to(n-1)} H(n,s) \right\} = \lim_{n\to\infty} \frac{\alpha(n+4)(n-2)}{9b(n+1)^2} = \left(\frac{\alpha}{3\sqrt{b}} \right)^2$. Therefore, a non-cooperative appropriator benefits more from the existence of a cooperative group because it causes him/her a positive externality. Furthermore, for a given number of appropriators in the community, the function $H(\cdot)$ is increasing in the number of cooperative appropriators, since $\frac{\partial H(n,s)}{\partial s} = \frac{2\alpha^2}{b(n-s+2)^3} > 0$, implying that as the number of cooperative appropriators in group S grows, $H(\cdot)$ becomes larger. With this we also observe that the benefits of the non-cooperative appropriator are increasing with the size of the cooperative group. In other words, an appropriator who does belong to a group that acts as a single appropriator would prefer that such a group to form due to the increase in her/his payoff. Thus, even in this situation, although the formation of the cooperative group reduces the amount of extraction that each member would obtain, it is not enough by itself to avoid falling into the tragedy.

Given the above results, we can now see which is the most favorable scenario for each type of appropriator. That is, will a cooperator prefer that her/his group extracts that prespecified amount of resource derived from the division of the social optimum, or will this agent go in for a group that behaves as a single appropriator? In parallel, if a non-cooperative appropriator had a choice of which scenario to be in, which one would yield better benefits? Recall, the payoff (Equation (3.25)) of the non-cooperative appropriator $j \notin S$ when the cooperative group S agrees to extract the resource according to the socially optimal extraction division is $u_{j\in N\setminus S}\left(x^*_{j\in N\setminus S}, x^c_{i\in S}\right)$, whereas the payoff of the non-cooperative appropriator (Equation (3.40)) when the cooperative group behaves as if it were a single appropriator is $u_{j\notin S}\left(x^{**}_{j\notin S}, x^{**}_{i\in S}\right)$. Then we put them in terms of s and n, so

$$u_{j\in N\backslash S}\left(x^{*}_{j\in N\backslash S},x^{c}_{i\in S}\right):=u^{1}_{j\in N\backslash S}\left(n,s\right) \tag{3.42}$$

$$u_{j\notin S}\left(x^{**}_{j\notin S},x^{**}_{i\in S}\right):=u^{2}_{j\notin S}\left(s,n\right) \tag{3.43}$$

where the superscripts $\{1,2\}$ tell us the scenario from which the utility is obtained. We set now the difference between these two utilities so that we can see under what conditions over n and s one is greater than the other. First, we take the limit of it as the size of the cooperative group grows, and then as the community does. We observe that this difference will be strictly positive.

$$\lim_{n\to\infty}\left\langle \lim_{s\to(n-1)}\left\{u^{2}_{j\notin S}\left(s,n\right)-u^{1}_{j\in N\backslash S}\left(n,s\right)\right\}\right\rangle=\frac{\alpha^{2}}{144b}>0 \tag{3.44}$$

The non-cooperative appropriators benefit the most from the situation in which the cooperative group acts as a single appropriator. Then, they would prefer to find themselves in this scenario. Reversely, we will see that the cooperative appropriators would prefer being in a situation where the cooperative group they belong to extracts a share of the socially efficient level (Equation (3.15)). Again, the individual payoff of a cooperative member when the socially optimal share resource extraction is exerted is given by $u_{i\in S}\left(x^{c}_{i\in S},x^{*}_{j\in N\backslash S}\right)$ (Equation (3.26)), and the individual payoff of $i\in S$ when S acts as a single appropriator is $u_{i\in S}\left(x^{**}_{i\in S},x^{**}_{j\notin S}\right)$(Equation (3.39)). Similarly, we write them as functions of n and s:

$$u_{i\in S}\left(x^{c}_{i\in S},x^{*}_{j\in N\backslash S}\right):=u^{1}_{i\in S}\left(n,s\right) \tag{3.45}$$

$$u_{i\in S}\left(x^{**}_{i\in S},x^{**}_{j\notin S}\right):=u^{2}_{i\in S}\left(n,s\right) \tag{3.46}$$

The difference between these two terms is positive when we subtract the $u^{2}_{i\in S}\left(n,s\right)$ from the $u^{1}_{i\in S}\left(n,s\right)$. And it holds as the number of cooperative appropriators is high enough in the community, and when there are at least three appropriators in the community:

$$\lim_{s \to n-1} \left\langle u^1_{i \in S}(n,s) - u^2_{i \in S}(n,s) \right\rangle = \frac{\alpha^2}{72b} \frac{n^2-9}{(n-1)^2} \geqslant 0 \iff n \geqslant 3 \tag{3.47}$$

The appropriators as a cooperative group get better payoff when they just comply with the community social optimum. That is, if non-cooperative appropriators will always act individually following a behavioral protocol dictated by the Nash criterion and if the cooperative appropriators had the decision to choose how to behave as a group, the best they could do is to consider the situation described in the first scenario. Notice, however, that this difference is very small as the size of the whole community is very large:

$$\lim_{n \to +\infty} \left\langle \frac{n^2-9}{(n-1)^2} \frac{\alpha^2}{72b} \right\rangle = 0$$

Therefore, being in one situation or another will be almost indifferent for a cooperative appropriator in a large community.

3.1.5 Scenario 3

Let us now say that within the community of n appropriators m cooperative groups might form: $\{S_1, S_2 \ldots, S_m\}$. Each group now would behave as if it were a single appropriator playing the CPR game. The payoff extraction of each group will be the sum of the utility functions of each appropriator in the group. Then group S_i will solve:

$$\begin{aligned} \underset{x_{S_i}}{\text{maximize}} \quad & se + \alpha x_{S_i} - x_{S_i} b \left[\sum x_{S_i} \right] \\ \text{subject to} \quad & 0 \leqslant x_{S_i} \end{aligned} \tag{3.48}$$

f.o.c

$$\alpha - x_{S_i} b - \sum x_{S_i} b = 0 \tag{3.49}$$

Thus, the optimal extraction strategy of S_i will be:

$$x_{S_i} = \frac{b \sum_{S_i=1}^{m} x_{S_i} - \alpha}{b} \tag{3.50}$$

Again, by symmetry over the strategy space, we can assume that $\sum x_{S_i} = m x^*_{S_i}$, where $x^*_{S_i}$ is the selected Nash equilibrium extraction strategy of the CPR game played among the m groups. Then, the level of extraction at this equilibrium is given by

$$x^*_{S_i} = \frac{\alpha}{b\,(m+1)} \tag{3.51}$$

which leads to the total payoff for group S_i:

$$u_{S_i}\left(x_{S_i}, x_{S_{-i}}\right) = s_i e + \frac{\alpha^2}{b\,(m+1)^2}, \tag{3.52}$$

assuming the equal sharing rule within the group S_i, each appropriator $j \in S_i$ will get the next payoff:

$$u_{j\in S_i}\left(x_{S_i}, x_{S_{-i}}\right) = e + \frac{\alpha^2}{b\,(m+1)^2\, s_i}. \tag{3.53}$$

The question looms over us: does it pay for an appropriator to belong to this group while the rest remains equal? Let us say that j drops out from S_i, so under this new situation we will have a game of $m+1$ players, then the appropriator j will choose optimally accordingly:

$$\begin{aligned} \underset{x_j}{\text{maximize}} \quad & se + \alpha x_j - x_j b\left[\sum x^{m+1}_{S_i}\right] \\ \text{subject to} \quad & 0 \leqslant x_j \end{aligned} \tag{3.54}$$

f.o.c

$$\alpha - x_j b - \sum x^{m+1}_{S_i} b = 0 \tag{3.55}$$

Same as before, we impose a symmetric equilibrium among the $m+1$ players, call it $x^\star$. Then, $\sum x^{m+1}_{S_{i=1}} = (m+1)\,x^\star$. And the level of exploitation/extraction of individual j at this equilibrium will be the next:

$$x^\star = \frac{\alpha}{b\,(m+2)} \tag{3.56}$$

and the corresponding payoff will be:

$$u\left(x^\star_j, x^\star_{S_{i\neq j}}\right) = e + \frac{\alpha^2}{b\,(m+2)^2} \tag{3.57}$$

Should individual j stay in the group S_i?

We can contrast Equation (3.53) with Equation (3.57), so given that $1 < s_i < m$, we have that $\frac{\alpha^2}{b(m+1)^2 s_i} < \frac{\alpha^2}{b(m+2)^2}$ holds as long as $\left(\frac{m+2}{m+1}\right)^2 < s_i$, which is true as $m \geqslant 3$. Now one can readily notice that appropriate j stays out of the group S_i. The same reasoning applies to the rest of the members of the community. So even with the possible formation of cooperative groups within the community and under the conditions of the CPR game, the *tragedy* is persistent. In the first scenario the formation of a cooperative group helps to cope with the tragedy and sets an example to the community. However, the group by itself is not enough to induce the whole community to cooperate; rather, what occurs is that the uncooperative appropriators take advantage of the fact that the formation of a cooperative group implies foregoing a part of the resource that can be taken by the non-cooperative appropriators. Hence, the non-cooperative appropriators prefer to coexist with a cooperative group. Thus, it seems that in these conditions the tragedy is reluctant to be bested by cooperative people who face non cooperative individuals. In the following section we will consider the CPR game from the approach of cooperative game theory and coalition formation. Could it be that under this framework we could explain situations such that the communities overcome the *tragedy*?

3.2 Coalitions and Cooperative Game Theory

As stated earlier in Chapter 2, there are common variables that help to explain cooperative behavior. Some of which can be accounted for by cooperative game theory and coalition formation, such as communication mechanisms, bargain, homogeneity of the participants, and group size. What follows now is the relation of CPRs and coalitions through some case studies. Further on, we present a cooperative model in partition function form derived of a general strategic game introduced by Chander (2019) and then study it in our context of the CPRs. That is, we transform the strategic CPRs game into a game in partition function form. And, in this line, we also apply a game of coalition formation of the same author to the case that concerns us here.

3.2.1 Coalitions and Common Pool Resources

Consider the case where players are able to form groups that act as single entities. This alternative scenario implies additional examination beyond the mere formation of a cooperative group studied in Section 3.1.2. Since cooperation among appropriators through the formation of those entities is permitted, the analysis of CPR situations changes. They are allowed to negotiate. There are different possibilities in terms of what groups may come up. Indeed, the basic objects of study now are those groups, which in the literature are termed as *coalitions*. In this sense, players may be involved in a bargaining process. If players perceive that by cooperating with other players they receive more than what they would be able to get by themselves, they might want to enter into negotiations. The result of such negotiation processes aim at some stable coalitions where players have no incentive to deviate from the establishing agreement. We look at this issue within the CPRs setting. In this connection, the literature shows cases where the effects of groups size in the management of CPRs are studied together with those of coalition formation. Wilson and Thompson (1993) study the reasons behind a breakdown in productivity of communally held Mexican lands called ejidos.[4] They attribute such reasons to a deterioration in property management at the community level. According to their work, rights, duties, functions, and obligations of individual herders had not been clearly specified or enforced by *ejido* authorities. Nevertheless, *failure* of group management —they argue—has led to the formation of coalitions within smaller groups where cooperation is assured and benefits are enjoyed under severe ecological conditions. They call "compensating coalitions" in the sense that they recognize the failure of the *ejido*, and in response try to make up for it by forming a group with enough structure to make a collective decision that benefits its members. The uncertainty of others' behaviors is reduced in these coalitions, which enables them to reach a partial level of cooperation.

[4]An *ejido* combines communal ownership with individual use. It consists of cultivated land, pastureland, other uncultivated lands, and the *fundo legal* (town-site) Britannica (2011). The *ejidos* controls a substantial share of the Mexican agricultural land.

Perez-Verdin et al. (2009) conduct an empirical analysis on the relationship between common-based property regimes and the conservation of natural resources. Specifically, they study the effect of group size and heterogeneity upon the performance of *ejidos*, in protecting forest resources in Northern Mexico. They conclude that, in general, group size and heterogeneity have no significant effect on deforestation. Deforestation would be driven not by the characteristics of the *ejidos* like total area or number of members but by resource-specific characteristics such as location and soil productivity. In this vein, Ostrom and Poteete (2004) approach the research of the International Forestry Resources and Institutions related to, among other aspects, the interrelations among group size, heterogeneity, and institutions. They show that group size and some forms of collective action exhibit a non-linear relationship.

On the other hand, in the example by Ostrom et al. (1990c) concerning a fishery in Sri Lanka, in addition to the analysis of the dynamic adjustment from a partially solved CPR dilemma to a failed one the authors describe, I highlight how in certain situations the formation of groups emerges as a way of managing the exploitation of a resource. People in this small fishing village used beach seines as a catching fish technology, but as each net was expensive and at least eight men were needed to cast it and draw it ashore, they decided to split the ownership of a single net into eight shares. Then, they used approximately twenty jointly owned beach-seines. And each share was single-handedly worked by a fisher. The catch then was divided equally among the eight owners. In this case, factors such as the characteristics of the resource (size and availability of it) as well as of the used technology (the size, weight and costs of the beach seine) led people to form groups and devise a way of collective exploitation (at least until a certain point).

3.2.2 A Coalition Approach to CPR

Let us now introduce the cooperative model for the CPRs problems of appropriation. As said, the analysis changes slightly since the entity of study is now *coalitions*. This does not mean I disregard cases in which individuals just want to act singly. We set the problem of the CPRs into a particular form of cooperative games[5]: the partition function form. This form takes into account possible externalities that coalitions impose on each other (recall: what I subtract from the resource, you can not).

Basically, by setting the CPRs issue under coalition structures we explore the formation of coalitions and the allocation of the coalition worth to its members. In this sense, we study situations in which also extreme cases of cooperation (no one forms a coalition or all players join) may arise as well as intermediate cases.

3.2.3 The Partition Function and the γ-Core of the CPRs

Let N be the finite set of players. Formally, a group of players $S \subset N$ is called a coalition. Accordingly, a *coalition* has to be thought of in a broad sense. It has a purpose and is assumed to be able to formulate and execute collective action. This entails that the members of a coalition are provided with a collective decision mechanism or a governance structure [Gilles (2010).] The reason behind studying the CPRs appropriation issue with a coalition approach lies in the fact that players are allowed to plan, formulate and execute collective actions through institutions, behavioral norms, and communication structures. In this light, this argument links up with the lines advanced by [Ostrom (2010) and Ostrom (2002a).] These studies posit that participants involved in a CPRs situation do undertake efforts to design their own governance arrangements, and substantial empirical evidence supports it.

3.2.3.1 The Partition Function Form.

In this section, we proceed to formally transform the Γ game introduced in Section 3.1.1.1 into a partition function game by relying upon the method proposed by Chander (2019), who, at the same time, uses the notion of *games in partition function form* presented by Thrall and Lucas (1963). For that matter, we being our analysis by defining what a

[5]Here we work on the grounds of n-person cooperative games with transferable utility (TU). In these games, it is assumed that the members of a coalition, if formed, enjoy of an utility or a commodity -say money- that can freely be transferred among them (Peters (2015)).

partition is.

Definition 1 (*Partition*)

A partition $P = \{S_1, S_2, \ldots, S_m\}$ of N, the finite set of players, is a set of subsets such that $S_i \neq \varnothing$, $S_i \cap S_j = \varnothing\ \forall i \neq j$, and $\bigcup_{i=1}^{m} S_i = N$.

In words, a partition of a set is a collection of disjoint and non-empty subsets where every element belonging to one subset is not included in another one. In our context, naturally, we dub those subsets as *coalitions*. Next, we call on the concept of embedded coalition offered by Kóczy (2018).

Definition 2 (*Embedded Coalition*)

An embedded coalition is the pair (S_i, P), where the coalition S_i is embedded in partition P if $S_i \in P$. The set of embedded coalitions is then $\mathscr{E} = \{(S_i, P) : S_i \in P, P \in \mathcal{P}(N)\}$, where $\mathcal{P}(N)$ is the set of partitions of N.

Now we are in conditions to define a game in partition function form.

Definition 3 (*Game in Partition Function Form*)

A game in partition function form is a pair (N, v), where v denotes the partition function

$$v : \mathscr{E} \mapsto \mathbb{R}$$

which associates each embedded coalition to a real value (payoff).

Following Chander (2019), the game CPR game Γ can be converted into a partition function game induced by a partition P in which each coalition behaves as a single player (appropriator), since it admits a unique symmetric Nash equilibrium. Then, the extraction strategy of S_i is chosen by the players who are part of it, and whose sum of individual payoffs is aimed to be maximized by joining their strategies, given the strategies of the other coalitions. Naturally, the reader may note that this is a way of formalizing the scenario three presented in Section 3.1.5. Let $\Gamma^p = (P, \chi^p, u^p)$ denote the CPRs game induced by a partition P, where

- P is a partition of N as established in Definition 1.
- $\chi^p = \times_{i \in m} \chi_{S_i}$ is the set of all vector strategies, where $\chi_{S_i} = \times_{j \in S_i} x_j$ is the appropriation strategy set of coalition S_i, for every $S_i \in P$, so $x_{S_i} \in \chi_{S_i}$ represents the amount of the resource (joint appropriation strategies of appropriators) collected by coalition S_i.
- $u^p = \left(u_{S_i}, \ldots, u_{S_m}\right)$ is the vector of payoff function, where $u_{S_i}\left(x_{S_i}, x_{S_{-i}}\right) = \sum_{j \in S_i} u_j\left(x_{S_i}, x_{S_{-i}}\right)$ with $x_{S_{-i}}$ is, as usual, the vector of all extraction strategies excluding S_i's

Then, the worth of coalition S_i is the symmetric Nash equilibrium payoff of the game Γ^p. Specifically, the partition function v of this game is

$$v\left(S_i; P\right) = \max_{x_{S_i}} \left\{ \sum_{j \in S_i} u_j\left(x_{S_i}, x_{S_{-i}}\right) \right\}. \tag{3.58}$$

It is worth noting that under this framework the appropriating coalition might choose, at worst, the same strategies as when playing singleton. The grand coalition $\{N\}$ is then itself an efficient partition in terms of the disposal of strategies over the space strategies appropriators enjoy. That is, the fact that the grand coalition is itself the largest coalition in a partition implies that players in here have a set of strategies that range from the strategies that each single player has individually to the set of strategies generated by the union of all of them. In other words, appropriators in the grand coalition will always have the possibility to choose at least those strategies that could be implemented in any other partition, and players in a partition $P \neq \{N\}$ would not be able to choose strategies associated with the formation of the grand coalition. We stick with the author as regards this point, so we assume that the worth generated by forming the grand coalition is greater or equal that the worth generated by every other partition different from it:

$$v\left(N; \{N\}\right) \geqslant \sum_{S_{i \in P}} v\left(S_i; P\right) \; \forall \; P = \{S_1, S_2, \ldots S_m\} \neq \{N\} \tag{3.59}$$

where $v\left(N;\{N\}\right) = \max_{0\leqslant x(N)} U\left(x\left(N\right)\right)$, and from Equation (3.11), Equation (3.12), and Equation (3.13), it is explicitly give by the next expression:

$$v\left(N;\{N\}\right) = ne + \frac{\alpha^2}{4b}. \tag{3.60}$$

In terms of the resource appropriation, when all appropriators unite in a single coalition, they have the option of choosing that level of resource extraction equal to the one they would choose should they are in any other partition-induced CPR game. Indeed, making allowance for the grand coalition to be the efficient partition in the given terms of inequality (3.59) also entails that the decision of bringing about a coalition will depend on the extraction strategies chosen by the appropriators. The level of extraction each player can get in the induced CPR game when they are singleton $(s_i = 1)$ is a possible level of extraction that any other larger coalition can garner. Therefore, we can assume, as Chander (2019) does, that when appropriators in a coalition pick out the same appropriation strategy they would go for as if they were singleton and given the appropriation strategies of the other appropriators, this coalition simply does not arise with the agreement of all involved.

Having presented the above, the question arises as to how the worth generated by each coalition should be allocated among its members as a matter of course. More precisely, if the best cooperative outcome is the grand coalition, how can we assure that everyone in gets a fair payoff. Solution concepts such as the core are called upon as they are a way of deciding on the basis of diverse fairness criteria as how to distribute the worth of the grand coalition. That way, we study what coalitions we might expect to be formed or split off. The related literature on cooperative CPRs games (characteristic function form) applies conventional core concepts such as the α-core and β-core (Meinhardt (2012)). And in Kóczy (2018) we find recent solution concepts applied to the game of the CPRs, but to the best of our knowledge, the concept solution we consider here have not been explored in the context of CPRs. In the next section, we start off our analysis with the one proposed by Chander (2019) —the γ-core. This concept presumes, as his proposer explains, that a coalition that deviate from the grand coalition expects that the coalition structure (partition) that might form in the complement upon its deviation is the worse possible form from its point of view. That is, that the complement is such that all coalitions are singletons. It might be a pessimistic expectation, but given that the deviating coalition does not know how the other coalition are going to behave, it adopts the worse case scenario (Chander (2018b)).

3.2.3.2 The γ-Core

In order to understand its definition, we need to know the notion of feasible payoffs. Precisely, those payoffs that result from the division of the grand coalition's worth.

Definition 4 (*Feasible payoff*)

Let (N, v) be a a partition function game. A vector of payoffs $(z_i, ..., z_n)$ is feasible if $\sum_{i \in N} z_i = v(N; \{N\})$.

Next, the γ-core is defined drawing on Definition 4,

Definition 5 (γ-*core*)

The γ-core of a partition function (N, v) is the set of feasible payoff vectors $(z_1, ..., z_n)$ such that $\sum_{i \in S} z_i \geqslant v\left(S; \{S, [N \backslash S]\}\right)$ for all $S \subset N$.

where $[N]$ and $[N \backslash S]$ indicate the finest partitions of the N and $N \backslash S$, respectively. For a feasible payoff vector to belong to the γ-core of a partition function game, any coalition S must not get more at the Nash equilibrium of the game in which the players of coalition S —acting as a single entity—believe that those who do not belong to it play individually. In other words, under Definition 5 we have $n - s + 1$ players playing the associated strategic game. Chander (2019)[6] proves that the γ-core of symmetric partition function games with the following two proprieties is nonempty. First, the individual payoffs of players belonging to the larger coalitions in each partition are smaller than the individual payoffs received in smaller coalitions. Second, the unique efficient partition is the grand coalition. Suitably, the key is that the partition function

[6] See proposition 2 of Chander (2019)

is symmetric as defined below.

Definition 6 (***A Symmetric Partition Function Game***)

Let (N, v) be a partition function game such that for every partition $P = \{S_i, \dots, S_m\}$, if $s_i = s_j$ implies that $v(S_i; P) = v(S_j; P)$, then it is symmetric.

That is, given a partition of a symmetric partition function, two or more coalitions with the same number of members each will get the same worth. So the next step now is to verify that the CPRs partition function on treatment (Equation (3.58)) is symmetric under definition 6, and if so, we check that it has the two properties above mentioned, which raises the question whether or not the γ-core of the CPRs partition function game is nonempty. In this manner, we can study the implications of this for the CPRs problem. What is the significance of the non-emptiness of the γ-core in terms of explaining under this approach the overcoming of the *tragedy of the commons*? Could it be the case that we are in the imminent occurrence of a stable coalition capable of eluding it? Before answering those questions, remember that CPR game is a symmetric game since we assumed that the appropriators have the same strategies, costs of extraction, endowments, and utility functions. Which suggest that partition function form of CPRs game is symmetric. Let us find out.

3.2.3.3 The symmetry of the Partition Function CPRs Game

Assume a partition $P = \{S_i, \dots, S_m\}$, with coalitions S_i and S_j such that $s_i = s_j$, $i, j \in 1, 2, \dots, m$. Le us say that those coalitions are involved in the CPRs game. Each coalition now choose a level of appropriation x_{S_i} from the resource. Then, under the partition function game and from Equation (3.4) and Equation (3.58) our examined CPRs game yields a worth for each coalition which is computed as follows:

$$v(S_i; P) = \max_{0 \leqslant x_{S_i}} \quad s_i e + \alpha x_{S_i} - x_{S_i} b \left[\sum x_{S_i \in P} \right] \tag{3.61}$$

This is the maximum of the total sum of the individual utilities of each of the appropriating members in coalition S_i, which solves Equation (3.61). The first-order condition is

$$\alpha - x_{S_i} b - \left(\sum x_{S_i \in P}\right) b = 0 \tag{3.62}$$

Term Equation (3.62) is the same for the rest $m-1$ coalitions, so we face m first-order conditions that define a system of m equations. As in the original CPRs game, we can take advantage of the symmetry of the game, so we set the total level of appropriation accordingly as

$$\sum x_{S_i \in P} = m x^*_{S_i} \tag{3.63}$$

where $x^*_{S_i}$ is the Nash equilibrium of the CPRs game played within the partition P. Every coalition's Nash equilibrium resource extraction, so we plug Equation (3.63) into Equation (3.62)

$$\alpha - x^*_{S_i} b - \left(m x^*_{S_i}\right) b = 0$$

isolating $x^*_{S_i}$

$$x^*_{S_i} = \frac{\alpha}{b\,(m+1)} \tag{3.64}$$

The worth of coalition S_i is then

$$v\,(S_i; P) = s_i e + \alpha \left[\frac{\alpha}{b\,(m+1)}\right] - \frac{\alpha}{b\,(m+1)} \left[\frac{m\alpha}{b\,(m+1)}\right] b = s_i e + \frac{\alpha^2}{b(m+1)^2} \tag{3.65}$$

Analogously, we can compute the worth of coalition S_j,

$$v\left(S_j; P\right) = s_j e + \frac{\alpha^2}{b(m+1)^2} \tag{3.66}$$

Since $s_i = s_i$, then

$$v\left(S_j; P\right) = v\,(S_i; P)\,. \tag{3.67}$$

Thus the partition function associated with our CPRs game given by Equation (3.58) is symmetric. The next thing is to see whether or not it is part of those classes of symmetric partition function games where in each partition when allocating their worth equally, the members of the large coalitions (different from the grand coalition, being itself an efficient partition (Chander (2014)) obtain lower payoffs relative to the payoffs they would obtain if they were in a smaller coalition. Proposition 1 sheds light on the matter.

Proposition 1 (*Members of Larger Coalitions Get Lower Payoffs*)

In the partition function form associated with the CPRs game, the appropriating members of larger coalitions in any given partition different from the grand coalition receive individually lower payoffs than if they were in a smaller coalition. And the greater the number of appropriating members, the lower the payoffs they gain.

Proof: See Appendix A Section 3.4.1 ■

That is to say, if we have a CPRs game played among the m coalitions contained in the partition P, and in which coalition S_i tries to maximizes its worth (given by the sum of individual utilities of each appropriating members) choosing x_{S_i} as its quote resource extraction, what each appropriator gets is a an equal share of the worth that turns out to be greater as the coalition size is small relative to other coalitions. This occurs of any partition different from the partition that contains the grand coalition (coalitions of size less than or equal to $n-1$).

On the other hand, given that the CPRs game in partition function form is symmetric and fulfills the result of proposition 1, it follows from Chander (2019)[7] that it will have a non-empty γ-core as long as the grand coalition is an efficient partition. Then the next step is to demonstrate that the above is indeed true. Namely, we have to make sure that the vector of feasible payoffs with equal shares belongs to the γ-core and that the largest coalition in each partition is worse-off relative to that feasible payoff. Proposition 2 confirms it.

[7] See proposition 2 (Chander (2019))

Proposition 2 (***The Belonging of the Equal Share Payoffs to the γ-Core***)

In the partition function form CPRs game, the feasible payoff vector with equal shares is in the γ-core, and the largest coalition in each partition is worse-off relative to this feasible payoff vector as long as the grand coalition is the efficient partition.

Proof: See Appendix A Section 3.4.2 ■

Until now, we have seen that the gains from cooperation that the individuals involved in a problem of CPRs can obtain through the worth of coalitions. In this setting, the game is symmetric, which means that a reasonable way of sharing the value of a coalition is just splitting it off by the number of its members. That way, every member of a coalition gets the same share of the total worth. Thus, in this game a coalition with more members has lower-per members payoffs in each partition. This implies that given a partition different from the grand coalition, the coalition with more members willing to cooperate may not form, since their individual payoff is lower than if they were in a smaller coalition or singleton. In the context of CPRs, this suggests that, when players form a partition or a coalition structure, the largest coalition, is paradoxically the coalition least stable; notwithstanding it being the coalition with greatest value as such. In fact, this result is reminiscent of the "paradox of cooperation," conceived in the literature on coalition formation in the context of international environmental agreements, which states that cooperation emerges when its benefits are small or nearly equal to the benefits of a totally non-cooperative situation, rendering cooperation unattractive to the involved agents (Barrett (1994), Submitter et al. (2021)). Consider a case in which a partition that consists of two coalitions, one with $n-1$ players and a singleton coalition. Even when the majority is willing to cooperate, this partition disintegrates. A greater size of a coalition relative to the size of other coalitions discourages its formation in favor of the grand coalition. See the following example.

Example 1

Say that $e = 25$, $c = 5$, $n = 9$, and that the total revenue is given by $f\left(\sum x_i\right) =$

$23x_i - 0.25\left(\sum x_i\right)^2$. *Consider the following partition,*

$$P = \left\{\{1,2,3,4,5\},\{6\},\{7\},\{8\},\{9\}\right\}$$

Thus, the worth of the five coalitions are

$$v\left(\{1,2,3,4,5\};P\right) = 161$$

$$v\left(\{i\};P\right) = 61, i \in 6,7,8,9$$

Notice that if the value of the largest coalition is shared evenly among its members, then each would get 32.2 *which is less than* $v\left(\{i\};P\right)$. *Moreover, if player* 5 *withdraws from the coalition (s)he belonged to and decides to be singleton, a new partition* P' *would appear.*

$$P' = \left\{\{1,2,3,4\},\{5\},\{6\},\{7\},\{8\},\{9\}\right\}$$

Accordingly, under this new configuration the worth of coalitions in P' *that re-optimize their extraction strategies is the following,*

$$v\left(\{1,2,3,4\};P\right) \approx 126.44$$

$$v\left(\{i\};P\right) \approx 51.44, i \in 5,6,7,8,9$$

Which means that were the worth of the coalition from which player 5 *withdrew to split up into its actual cardinality, every member would get* $\dfrac{126.44}{4} \approx 31.6$*, which is less than the individual value of* 32.2 *when player* 5 *stays in. Thus, the withdrawal of this player affects negatively the worth of the remaining players as it entails a lost of* 0.6*. However, (s)he gets now* 51.4*, which is greater than the individual value (s)he gains in the original partition* P*. In this sense, partition* P *is even less stable than* P'*.*

In this vein, the γ-core of this game exists —as mentioned above —the equal payoff sharing rule belongs to it, and the largest coalition in any partition is in a worse position relative to it. A new, given the symmetry of the game —which, in passing, is due to the homogeneity of the players—equal sharing rule of the grand coalition is fair and comes up naturally. Each of the players gains the same amount. Under these circumstances, applying this rule boots the players to move towards the grand coalition, since if they abide in any other partition, their cooperative gains will be smaller or equal than that one of that rule. Look at example 2. This is in line with empirical studies that show that when the group is relatively homogeneous, the individuals tend towards cooperation in terms of self-governing the resource, Bardhan et al. (1993), Libecap (1994), Lam et al. (1998), Ostrom and Varughese (2001), Bardhan et al. (2002).

Example 2

Going back to the example 2. The equal sharing rule pungles up $\frac{v\left(N;\{N\}\right)}{9} = \frac{549}{9} = 61$ *to each individual. Thus there are incentives to form the grand coalition.*

3.2.4 Coalition Formation in CPRs through The payoff of Sharing Game

On the other hand, in the light of results of Chander (2019), the γ-core as a cooperative solution concept can be supported as an equilibrium outcome of the so-called *payoff sharing game*, which we introduce below. Also, the grand coalition is the unique equilibrium outcome if and only if the γ-core is non empty. This is another way of conceiving the formation of coalitions. Since we are interested in understanding this issue in the CPRs setting, we explore these results in relation to our problem.

3.2.4.1 Infinitely Repeated Games

The payoff sharing game is a game in two stages. It is played infinity. The stages are:

- First Stage
  - It departs from the non-cooperative *status quo*; i.e., the finest partition $[N]$,

and each player announces some nonnegative integer from 0 to n.

- Second State
  - All those players who announced the same positive integer in the first stage form a coalition and may either give effect or dissolve it. All those players who announced 0 remain singletons. That is, here the appropriators maximize their payoffs of their coalitions by choosing their optimal levels of extractions given the extractions of the others.
- If the outcome of the second stage is not the finest partition, the game ends and the partition formed remains forever. But if the outcome in the second stage is the finest partition, the two stages are repeated, until some nontrivial partition is formed in a future period. In either case, the outcome of the second stage is a partition in which players receive payoffs, in each period, in a proportion to a pre-specified feasible payoff vector $(z_i^*, \ldots, z_n^*)$

Suppose that the community of n individuals is interested in the preservation and in moderate extraction of the resource, so they have to meet in order to decide upon how to coordinate and who works with whom knowing in advance what their payoffs will be in each partition, those derived from the CPRs game. If the players agree upon forming a partition different from the finest one, the meeting ends. And they get payoffs according to a predetermined rule. Otherwise, the meeting lasts until a partition different from the finest one takes place. That is to say, the meeting comes off with participation through an agreement. Related to this, there are field cases where people meet with management and extract a CPR. As an example of this situation, the study of indigenous people in the state of Oaxaca, Mexico, where under the framework of *usos y costumbres* (customs and practices) program[8], they have regular meetings to deliberate responsibilities, charges, and duties regarding the extraction and management of their

[8] *Usos y costumbres* is a unique legally recognized program which enables its municipalities, the basic entity of its political-administrative division that possesses full autonomy through its own legislative and executive power, being ruled by traditional governance practices. This program coexists with formal institutions in certain municipalities with high indigenous populations.

resources. Then, they form work groups[9]. In this sense, the payoff sharing game is a useful framework to understanding processes of formation of coalitions, as in the example of the *usos y costumbres* program.

In this game, the specified payoff vector plays a significant role, since the players will anchor their strategies to this. A priori, any partition could be a possible outcome of the second stage. Chander (2019) proves specifically that as long as a partition function game is partially super-additive with nonempty γ-core, each payoff vector $(z_1^*, \ldots, z_n^*)$ that belongs to this core is actually an equilibrium payoff vector of the payoff sharing game in which payoffs are assigned in proportion to this vector. A partially super-additive partition function means that combining only all non-singleton coalitions in a partition increases their total worth. See the formal definition right below.

Definition 7 (*Partially Super-additive Partition Function*)

A partition function (N, v) *is partially super-additive if for any partition* $P = \left\{S, \left[N \backslash S\right]\right\}$ *and* $\{S_1, \ldots, S_k\}$ *such that* $\bigcup_{i=1}^k S_i = S,\ s_i > 1, i = 1, \ldots, k,\ \sum_{i=1}^k v\left(S_i; P'\right) \leqslant v\left(S; P\right)$*, where* $P' = P \backslash S \cup \{S_1, \ldots, S_k\}$.

In order to prove that γ-core payoff vectors can be equilibrium payoff vectors, the author shows that to dissolve a coalition if it does not include all players is an equilibrium strategy of each player, and that the grand coalition N is an equilibrium outcome resulting in per-period equilibrium payoffs equal to $(z_1^*, \ldots, z_n^n)$. Also, he characterizes the equilibrium of the repeated game by comparing each period payoffs of the players.

So, a natural question comes to my mind. What implications would entail for the players involved in a CPRs issue to play the payoff sharing game where the partition function is Equation (3.58)? First of all, this game is a way of incorporating a mechanism of communication and observation, since they have the possibly of forming, or not, a coalition in the second stage. Allowing for communication might improve results from group interaction, Ostrom et al. (1992). Second, under “round bargains" their efforts will be in favor of forming the grand coalition. And third, that the coalitions

[9] For instance, young women carry out activities different from those of young men, who typically do the hard work whereas others chose not to be part of it but to make up for it by paying a fine

different from it will not be stable in the sense that, it is not an equilibrium strategy for each player to materialize them. That said, we know that the partition function of the CPRs game is symmetric and that the grand coalition is the efficient partition, so its γ-core is nonempty. Next, we have to verify whether it is partially super-additive. See the next example, which in turn uses the same parameter values of the endowments, costs, number of players and the total revenue function that haven been used throughout this chapter.

Example 3

Say that $e = 25$, $c = 5$, $n = 9$,*the total revenue is given by* $f\left(\sum x_i\right) = 23\sum x_i - 0.25\left(\sum x_i\right)^2$. *Consider the following partition,*

$$P = \left\{\{1,2,3,4,5\},\{6\},\{7\},\{8\},\{9\}\right\}$$

Now say that $S = \{1,2,3,4,5\}$ *and that* $S_1 = \{1,2\}$ *and that* $S_2 = \{3,4,5\}$*, then* $S_1 \cup S_2 = S$ *and*

$$P' = \left\{\left\{P \backslash S\right\} \cup \{S_1, S_2\}\right\} \tag{3.68}$$

$$P' = \left\{\left[N \backslash S\right] \cup \left\{\{1,2\},\{3,4,5\}\right\}\right\}$$

$$P' = \left\{\{1,2\},\{3,4,5\},\{6\},\{7\},\{8\},\{9\}\right\}$$

The worth of coalitions S, S_1, S_2 *are the next,*

$$v\left(\{S\};P\right) = 161$$

$$v\left(\{S_1\};P'\right) \approx 76.44$$

$$v\left(\{S_2\};P'\right) \approx 101.44$$

then

$$v\left(\{S\};P\right) < v\left(\{S_1\};P'\right) + v\left(\{S_2\};P'\right).$$

This counterexample shows that the partition function of the CPRs game is not partially super additive. However, in the payoff sharing game when the number of members of a group involved in a CPRs issue is relatively small (three or four) they will end up grouping as the grand coalition is an equilibrium outcome. Moreover, as Chander (2019) shows, the grand coalition is the only equilibrium outcome if the players believe that the finest partition (every one single) is not a strategically relevant equilibrium outcome. Also when the game is played once, the grand coalition remains as an equilibrium outcome in the case of three players. See the following example.

Example 4

Say that $n = 3$, player i may consider a deviation of the grand coalition to the partition $P = \{\{i\}\{j,k\}\}$, which will be strategically relevant rather than the finest partition if the payoffs of the other two players are higher in partition P than in the finest one.

$$v\left(\{N\};\{N\}\right) = 3e + \frac{\alpha^2}{4b}$$

$$v\Big(\{i\};\{\{i\},\{j,k\}\}\Big) = e + \frac{\alpha^2}{9b}$$

$$v\Big(\{j,k\};\{\{i\},\{j,k\}\}\Big) = 2e + \frac{\alpha^2}{9b}$$

$$v\Big(\{i\};\{\{i\},\{j\},\{k\}\}\Big) = e + \frac{\alpha^2}{16b}$$

Under this structure, as the game is symmetric, then the feasible payoff vector with equal shares belongs to the γ-core of this game, then it can be the pre-specified payoff vector. Recall that payoffs are assigned in proportion to this vector. Thus,

$$z_i^* = z_j^* = z_k^* = e + \frac{\alpha^2}{12b},$$

and the individual payoffs if partition P is formed are $e+\frac{\alpha^2}{9b}$ for player i, and $e+\frac{\alpha^2}{18b}$ for players j and k. Players j and k have no incentives to deviate from the grand coalition towards coalition P, since they get better payoffs. In contrast, player i finds it attractive to move to partition P, but (s)he knows that for the others it is not. Then the equilibrium outcome is the grand coalition.

The above results contribute in favor of small group size as a facilitator of cooperation in the debate regarding the size of the group and cooperation in CPRs. Studies show that when the size of a group is relatively small, cooperation is easier to achieve

(Wilson and Thompson (1993), Franzen (1984), Fujiie et al. (2005)). In this matter, Olson (2017) explains why small group sizes promote cooperation with the help an illustrative example about meetings of people who have to make decisions on whatsoever issues involving a large number individuals:

> When the number of participants is large, the typical participant will know that his own efforts will probably not make much difference to the outcome, and that he will be affected by the meeting's decision in much the same way no matter how much or how little effort he puts into studying the issues. Accordingly, the typical participant may not take the trouble to study the issues as carefully as he would have if he had been able to make the decision by himself. The decisions of the meeting are thus public goods to the participants (and perhaps others), and the contribution that each participant will make toward achieving or improving these public goods will become smaller as the meeting becomes larger.

In this line, Isaac and Walker (1988) suggest that increasing group size of a group in a public good makes it difficult to sustain cooperation since larger groups tend to reduce the efficiency of allocation together with a lower marginal return from the public good. For other authors, however, it is not clear if larger groups prevent cooperation. Capraro and Barcelo (2015a) show experimentally that it will depend on the strategic situation. In the public good game, larger groups are cooperative, whereas in n-player prisoner's dilemma games, agents are less cooperative. Also, Capraro and Barcelo (2015b) conduct a large lab experiment of a class of the public good games finding that the effect of group size on cooperation is curvilinear. For which, they set three “sizes" of groups: smaller, intermediate and larger. Accordingly, intermediate group sizes are more cooperative than the other two sizes. In our case we have that when the size n is three the gran coalition is the only equilibrium outcome, but what about cases in which the number of involved players are more than this number? It it is not clear so far if an element of the γ-core will be an equilibrium payoff vector of the game, since the the function is not partially super additive. Nevertheless, Chander (2019) states that if a partition function fulfills the property that in any possible partition $P = \{S_1, S_2, \ldots, S_m\} \neq \{N\}$, $[N]$, $\sum_{j \in S_i} z_j \geqslant v(S_i; P)$ for at least one non-singleton coalition $S_i \in P$, then for symmetric partition function games the γ-core payoff vector with equal shares $(z^*, \ldots, z^*)$ is an equilibrium payoff vector of the repeated game in which payoffs are assigned in proportion to $(z^*, \ldots, z^*)$ even if the game is not partially super additive. Then we have that the partition function of CPRs game is not partially super-additive but symmetric, then the payoff vector with equal shares is an equilibrium payoff vector. We next study that such a result is true for our partition function under treatment. For which we test the strategy presented by Chander (2019) that *dissolving a coalition if it does not include all players* $(\circlearrowleft)$. So Proposition 3 shows that this strategy might be an equilibrium strategy of each appropriator when we consider the structure of the CPRs game, which implies that, under certain circumstances, the grand coalition N can be an equilibrium outcome resulting in per-period equilibrium payoffs equal to $(z^*, \ldots, z^*)$.

Proposition 3

Subject to particular conditions on the size of coalitions, the strategy $\circlearrowleft$ *is an equilibrium strategy of each appropriator playing the repeated payoff sharing game under the payoff structure of the CPRs game, which leads appropriators to form the gran coalition* N *resulting in per-period equilibrium payoffs equal to* $(z_1^*, \ldots, z_n^*)$ *when they are patient enough with respect future payoffs.*

Proof: See Appendix A Section 3.4.3 ■

Proposition 3 states that when appropriators consider their future payoffs to be of almost equal importance to their present time payoffs, the disintegration of one coalition will cause a domino effect in the sense that it will cause the other coalitions to disintegrate until the finest partition (all singletons) is reached as long as the size of the involved coalitions are bounded. Remember that the individual payoff in the finest partition will always be less than the payoff received in the grand coalition (the feasible payoff vector with equal shares) —e.g., $e + \frac{\alpha^2}{b(n+1)^2} < e + \frac{\alpha^2}{4bn} \iff n > 1$. This means that appropriators will stick with the strategy $\backepsilon$ if, besides considering the size of coalitions, the individual payoff received in the grand coalition is greater than even the payoff received in any other non-trivial partition in the repeated game, which is true as we assumed that the grand coalition is the efficient partition. Let us see this issue. Take the partition $P^m = \{S_1, S_2, \ldots, S_m\}$ with the assumption that $s_1 \leqslant s_2 \leqslant \cdots \leqslant s_m$. And recall that the equal payoff is $z_1^* = z_2^* =, \ldots z_n^* = z^* = e + \frac{\alpha^2}{4bn}$, and since the grand coalition is the efficient partition (inequality (3.59)) , so

$$v\left(N; \{N\}\right) \geqslant \sum_{S_i \in P^m} v\left(S_i; P^m\right)$$

where $v\left(N; \{N\}\right) = z_1^* + z_2^* + \cdots + z_n^* = nz^* = ne + \frac{\alpha^2}{4b}$, then

$$nz^* \geqslant \sum_{S_i \in P^m} v\left(S_i; P^m\right). \tag{3.69}$$

The left side inequality (3.69) can be written as $z^*\left(s_1 + s_2 + \cdots + s_m\right)$. Then it is easy to see that $s_i z^* \geqslant v\left(S_i; P^m\right)$, that is

$$z^* \geqslant \frac{v\left(S_i; P^m\right)}{s_i}$$

Now, as $s_1 \leqslant s_2 \leqslant \cdots \leqslant s_m$, then

$$\frac{v\left(S_m; P^m\right)}{s_m} \leqslant \frac{v\left(S_{m-1}; P^m\right)}{s_{m-1}} \leqslant \cdots \leqslant \frac{v\left(S_1; P^m\right)}{s_1} \leqslant z^*.$$

Then the feasible equal payoff stemmed from the gran coalition is greater or equal than the per-member payoff received in any other partition. Then, if in a partition $P^k = \{S_1, S_2, \ldots, S_k, [S_{k+1}], \ldots, [S_m]\}$, where $k = 1, 2, \ldots, m$ and the coalitions $k+1$ up m are broken apart into singletons, the non-singleton coalitions decided not to dissolve, each of its members would receive a payoffs that is lower than $z^*_{i \in S_i}$. However, if, as we have seen, they all dissolve, the game will be repeated and in the next period they will start with a discounted payoff of $u^*_{i \in S_i} = v\left(\{i\}; [N]\right)$, and this time they will decide to cooperate -if the conditions over their size allowed it- by announcing the same integer, since in this way they will guarantee to themselves the best possible payoff. Then in this case the feasible payoff derived from the formation of the grand coalition is an equilibrium payoff of the repeated game with payoffs derived from the CPRs game structure. The difference between this result here and that one of Parkash (2019) is that dissolving coalitions other than the gran coalition is an ex post optimal strategy under the conditions stated in Proposition 3. That is, this strategy is going to be ex post optimal when the size of the involved coalitions lies a certain interval values. Hence, the conditions on coalition sizes are crucial to determine whether a coalition will always prefer to dissolve given the dissolution of another, since it may happen that its payoffs in a coalition structure where some coalition disintegrated are higher than the payoffs it receives in the finest partition in the next period. Moreover, the role of the patience of appropriators is also crucial for the above to hold, since the analysis considered that appropriators give almost the same value to present and future payoffs.

Let us now see a case where, assuming patience enough appropriators, the dissolution of one coalition might have both effects on another coalition, to dissolve so that the finest partition is reached, the game is repeated and the grand coalition is formed, and not to dissolve so the partition structure they arrived at is that one where just one coalition is dissolved. We will see that a coalition will decide to disintegrate depending on the number of appropriating members that constitute it. Besides the above, we will study the effects the disintegration of a coalition has in terms of the per member payoffs of the other coalitions as well as the level of the extracted resourced. Then, suppose two partitions, the first one composed by two coalitions, whereas the second is formed out of the total disintegration of one of the coalitions of the first partition. Thus, $P = \{S_1, S_2\}$, with $S_2 > 1$, and $P' = \big\{S_1, [S_2]\big\}$, where coalition S_2 breaks apart into singletons. Then, $n = s_1 + s_2$ and $n \geqslant 3$. From Equation (3.64) and Equation (3.65), and under coalition structure $P = \{S_1, S_2\}$, the per-members extraction of S_1 is

$$x_{i\in S_1}\left(S_1;P\right)=\frac{\alpha}{3bs_1} \tag{3.70}$$

so the total level of extraction under coalition structure P is $X\left(P\right)=\frac{2\alpha}{3b}$, and the per-member payoff is

$$\frac{v\left(S_1;P\right)}{s_1}=e+\frac{\alpha^2}{9bs_1} \tag{3.71}$$

whereas under coalition structure P', the level of extraction of each appropriator in S_1 is

$$x_{i\in S_1}\left(S_1;P'\right)=\frac{\alpha}{b\left(p'+1\right)s_1}=\frac{\alpha}{b\left(s_2+2\right)s_1}$$

so the total level of extraction is $X\left(P'\right)=\frac{(s_2+1)\alpha}{(s_2+2)b}$, and the respective per-member payoff is

$$\frac{v\left(S_1;P'\right)}{s_1}=e+\frac{\alpha^2}{b\left(s_2+2\right)^2 s_1}$$

The fact that all the members of the coalition S_2 decide to separate has the following effects. First, individually each member in S_1 would get a smaller share of the common resource as[10]

$$x_{i\in S_1}\left(S_1;P\right)-x_{i\in S_1}\left(S_1;P'\right)=\frac{\alpha}{bs_1}\left[\frac{1}{3}-\frac{1}{s_2+2}\right]>0.$$

However, this does not mean that the level of extracted resource as a whole decreases, but rather that it increases because more resource is taken by the singletons ex members of S_2:

$$X\left(P'\right)>X\left(P\right)=\frac{\alpha}{b}\left[\frac{s_2+1}{s_2+2}-\frac{2}{3}\right]>0.$$

Next, as it can be seen below, the disintegration of coalition S_2 also causes the individual payoffs of the members of coalition S_1 to be reduced,

[10]To better understand the effects of the dissolution of S_2 we compare the amounts of appropriate resource of each situation. Yet, when we are in a cooperative game theory framework, we try to understand cooperation from the gains that each appropriator achieves by forming a coalition with other appropriators, so what amount of resource each one ends up with would not be the first target when we use this theory. It is true, however, that when each coalition acts as a single individual that chooses the amount of resource to extract, cooperation will occur when precisely the amount extracted is such that they obtain the best possible gains.

$$\frac{v\left(S_1;P'\right)}{s_1} \leqslant \frac{v\left(S_1;P\right)}{s_1} \iff 9 \leqslant \left(s_2+2\right)^2$$

Also, notice that $\frac{d}{ds_2}\left(\frac{v\left(S_1;P'\right)}{s_1}\right) < 0$ and that $\frac{d}{ds_2}\left(x_{i \in S_2}\left(S_1;P'\right)\right) < 0$. The larger the number of members of the coalition that disintegrates completely (S_2), the greater the negative effect it has on the payoff of coalition (S_1) members. So the fact that one coalition disintegrates somehow would discourage the other coalitions from doing the same thing due to the decrease on of their payoffs. As coalition S_2 comes apart, the individual payoff received by the members of coalition S_1 is reduced. Parkash draws attention to this issue. Coalition members might have incentives to disintegrate if, by doing so, they can deter other coalitions to dissolve. This is especially true in the case of the environmental game he treats, since the disintegration of a coalition causes a negative externality on other coalitions. That is to say, Chander (2018b) shows that in the context of an environmental game where agents decide the level of emissions of greenhouse gases (GHGs) they want to emit, it is possible for coalitions to develop incentives to disintegrate with the aim of deterring other coalitions from leaving the grand coalition. And this is not unlike our CPRs game. As we have seen throughout the chapter, appropriating, by definition, generates a negative externality on the members of the community. Foresighted coalitions may have incentives to disintegrate if they can thereby prevent other coalitions from turning away from the grand coalition.

On the other hand, if coalition S_1 also disintegrates, the new coalition structure will be $P'' = \left\{\left[S_1\right], \left[S_2\right]\right\}$, and in the next period the payoff of appropriators in S_1 will be:

$$u_{i \in S_1}\left(P''\right) = e + \frac{\alpha^2}{b\left(s_1+s_2+1\right)^2} = e + \frac{\alpha^2}{b\left(n+1\right)^2}.$$

Then, for coalition S_1 to disintegrate it is necessary that the following inequality be satisfied,

$$u_{i \in S_1}\left(P'\right) \leqslant \delta u_{i \in S_1}\left(P''\right) \tag{3.72}$$

where $P'' = \left\{\left[S_1\right], \left[S_2\right]\right\} = \{N\}$, $P' = \{S_1, \left[S_2\right]\}$; $u_{i \in S_1}\left(P'\right) = e + \frac{\alpha^2}{b\left(s_2+2\right)^2 s_1} =$

$e + \frac{\alpha^2}{b(n-s_1+2)^2 s_1}$, and $u_{i \in S_1}(P'') = e + \frac{\alpha^2}{b(n+1)^2}$.

Assuming $\delta \to 1$, the disintegration of coalition S_2 is a strategic disintegration if by doing so induces coalition S_1 to dissolve, which subsequently would lead appropriators to reach the finest coalition structure $[N]$. Then, the game will be repeated, and consequently the formation of the grand coalition will take place. However, although in most cases this is true, there are some exceptions in which the disintegration of coalition two has no effect on coalition one to disintegrate, as we will see in the following proposition.

Proposition 4 (*The Induced Dissolution of S_1 by S_2*)

When appropriators are patience enough, the dissolution of S_2 will induce the members of S_1 to break apart into singletons if the community size is $3 \leqslant n \leqslant 11$. For a community of $n > 11$ user members, the dissolution of S_2 will induce the members of S_1 to disband if and only if $1 \leqslant s_1 \leqslant \frac{1}{2}\left[2n - \sqrt{4n+5} + 3\right] \leqslant (n-2)$.

Proof: See Appendix A Section 3.4.4 ■

In contrast to the game of pollutant emissions Parkash presents, here given the disintegration of S_2, the disbandment of S_1 is not imminent. Proposition 4 tells that when the number of appropriators with access to the common resource ranges from 3 to 11, coalition S_1 wants to break apart into singletons. However, when the number of appropriators in the community is greater than 11, things change slightly. In this case, whether coalition S_1 responds to the dissolution of S_2 by dissolving itself will depend substantially on the number of appropriators within S_1. This proposition tells us that there is a threshold number of members of S_1 that allows it to dissolve in response to the dissolution S_2 when $n > 11$. In other words, the proposition states that for $n \in [3, 11]$, the dissolution of the coalition is the only alternative to improve the payoff of its members and that this decision is not affected if S_1's size grows and as long as n lies on the aforementioned mentioned interval, and of course $s_1 \in [1, n-2]$. As for the case $n > 11$, we will see that the action of disbanding on the part of S_1 is extinguished if the size of the coalition grows and exceeds a certain point. For example, if $n = 15$, we know that, at first, $s_1 \in [1, 13]$, but according to the results of the proposition we have that if $s_1 = 13$, members of S_1 stay in.

3.2.4.2 The Stability of the Grand Coalition under the CPR structure

Next, we test now the stability of the grand coalition of the payoff sharing game played among the appropriators within the context of CPRs. Chander (2018b) defines a coalition structure $P = \{S_1, S_2, \ldots, S_m\}$ as stable *if no player or group of players can strictly improve its final payoff by leaving a coalition in the coalition structure*. So we adopt this definition.

Proposition 5 (***The Stability of the Grand Coalition of the CPRs game***)

The gran coalition of the Common-Pool Resources game is stable.

Proof : See Appendix A Section 3.4.5 ■

3.3 Summary

In this chapter we drew from a CPRs strategic game [Ostrom et al. (1994)] so as to study a cooperative version of it. Based on the awareness of cooperation that some individuals may have and under the assumptions of the model, we studied conditions in which forming a group of cooperative members may be advantageous to them. We show three scenarios in which this could be possible. We observe that the decision of a cooperative agent might affect the decision extraction of a non-cooperative member. That is, given that the cooperative members are somehow foregoing extraction of resource units, the non-cooperative members might want to increase their extractions.

In the last part of the chapter, we transformed the original CPRs game into a partition function game. Since we studied the formation of coalition structures and the gains of cooperation, we started out by applying some recent results regarding strategic games in partition form. So far, we noticed that given the symmetry of the original game, its partition function version is symmetric and the grand coalition is an efficient partition in the sense that it maximizes the total payoff of all players. These two properties are fundamental, since they guarantee that the so called γ-core is not empty, and an element of it is the equal payoff sharing. Also we found that partitions different from the grand coalition partition will not be stable, so the efforts of the players move towards full cooperation. In addition to that, we studied a game of two stages for formation of coalition structures called the payoff sharing game in relation to the γ-core of the CPRs game. When appropriators play this game, an equilibrium outcome of the payoff sharing game is the grand coalition if appropriators play the strategy of dissolving a coalition if it is not the grand coalition. However, we observed specific cases in which, depending on the size of the community and the size of the coalitions, this strategy does not result in the formation of the grand coalition. Despite this, the grand coalition is always stable. Once reached, players have no incentive to deviate from it.

Hence, taking into consideration all the above, the cooperative approach (the solution concept) and the game of coalition formation both proposed by (Chander (2019)) have proven to be effective in explaining cooperative outcomes in our CPRs problem, therefore achieving, albeit with some limitations, a way of reconciling theory and observed and well studied praxis in CPRs. That is, by passing from a strategic environment to a game in a partition function form, explaining cooperation here is somehow possible. Nonetheless, the theory, to the extend it was applied, allows only for full cooperation.

Finally some caveats are worth mentioning. The results obtained hinge highly on the symmetry assumption on the players. More accurate explanations of partial cooperation might be arrived at by abandoning that assumption. Concomitantly, notice that a solution to a cooperative game like the core, continues to rely upon the notion of pure self-interest and competitive individuals. This motivates the search for more complex frameworks that capture formation of groups in which the community is made up of heterogeneous agents. Of course, agents may be self-interested, selfish, or act as utilitarians, but they may be morally driven as well. How does the introduction of this form of heterogeneity explain cooperation in CPRs scenarios? This will be the object of study for the next chapter.

3.4 Appendix A

3.4.1 Proof of Proposition 1: Members of Larger Coalitions Get Lower Payoffs

Proof: Consider the situation represented in Section 3.2.3.3, where m coalitions in partition $P = \{S_i, \ldots, S_m\} \neq N$ with S_i and S_j such that $s_i = s_j$, $i, j \in 1, 2, \ldots, m$ playing the CPRs game. Then, the worth of S_i and S_j is the same as stated in Equation (3.67). Next, assume that those coalitions allocate a share of their respective generated worth to their members, so equal sharing rule comes out naturally, and the per-members payoff in each one of these coalitions are

$$\frac{v(S_i; P)}{s_i} = \frac{v(S_j; P)}{s_j} = e + \frac{\alpha^2}{b(m+1)^2 s_i} \tag{3.73}$$

Now, if $s_i < s_j$, then

$$\frac{v(S_i; P)}{s_i} = e + \frac{\alpha^2}{b(m+1)^2 s_i} > e + \frac{\alpha^2}{b(m+1)^2 s_j} = \frac{v(S_j; P)}{s_j} \tag{3.74}$$

Thus, the members of larger coalitions in any given partition different from the grand coalition get individually lower payoffs. Moreover, from inequality (3.74) we can notice that as the number of members of a coalition increases, the lower the individual payoff they can ensure for themselves. ■

3.4.2 Proof of Proposition 2: The Belonging of the Equal Share Payoffs to the γ-Core

Proof: This proof consists of two parts. On one hand we prove that the feasible payoff vector with equal shares,$(z_1, \ldots, z_n)$, belongs to the γ-core of (N, v), where v is given by Equation (3.58) from Section 3.2.3.1, and it is the following

$$v(S_i; P) = \max_{x_{S_i}} \left\{ \sum_{j \in S_i} u_j \left(x_{S_i}, x_{S_{-i}} \right) \right\} \tag{3.75}$$

On the other hand, we show that the largest coalition in each partition is worse-off relative to $(z_1, \ldots, z_n)$.

First Part: The Feasible Payoff Vector with Equal Shares Belongs to the γ-core of the CPRs Game

Then, the feasible payoff vector with equal shares is $(z_i, \ldots, z_n)$, so from Equation (3.60):

$$\sum_{i \in N} z_i = v(N; \{N\}) = ne + \frac{\alpha^2}{4b} \tag{3.76}$$

I check that this payoff belongs to the γ-core of this game. Which is the same as verifying

$$\sum_{i\in S} z_i \geqslant v\left(S; \left\{S, [N\backslash S]\right\}\right) \forall S \subset N. \tag{3.77}$$

The worth of a coalition S in the CPR game induced by the partition $\left\{S, [N\backslash S]\right\}$ is

$$v\left(S; \left\{S, [N\backslash S]\right\}\right) = \max_{0\leqslant x_S} \left\{\sum u_{i\in S}\left(x_S, x_{i\notin S}\right) = se + \alpha x_{S_i} - x_{S_i} b\left[\sum x_{S_i}\right]\right\} \tag{3.78}$$

So, S solves

$$\begin{aligned} \underset{x_S}{\text{maximize}} \quad & se + \alpha x_S - x_S b\left(\sum x_{i\in N}\right) \\ \text{subject to} \quad & 0 \leqslant x_S \end{aligned}$$

$$\text{f.o.c: } \alpha - x_S b - \left(\sum x_{i\in N}\right) b = 0 \tag{3.79}$$

whereas a non-cooperative appropriator $i \notin S$ will solve

$$\begin{aligned} \underset{x_{j\in N\backslash S}}{\text{maximize}} \quad & e + \alpha x_{j\notin S} - x_{j\notin N\backslash S} b\left[\sum x_{i\in N}\right] \\ \text{subject to} \quad & 0 \leqslant x_{j\notin S} \end{aligned}$$

$$\text{f.o.c: } \alpha - x_{i\notin S} b - \left(\sum x_{i\in N}\right) b = 0 \tag{3.80}$$

Equation (3.79) and Equation (3.80) together define a two-equation system. The former gives the best response of the coalition to $\sum x_{i\notin S}$ whereas the second equation gives the best response of j to $\sum x_{i\in S} + \sum x_{j\in N-S-\{j\}}$. Now, given the symmetry of players in terms of the strategies, we assume and selected $\tilde{x}$ as a symmetric Nash equilibrium, so the total amount of resource then at this equilibrium is

$$\sum x_{i\in N} = x_S + \sum x_{i\notin S} = (n - s + 1)\,\tilde{x} \tag{3.81}$$

That is, $\tilde{x}$ is the Nash equilibrium level of CPRs game between the appropriating coalition S and $n - s$ singleton appropriating coalitions. Then, we plug Equation (3.81) into both Equation (3.79) and Equation (3.80) and solve the system, so

$$\tilde{x} = \frac{\alpha}{b\,(n - s + 2)} \tag{3.82}$$

Then, the worth of coalition S at $\tilde{x}$ is

$$\begin{aligned} v\left(S; \left\{S, [N\backslash S]\right\}\right) &= se + \alpha\tilde{x} - \tilde{x}^2\,(n - s + 1)\, b = \\ & se + \alpha\left[\frac{\alpha}{b\,(n - s + 2)}\right] - \left[\frac{\alpha}{b\,(n - s + 2)}\right]^2 [n - s + 1]\, b = \\ & se + \frac{\alpha^2}{b\,(n - s + 2)^2}. \end{aligned} \tag{3.83}$$

Then we obtain that $\sum_{i \in S} z_i \geqslant se + \frac{\alpha^2}{b(n-s+2)^2}$, namely

$$se + \frac{s\alpha^2}{4bn} \geqslant se + \frac{\alpha^2}{b(n-s+2)^2}. \tag{3.84}$$

Rearranging inequality (3.84),

$$(n-s+2)^2 s \geqslant 4n \ \ \forall\, s \in \{1, \ldots, n-1\} \tag{3.85}$$

One can readily notice by substitution that for $s = 1$ and $s = n-1$ the inequality (3.85) holds:

i If $s = 1$, then Inequality (3.85) becomes $(n-1)^2 \geqslant 0$, which obviously holds true.

ii If $s = n-1$, then Inequality (3.85) holds $\iff n \geqslant \frac{9}{5}$, which again is true because of part *(i)*.

Next, in view of parts (i) and (ii), what remains to prove is that $(n-s+2)^2 s \geqslant 4n$ is true for $s \in \{2, \ldots, n-2\}$ and $n \geqslant 3$. One can immediately see that inequality (3.85) holds for $s = 2$ and for $s = n-2$. Later, we define the following continuous function:

$$f(s_r) := (n - s_r + 2)^2 s_r, \tag{3.86}$$

where n is any given integer and $s_r \in \mathbb{R}$, with $s_r \in [2, n-2]$. It can easily be checked that $f'(s)_{s=2} \geqslant 0$ (equality only holding for $n = 4$), whereas $f'(s)_{s=n-2} \leqslant 0 \iff n \geqslant 4$ (equality only holding for $n = 4$). By the same token, one can also readily see that, if $f'(s) \leqslant 0$ (some $s < n-1$ then $f'(s') < 0$ for some $s' = s+1$); therefore $f(s)$ is quasi-concave. Thus, we are done for $n \geqslant 4$. In Figure 3.3 we observe that the curve $f(s_r)$ lies above the horizontal line $4n$, for any $s_r \in [2, n-2]$.

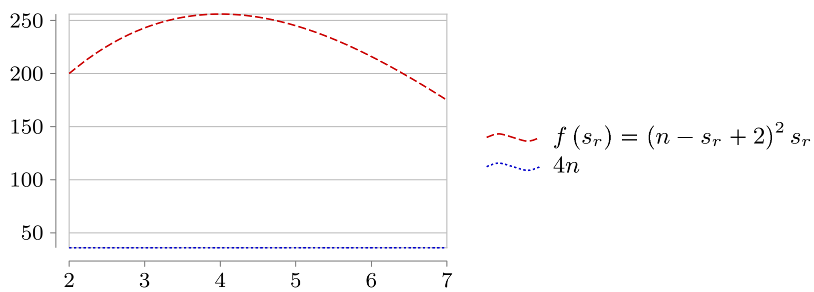

Figure 3.3: *Graphic representation of* $4n \leqslant f(s_r)$ *for any given* $n \in \mathbb{Z}_+$, *and* s_r *over* $[2, 2n]$

Second Part: The largest Coalition in each Partition is Worse-off Relative to $(z_1, \ldots, z_n)$

Next, we verify that in the partition function of the CPRs game the largest coalition in each partition is worse-off with respect to the feasible payoff vector with equal shares if the efficient partition is the grand coalition. Now, given that $z_i = z_j$, $i, j \in N$, suppose $P = \{S_1, \ldots, S_m\}$ a partition of N different from both the trivial coalition $[N]$ and the grand coalition $\{N\}$. Then, it is sufficient to prove that assuming $s_m \leqslant s_{m-1} \dots s_2 \leqslant s_1$, inequality (3.87) is true.

$$v\left(S_1; P\right) < \sum_{i \in S_1} z_i. \tag{3.87}$$

We know that $\sum_{i \in N} z_i = ne + \frac{\alpha^2}{4b}$, and since $(z_1, \dots, z_n)$ is the feasible vector with equal shares, we have the next expression:

$$\sum_{i \in S_1} z_i = s_1 e + \frac{\alpha^2 s_1}{4bn}, \tag{3.88}$$

and from Equation (3.66), $v\left(S_1; P\right)$ is given by

$$s_1 e + \frac{\alpha^2}{b(m+1)^2}. \tag{3.89}$$

Then, inequality (3.87) becomes:

$$s_1 e + \frac{\alpha^2}{b\left(m+1\right)^2} < s_1 e + \frac{\alpha^2 s_1}{4bn}, \tag{3.90}$$

simplifying

$$\frac{s_1}{n} - \frac{4}{\left(m+1\right)^2} > 0. \tag{3.91}$$

We know that S_1 is the largest coalition in any partition P that is neither the grand coalition nor the finest partition, we also know that if we assume that

$$s_m \leqslant s_{m-1} \dots s_2 \leqslant s_1, \tag{3.92}$$

then the number of coalitions in the partition will be such that $2 \leqslant m < n$. Thus, for any value of n and m satisfying inequality (3.92), the size of S_1 that makes it the largest coalition does necessarily lie in the following interval:

$$\left[\frac{n}{m}, n - m + 1\right],$$

where in the case that $\frac{n}{m}$ is a decimal number, we will assume w.l.o.g that s_1 takes the nearest largest integer number. Let us denote the left part of inequality (3.91) by $Q\left(S_1\right)$. Therefore, we are going to prove by induction that $Q\left(s_1\right) > 0$ holds for any s_1 such that $\frac{n}{m} \leqslant s_1 \leqslant n - m + 1$. That said, we begin by showing that is true for the endpoints of the interval. First, when $s_1 = \frac{n}{m}$,

$$Q\left(s_1\right)_{s_1 = \frac{n}{m}} = \left(\frac{m-1}{m+1}\right)^2 \frac{1}{m}, \tag{3.93}$$

And $Q(s_1)_{s_1=\frac{n}{m}} > 0$ is true since $2 \leqslant m$. Further, we can define a continuous function $f(m_r) = \left(\frac{m_r-1}{m_r+1}\right)^2 \frac{1}{m_r}$, where m_r is a real number, with $2 \leqslant m_r \leqslant n-1$, and $n \in \mathbb{Z}_+$, so we can check graphically that although $f(m_r)$ is decreasing in m_r, it is is non-negative. See Figure 3.4.

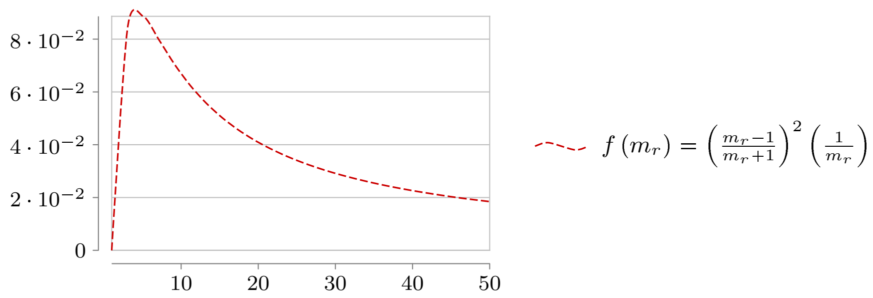

Figure 3.4: *Graphic representation of* $f(m_r)$, $m_r \in [2, n)$

Next, we now prove that $Q(s_1) > 0$ is true for $n = n - m + 1$, so

$$Q(s_1)_{s_1=n-m+1} = [m-1]\left[\frac{m+3}{(m+1)^2} - \frac{1}{n}\right]. \tag{3.94}$$

Equation (3.94) is non-negative as we see that $\lim_{m \to (n-1)} Q(s_1)_{s_1=n-m+1} = \frac{2(n-2)}{n^2} > 0$.

Now, assume that $Q(s_1)$ holds for some number $\frac{n}{m} + 1$. Then, the inequality in this case reads as follows

$$Q(s_1)_{s_1=\frac{n+m}{m}} = \frac{n+m}{nm} - \frac{4}{(m+1)^2} > 0. \tag{3.95}$$

We need to show now that $Q(s_1) > 0$ holds for some number $\frac{n+m}{m} + 1$. We compute $Q(s_1)_{s_1=\frac{n+2m}{m}}$:

$$Q(s_1)_{s_1=\frac{n+2m}{m}} = \frac{n+2m}{nm} - \frac{4}{(m+1)^2}, \tag{3.96}$$

which can be written as $\left(\frac{n+m}{m} + 1\right)\left(\frac{1}{n}\right) - \frac{4}{(m+1)^2}$, or more conveniently as

$$\frac{n+m}{nm} - \frac{4}{(m+1)^2} + \frac{1}{n}, \tag{3.97}$$

combined with inequality Inequality (3.95), this expression gives

$$Q(s_1)_{s_1=\frac{n+m}{m}} + \frac{1}{n}. \tag{3.98}$$

The second part of the above term is greater than zero as $n \geqslant 3$, whereas the first one is true by induction hypothesis. It follows that $Q(s_1)_{s_1=\frac{n+2m}{m}} > 0$ holds. Therefore, we conclude that $Q(s_1) > 0$ is true for all s_1 such that $\frac{n}{m} \leqslant s_1 \leqslant n - m + 1$. ∎

3.4.3 Proof of Proposition 3

Proof: We resort to Chander (2018b) procedure, who studies the implications of this repeated payoff sharing game in the context of an environmental game of carbon emissions. In our case, we study the implications of this game in the context of the CPRs. Let $\{S_1, \ldots, S_m\} \neq \{N\}$ some coalition structure. W.o.l.g. assume that

$$s_1 \leqslant s_2 \leqslant \cdots \leqslant s_m$$

Now, we consider a finite sequence of coalition structures gotten if the largest partition in each subsequent coalition structure disband into single coalitions, starting from the largest coalition $S_m \in P^m$, so

$$\begin{aligned} P^m &\equiv \{S_1, \ldots, S_m\} \\ P^{m-1} &\equiv \{S_1, \ldots, S_{m-1}, [S_m]\} \\ P^{m-2} &\equiv \{S_1, \ldots, S_{m-2}, [S_{m-1}], [S_m]\} \\ P^{m-3} &\equiv \{S_1, \ldots, S_{m-3}, [S_{m-2}], [S_{m-1}], [S_m]\} \\ &\vdots \\ P^1 &\equiv \{S_1, [S_2], \ldots, [S_m]\} \\ P^0 &\equiv \{[S_1], [S_2], \ldots, [S_m]\} = [N] \end{aligned} \tag{3.99}$$

Suppose that we are in $P^1 \equiv \{S_1, [S_2], \ldots, [S_m]\} = \left\{S_1, \left[N \backslash S_1\right]\right\}$.

So now the appropriating members of coalition S_1 have to made a decision on either giving effect to in or dissolving it. Assume that they do not follow the strategy of dissolving a coalition if it is not the grand coalition when the rest of the players have announced 0. Then, the outcome of the second stage is effectively P^1, then the worth of coalition S_1 is $v\left(S_1; P^1\right) = s_1 e + \frac{\alpha^2}{b(n-s_1+2)^2}$, and the payoff of appropriator $i \in S_1$ will be:

$$\frac{v\left(S_1; P^1\right)}{s_1} = e + \frac{\alpha^2}{b\left(n - s_1 + 2\right)^2 s_1} \tag{3.100}$$

If they break up their S_1, the outcome of the second stage will be the finest partition, the two-stages are repeated and the players in S_1 will receive the Nash equilibrium extraction payoff when players are singleton (see Equation (3.9) and (Equation (3.9))). That is,

$$u^*_{i \in S_1} = \left(x_i^*, x_{-i}^*\right) = v\left(\{i\}; [N]\right) = e + \frac{\alpha^2}{b\left(n+1\right)^2}. \tag{3.101}$$

In order to compare Equation (3.101) and Equation (3.100) we need to bring $u^*_{i \in S_1}$ into present value since it will be effective the following period. Let be $\delta \in (0, 1)$ the discount factor, and suppose for a moment that the appropriators are patient enough, so they attach almost the same importance to future payoffs as present payoffs, and δ is close to one. Under this case, we have that

$$\frac{v\left(S_1; P^1\right)}{s_1} < \delta u^{*}_{i \in S_1}, \; i \in S_1 \tag{3.102}$$

and dissolving the coalition S_1 will be ex post optimal. To see this, we need to show that $e + \frac{\alpha^2}{b(n-s_1+2)^2 s_1} \leqslant e + \frac{\alpha^2}{b(n+1)^2}$ holds, so we re-write this inequality as follows:

$$(n - s_1 + 2)^2 \, s_1 \geqslant (n+1)^2 \, . \tag{3.103}$$

Next, as in the proof of Proposition 2, define the following continuous function: $f\left(s_{1_r}\right) := \left(n - s_{1_r} + 2\right)^2 s_{1_r} - (n+1)^2$, where $n \in \mathbb{Z}_+$ and s_{1_r} lives in an interval of $\mathbb{R}$. Specifically, since S_1 is the smallest partition in P^m and becomes the largest coalition in P^1, then its cardinality is bounded, so we can assume that $s_{r_1} \in \left[1, \frac{n}{m}\right]$[11] for any given $m < n$. Then, $f\left(s_{1_r}\right) \geqslant 0 \iff 1 \leqslant s_{1_r} \leqslant \frac{1}{2}\left(2n - \sqrt{4n+5} + 3\right)$ and $n > 1$. Look at Figure 3.5 for an example. This means that necessarily $\frac{n}{m} < \frac{1}{2}\left(2n - \sqrt{4n+5} + 3\right)$. We rearrange this inequality, so $m > \frac{2n}{2n - \sqrt{4n+5} + 3}$, which is true since the limit of the right side as $n \to \infty$ is 1.

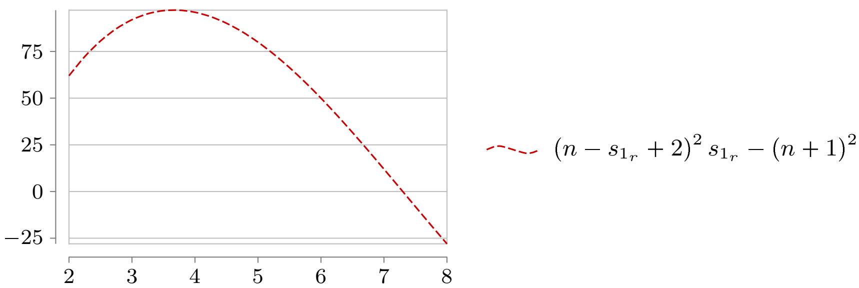

Figure 3.5: *A Graphic representation of* $\left(n - s_{1_r} + 2\right)^2 s_{1_r} - (n+1)^2$ *for* $n \in \mathbb{Z}_+$, $m < n$, *and* s_r *over* $\left[1, \frac{n}{m}\right]$

Having seen that $f\left(s_{1_r}\right) \geqslant 0$ holds, then inequality (3.102) is true as long as $\delta \to 1$. Then, dissolving S_1 is ex post optimal. Next, let us move to the case of P^2,

$$P^2 \equiv \{S_1, S_2, [S_3], \ldots, [S_m]\} = \left\{S_1, S_2, \left[N \backslash \{S_1 \cup S_2\}\right]\right\}$$

In this case, we have four possible options regarding the course of individuals in each coalitions. That is, once arrived a P^2 they decide whether or not to form their respective coalition. We proceed to analyze the following decision scenarios.

- Both coalitions, S_1 and S_2, decide to dissolve.
- One coalition dissolves while the other does not.

[11] Notice, however, that if $s_1 = 1$, $P^1 \equiv P^0$.

- No coalition is dissolved.

Firstly, if appropriating members of both coalitions, S_1 and S_2 decide to disintegrate, the game is repeated since the outcome of the second stage is the finest partition $\{[N]\}$, and each member in the community will start the next period with the following payoff:

$$v\left(\{i\};[N]\right) = e + \frac{\alpha^2}{b\,(n+1)^2} \tag{3.104}$$

Second, if appropriating members in $S_2\,(S_1)$ follow the strategy $\circlearrowleft$ while appropriating members in $S_1\,(S_2)$ do not, the resulting partition is $\left\{S_{1(2)},\left[N\backslash S_{1(2)}\right]\right\}$, but then, if it is the case, $S_1(S_2)$ will dissolve their coalitions as we just saw above. Again, the resulting partition is the finest one, the game is repeated, and the individual payoffs will be as mentioned above. Third, if none of the above coalitions fall apart, we have the following per member payoffs of S_1 and S_2 respectively,

$$\frac{v\left(S_1;P^2\right)}{s_1} = e + \frac{\alpha^2}{b\,(n-s_1-s_2+3)^2\,s_1}$$

$$\frac{v\left(S_1;P^2\right)}{s_2} = e + \frac{\alpha^2}{b\,(n-s_1-s_2+3)^2\,s_2}$$

Clearly, since $s_1 \leqslant s_2$, (Proposition 1), $\frac{v\left(S_1;P^2\right)}{s_2} \leqslant \frac{v\left(S_1;P^2\right)}{s_1}$. Next, we verify that $\frac{v\left(S_1;P^2\right)}{s_1} \leqslant e + \frac{\alpha^2}{b(n+1)^2}$ holds, that is $e + \frac{\alpha^2}{b(n-s_1-s_2+3)^2 s_1} \leqslant e + \frac{\alpha^2}{b(n+1)^2}$, or reduced:

$$(n+1)^2 \leqslant (n-s_1-s_2+3)^2\,s_1 \tag{3.105}$$

$$f\left(s_{1_r},s_{2_r}\right) := \left(n-s_{1_r}-s_{2_r}+3\right)^2 s_{1_r} - (n+1)^2 \geqslant 0 \tag{3.106}$$

where $1 \leqslant s_{1_r} \leqslant s_{2_r},\ s_{2_r} < n - \sum_{i_r=3}^{m} s_{i_r},\ n > 0,$
$n > \sum_{i_r=3}^{m} s_{i_r},\ \sum_{i_r=3}^{m} s_{i_r} \geqslant s_{1_r} + s_{2_r},\ \sum_{i_r=1}^{m} s_{i_r} = n$. Then the solution to this system is,

$$n > 4,\ \frac{4\,(n+1)^2}{(n+6)^2} \leqslant s_{1_r} < \frac{n}{4},\ s_{1_r} \leqslant s_{2_r} \leqslant \frac{1}{2}\left(n-2s_{1_r}\right),\ n = \sum_{i_r=1}^{m} s_{i_r}$$

Under these conditions over the size of S_1 and S_2, inequality (3.105) will hold. Then for values of δ close to one, the following inequality is true,

$$\frac{v\left(S_1;P^2\right)}{s_1} \leqslant \delta v\left(\{i\};[N]\right). \tag{3.107}$$

Then the appropriators in the coalitions belong to P^2 will end up breaking their coalitions apart, and the game will be repeated, and they will start the next period with a payoff of $v\left(\{i\};[N]\right)$.

In general, we have the partition

$$P^k = \{S_1, S_2, \ldots, S_k, [S_{k+1}], \ldots, [S_m]\}$$
$$k = 1, 2, \ldots m.$$

If coalition P^k stands as it is (none of the non-singleton coalitions is broken up), the individual payoffs of members of coalition S_k will be as follows:

$$\frac{v\left(S_k; P^k\right)}{s_k} = s_k e + \frac{\alpha^2}{b\left(p^k+1\right)^2 s_k}$$

where $p^k = n - \sum_{i=1}^{k} s_i + k = \sum_{i=k+1}^{m} s_i + k$. Since coalition S_1 here is the smallest coalition in P^k, it means that the payoffs that each of its members will receive under this partition will be larger than the per member payoffs that the larger coalitions will receive. We then proceed to compare this payoff with the payoff that would be obtained if all the coalitions were to break up (the next period payoff):

$$\frac{v\left(S_1; P^k\right)}{s_1} \leqslant \delta v\left(\{i\}; [N]\right) \tag{3.108}$$

Again, let us assume that δ is sufficiently high —e.g., $\delta \to \infty$. Then we have that $e + \frac{\alpha^2}{b\left(\sum_{i=k+1}^{m} s_i + k + 1\right)^2 s_1} \leqslant e + \frac{\alpha^2}{b(n+1)^2}$, rearranging,

$$(n+1)^2 \leqslant \left(\sum_{i=k+1}^{m} s_i + k + 1\right)^2 s_1 \tag{3.109}$$

Name $\sum_{i=k+1}^{m} s_i := \sigma$, then we need to prove that

$$(n+1)^2 \leqslant (\sigma + k + 1)^2 s_1$$

Define $f(\sigma_r, s_{1_r}) := (\sigma_r + k + 1)^2 s_{1_r} - (n+1)^2$, where $s_{1_r} < \sigma_r < n,\ 1 \leqslant k \leqslant m,\ 2 \leqslant m < n,\ 1 < \sigma_r + s_{1_r} < n$. Hence, under this constraints,

$$f(\sigma_r, s_{1_r}) \geqslant 0 \iff$$

$$\frac{1}{3\sqrt[3]{2}}\left\{2k^2(k+3) + \sqrt{-4(k+1)^6 + \left[2(k+1)^3 + 27(n+1)^2\right]^2} + 6k + 27(n+1)^2 + 2\right\}^{\frac{1}{3}} - \frac{-\sqrt[3]{2}(k+1)^6}{3\left\{2k^2(k+3) + \sqrt{-4(k+1)^6 + \left[2(k+1)^3 + 27(n+1)^2\right]^2} + 6k + 27(n+1)^2 + 2\right\}^{\frac{1}{3}}}$$
$$-\frac{2}{3}(k+1) < \sigma_r < \frac{n}{2},$$
$$\frac{(n+1)^2}{(k+\sigma_r+1)^2} \leqslant s_{1_r} < \sigma_r$$

When the above conditions over the size of coalitions are met, inequality (3.109) will check. Then, for values of δ close to 1 (the appropriators are patient, so they attach almost the same importance to future payoffs as present payoffs), inequality (3.108) holds. ∎

3.4.4 Proof of Proposition 4: The Induced Dissolution of S_1 by S_2

Proof: We need to prove

$$u_{i \in S_1}\left(P'\right) \leqslant \delta u_{i \in S_1}\left(P''\right) \tag{3.110}$$

where $P'' = \{[S_1], [S_2]\} = \{N\}$, $P' = \{S_1, [S_2]\}$; $u_{i \in S_1}\left(P'\right) = e + \frac{\alpha^2}{b(s_2+2)^2 s_1} = e + \frac{\alpha^2}{b(n-s_1+2)^2 s_1}$, and $u_{i \in S_1}\left(P''\right) = e + \frac{\alpha^2}{b(n+1)^2}$, and $\delta \to 1$. Simplifying inequality (3.110)

$$(n+1)^2 \leqslant (n-s_1+2)^2 s_1 \tag{3.111}$$

We know that $s_2 > 1$, so this means that $2 \leqslant s_2 \leqslant n-1$; $1 \leqslant s_1 \leqslant n-2$; and consequently $3 \leqslant n$. Note that for case $s_1 = 1$, it is clear that inequality (3.111) is fulfilled since its right hand side becomes $(n+1)^2$. This point corresponds to the point A of the graph in Figure 3.6.

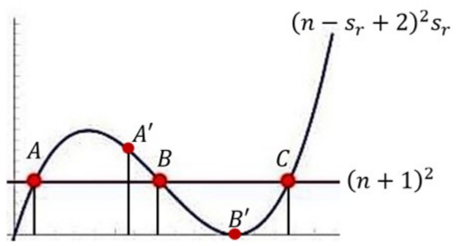

Figure 3.6: $(n-s+2)^2\, s$, $(n+1)^2$

Let us now examine what happens over the interval of possible values of s_1, that is $[1, n-2]$. For this purpose, it will be useful to find point B of the graph in Figure 3.6. Let us proceed. We solve $(n-s_r+2)^2\, s_r = (n+1)^2$ for s_r:

Expand and collect in terms of s_r:

$$s_r\left(n^2+4n+4\right)+s_r^2(-2n-4)+s_r^3=(n+1)^2$$

subtract $(n+1)^2$ from both sides:

$$-(n+1)^2+s_r\left(n^2+4n+4\right)+s_r^2(-2n-4)+s_r^3=0$$

The left hand side factors into a product with two terms:

$$(s_r-1)\left(s_r^2-2ns_r-3s_r+n^2+2n+1\right)=0$$

Split into two equations:

$s_r-1=0$ or $s_r^2-2ns_r-3s_r+n^2+2n+1=0$

The first equation is already known beforehand. We will concentrate on the second equation then.

Collect in terms of s_r

$$1+2n+n^2+s_r(-2n-3)+s_r^2=0$$

Subtract n^2+2n+1 from both sides:

$$s_r(-2n-3)+s_r^2=-n^2-2n-1$$

Add $\frac{1}{4}(-2n-3)^2$ to both sides:

$$\frac{1}{4}(-2n-3)^2+s_r(-2n-3)+s_r^2=-1+\frac{1}{4}(-2n-3)^2-2n-n^2$$

Write the left hand side as a square:

$$\left[\frac{1}{2}(-2n-3)+s_r\right]^2=-1+\frac{1}{4}[-2n-3]^2-2n-n^2$$

take the square root from both sides:

$$\frac{1}{2}(-2n-3)+s_r\pm\sqrt{-1+\frac{1}{4}(-2n-3)^2-2n-n^2}$$

Subtract $\frac{1}{2}(-2n-3)$ from both sides:

$$s_r=\frac{1}{2}(2n+3)\pm\sqrt{-1+\frac{1}{4}(-2n-3)^2-2n-n^2}$$

Given that $-1+\frac{1}{4}(-2n-3)^2-2n-n^2=\left(n+\frac{5}{4}\right)$

$$s_r=\frac{1}{2}(2n+3)\pm\sqrt{n+\frac{5}{4}}=\frac{1}{2}\left[2n\pm\sqrt{4n+5}+3\right]$$

Then, the point B is given by $\left(\hat{s}_1, f(\hat{s}_1)=(n+1)^2\right)$, where $\hat{s}_1:=\frac{1}{2}\left[2n-\sqrt{4n+5}+3\right]$. Moreover, we know from Proposition 2 that the function $f(s_r)$ (Equation (3.86)) is quasi-concave in $[1,n-1]$. Then, by the same argument, the function is quasi-concave in $[1,n-2]$. But, here we have to check if indeed the curve of the function will always be above the horizontal line given by $(n+1)^2$. Then, we analyze the function closely. As we have observed, it is cubic, so in addition to having a concave downwards part, it will have a concave upwards. We should then study its concavity in detail and identify its points of inflection. Then, $f'(s_r) = (2 + n - 3s_r)(n - s_r + 2)$ and $f''(s_r) = -2(2n - 3s_r + 4)$, so

$$f''(s_r) = 0 \iff s_r = \frac{2(n+2)}{3}$$

Testing the intervals to the left and right of $s_r = \frac{2(n+2)}{3}$ for $f''(s_r)$, we found that $f''(s_r) < 0$ on $\left(-\infty, \frac{2(n+2)}{3}\right)$ and$f''(s_r)$ on $\left(\frac{2(n+2)}{3}, +\infty\right)$. Hence, $f(s_r)$ is concave downward on $\left(-\infty, \frac{2(n+2)}{3}\right)$ and concave upward on $\left(\frac{2(n+2)}{3}, +\infty\right)$, and it has a point of inflection at $\left(\frac{2(n+2)}{3}, \frac{2(n+2)^3}{27}\right)$. Also, we noted the turning points of this function, $f'(s_r) = 0 \iff \bar{s}_{r_1} = \frac{n+2}{3}$ and $\bar{s}_{r_2} = n + 2$. Likewise, we observe that $f''(\bar{s}_{r_1}) < 0$ and $f''(\bar{s}_{r_2}) > 0$. Then, $f(s_r)$ has the local maximum at $\bar{s}_{r_1}$ and the local minimum at $\bar{s}_{r_2}$. Considering all the above, taken together the point of inflection, $\hat{s}_1$, the study interval, and the argmax, the following inequalities can be inferred.

i $1 \leqslant s_1 \leqslant n - 2$ and $n \geqslant 3$

ii $\hat{s_1} \leqslant \frac{2}{3}(n+2) \iff n \in \left[2, 4 + 3\sqrt{3}\right]$

iii $\hat{s_1} > \frac{2}{3}(n+2) \iff n > 4 + 3\sqrt{3}$

iv $\hat{s_1} \geqslant (n-2) \iff n \in [3, 11]$

v $\hat{s_1} < (n-2) \iff n > 11$

vi $\frac{n+2}{3} \leqslant n - 2 \iff 4 \leqslant n$

vii $\frac{n+2}{3} > (n-2) \iff n = 3$

viii $n - 2 \leqslant \frac{2}{3}(n+2) \iff n \in [3, 10]$

ix $n - 2 > \frac{2}{3}(n+2) \iff n > 10$

x $\frac{n+2}{3} \leqslant \hat{s}_1$

From *i* through *ix* we narrow down the values of n over which some of them hold simultaneously, that is,

(a) Together *i, ii, iv, vi, viii* hold as long as $n \in [4, 9]$. That is $\frac{n+2}{3} \leqslant (n-2) \leqslant \hat{s}_1 \leqslant \frac{2}{3}(n+2)$.

(b) If $n = 3$, *i, ii,* and *iv* will hold simultaneously[12].

(c) For $n = 10$, *i,iii,iv, vi, viii,* and *x* will hold at the same time[13].

(d) For $n = 11$, we see that *i, iii, iv, vi, ix,* and *x* hold[14]

(e) Whereas for the case in which $11 < n$, *i, iii, v, vi, ix,* and *x* will compose the following inequality: $(n-2) \geqslant \hat{s}_1 \geqslant \frac{2}{3}(n+2) > \frac{(n+2)}{3}$

12 $\frac{2}{3}(3+2) > \hat{s}_{1_{n=3}} > \frac{3+2}{3} > 1$
13 $\frac{10+2}{3} \leqslant (10-2) \leqslant \frac{2}{3}(n+2) < \hat{s}_{1_{n=10}}$
14 $\frac{11+2}{3} < \frac{2}{3}(11+2) < (11-2) \leqslant \hat{s}_{1_{n=11}}$

To illustrate this, we can plot together the point $\hat{s}_1$, the inflection point $\left(\frac{2}{3}(n+2)\right)$, the $s_{r_{argmax}} = \left(\frac{n+2}{3}\right)$, and the right endpoint of the interval under consideration $(n-2)$ as functions of n, thus, we can appreciate the values of n for which the inequalities are satisfied. See Figure 3.7.

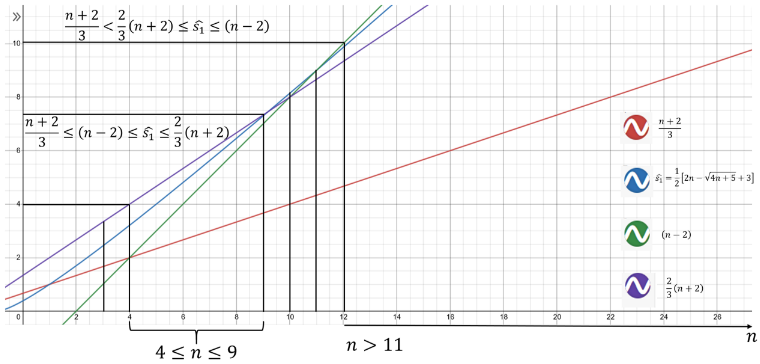

Figure 3.7: $\hat{s}_1$, *inflection point* $\left(\frac{2}{3}(n+2)\right)$, $s_{r_{argmax}} = \left(\frac{n+2}{3}\right)$, *and* $(n-2)$ *as functions of* n

Next, inequalities given by the points (a)-(e) and the conditions over s_1 and n lead us to the cases presented below. What we will do next is to study inequality (3.111) in each of them.

3.4.4.1 Case $n = 3$

We start with the case of $n = 3$, so here it is easy to see that the only value s_r takes is 1 as $(5 - s_r)^2 s_r = 16 \iff s_r = 1$. Observe Figure 3.8.

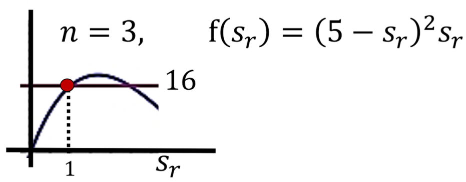

Figure 3.8: *Function* $f(s_r)$ *for the case* $n = 3$

3.4.4.2 Case $n \in [4, 9]$

Here, the interval $[1, n-2]$ falls in the concave-downward part of the function. Also, the right end-point is less than or equal to $\hat{s}_1$, which also falls in the concave-downwards part. Then, the curve of the function will lie above the line given by $(n+1)^2$. See Figure 3.9.

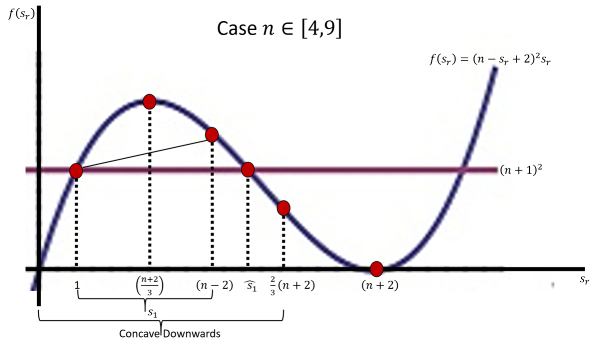

Figure 3.9: *Function $f(s_r)$ for the case $n \in [1, n-2]$*

3.4.4.3 Case $n = 10$

Notice $(12 - s_r)^2 s_r = 121 \iff s_{r_1} = 1$ and $s_{r_2} = \frac{1}{2}\left(23 - 3\sqrt{5}\right)$, but s_{r_2} is not of our interest as the interval we look at is $[1, 8]$. We have that the inflection point of the function is exactly the end point of this interval, so the function is concave downward until $(8, f(8))$. Therefore, $f(8) > 121$. And as we see in the graph depicted in Figure 3.10, $f(s_r) >= 121$ for all $s_r \in [1, 8]$.

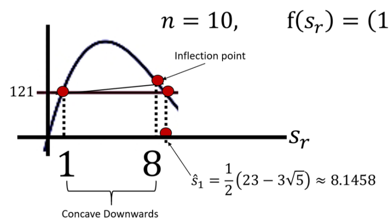

Figure 3.10: *Function $f(s_r)$ for the case $n = 10$*

3.4.4.4 Case $n = 11$

In this case we observe that the values s_1 can take falls within the interval $[1, 9]$, which are precisely the points that define the line that cuts the curve of the function in such interval. That is, $(13 - s_r)^2 s_r = 144 \iff s_r = 1$ and $s_r = 9$ as shown in Figure 3.11. Then, due to the upward concavity of the the curve, it will lie above the straight line given by (144). Note, however, that the function is concave downward at up to the point $\frac{26}{3} < 9$. It means that the right endpoint of the interval falls within upwardly concave part. However, the the point of interest that lies on this part is 9, which is precisely $\hat{s}_1$. Thus, when $n = 11$ $f(s_r) \geqslant 144$ holds.

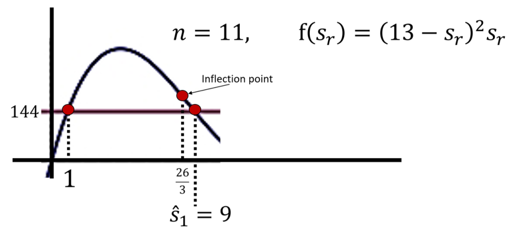

Figure 3.11: *Function $f(s_r)$ for the case $n = 11$*

3.4.4.5 Case $n > 11$

Here, the curve of the function will be above the dividing line —defined by the interval of values that s_r admits—and up to exactly $\hat{s}_1$. Unlike all the previous cases, we find cases where the inequality is reversed when $n > 11$. Let us see, the interval we are interested in is given by $[1, n-2]$, and we observe in Figure 3.12 that both point $\hat{s}_1$ and point $n-2$ are located the right side of the inflection point —the part of the function where it is concave upwards. So, the downward concavity of this function on the interval helps us partially since there exists a small interval, $\left(\frac{2}{3}(n+2), \hat{s}_1\right)$, where the function is concave upwards but on which the curve is still above the straight line defined by $(n-1)^2$. Nevertheless, we also found values of $s_1 \in (\hat{s}_1, n-2]$ where the inequality (3.111) is not satisfied, but rather reversed. For example, if $n = 15$, the function will be $f(s_r) = (17 - s_r)^2 s_r$, the horizontal line will be given by 256, and $s_1 \in [1, 13]$, so $f(13) = 208 < (15+1)^2 = 256$. The number of values of s_1 on which the inequality is inverted will obviously depend on the value of n we may take, but we can observe already that such a number will grow as n does, since the number of values between $\hat{s}_1$ and $(n-2)$ is given by $(n-2) - \frac{1}{2}\left[2n - \sqrt{4n+5} + 3\right] = \frac{1}{2}\left[\sqrt{4n+5} - 7\right]$ and $\lim_{n\to\infty}\left\{\frac{1}{2}\left[\sqrt{4n+5} - 7\right]\right\} = \infty$. See Table 3.1 for some example values of n for which the inequality (3.111) does not hold.

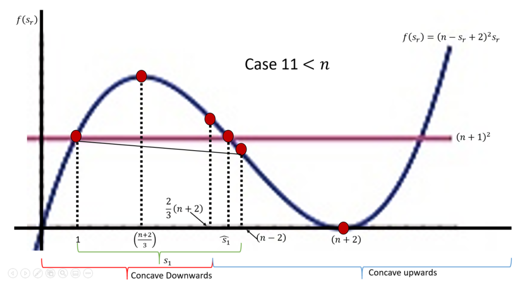

Figure 3.12: *Function $f(s_r)$ for the case $n = 11$*

n	**Number of values between $\hat{s}_1$ and $(n-2)$ for which inequality (3.111) is not satisfied:** $\frac{1}{2}\left(\sqrt{4n+5}-7\right)$
19	1
29	2
41	3
55	4
71	5
89	6
109	7
131	8
155	9
181	10
⋮	⋮

Table 3.1: *Some values of n for which* $f(s_r) \leqslant (n+1)^2$

■

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3.4.5 Proof of Proposition 5:The Stability of the Grand Coalition of the CPRs game

Proof: Let us say that a coalition S decides to opt out of the grand coalition. The new coalition structure that emerges from is $P = \{S, N\backslash S\}$. Then, depending on the number of appropriators in S, we will have either that it is larger than the *residual coalition*, $N\backslash S$, or that this latter is larger than S. Let us see both cases.

3.4.6 Case $s < n - s$

If $s < n - s$, then it is clear that $1 \leqslant s \leqslant \frac{n}{2}$. Then, under partition P, we know from Proposition 1 that members of larger coalitions get lower payoff, and moreover, from the second part of Proposition 2 we know that the largest coalition in each partition is worse-off relative to the per-member payoff received in the grand coalition, then

$$\frac{v\left(N\backslash S;\{P\}\right)}{n-s} < z_i \quad \forall i \in N\backslash S.$$

That is, if the coalition S withdraws from the gran coalition, partition P is formed, but how can the members of the complementary/residual partition, $N \backslash S$ response? Well, if they are far-sighted, they just simply break apart into singletons, which would lead to the formation of the finest partition and thus the repetition of the game, and the gran coalition would be re-reached. Let us see, assume that the residual coalition breaks apart into singletons, then we have the partition $P' = \Big\{S, \big[N \backslash S\big]\Big\}$. Under this partition, each member $i \in S$ gets

$$\frac{v\left(S; \{P'\}\right)}{s} < z_i,$$

We know that this inequality holds by the first part of the proof of Proposition 2 where we proved that $4n < (n - s + 2)$. Then, given the partition P', the members of S will decide to dissolve into singleton coalitions as well, so we arrive at the finest partition, the game will be repeated and the gran coalition will be reached. The move of breaking apart on the part of the members of the complementary coalition deters the members of S from moving away from the grand coalition.

3.4.7 Case $s > n - s$

By the same reasoning of the previous case, the members of the coalition S further dissolve into singletons in order to have the complementary coalition to break apart into singletons as well, so the game is repeated, and the grand coalition is reached. Hence following the logic of Parkash of far-sighting coalitions, the grand coalition is stable under the CPRs structure. In words, it is stable because of the incentives to disintegrate coalitions have, second because of the fact that members of larger coalition in non-trivial partitions get lower individual payoffs, and third, the gran coalition payoffs are greater than the payoffs of largest coalitions in each partition, ■

CHAPTER 4

Evolution and Kantian Optimization in an Extended Version of the CPRs Game

In writing about the effect of earth (or physical environment of it) on human activity, Ritter (1822), co-founder of modern geography, posited that if it is recognized that every moral person, for the fulfillment of his calling, and everyone who is to succeed in doing right in something, must bear in consciousness the measure of his powers and know what is given outside him, or his surroundings, as well as his relation to them; then it is clear that every human association, every person, should also become aware of his or her own inner and outer powers as of those of their neighbors, and those positions in relation to others acting on those from the outside, in order not to miss its true aim. Natural Common-Pool Resources (CPRs) are clear-cut representations of this human-environment relationship. One's behavior and decisions, swayed by the environment, resonates beyond us. Let me explain. In a CPR situation, the actions of individual members of a community in relation to a resource affect both other users and the resource itself. And, at the same time, what one uses from the common resource others can not, but the degree of exclusion others from using it is typically low. Then, when agents in this situation come truly to realize that the consequences that their actions have on others might ultimately impact on themselves, their behavior can certainly change for a better choice for them, and consequently for others. Precisely, those people who understand that by cooperating they are better off explicitly recognize that a situation such as the one constituted by the use and appropriation of a common resource demands a common effort.

Elinor Ostrom's initial work focuses on and explains the elements that support long-term cooperation and coordination among appropriators followed by the identification of the conditions under which appropriators are likely to cooperate to devise governing arrangements, Schlager (2004) tells us. Schlager mentions as well that Ostrom identifies the attributes of both CRPs and appropriators that are conducive to the emergence of cooperation, and then states the well known institutional design principles[1] that characterize robust institutions for the management of CPR. In this respect, the model of the individuals, in which Ostrom (2015) relies on, consisted of "fallible, norm-adopting individuals who pursue contingent strategies in complex and uncertain environments." Yet the inner motives of involved people to decide upon following the norms or doing what they do are still being studied. In this matter, DeCaro (2019) tries to disentangles the various drivers of cooperation by developing a humanistic rational theory. He is straightforward:

[1]Other authors have reviewed the principles, for instance Cox et al. (2010), which analyze studies to evaluate the principles empirically and to consider what theoretical issues have arisen since their introduction. They conclude that the principles are well supported empirically and that several important theoretical issues warrant discussion. They provide a reformulation of the design principles, drawing from commonalities found in the studies.

> [T]he Ostrom School of Political Economy demonstrates that self-organization and cooperation are possible and identifies key factors (Cole and McGinnis (2014)). However, Ostrom's perspective still adopts (boundedly) rational self-interest as its motivational starting point and assumes stakeholders are externally motivated (Ostrom (2010)). Therefore, it cannot explain why people decide to self-govern in the first place, or continue to voluntary cooperate long term, especially when cooperation is risky or personally costly, as is often the case (Ostrom (1990); Ostrom et al. (2010): appendix 9.1).

This method of analysis, nevertheless, has a number of limitations as it does not include other aspects of human cognition, such as prejudice, formation of in-groups, loss aversion, and morals. In Roemer (2019a) I find a moral glance to explain co-operation. He states a motivational foundation for cooperation focusing on a sort of Kantian thinking consisting of altering the way agents optimize without focusing on the preferences side of the individuals but on their ethos. The theory of cooperation he formally develops is based upon a concept of optimization inspired by the *Categorical Imperative* posited by Kant (1785). There is a moral reason that may drive agents to make cooperative decisions, and here it is formalized. While in the Nash way of doing a player individually undertakes an action considering that the actions of others remain unchanged in a competitive setting. In the Kant way of doing a player thinks individually before undertaking an action, but considering at the same time, that it can be collectively undertaken in a cooperative environment. In this sense, it is said that the former theory decentralizes competition, whereas Kantian optimization decentralizes cooperation. The *tragedy* and *free-rider* problems are efficient in Pareto's sense under the latter framework.

Research on CPRs includes also work using evolutionary game theory tools (Sethi and Somanathan (1996)). These studies have relied mainly on certain mechanisms of action for the evolution of cooperation through social preferences, and/or sanctions that can either be punitive, like pro-social punishments, and/or incentive, like ostracism (Tavoni et al. (2012)) as a case of social shunning (exclusion of non-cooperators from community privileges or that the resource stock and the non-excessive extraction are held via exclusion by general consent from the right of use the resource). On the same page, other mechanisms include group patterns within populations be they unstructured or structured —e.g., networks of agents like lattices, or parochial networks. (Bowles and Gintis (2004)).

In modeling the evolution of cooperation in these environments, we have to note something. Typically, we assume the presence of cooperative and non cooperative agents. The former can be cooperative with some sort of social preferences. The quote of appropriation/extraction from the CPR corresponding to the non-cooperative members usually is taken from that of the traditional maximization criterion, while the decision of the level of extraction/appropriation of cooperative members that is assumed comes from a process that seeks to maximize the social welfare. However, we can also consider a protocol of individual optimization to determine a cooperative level of appropriation, as we shall see in this chapter.

Also, we observe the recurrent studied paradigm of explaining cooperation through the existence of an infliction or imposition of a penalty. Sethi and Somanathan (2005) study how reciprocators agents survive in competition with opportunist agents by means of punishments in a community that extracts a common property resource. Moreover, ostracism, monitoring, imperfect monitoring (Tavoni et al. (2020)), and other means or forms of punishments represent, although in a difference sense, a cost that both cooperators and non-cooperators face. The former might even choose deliberately to bear a personal cost to ground opportunistic individuals, whereas the later are compelled to pay for breaking the rules. There is an inclination among CPRs scholars towards focusing on the natural tendency of individuals to punish free-rider behavior. Sanctions (particularly, monetary punishments) nonetheless may undermine long term cooperation, by making group members think that no one wants to cooperate intrinsically (Mulder et al. (2006)). Little thought has been given to intrinsic motivations as one of the drivers for cooperation that some people might have in CPR contexts. **Moral inner motivations do matter** (Bowles (2016)). Indeed, moral decisions were acknowledged in a second-generation of models examined by Ostrom (1998). Although those models explain cooperation, they are not approached under the criterion of optimization Roamer suggests. Kantian Optimization is a theory of Cooperation in the first place. It better suits observed cooperative behavior. Under this view, morality —which is not an object of preferences—does not consist of caring about others but rather in understanding that a particular problem, as it happens in CPRs, is one of solidarity that requires cooperation and trust (Roemer (2019b)).

Nash behavior, nevertheless, should not be neglected entirely. As explained in the previous chapter, there are cases of CPRs in which individuals are caught in non efficient Nash equilibria. Individual actors do not always beget cooperation, cooperative appropriators live together with non-cooperative appropriators. Admittedly, an over-explotation is readily possible. And in fact, as highlighted by Elsner et al. (2015), it has often been identified as a likely result of the common use of resources when agents purse an individualistic *ethos*. Thus, the theory proposed by Roemer might be used together with the traditional strategic way so as to explain the evolution of cooperation in CPRs settings from a moral standpoint.

Moreover, most studies in the field of CPRs, particularly natural CPRs, have mainly focused on the *appropriation externalities* the users impose upon each other, disregarding the impact of appropriation on the public good feature of the resource, to which Blanco and Walker (2019) refer as *degradation externalities* (i.e. the loss of public good benefits of conservation of the CPRs). When this kind of externalities are taken into consideration, it becomes apparent that appropriators from CPRs might decrease the value of the public good as they increase the extraction of the resource. Notice that such a degradation might affect not only individuals involved in the appropriation but also those without access to it. The Amazon forest is a prime example of this, since it is a CPR whose degradation externalities affect the entire planet. People who are not involved in a CPR might partake in the conservation benefits seen as a sort of public good. In this respect, Ottone and Sacconi (2015) highlights that negative externalities together with positive externalities from consumption/use of the same resource can be created either directly and/or indirectly. As they posit, while collective use of a resource might congest or deplete it temporarily, the use of this resource might imply positive things for others, even involving outsiders. Access to water reduce it, but at the same time, it improves quality of life of both direct and indirect users. In other words, the so created positive externality is of second order.

In this line, in an attempt to reconcile cooperative behavior with individual rational behavior, and considering, although not simultaneously but sequentially, the problems of appropriation and conservation in CPR harvest environment, Solstad and Brekke (2011) argue that cooperation can be enhanced when individuals have a unified purpose, captured as the joint provision of a PG. The contribution is created from what they harvest from the common resource. That is, appropriators receive an income from the harvest, which is then used for private consumption and for contributing to a PG like the maintenance of the CPR. The situation is, then, modeled as a two stages game. This allows individually rational appropriators to share a common interest (the contribution), so cooperation will emerge. However, the analytic framework they stick with is non-cooperative game theory, so the Nash solution prevails. We can have individually rational, non-altruistic cooperative rational individuals guided by a *common purpose* without calling upon competitive concepts as well.

Having regarded all the above, in this chapter we want to know whether groups of agents evolve towards a community with a hardwired sense of morality that exploits a CPR socially and efficiently. Cooperative behavior here is followed by those individuals driven intrinsically by a sort of a quasi-moral norm captured through the Kantian optimization. With that in mind, we shall explore the conditions under which Kantian agents can survive and spread in evolutionary competition with Nash agents within a setting where the problems of appropriation and conservation of a CPR are considered simultaneously. Accordingly, we explore that idea that a quasi-moral norm might act as a mechanism for the evolution of cooperation. We show that a Kantian population is just as robust as a Nash population is. That is, both populations are stable. The selection dynamics in this case will depend on the initial conditions.

The exposition is in four sections. In Section 4.1 we survey the concept of Kantian equilibrium and its linkage with CPRs. The problem of appropriation and conservation are introduced in Section 4.2. Then, in section Section 4.3 the Evolutionary stability of Kantian versus Nasher appropriators is studied. Finally, Section 4.5 sums it up.

4.1 Cooperation in CPRs: Social Preferences, Morals, and Kantian Optimization

4.1.1 Social Preferences and CPRs

Some studies in CPRs settings resort to social preferences to account for cooperation. They present models that modify the utility function of individuals to capture preferences such as altruism, fairness, warm-glow, and cold-prickle (Andreoni (1995)), to name but a few. Let me explain how some of them work. Altruism (subjects care about the payoffs of other subjects) is usually introduced into the agents' preferences as a parameter that captures the degree of care an agent has in regard to the total level of extraction/appropriation of the resource in common. In the same manner, reciprocity is often part of the argument of utility function in a way that captures this type of relationship among agents (Kolm (2008)).

On the other hand, Andreoni (1989) and Andreoni (1990) propose the term "warm-glow" or *impure altruism* to refer to the motivation of subjects who care about the act of doing good for others. In other words, warm glow can be understood as the private benefit an individual gets from the act of doing good or giving to others, or the good feeling one experiences for doing the right thing. In terms of public goods and common pool resources, an individual wants to contribute to the public good because in this way, (s)he guarantees somehow a personal satisfaction from that decision. Analogously, an individual who enjoys of warm glow does not want to increase his or her level of extraction/appropriation too much because by doing so, (s)he obtains better personal benefits.

Conversely, cold-prickle is another term the same proposer of the warm-glow concept suggests to name the negative sensation for doing the bad thing. And it might be captured by the disutility individuals might get from the act of doing bad (Andreoni (1995)). Agents who do not contribute to the creation of a PG feel bad for making this decision, so this action fails to give them satisfaction. In this vein, appropriators who increase too much their level of extraction of a CPR might experience this sort of negative feeling as well.

Warm-glow and cold prickle are opposite experiences of contrary behaviors. They influence agents' decisions to different extends depending on the frame in which they occur. Experimental studies show that some people are more likely to cooperate when the frame is such that they generate a positive externality derived from the warm glow. Andreoni (1995) tells us "the warm-glow of creating a positive externality appears to be stronger that the cold-prickle of creating a negative externality[.]"Although more recently Grossman et al. (2012) shows that framing can also be irrelevant.

I do not completely agree with the view that there is an *impure altruism* or that warm-glow is a means to an end —individual interests. Altruist people do not need to seek personal rewards in the first place. I concur with the position of Roemer (2019a) that warm-glow is a perceived sensation *post facto*. It is an effect after doing what is right. Doing the right thing generates the *reward* called *warm-glow*, but if one sees this as the driver, then it is just selfishness —hence the name "impure altruism." Besides, this expression is actually an oxymoron. Altruism[2] means disinterested and selfless concern for the well-being of others, or unselfish regard for or devotion to the welfare of others.[3] My thoughts are that when altruism is modeled as a part of the utility function of the individuals, what we actually mean is that individuals focus on the maximization of their utilities, so for them to do the right thing, they *ex-ante* single out their own welfare over the others'. That way they increase their own satisfaction. This approach is not necessary wrong, but again, altruism or impure altruism are inaccurate terms.

4.1.2 Kantian Optimization and CPRs

Steering away from altering preferences, Roemer (2019a) proposes to change the way people optimize[4] in order to explain cooperation. Accordingly, when deciding upon a set of actions, an individual member of a society with a *cooperative ethos* considers the consequences of her[his] action upon herself[himself] as if it were taken by others. In this sense, the proposed solution concept is inspired on the Categorical Imperative of Immanuel Kant (1785):

> *Act only on that maxim through which you can concomitantly will that it should become a universal law.*[a]
>
> [a]Translation from Kitcher (2004)

Before proceeding further, it is worthwhile to furnish some clarifications on this point. When Roemer draws on Kant's categorical imperative, he is aware that strictly speaking Kant's imperative is absolute and resolute, *"what ought to be for the sake of duty."*. In contrast, the optimization protocol is conditional, so a more fitting term —the argument goes—is a "quasi-moral optimization." Nonetheless, he sticks to the term *Kantian optimization* because, in his words, " there is a history of using it in economics, and because it is aptly described by Kant's phrase [above quoted], even if Kant meant this is

[2]The word altruism travels from Italian language "altri," 'somebody else,' from Latin *alteri huic* "to this other." Thus, disinterested and selfless concern for the well-being of others "Altruism." Lexico.com Diccionary, Oxford English, https://www.lexico.com/definition/altruism. Accessed 22 Dec. 2020.

[3]"Altruism." Merriam-Webster.com Dictionary, Merriam-Webster, https://www.merriam-webster.com/dictionary/altruism. Accessed 22 Dec. 2020.

[4]Of course, both alterations are not mutually exclusive.

an unconditional way." Then he adds:"[i]t may be more textually accurate to justify the Kantian nomenclature by invoking Kant's hypothetical imperative. I use the term for its suggestive meaning and do not wish to imply that there is a deeper, Kantian justification of my proposal."

That said, we bring forward now this notion of Kantian optimization in game theory. Kantian-kine individuals prefer a situation in which they see their action universalized. Formally, following Roemer (2020), let $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ be a game in normal form with n players, where the payoff functions $u_i : I^n \mapsto \mathbb{R}$ and I is an interval in $\mathbb{R}_+$, the strategy space for each player. The strategies $x_i \in I$ can be seen as the appropriations. A game is strictly monotone increasing(decreasing) if each payoff function u_i is strictly increasing(decreasing) function in the contributions of the players other than i.

Definition 8 (*Kantian Equilibrium*)

A strategy profile $(x_i, \ldots, x_n)$ *is Kantian equilibrium if*

$$(\forall i) \ \arg\max_{r} u_i\,(rx_1, \ldots, rx_n) = 1$$

In contrast to the Nash equilibrium in which no appropriator can do better by individually deviating from a certain strategy, in a Kantian equilibrium, no appropriator wants to scale her or his appropriation by a non-negative factor r under the assumption that if [s]he does so, other counterpart players might want to scale their appropriation in the same way [s]he is contemplating to scale.

Roemer (2019a) proves that in any strictly monotone game, any multiplicative Kantian equilibrium is Pareto efficient. The CPR game is a strictly monotone decreasing game. Remember that what one gets from the resource takes away that amount available to others, which implies a negative externality for them. That is, if appropriators increase their level of extraction, their individual level of extracted resources will drop. Cooperation is then begotten by this Kantian-like way of thinking. And, unlike the Nash optimization, under Kantian optimization, the *tragedy* is surmounted. At the same time, one important thing to notice is that the *morality of a Kantian optimizer consists not in caring explicitly about the payoff accruing to other players but rather in playing that one strategy that [s]he would be happy if everyone played it* (Roemer (2019b)).

Next, I want to stress a sort of a relationship I found between Kantian optimization assumptions and some of the attributes of appropriators that promote cooperation distinguished by Ostrom (2002b, pp. 5). These are *common understanding* and *trust*. Meaning that appropriators have a shared image of how the resource system operates and how their actions affect each other and the resource system and that they trust one another to keep promises. Thus, such attributes are captured by the rationality involved in the Kantian protocol: *humans ability of cooperation is rooted in their joint intentionally [common understanding] that, in turn, is built upon common knowledge and trust[I know others are kantians, so I trust they are going to behave as such].* Precisely, Roemer (2020, pg. 5) mentions “...in real life, we are very often in situations where trust is warranted, either because of past personal experience with potential partners, or because of social conventions, of culture. In these situations, trust exists, and the Kantian question is a natural one to ask."

4.1.3 Cooperative and Non-Cooperative Mindsets: Comments on Some Study CPRs Cases

In this spirit, Timilsina et al. (2017) design and implement a set of dynamic CPR games and experiments in two types of Nepalese areas, which they dub urban (capitalistic) and rural (non-capitalistic) areas. According to the authors, sustainability is jeopardized by competitive societies due to the influence of a capitalistic oriented system on human nature for using CPRs. They show that “when societies move toward more capitalistic environments, the[*sic*] sustainability of common pool resources tends to decrease with the changes in individual preferences, social norms, customs and views of others through human interactions" suggesting that in capitalistic societies, individuals are somehow biased to the extend to which they may lose the ability to coordinate a sustainable use of common resources. In this respect, one point to be highlighted comes from the answer of a post-question of their experiment —*How did you want to play?* Most of the urban subjects answered — *I really wanted to play the game for longer, but I was not sure whether the other group members were motivated to do the same.*

This response evidences a lack of trust in others and a sort of Nash reasoning among urban individuals. Moreover, they found that the proportion of pro-social individuals in rural areas is greater than pro-social individuals in urban areas. In rural areas most individuals still engage in agriculture and in natural resource management based on indigenous knowledge and traditional practices where cooperation and sharing are quite common among individuals, the study explains. The fact that rural subjects are more willing to cooperate reveals that these groups might have discovered a reasoning in accordance with the Kantian equilibrium. Accordingly, they understand that they are in the same boat, so they row in the same direction. An example from the same work [Timilsina et al. (2017, p. 10)] underlies this claim:

> *Mela pat and Parma are well known as voluntary and cooperative farming practices that prevail in rural Nepalese culture. Individuals exchange or offer farming and forestry services without monetary rewards. Such forms of voluntary cooperation remain common of Nepalese rural areas, as rural residents are vulnerable to natural uncertainties and calamities, and cannot sustain their lives without mutual cooperation.*

Naturally, a cooperative ethos can be found to a greater or lesser extent in diverse societies, and indigenous communities are no exception. Monterroso et al. (2019) point out that one challenge currently faced by indigenous commons is the assumption made at an authority level that indigenous people will manage and govern common resources through collective action. It is normally assumed that distribution of shared resources, rights and benefits is always equal. However, this will not always be the case. As a matter of fact, groups of indigenous people have different identities —values, beliefs and customs. Thus, the management of common resources might vary within and across indigenous communities. Besides, although the presence of a Kantian way of thinking in these societies might explain why they are able to achieve cooperative outcomes, the Nash thinking might also be present. Indigenous societies might evolve over time or might be influenced by modern societies. Shahrier et al. (2016) demonstrate that with evolution from rural to capitalistic societies, people are likely to be less pro-social and more likely to be competitive. By the same token, in the state of Oaxaca, Mexico, there are some indigenous communities ruled by traditional governance practices, and one of the mechanism of cooperation they implement is the so-called "*tequio* and *servicio*" whereby they farm out tasks or assignments of public good provision character to a member or group of members. People there, however, are less involve in those activities nowadays. Particularly, young people are more reluctant to be told off duties. They do not longer cooperate in the *tequio* and *servicio* to the point that non-cooperative individuals get punished (Magaloni et al. (2019)). Now, they pay a fine, something that long before was not even considered in their arrangements.

4.2 Appropriation and Conservation of CPR

Additionally, substantial literature of CPRs typically dwells on the problem of appropriation of the CPR. In the traditional CPR game set forth by Ostrom et al. (1994) conventional self-interested individuals with the right to appropriate have incentives to increase their level of extraction, x_i, to the extend of appropriating the resource beyond the social optimum level, which generates the well-know overuse or congestion externality —individual efforts to secure more benefits from the CPR have the effect of reducing the benefits received by others who have appropriation access right to the resource Holt (2019, p. 331). In this context, the conservation aspect of CPR is not included. Yet, both problems are complementary, particularly as for natural CPRs and sustainability, since conservation of natural CPRs provide benefits of public good character. Natural CPRs are ecosystems that, when conserved, provide benefits such as the so-called *ecosystem services* (Reid et al. (2005)). Therefore, the conservation part of CPRs implies the analysis of degradation externalities -the loss of public good benefits of conservation of natural resources (Blanco and Walker (2019)). In this direction, Bednarik et al. (2019) set a CPR experiment game in which players harvest trees to generate income, but they also consider the protection against floods provided by forests. This study highlights the importance of taking into account additional group level benefits, since they found that reducing the harvest rate improves the group outcomes in terms of the forest sustainability.

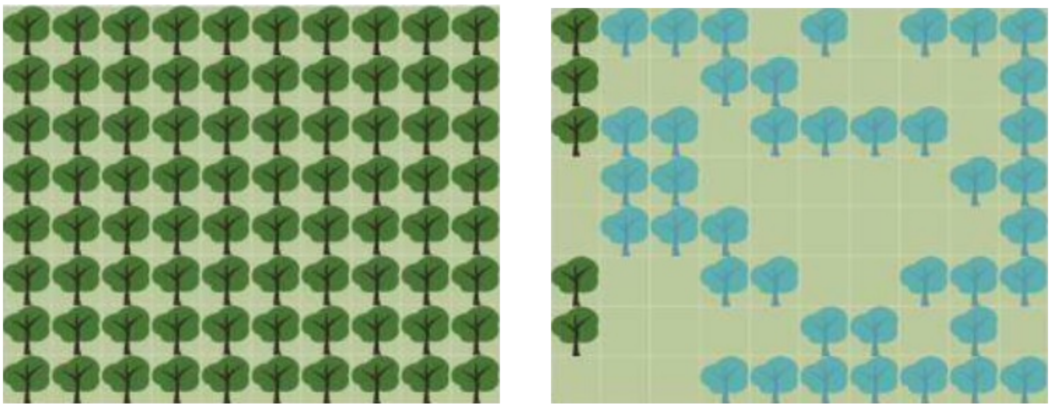
In light of this, degradation externalities can be captured by setting the total loss in conservation value[5] of the resource based on the total appropriation,$\sum x_i = X$. Each unit of appropriation reduces its conservation value, generating a loss to the community. This loss to the group from appropriation is the so-called *degradation externality*. In other words, forgoing the appropriation has a value of g for a community, which means a value of $\frac{g}{n}$ for each community member. Thus the conservation value of the CPR is formulated as

$$g\left[T - X\right] \tag{4.1}$$

where T is the available resource stock. Conservation value can be seen as an *existence value*, which "corresponds to the value attached to the existence of a resource, compared to its loss, by people who do not use it and never intend to"(Chander (2018a)). To illustrate this, in forest common pool resources, trees are one of the units derived. Thus, T here would be the total number of trees in a CPRs forest. And each tree that is not cut down generates a benefit for the community. Indeed, CPRs forest provide several benefit at local and global scales, such is the case of global public good of carbon sequestration and local national level contributions to livelihoods, see Chhatre and

[5]Conservation value will be understood as that value generated from the ecosystem services when the resource is preserved. See Capmourteres and Anand (2016) for a review of the concept of Conservation value. They argue that this term is somehow an umbrella concept, and that its meaning depends on the context in which it is used.

Agrawal (2009). Besides, as mentioned before, some *CPRs* forests can help to prevent floods and droughts EEA (2015). Hence, the public benefit stems from the unfelled trees is captured by g, which likewise generates the conservation value of this CPR. Hence, $\frac{g}{n}T$ represents the maximum value an individual can receive from the conservation component of the CPR seen as having a public good character. Accordingly, the maximum value of conservation occurs in the case of no appropriation, and the conservation value decreases in aggregate group appropriation from the CPR (Blanco and Walker (2019)). Continuing with the case of CPRs forest, see Figure 4.1 for an example of the conservation value of preserving trees.



Conservation Value: $80g$ *Conservation Value:* $5g$

Figure 4.1: *Conservation of CPRs generates a public good captured by* $g\,[T - X]$.

Image source: Bednarik et al. (2019)

In addition, the degradation of the resource affects individuals who actively appropriate, as well as those who could appropriate but choose not to do so, Blanco and Walker (2019). Similarly, it might affect external actors. Again, some *CPRs* forest management depends largely on some locals, but their benefits encompass outsiders indirectly. In this sense, the conservation part of the CPR can be seen as a public good that loses its total conservation value due to the levels of appropriation.

4.2.1 Appropriation and Conservation Externality: A Two Players Normal-Form CPRs Game

The role of the degradation externality can be brought to light with the following two player version normal-form CPRs game adapted from van der Heide and Heijman (2019).There is an amount of commons resource T available to two appropriators A and B. Each might determine between appropriating (extracting, grazing) a small number of units of the resource x_1 and a large number of it x_2, thus $x_1 < x_2$. The CPR extraction yields a gross revenue equals to I. A player's share of the CPR revenue is proportional to his or her appropriation. Thus, when an appropriator decides to appropriate x_i, (s)he gets $\frac{I}{x_i+x_j}x_i$, where $i = 1, 2$ and $j = 1, 2$. Also, there is a cost c per unit of extracted resource. Moreover, the conservation of this CPR implies a benefit for all players that can be stated in the same way as (Equation (4.1)). Each conserved unit of the resource generates a value of $g \in \mathbb{R}_+$, so each player gets a value of $\frac{g}{2}T$ when the CPR is totally conserved. In this sense, appropriation leads to a loss in the total value of the conservation of CPR considered as a public good:

$$gT - g\left[T - X\right] = g\sum x_i \tag{4.2}$$

The above situation shapes the game portrayed in Table 4.1. In the rows we represent the strategies and their associated payoffs of appropriator A, while in the columns we represented the strategies and associated payoffs of appropriator B.

		Appropriator B	
		x_1	x_2
Appropriator A	x_1	(R, R)	(S, T)
	x_2	(T, S)	(P, P)

Table 4.1: *A two players normal-form version of an extended CPR game*

where:

$$\begin{aligned}
R &:= u_A(x_1, x_1) = u_B(x_1, x_1) = \frac{I}{2} - cx_1 + \frac{g}{2}[T - 2x_1] \\
S &:= u_A(x_1, x_2) = u_B(x_1, x_2) = \frac{I}{x_1 + x_2}x_1 - cx_1 + \frac{g}{2}[T - (x_1 + x_2)] \\
T &:= u_A(x_2, x_1) = u_B(x_2, x_1) = \frac{I}{x_1 + x_2}x_2 - cx_2 + \frac{g}{2}[T - (x_1 + x_2)] \\
P &:= u_A(x_2, x_2) = u_B(x_2, x_2) = \frac{I}{2} - cx_2 + \frac{g}{2}[T - 2x_2]
\end{aligned} \tag{4.3}$$

Since we assume that $x_1 < x_2$, then $P < R$ holds. Notice that even in absence of costs, the inequality holds. Mutual low levels of extractions leads to greater payoffs than mutual high levels of extraction. The strategy pair (x_2, x_2) is a Pareto sub-optimum strategy, and the degradation of the common resource is higher. This strategy outcome is the *tragedy*. Appropriator B and appropriator A will be better off when both choose x_1. In (x_2, x_2) both players get half of the gross income I at higher costs and lower conversation value than in (x_1, x_1), where appropriators get half of the gross income at lower costs and higher conservation value. Now, in a Prisoner Dilemma (PD) game each player has a strict dominant strategy, and the outcome of the game is a Pareto sub-optimum. This means that in order to set up a commons PD, the payoff relationships (eq. (4.3)) must meet the next inequalities:

$$T > R \tag{4.4}$$

$$P > S \tag{4.5}$$

which guarantees that x_2 is the dominant strategy for both appropriators. Then we have that eq. (4.4) holds when the next inequality holds:

$$x_2 < \frac{I}{2\left(c + \frac{g}{2}\right)} - x_1 \tag{4.6}$$

Assume that $x_1 = \lambda x_2$ for $\lambda \in (0, 1)$, so we obtain the next inequality for x_2

$$x_2 < \frac{I}{\left(c + \frac{g}{2}\right)(1 + \lambda)} \tag{4.7}$$

Observe that when $g = 0$, eq. (4.7) we do not have conservation value at all, so we are in the traditional CPR setting presented by van der Heide and Heijman (2019). Next, what would be the role of g for having or not a prisoner dilemma tragedy? Let us find it out. For the PD holds, g should lies in the next interval:

$$0 < g < \frac{I}{x_2\,(1+\lambda)} - 2c \tag{4.8}$$

which implies that

$$\frac{I}{2c\,(1+\lambda)} > x_2$$

On the other hand, if there are no cost at all, we have three cases (assuming again that $x_1 = \lambda x_2$).

4.2.1.1 Case 1:$R > T$ and $S > P :\Leftrightarrow \frac{I}{x_2(\lambda+1)} > g$

In this situation x_1 strictly dominates x_2 and the NE of this game is (x_1^*, x_1^*).Although there is no tragedy at all, there is a certain degradation of the resource.

4.2.1.2 Case 2:$R < T$ and $S < P :\Leftrightarrow \frac{I}{x_2(\lambda+1)} < g$

This is the case eq. (4.4). The tragedy is a PD

4.2.1.3 Case 3: $R = T$ and $S = P :\Leftrightarrow \frac{I}{x_2(\lambda+1)} = g$

In this case, anything can happen. We found a proliferation of Nash equilibria, $\{(x_1^*, x_1^*)\,, (x_1^*, x_2^*)\,, (x_2^*, x_1^*$
Notice that (x_1^*, x_2^*) and (x_2^*, x_1^*) are somehow *tragedy outcomes* derived from a CPRs dilemma that is not a PD. One appropriator gets more from the resource, so (s)he receives a better payoff. This in line with (van der Heide and Heijman (2019) , Ostrom et al. (1994)). Not all "tragedy outcomes" derived from CPR dilemmas are, *sensu stricto*, prisoner dilemmas.

4.3 The Common-Pool Resource Public Good Game

The relationship between the overused externality and degradation externality (the loss of public good benefits derived from conservation) can be included in the standard n-person CPRs game as well. For that matter, we consider the extended version proposed by Blanco and Walker (2019). In essence, the conservation value (Equation (4.1)) is added to each individual payoff so as to have a formal game that includes the CPR traditional problem and a public good conservation problem, thus Common-Pool Resource - Public Good Game (henceforth, CPR-PG). In contrast to the game of Section 4.2.1, the setting we will now consider, beside involving n appropriators, assumes that the gross revenue will be a concave function on the total level of appropriation/extraction. More precisely, we take the same standard version of the CPR game and assumptions described in the previous chapter (Section 3.1.1.1). We recall it down below. The reader who has studied the previous chapter can skip the description of the game and go forward to section Section 4.3.1.

The Standard CPRs Game

There is a community in which each of its n members, possessing an initial endowment e, extracts or appropriates[a] a part of a limited CPR for personal benefits. They decide independently and simultaneously how much they want to take from the CPR. Although the appropriation of the resource yields a revenue for the community that depends on the total level of appropriation, it involves an individual cost $c \in \mathbb{R}$ per appropriation unit irrespective of the decisions of all other community members. Moreover, for low levels of the amount of total appropriation, the revenue from the resource is positive and increases —up to a certain level—as the total amount appropriated does. After that point, when individuals appropriate too much, the outcome is detrimental. Also, each appropriator i retains a share of the total revenue obtained as a community. Then, the allocation rule is that they keep a part of revenue in proportion to their share in the total amount of appropriation, which leads the community to implement a proportional sharing rule. This situation defines a game in strategic form (or in normal form) $\Gamma = (N, \chi, u)$, in which:

- $N = \{1, \ldots, n\}$ is a finite set of players/appropriators.
- χ_i is the strategy set of player/appropriator i, for every player $i \in N$. $\chi = \times \chi_i$ denotes the set of all vectors of strategy profiles. A strategy profile is denoted by $x = (x_i, \ldots, x_n) \in \chi$, where x_i corresponds to the amount of the appropriate resource (units of appropriation).
- $u_i : \chi \mapsto \mathbb{R}$ is the payoff function of player/appropriator i, so $u = (u_i, \ldots, u_n)$ is the vector of payoff functions.
  - The payoff function of i is given by:

$$u_i(x_i, x_{-i}) = e - cx_i + \left[\frac{x_i}{x(N)}\right] f\left(x\left(N\right)\right) \tag{4.9}$$

  where $x_{-i} = (x_1, ...x_{i-1}, ..., x_n)$, and
    * $x(N) = \sum_{i \in N} x_i$ is the amount of total appropriation.
    * $\dfrac{x_i}{x(N)}$ is the sharing rule: an individual appropriator i gets a fraction of the total revenue according to her or his share in total appropriation.
    * $f\left(x\left(N\right)\right)$ is a strictly concave function that governs the total revenue with $f\left(0\right) = 0$ and $f'\left(0\right) > c$. Accordingly, say that $\hat{x}$ is the level of $x\left(N\right)$ such that $f'\left(\hat{x}\right) = 0$, so for a $\bar{x} > \hat{x}$ we have that $f'\left(\bar{x}\right) < 0$.

[a] Depending on the context, we also say harvest, fish, extract, or graze.

4.3.1 The Model

The CPR-PG is built simply by adding the conservation value, $g\left[T-\sum x_i\right]$, of the resource to the appropriators payoffs. Thus, the payoff to an individual i is given by:

$$u(x_i,x_{-i}) = \begin{cases} e + \frac{g}{n}\left[T-\sum x_i\right], & \text{if } x_i = 0 \\ \underbrace{e - cx_i + \left[\frac{x_i}{\sum x_i}\right] f\left(\sum x_i\right)}_{Appropriation\ Value:\ CPR} + \overbrace{\frac{g}{n}\left[T-\sum x_i\right]}^{Conservation\ Value:\ PG}, & \text{if } x_i > 0 \end{cases} \tag{4.10}$$

Where g will be a proper/reasonable value such that the social dilemma captured by the classical CPR holds. In this manner, this extended version factors in the degradation component in a way that captures the externality the classical appropriation model disregards. Anew, we use the same revenue function, parameters, and assumptions as in chapter three (Section 3.1.1.1), namely

$$f(x(N)) = ax(N) - b\left[x(N)\right]^2 \tag{4.11}$$

with $c < a = f'(0)$, and $f'(\bar{x}) = a - 2b\bar{x} < 0$ where $\bar{x} > \hat{x}$ and $f'(\hat{x}) = 0$.

Thus the payoff of individual i becomes the following,

$$u(x_i,x_{-i}) = \begin{cases} e + \frac{g}{n}\left[T-\sum_{j\neq i} x_j\right] & \text{if } x_i = 0 \\ e + x_i\left[\beta - b\left(\sum x_i\right)\right] + \frac{g}{n}\left[T-\left(\sum_{j\neq i} x_j\right)\right] & \text{if } x_i > 0 \end{cases} \tag{4.12}$$

where $\alpha := (a-c)$ and $\beta := \left(\alpha - \frac{g}{n}\right)$

The **Nash Equilibrium** (NE) of this game is given by

$$x_i^* = \frac{\beta}{b\,(n+1)} = \frac{a-c-\frac{g}{n}}{b\,(n+1)} = \frac{n\alpha - g}{bn\,(n+1)} \tag{4.13}$$

Under this equilibrium, people act in such a way as to end up being worse off individually than if they acted collectively. Individuals' Nash behavior is not collectively optimal. The thesis of the tragedy of the commons is captured (Hardin (2017)) by this result. They could make it better by finding a way to cooperate or by drifting away from the standard strategic reasoning.

Now, assume that individual actors reason à la Kant: *—I would like to increase my extraction so that I get as much as I can from the resource, but I should do so only if all others could similarly increase their efforts, and that I would not like*, Roemer (2015).Let us see this this optimization at works:

The F.O.C defining multiplicative KE of the CPR-PG game are

$$\alpha x^{\star} - 2bx^{\star}\left[x^{\star} + \sum x_{j\neq i}\right] - \frac{g}{n}\left[x^{\star} + \sum x_{j\neq i}\right] = 0$$

By symmetry,

$$(\alpha - g)\, x^{\star} - 2bnx^{\star 2} = 0$$

$$x_i^{\star} = \frac{\alpha - g}{2bn} \tag{4.14}$$

As the CPR-PG game is a strictly monotone game,[6] thus $x^{\star}$ is Pareto efficient. Moreover, given the symmetry of this game, this equilibrium is actually a *simple Kantian equilibrium*. In other words, among all strategy profiles belonging to the *isopraxis* set —the set of strategy profiles where all players play the same strategy, $x^{\star}$ is that strategy a Kantian player would like everyone to play (Roemer (2019b)). On another note, observe that both equilibira, (Equation (4.13)) and (Equation (4.14)) decrease on g, as the conservation externality increases, players will reduce their level of appropriation. Also, for any $n > 1$, the reader can easily check that (Equation (4.14)) is lower than (Equation (4.13)). Kantians appropriate less than Nashers.

[6] The more a person appropriates from the resource, the less yield (s)he will receive.

4.4 Evolutionary Dynamics

Let us say now that there large population of appropriators, a fraction of θ appropriators are motivated by doing what is right. Which, in turn, depends mainly on what others do (Elster (2017)). Say that the appropriators are driven by the inner quasi-moral norm that states "to extract that part of the common resource you would like other appropriators to extract." We call those appropriators "Kantian appropriators or simply Kantians" who optimize according to the Kantian protocol. Notice that for a quasi-moral norm to be triggered, the agent need not have individual-level knowledge about what others are doing: aggregate information may be sufficient. Its efficacy depends on the agent seeing (or getting to know about) what other people do (Elster (2017)). Then, appropriators follow a quasi-moral norm when reducing their level of extraction even without knowing individual extractions of others but knowing aggregate extraction.

Suppose as well that the other fraction, $1-\theta$, is made of Nash appropriators (Nashers) who optimize in the standard way. Now, in the same spirit of Roemer and Curry (2012), we consider the population of appropriators(Kantians and Nashers) playing the CPR-PG game in pairs formed randomly each period so that we can understand the role of a quasi-moral norm as a mechanism that promotes cooperation from an evolutionary perspective. Let us say that an appropriator knows that with probability θ [s]he will play with a Kantian appropriator and with probability $1-\theta$ [s]he will play with a Nash appropriator. Also, say that Nashers again will play according to their ethos. Under this scenario the expected payoff function of the Kantian individual taking x_j as distinct from x_i is

$$\arg\max_{r>0}\left\{\theta\left[rx_i\beta \ -br^2x_i\left(x_i+x_j\right)+\tfrac{g}{2}\left(T-rx_j\right)\right]+(1-\theta)\left[rx_i\beta-r^2x_i^2b-brx_iy+\tfrac{g}{2}\left(T-y\right)\right]\right\}=1. \tag{4.15}$$

An analogous maximization problem must be solved for individual j, if j too is a Kantian maximizer:

$$\arg\max_{r>0}\left\{\theta\left[rx_j\beta \ -br^2x_j\left(x_i+x_j\right)+\tfrac{g}{2}\left(T-rx_i\right)\right]+\left(1-\theta\right)\left[rx_j\beta-r^2x_j^2b-brx_jy+\tfrac{g}{2}\left(T-y\right)\right]\right\}=1. \tag{4.16}$$

By Solving now Equation (4.15) is obtained the following:

$$x_i=\frac{2\beta-2by\left(1-\theta\right)-\theta g}{4b}-\theta x_j. \tag{4.17}$$

same procedure for Equation (4.16),

$$x_j=\frac{2\beta-2by\left(1-\theta\right)-\theta g}{4b}-\theta x_i. \tag{4.18}$$

Then we solve the system given by Equation (4.17) and Equation (4.16) and obtain the solution

$$\left(x_i^\star,x_j^\star\right)=\left(\frac{2\beta-\theta g-2by\left(1-\theta\right)}{4b\left(\theta+1\right)},\frac{2\beta-\theta g-2by\left(1-\theta\right)}{4b\left(\theta+1\right)}\right)$$

so $x_i^\star=x_j^\star:=x$, then we have

$$x=\frac{2\beta-\theta g-2by\left(1-\theta\right)}{4b\left(\theta+1\right)} \tag{4.19}$$

Which is the Kantian level of extraction as a function of the level of extraction y.

Analogously, the payoff function of a **Nasher** individual i is given by:

$$u_i\left(y_i,x_j;\theta\right)=\theta\left[e+y_i\left(\beta-b\left(y_i+x_j\right)\right)+\tfrac{g}{2}\left(T-x_j\right)\right]+\left(1-\theta\right)\left[e+y_i\left(\beta-b\left(y_i+y_j\right)\right)+\tfrac{g}{2}\left(T-y_j\right)\right]. \tag{4.20}$$

where x_j and y_j are the appropriation levels chosen by his opponent j, if Kantian and Nasher, respectively. Next, Nasher individual i maximizes Equation (4.20) wrt y_i, so

$$\frac{\partial u_i\left(y_i,x_j;\theta\right)}{\partial y_i}=0 \tag{4.21}$$

solving Equation (4.21), we obtain:

$$y_i=\frac{\beta+\theta b\left(y_j-x_j\right)-by_j}{2b} \tag{4.22}$$

We proceed now in the same manner for individual j is a Nasher and (s)he believes that there is probability θ that i is a Kantian optimizer, so

$$y_j = \frac{\beta + \theta b\left(y_i - x_i\right) - b y_i}{2b} \tag{4.23}$$

so let us put $y_i = y_j$ and $x_i = x_j = x$. From Equation (4.22)-Equation (4.23), we get:

$$y = \frac{\beta - \delta b x}{b\left(3 - \theta\right)} \tag{4.24}$$

so Equation (4.24) is the level of appropriation of a Nasher optimizer as a function of the level of extraction x. Together Equation (4.19) and Equation (4.24) define a system of equations with two unknowns. They are a sort of reaction functions of both type of players, so by solving this system we obtain:

$$x^k = \frac{\left(3 - \theta\right)\theta g - 4\beta}{b\left(\theta\left(2\theta - 6\right) - 12\right)} \qquad y^\eta = \frac{\theta^2 g + 2\beta\left(\theta + 2\right)}{2b\left(6 + 3\theta - \theta^2\right)} \tag{4.25}$$

We take the terms in eq. (4.25) to determine the fitness of each kind of agent. Then, the **fitness** of a **Kantian appropriator** is

$$e + x^\kappa \beta - (x^\kappa)^2 b - b x^\kappa y + \frac{g}{2}(T - y) + \frac{\theta}{2}(y - x^\kappa)(2bx^\kappa + g) \tag{4.26}$$

and the **fitness** of a **Nash appropriator** is given by:

$$e + y(\beta - 2by) + \frac{g}{2}\left[T - y\right] + \theta(y - x^\kappa)(yb + \frac{g}{2}) \tag{4.27}$$

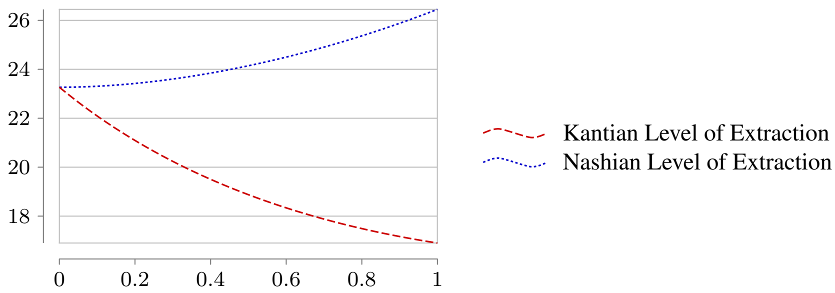
The question begs: will natural selection oppose an invasion of Nashers appropriators into Kantian appropriators population? Let us assume that there is an infinitesimally small quantity of Nasher invaders, thus the frequency of Kantian appropriators approaches one. Then, we examine whether, under the concept of Smith (1982), Smith et al. (1989), Kantian protocol of optimization represents an Evolutionarily Stable Strategy (ESS). Proposition 6 states that it is actually not the case.

Proposition 6 (*Extinction*)

In the randomly matched appropriators CPR-PG game Kantian appropriators become to extinct by Nash appropriators.

Proof: See Appendix A, Section 4.6.1 ■

As θ (share of Kantians in the population) approaches to 1, the Nash appropriators fitness is greater than the Kant appropriators fitness. Hence, the former invades the latter. Kantians are not evolutionarily stable, so they are decimated. From Equation (4.25) it can be shown that the level of extraction of Kantians decreases as they increase in number, whereas the level of the extraction of Nashers exhibits the opposite effect: it increases as the number of Kantiasn increases. Kantians get lower payoffs with respect Nashers, since the former extract less than the former. See the next plot for an example. The horizontal axis is the share of Kantian in the population, and the vertical axis is the level of extraction for each type of individual.



4.4.1 Formation of Groups

In the previous setting, the expected payoff differential exerts evolutionary pressure on the population composition to the point in which Nashers invade Kants. This takes us to inquire about other possible scenarios. Under what other circustances can Kantian appropriators become *stable*? Does the conservation externality g play a role on it?

4.4.1.1 Fixed Group Formation

Consider again a group or community of n appropriators with access to the common pool resource X, k of these are Kantians, and $n - k$ are Nashers. What would be their respective appropriation levels? Let us find out.

Proposition 7 (***Kantian Optimal Level of Appropriation Response***)

The Kantian optimal level of appropriation response of a group that is made up of k Kantians, so $n-k$ Nashers in the CPR-PG game is given by:

$$x_\kappa = \frac{\alpha \quad - b\,(n-k)\,x_\eta - \frac{g}{n}k}{2bk} \tag{4.28}$$

Proof : See Appendix A, Section 4.6.2 ■

Remark 1

Notice that when $k = n$, $x_\kappa = x^\star$, which is the original KE (Equation (4.14))

Analogously,

Proposition 8 (***Nash Optimal Level of Appropriation Response***)

The Nash optimal level of appropriation response of a group that consists of k Kantians, so $n-k$ Nashers in the CPR-PG game is given as follows

$$x_\eta = \frac{\alpha - \frac{g}{n} - bkx_\kappa}{b\,(n-k+1)} \tag{4.29}$$

Proof : See Appendix A, Section 4.6.3 ■

Remark 2

Notice that when $k = 0$, $x_\eta = x^*$, which is the original NE (Equation (4.13))

Thus, the actual level of appropriations corresponding to each kind of appropriators are stemmed from Equation (4.28) and Equation (4.29) as the next proposition states.

Proposition 9 (***Kantian and Nash Best Responses***)

In the CPR-PG game, in a community constituted of k Kantians and $n-k$ Nashers, the level of appropriation of a subject member of former group is $x^\star_\kappa = \left(\frac{1}{b}\right)\left[\frac{\alpha+g}{k(n+2-k)} - \frac{g}{n}\right]$, whereas the level of appropriation of a subject member of the latter group is $x^*_\eta = \frac{\alpha n + g(k-2)}{bn(n+2-k)}$ for $0 < \alpha$, $0 < b$, and $0 < k < n$.

Proof: See Appendix A, Section 4.6.4 ■

Given the corresponding level of appropriation stated in Proposition 9, we get now the common aggregate extraction level, $X = kx_\kappa + (n-k)\,x_\eta$, and so we get the Kantian and Nash payoffs.

$$X = \frac{k}{b}\left[\frac{\alpha + g}{k\,(n-k+2)} - \frac{g}{n}\right] + (n-k)\left[\frac{\alpha n + g\,(k-2)}{bn\,(n-k+2)}\right] = \frac{\alpha\,(n-k+1) - g}{b\,(n-k+2)} \tag{4.30}$$

4.4.1.1.1 Payoffs

Nashers then will get:

$$u_\eta\left(x_\eta^*, x_\kappa^\star\right) = e + x_\eta^*\left[\beta - bX\right] + \frac{g}{n}\left[T - \left(X - x_\eta^*\right)\right] = e + \frac{1}{b}\left[\left(\frac{\alpha + g}{n+2-k}\right)^2 - \frac{\alpha g}{n}\right] + \left(\frac{g}{n}\right)T \tag{4.31}$$

and Kantians will get:

$$u_\kappa\left(x_\kappa^\star, x_\eta^*\right) = e + x_\kappa^\star\left[\beta - bX\right] + \frac{g}{n}\left[T - \left(X - x_\kappa^\star\right)\right] = e + \frac{1}{b}\left[\frac{1}{k}\left(\frac{\alpha + g}{n+2-k}\right)^2 - \frac{\alpha g}{n}\right] + \left(\frac{g}{n}\right)T \tag{4.32}$$

Observe that $u_\kappa\left(x_\kappa^\star, x_\eta^*\right) \leqslant u_\eta\left(x_\eta^*, x_\kappa^\star\right)$. By optimizing in the traditional way of doing, Nashers do better in this situation. On the other hand, notice that if the whole group were composed of only Nashers (i.e. $k = 0$), a player would get the following payoff:

$$u_\eta\left(x_i^*, x_{j\neq i}^*\right) = e + \frac{(g - \alpha\,n)\,(gn - \alpha)}{bn\,(n+1)^2} + \frac{g}{n}T \tag{4.33}$$

Whereas if the whole community were consisted of only Kantians, (i.e. $k = n$), each member would obtain the next payoff:

$$u_\kappa\left(x_i^\star, x_{j\neq i}^\star\right) = e + \frac{(\alpha - g)^2}{4bn} + \frac{g}{n}T \tag{4.34}$$

Clearly, $u_\eta\left(x_i^*, x_{j\neq i}^*\right) \leqslant u_\kappa\left(x_i^\star, x_{j\neq i}^\star\right)$. And it is easy to see that by comparing eq. (4.31), eq. (4.32), eq. (4.33), and eq. (4.34), the following train of inequalities 4.35 holds.

$$u_\eta\left(x_i^*, x_{j\neq i}^*\right) \leqslant u_\kappa\left(x_\kappa^\star, x_\eta^*\right) \leqslant u_\eta\left(x_\eta^*, x_\kappa^\star\right) \leqslant u_\kappa\left(x_i^\star, x_{j\neq i}^\star\right) \tag{4.35}$$

From the above it follows that a Nash player would prefer to be in a group with some Kantians than in a community of only Nashers. In contrast, a Kantian player would prefer finding her[him]self appropriating the common resource in a group of only Kantians instead of doing it together with other Nashers. And Nashers do better when they encounter Kantians than when they actually face other Nashers. Conversely, Kantians do worse when they enter into play with Nashers than when they do it with other Kantians.

4.4.1.2 Overall Welfare

Let us compare now the overall welfare for each type of group community —Nasher, Kantian, Kantian-Nash.

Nash Community

$$U_{\eta=n} = n\left[u_\eta\left(x_i^*, x_{j\neq i}^*\right)\right] \tag{4.36}$$

Kantian-Nash Community

$$U_{\eta+k=n} = (n-k)\left[u_\eta\left(x_\eta^*, x_\kappa^\star\right)\right] + k\left[u_\kappa\left(x_\kappa^\star, x_\eta^*\right)\right] \tag{4.37}$$

Kantian Community

$$U_{k=n} = n\left[u_\kappa\left(x_i^\star, x_{j\neq i}^\star\right)\right] \tag{4.38}$$

Table 4.2 summarizes the whole welfare for each type of community.

Nashers $[U_{\eta=n}]$	Nasher-Kantians $[U_{\eta+k=n}]$	Kantians $[U_{k=n}]$
$ne + \frac{(g-\alpha n)(gn-\alpha)}{b(n+1)^2} + gT$	$ne + \frac{1}{b}\left[\left(\frac{\alpha+g}{n-k+2}\right)^2 [n-k+1] - \alpha g\right] + gT$	$ne + \frac{(\alpha-g)^2}{4b} + gT$

Table 4.2: *Overall Welfare for each type of community*

Not surprisingly, for the given parameters, we see in Table 4.2 that $U_{\eta=n} < U_{\eta+k=n} < U_{k=n}$. A community constituted of Kantians is far better off than both a mixed community and a Nash community. At the same time, the mixed community is only better off than the Nash community. In any case, the mere presence of Kantian appropriators in a community improves the overall welfare of it with respect to the overall welfare of a Nash community.

4.4.1.3 Random Group Formation

We just observed that a fixed community (group) of Kantians and Nashers is able to improve individual benefits (Inequalities 4.35). And that, not surprisingly, this is explained by the mere presence of Kantian-like appropriators. These latter getting lower payoffs though. The next step now is to introduce randomness to the conformation of group appropriators. Thus, drawing upon Sethi and Somanathan (2005),we contemplate said formation by randomly and repeatedly sampling groups of n appropriators out of a large global population comprised of Kantians and Nashers. Random group formation allows us to work with a degree of heterogeneity in the formed group different from a fixed group. Can we expect to have a well mixed group of Nashers and Kants stable over time? Consider the share of Kantians θ in the global population. Thus, the probability that a community formed in this manner has exactly k *Kantians* (and so $\eta = [n-k]$ *Nashers*) is given by $p(k,\theta) = \binom{n}{k}\theta^k (1-\theta)^{n-k} := p_k$. Then, the payoff reached by Kantians (Nashers) in any given group is configured by its composition (number of appropriators of each *ethos*), so the average payoff to Kantians (Nashers) in the population as a whole is computed by taking a weighted average of Kantians (Nashers) payoffs. The weight is applied to each possible group in proportion to the probability with which it will come up.

4.4.1.3.1 Kantians Expected payoffs

The expected payoff of Kantians (k) in the population as a whole then will be:

$$\bar{u}_k(\theta) = \frac{\sum_{k=1}^{n} p_k u\left(kx_\kappa^\star, [n-k]\, x_\eta^*\right)}{\sum_{k=1}^{n} p(k,\theta)} \tag{4.39}$$

4.4.1.3.2 Nashers Expected payoffs

And the Nashers (η) expected payoff in the population as a whole will be the next:

$$\bar{u}_\eta(\theta) = \frac{\sum_{k=0}^{n-1} p_k u\left([n-k]\, x^*, kx^{\star\star}\right)}{\sum_{k=0}^{n-1} p(k,\theta)} \tag{4.40}$$

4.4.2 Dynamics

We can say now that when $\bar{u}_\eta(\theta)$ and $\bar{u}_k(\theta)$ are different, the population of Kantians and Nashers will vary as well. For which, we suppose the evolution of the population share θ is governed by the replicator dynamics:

$$\dot{\theta} = \theta\left[\bar{u}_k(\theta) - \bar{u}(\theta)\right] \tag{4.41}$$

rewriting eq. (4.41),

$$\boxed{\dot{\theta} = \theta(1-\theta)\left[\bar{u}_k(\theta) - \bar{u}_\eta(\theta)\right]}$$

where $\bar{u}(\theta)$ is the mean payoff of the population:

$$\bar{u}(\theta) = \theta\bar{u}_k(\theta) + (1-\theta)\,\bar{u}_\eta(\theta) \tag{4.42}$$

Under Equation (4.41) the proportion of Kantian appropriators in the population grows at a rate equal to the difference between the average payoff of playing Kant (the evolutionary potential of Kantians) and the average payoff of the population as a whole. In this setting, if an appropriator decides on what kind of strategy to take -i.e what behavior to adopt: being a Kantian or being a Nasher- [s]he might consider two things.

First, [s]he might want to see what others are doing (copying). In this sense, the odds of choosing a particular behavior optimization protocol are great when the proportion of appropriations following that strategy-behavior constitutes the majority in the whole population. In other terms, if the whole population is mainly comprised by Kantians, then such as appropriator player will be prone to behave Kantian-like with a probability associated to the frequency of Kantians. Whereas the behavior associated to the frequency of Nashers is ignored. Second, the appropriator might adopt the one behavior that yields higher payoffs (being traditionally rational). Thus, the replicator equation captures both things by considering this decision to be reliant on the frequency of a certain behavior (Kantian or Nash) times its associated payoff. In our context, this equation describes the dynamics of how the proportion of Nasher and Kants change over time, or which kind of behavior (cooperative or non-cooperative ethos) happens to be more widespread. As usual, the central point is that the share of appropriators who possess a greater fitness than the average fitness of the population (i.e. appropriators who are better suited) grows faster relative to the population with the lower fitness.

4.4.2.1 Stability Analysis

In this section we examine the stability of the equilibrium points of eq. (4.41). The underlying question is whether Kantians appropriators can survive and spread in evolutionary competition with Nash appropriators. And if so, under what conditions? Can the *Tragedy* be palliated under random group formation over time?

Let us begin with a situation in which the global population is mainly made up of Nash appropriators. This means that the frequency of Kantian appropriators is really small, i.e., approaching zero. Can Nash appropriators be invadable? Proposition 10 tells us the answer.

Proposition 10 (*Nash Appropriators is a Stable Population*)

Under the CPR-PG game, a population consisting of Nash appropriators alone is stable for all parameter values.

Proof: See Appendix A, Section 4.6.5 ■

As the global Kantian population share approaches zero, Nash appropriators find themselves in quasi Nasher homogeneous groups, meaning that Kantian appropriators will find themselves in groups such that they constitute a minority. That is to say, a Kantian appropriator will be in a group in which there are no other Kantians but [her]himself. Figure Figure 4.2 depicts such a case. Here, the presence of a Kantian does not induce Nasher appropriators to extract the resource at lower levels. Nash appropriators guarantee for themselves the most they can from the resources getting necessarily greater payoffs than Kantian appropriators. Thus, even when Kantian appropriators will appropriate a Pareto efficient levels, it is not enough for them to invade Nashers.

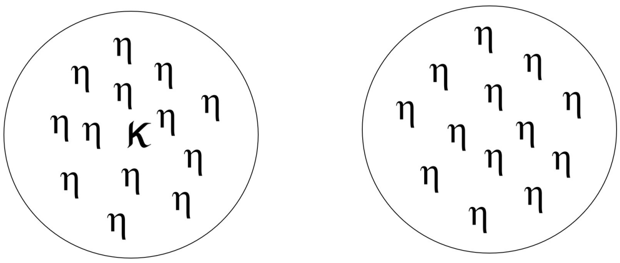


Figure 4.2: *Left Group: a Kantian appropriator among Nash appropriators. Right Group: only Nash appropriators.*

Let us now look at the opposite side. The global population is constituted mainly of Kantian appropriators, so the share of Nashers is very small. In this scenario, with a high probability those groups that might form will consist of only Kantian appropriators, whereas a Nash appropriator will be part of a community where [s]he is the only one of h[is]er *ethos*, see figure Figure 4.3. The question looms: does the presence of a Nash appropriator lead to instability of a population constituted of Kantian appropriators? The answer is no. As stated in proposition 11 a Kantian population will be uninvaded by Nash appropriators, meaning that the presence of a Nash appropriator in a group like that portrayed in the left side of figure Figure 4.3 does not assure that [s]he gets a greater payoff than a Kantian does when this latter is in a group consisting of only

Kantian appropriators —right side of figure Figure 4.3. Therefore, for groups formed when the Kantians constitute almost the totality of the global population, selection will favor them over Nashers. Therefore, Kantian population alone will be always stable in this context.

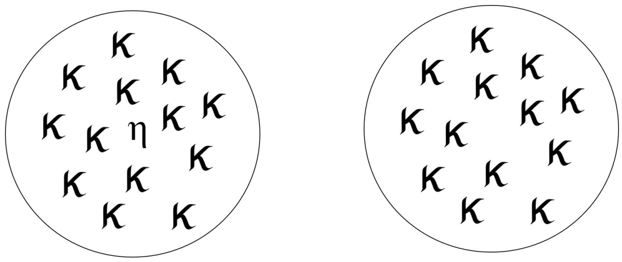


Figure 4.3: *Left Group: a Nash appropriator among Kantians appropriators. Right Group: only Kantian appropriators.*

Proposition 11 (***Kantian Appropriators is a Stable Population***)

In the CPR-PG game under random group formation, a population of Kantian appropriators alone will be stable for all parameters values.

Proof: See Appendix A, Section 4.6.6 ■

We have just studied the stability of the equilibria points. Interestingly, natural selection favors both types of appropriation behavior. For the given parameters, Nash and Kantians are bistable, see Figure 4.4. In other words, given that the stable rest points of this system are either zero or one, most of the formed groups will be made up of either Nashers or Kantians, so if a player were to decide as to what behavior protocol to abide by, [s]he should try to adopt the same behavior as others in the formed group. Thus, the decision ultimately hinges on what group [s]he falls into. Thus, being Kantian is the best response for a Kantian group. Conversely, being Nasher is the best response for a Nash group. Accordingly, the outcome of the selection dynamics within the population will depend on the initial condition (see figure Figure 4.5 and figure Figure 4.6 for an illustration). Therefore, following a Kantian protocol of optimization does not always constitute an evolutionary advantage relative to the Nash optimization and vice versa. On the other hand, stable polymorphic states were not found. Which suggests that, contrary to the facts, we should not expect to see a mix of Kantian and Nash appropriators coexisting over time. Also, worthy of note is that the conservation aspect of the resource,g, does not bear on preventing or allowing an encroachment of or by one type of appropritaors over the other as drawn from proposition 11 and proposition 10. That is say, for the values g might take, Nashers do not overrun Kantians; and conversely, Kantians do not invade Nashers.

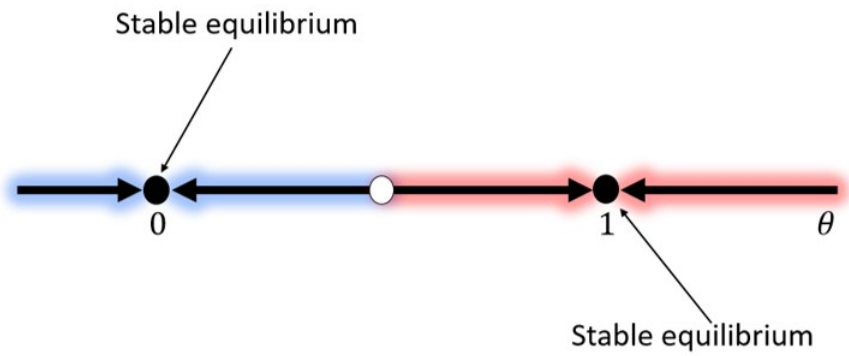


Figure 4.4: *Phase diagram of eq. (4.41), selection dynamics: Kantians and Nashers are bistable*

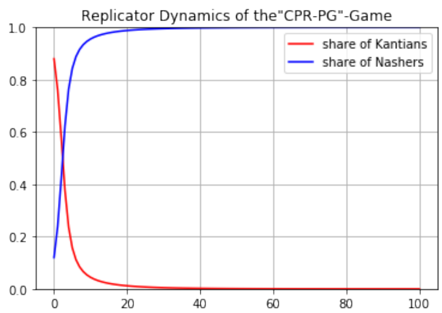


Figure 4.5: *Selection dynamics favors Nash appropriators over Kantian appropriators.*

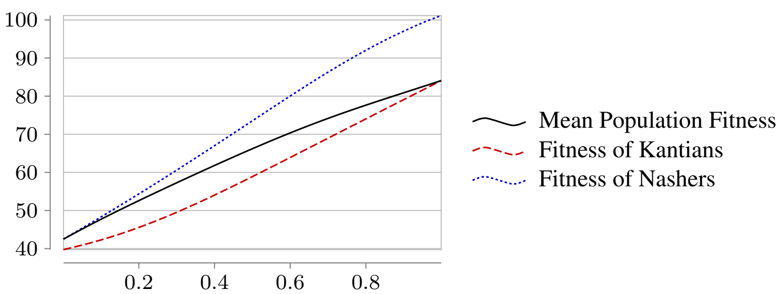


Figure 4.6: *Payoff Finesses and Mean Population Fitness*

4.5 Summary

To wrap up, in this chapter we have applied the Kantian optimization to an extension version of the classical Common-Pool Resources appropriation game of Ostrom et al. (1994). The model is simply built by adding the conservation value of the resource to the payoff of each player. This preservation feature together with the traditionally studied appropriation aspect are particularly relevant in natural CPRs, but typically cooperation in the two cases is studied separately. Thus, this work is somewhat novel in that both the appropriation problem and the conservation part regarded as a public good are considered. In this connection, some studies from *CPRs* literature, where individuals might have found out a reasoning akin to the Kantian equilibrium, were pointed out.

Then, in the second part of the work, the evolutionary stability of the Kantian optimization was studied using the aforementioned extended model as a baseline. When players are randomly paired from a large global population, appropriators who are following an appropriation strategy derive from the Kantian protocol are doomed to extinction by Nash appropriators.

In the last part of the work, we supposed the formation of a group of Kantians appropriators playing the CPR-PG game with some Nashers. The presence of Kantians in the community enhances the individual benefits of all members ; notwithstanding, Nash appropriators benefit the most. Thus, it is good for them to be in a mixed community rather than in a community of only Nashers. Conversely, it is not good for a Kantian to be with other Nashers. And naturally, (s)he would do better in a Kantian community. In this line of reasoning, we noted that, in terms of the overall welfare of each type of community, a mixed community does better than a Nash community, and a Kantian community is better off than both a mixed and a Nash community.

Next, random group formation was introduced into the picture. We supposed a Replicator Dynamics that governs the growth of Kantian appropriators ‘proportion.

The results show that nature will reward both kind of behaviors. Kantians alone are a stable group population just as Nashers alone are. The key difference with the previous scenario is that the introduction of randomness in the group formation allows to explain stable cooperative groups. Under the monomorphic equilibria points, the global population is comprised mostly by Nashers (Kantians).Thereof, the possible formed groups will consist of a really few Kantian (Nashers). Against this background, the results suggest that the reproductive success of Nashers (Kantians) will unavoidably excel that one of the Kantians (Nashers) in any given group when a large part of the percentage of the whole population is Nash (Kant). In other terms, few Kantians (Nashers) will not turn a group into its opposite *ethos*, meaning that the different formed groups are not invadable and that the players with a minority *ethos* will not survive.

To put it differently, if the users of the resource were to play the game in which the available strategies are "optimizing a la Kant (quasi-moral norm)" and "optimizing a la Nash," and choosing then their associated extraction strategies, our results shows that the outcome of the game is consistent with the notion of *quasi-moral norm* of Elster (2017), which is driven by the will of doing the right thing (take the action s(h)e would like all to take), yet the right thing conditioned largely to what others do. That is to say, an appropriator should try to play the same choice as the other appropriators: to play Kant when you are surrounded by Kantians and to play Nash when you are surround by Nashers. In this evolutionary scenario, therefore, that the tragedy be surmounted or not will be hinge on the outcome of the selection dynamics. Which, in turn, is subject to the initial condition of the system. This is in line with some results already in the literature. As Roemer and Curry (2012) explain, for Kantian appropriators always to survive they must learn to recognize who they are playing with. That is to say, if Kantian appropriators knew they are playing with a Nash appropriator, then they should simply play à la Nash. Then, Kantians would outperform Nash in expectation. Thereupon, a justification of why we observe that a moral behavior *per se* falls short of outcompeting a Nash behavior is the reflection made by Arrow that "one must not expect miraculous transformations in human behavior. Ethical codes, if they are viable should be limited in scope." Arrow (1973).

Finally, for future research directions that capture more realistic situations, one idea to be explored is that the resource itself might be changing over time according to the appropriations of players, and that they adapt to it. Then, a trade off between Kantian and Nashers might exist. New literate points toward this direction Hilbe et al. (2018). Also, another equilibrium concept can be applied to the CPRs problem, such as the generalized Kantian-Nash equilibrium proposed by Grafton et al. (2017).

4.6 Appendix A

4.6.1 Proof of Proposition 6: Extinction

Proof: We depart from term $(\alpha + g)^2$, which is strictly positive. Now we write the expanded form $\alpha^2 + 2\alpha g + g^2$, which is the same as

$$9\alpha^2 + 9g^2 - 14\alpha g - 8\alpha^2 + 16\alpha g - 8g^2 > 0 \tag{4.43}$$

rearranging inequality (4.43)

$9\left(\alpha^2 + g^2\right) - 14\alpha g - 8\left(\alpha - g\right)^2 > 0$

we can re-write:

$9\left(\alpha^2 + g^2\right) - 14\alpha g > 8\left(\alpha - g\right)^2$

the right-side can be rewrite as: $\frac{64}{8}\left(\alpha - g\right)^2$, then:

$\frac{9\left(\alpha^2 + g^2\right) - 14\alpha g}{64} > \frac{(\alpha - g)^2}{8}$

nothing change if we divide both sides by b, so

$\frac{9\left(\alpha^2 + g^2\right) - 14\alpha g}{64b} > \frac{(\alpha - g)^2}{8b}$

we can also add $e + \frac{g}{2}$, so

$$e + \frac{9\left(\alpha^2 + g^2\right) - 14\alpha g}{64b} + \frac{g}{2} > e\frac{(\alpha - g)^2}{8b} + \frac{g}{2} \tag{4.44}$$

The left side of inequality (4.44) is nothing more than:

$$\lim_{\theta \to 1}\left[e + y(\beta - 2by) + \frac{g}{2}\left[T - y\right] + \theta(y - x^{\kappa})(yb + \frac{g}{2})\right] \tag{4.45}$$

and the right side of inequality (4.44) is

$$\lim_{\theta \to 1}\left[e + x^{\kappa}\beta - (x^{\kappa})^2 b - bx^{\kappa}y + \frac{g}{2}(T - y) + \frac{\theta}{2}(y - x^{\kappa})(2bx^{\kappa} + g)\right] \tag{4.46}$$

Where the term in brackets in Equation (4.45) is the Nash appropriator fitness (Equation (4.27)), while the term in bracket in Equation (4.46)) is the Kantian appropriator fitness (Equation (4.26)). Thus, Nashers are better suit, and they drive Kantians to extinction. ■

4.6.2 Proof of Proposition 7: Kantian Optimal Level of Appropriation Response

Proof: Let us say that x_κ is a Kantian equilibrium level of appropriation. Thus, under thus protocol of optimization, it must hold that:

$$\arg\max_r u\left(kx_\kappa, [n-k]\, x_\eta\right) = 1 \tag{4.47}$$

Rewriting (Equation (4.47))

$$\arg\max_r e + rx_\kappa\beta - brx_\kappa\left[krx_\kappa + (n-k)\, x_\eta\right] + \frac{g}{n}\Big[T - \left[(k-1)\, rx_\kappa + (n-k)\, x_\eta\right]\Big] = 1 \tag{4.48}$$

Then

$$\max_r e + rx_\kappa\beta - brx_\kappa\left[krx_\kappa + (n-k)\, x_\eta\right] + \frac{g}{n}\Big[T - \left[(k-1)\, rx_\kappa + (n-k)\, x_\eta\right]\Big]$$

F.O.C

$$\left.\frac{du(\cdot)}{dr}\right|_{r=1} = 0$$

$$x_\kappa\beta - 2bx_\kappa^2 k - bx_\kappa\,(n-k)\, x_\eta - \frac{g}{n}(k-1)\, x_\kappa = 0$$

$$x_\kappa = \frac{\beta - b\,(n-k)\, x_\eta - \frac{g}{n}(k-1)}{2bk} \tag{4.49}$$

since $\beta = \alpha - \frac{g}{n}$, then

$$x_\kappa = \frac{\alpha \quad - b\,(n-k)\, x_\eta - \frac{g}{n}k}{2bk} \tag{4.50}$$

■

4.6.3 Proof of Proposition 8: Nash Optimal Level of Appropriation Response

Proof: A player j with a non-cooperative *ethos* appropriates at the level x_{η_j} that maximizes his payoff treating the levels of all others' extraction strategies as parameters.

$$\max_{x_\eta} e + x_{\eta_j}\left[\beta - bX\right] + \frac{g}{n}\left[T - \left(X - x_{\eta_j}\right)\right]$$

F.O.C

$$\beta - b\left[x_{\eta_j} + X\right] = 0$$

thus, a player who has opted for appropriating à la Nash must extract the resource at:

$$x_{\eta_j} = \frac{\beta - bX}{b} \tag{4.51}$$

And this is true for all $n - k$ non-cooperative appropriators. Also, given that X is the aggregate extraction and is common to all, each appropriator who behaves in a competitive way will thus appropriate at the same level, say x_η, so

$$X = kx_\kappa + (n-k)x_\eta \tag{4.52}$$

Plugin (Equation (4.52)) into (Equation (4.51)), we obtain

$$x_\eta = \frac{\beta - bkx_\kappa}{b\,(n-k+1)} \tag{4.53}$$

again, knowing that $\beta = \alpha - \frac{g}{n}$

$$x_\eta = \frac{\alpha - \frac{g}{n} - bkx_\kappa}{b\,(n-k+1)} \tag{4.54}$$

■

4.6.4 Proof of Proposition 9: Kantian and Nash Best Responses

Proof: (Equation (4.28)) and (Equation (4.29)) define a system of two linear equations with two unknowns,

$$\begin{cases} 2bkx_\kappa + \quad b\,(n-k)\,x_\eta = \alpha - \dfrac{g}{n}k \\ bkx_\kappa + b\,[n-k+1]\,x_\eta = \alpha - \dfrac{g}{n} \end{cases} \tag{4.55}$$

so after some algebraic manipulations, we have get the result.

$$x_\kappa^\star = \frac{\alpha n - g\left[k\,(n+2-k) - n\right]}{bkn\,(n+2-k)} = \left(\frac{1}{b}\right)\left[\frac{\alpha + g}{k\,(n+2-k)} - \frac{g}{n}\right] \qquad x_\eta^* = \frac{\alpha n \ + g\,(k-2)}{bn\,(n+2-k)}$$

Then, $x_\kappa^\star$ is a best Kantian response to a Nash appropriator, and x_η^* is a best Nash response to a Kantian appropriator. ■

4.6.5 Proof of Proposition 10: Nash Appropriators is a Stable Population

Proof: The equilibrium $\theta = 0$ is *stable* if $\bar{u}^k(0) < \bar{u}^\eta(0)$.

- $\lim_{\theta \to 0} \bar{u}^k(\theta) = e - \frac{\alpha g}{bn} + \frac{g}{n}T = \bar{u}^k(0)$ and • $\lim_{\theta \to 0} \bar{u}^\eta(\theta) = e + \frac{(g-\alpha n)\,(gn-\alpha)}{bn\,(n+1)^2} + \frac{g}{n}T = \bar{u}^\eta(0)$

Set $u(0) = \bar{u}_\eta(0) - \bar{u}_k(0)$.

$$u(0) = \frac{(g - \alpha\, n)\,(gn - \alpha)}{(n+1)^2} + \alpha g \tag{4.56}$$

eq. (4.56) can be rewritten as

$$(\alpha + g)^2 \, \frac{n}{(n+1)^2}$$

which is positive, then $\bar{u}^\eta(0) > \bar{u}^k(0)$ for all parameters given. Hence $\theta = 0$ is a *stable* rest point. ■

4.6.6 Proof of Proposition 11: Kantian Appropriators is a Stable Population

Proof : The equilibrium $\theta = 1$ is *stable* if $\bar{u}_\eta(1) < \bar{u}_k(1)$, so

- $\lim_{\theta \to 1} \bar{u}_k(\theta) = e + \frac{(\alpha - g)^2}{4bn} + \frac{g}{n} T = \bar{u}_k(1)$ and
- $\lim_{\theta \to 1} \bar{u}_\eta\,(\theta) = e - \frac{\alpha g}{bn} + \frac{g}{n} T = \bar{u}^\eta(1)$

Set $\bar{\nu}(1) = \bar{u}_k(1) - \bar{u}^\eta(1)$, then

$$\bar{\nu}(1) = \frac{(\alpha - g)^2}{4bn} + \frac{\alpha g}{bn} = (\alpha + g)^2 > 0$$

Katian population alone is stable. ■

CHAPTER 5

Conclusions

This dissertation consists of three stand-alone chapters that have the commons theme in common. Over the last few years, this topic and related matters have gained momentum in social sciences, especially in economics, due to its implications as an alternative or complementary framework that enables us to comprehend real-world scenarios that surpass the market-state approach. Also, the theory of commons has attracted attention of scholars across a range of disciplines considering that it has been proven useful to analyze a wide range of problems such as those of anthropogenic climate change, sharing knowledge, urbanity, ocean fishers, and global health (pandemics) to name but a few. This attractiveness, however, can lead to some misperceptions about their significance. For instance, according to some scholars, commons are shared resources within a self-governed community entitled by a sort of collective property regime, for others, commons are systems of local self-governance that imply more than shared resources. These differences were addressed in chapter two. We noticed that a broader concept of commons is given when they are conceived as a system where, resources (tangible or intangible), rights, boundaries, ways of actions, and arrangements are clearly established. We distinguished, based on Bollier and Helfrich (2019), commons from common and public goods as defined in economics, we also saw the difference between common pool resources (CPRs) and public goods as conceived from Ostrom (2010). Moreover, we noticed that CPRs can be categorized in open-access resources and common-property resources in contrast to private property resources. Also, we described the problems of appropriation and provision (conservation) typical of CPRs. In this direction, we presented the nature of CPRs, and we discuss the diverse ownership regimes and property rights identified in the literature. Additionally, when we talk about CPRs in economics, we necessary look at the incentives and conditions involved people have when it comes down to sustainable management of these resources. Here, we discussed the problem of overuse of CPRs stemmed from the lack of mechanisms that guarantee a sustainable use. The literature om CPRs puts emphasis on the fact that economic incentives stem from the state regulation or the privatization of the resource not only may not work but also could be counterproductive. Then we mentioned the importance of social capital in the governance of CPRs. Studies suggest that social capital plays a positive role in the management of CPRs. Next, we also reviewed some common variables involved in the emergence of cooperation in CPRs derived from experimental psychology. And we ended this initial chapter by putting forward the origins of peer-governance of Commons as systems identified by Boldlier and Helfrich, namely, spontaneous attraction, tradition, and conscious design.

On another note and inspired by cases in which cooperation is observed through the formation of groups of cooperative individuals within a community, in chapter three we focused on the formation of groups and coalitions using the well know model of Ostrom et al. (1994), which captures the problem of appropriation in a CPRs situation.

The setting is described by a strategic game in which players (appropriators) with access to a common resource decide upon their level of appropriation they want to take. The issue lies on the fact that each induvial level of apparition affects the total benefit derived from the extraction of the common resource, and that each rational member of the community will not appropriate at socially efficient levels. Appropriators then face a negative externality due to overuse of the resource. Against this backdrop, we studied the consequences, for the members of the community, of having a cooperative group. We analyzed the conditions under which this group may actually have interest in playing it out, and how it can affect the decision of non-cooperative people in terms of appropriating the resource. We found that the latter want to increase their extraction levels as a consequence of the group's cooperative action. Then we continued our study of group formation but now from another approach. Then, after having presented studies related to the formation of coalitions in real CPRs cases, we examined the same game of the first part of the chapter under the light of cooperative game theory, so we assumed that appropriators now are not involved in any kind of strategic interaction but rather know the possible gains should they join to a coalition -which will be determinant to explain cooperation. We transformed the original CPRs game into a game in partition function form. Therefore, each possible partition represents a possible way in which appropriators group, and each coalition now acts as if it were one single appropriator. Unlike the strategic game, this approach accepts appropriators to communicate in order to decide whether to form a coalition, and if so, to come to terms as of sharing the joined gains. The question is then how to share those benefits? We considered the solution concept introduced by Chander (2019) called γ-core. For which we showed that the partition function form of the CPRs game exhibits some important proprieties that guaranteed the existence of this core, and that the equal share payoffs vector belongs to it. First, it is symmetric in the sense that for every partition the worth of coalitions that contain the same number of members will be the same. Naturally, each appropriator receives an equal share of the value of the coalition (s)he belongs to. Second, the grand coalition -the coalition constituted of all involved appropriators - is the unique efficient partition in the sense that the worth of the grand coalition is greater or equal than the sum of the worth of the coalitions belonging to any partition other than the grand coalition. Third, the smaller the size of a coalition in each partition, the greater the payoffs its members get. Conversely, in coalitions with a larger number of members, each member receives a lower payoff than in small coalitions. Thus, we proved that when we have the strategic CPRs game transformed into its partition function form, the appropriators will prefer to create the grand coalition. By contrast with the outcome of the original game, this result tells us that the coalition approach here considered tackles successfully the *tragedy*. Cooperation is captured by the fact that it is in the interest of the appropriators to be part of the grand coalition because they know that in this way, they guarantee for themselves better payoffs. On the other hand, we also applied a game dubbed as payoff sharing game to our CRPs setting. It is a game of coalition formation in two stages repeated infinitely. Under this scenario, we showed that,under certain conditions on the size of coalitions, the γ-core payoff vectors of the partition function of the CPRs can be equilibrium payoff vectors of this game and that the grand coalition will be the equilibrium outcome. We concluded this chapter by pointing out that the coalition formation approach and the solution concept we used proved to be useful to explain the overcoming of the tragedy by rational appropriators. Somehow, it is a step forward in our intent to reconcile theory and some realistic CPRs contexts. Cooperation was supported through the appropriators' gains when they form a coalition. However, this approach, as it stands, has some limitations. It will be successful inasmuch as we focus on full cooperation. That is, partial cooperation, the emergence of group of appropriators other than the grand coalition, is not sustained.

In contrast to chapter three, chapter four puts strategic interaction among appropriators center stage. We investigated a sense of morality as one of the mottos for appropriators to cooperate, so we considered some microfoundations of cooperation currently in the literature. That is, we use the theory of Kantian optimization proposed by Roamer (2019) as our main framework. We opened the chapter by discussing how typically cooperation is modeled considering the role of social preferences. In general, some scholars consider social preferences such as altruism, reciprocity, and warm-glow as key mechanisms that encourage cooperation. And the case of CPRs is no exception. The way they do it is assuming that appropriators are rational individuals whose social preferences drive them to increase their own welfare. For instance, when a rational altruist individual extracts the common resource, (s)he obtains a better payoff out of the fact that others have some part of the said resource as well. Then, we stated why this way of modeling social preferences and cooperation conflicts with the real motives they intend to capture. Take the example of an altruist person. (S)he seeks to improve his or her payoff by doing an altruist act, so (s)he cooperates. However, these sequences of agencies do not represent altruism. Should an appropriator be altruist, (s)he does not need to seek personal benefits. In this context, the behavioral protocol used is one of competitive environments, so we explained that it is then a matter of looking into the decision-making process and mindset that is assumed about the appropriators. Then we briefly expounded the theory of Kantian optimization and its relationship with the CPRs. This theory, which as its name implies, is inspired in the categorical imperative of Kant. You take an action you would like to see universalized. Romer's theory offers to mind a decision according to this maxim, so appropriators optimize accordingly. In contrast to the traditional optimization, in this way of acting, appropriators think about the actions of others upon them as a result of their own actions. Thus, what the majority does will become a semi moral norm that can be captured by this optimization protocol. Then, this chapter applies the concept of Kantian protocol of optimization in a extended version of the Common Pool Resources (CPRs) game wherein the problems of appropriation and conservation of the resource are considered jointly. In this context, this work considers individuals that follow a moral behavior (Kantians) as well as those who follow the traditional strategic Nash behavior (Nashers). We explore the conditions under which the former agents can survive and spread in evolutionary competition with the latter. In general, a Kantian population is just as stable as a Nash population is.

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