Demand-led growth with endogenous innovation

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Abstract

This paper contributes to the recent macro-dynamics literature on demand-led growth, drawing upon J. Hicks (1950) seminal idea that the implications of Harrodian instability may be tamed by a source of autonomous expenditure in the economy. Contrary to the other contributions in this literature, real autonomous expenditure is not growing at an exogenously given rate, and partly consists of a flow of profit-seeking R&D and innovation expenditures raising labour productivity through time. If the state of distribution, hence the wage share, is exogenously fixed and constant, the model gives rise to dynamics in a two dimensional state space, that may converge to, or give rise to a limit cycle around, an endogenous growth path. An exogenous rise of the profit share exerts negative effects on long-run growth and employment, showing that growth is wage led.

Keywords: wage-led growth; endogenous autonomous expenditure; labour-saving technological progress; limit cycles.

JEL classifications: E11; E12; O41

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1 Introduction

Recent and less recent contributions to the macro-dynamics literature of demand-led growth (Freitas and Serrano, 2015; Allain, 2015; Lavoie, 2016; Serrano, 1995A,B) have revived the idea expressed long ago by Hicks (1950) that the implications of Harrodian instability may be tamed by a source of autonomous expenditure in the economy. Incidentally, this gave rise to a welcome convergence between different strands of thought in macrodynamics, of Sraffian and Kaleckian inspiration (Cesaratto, 2015; Trezzini and Palumbo, 2016; Serrano and Freitas, 2017; Lavoie, 2017). In these contributions, autonomous expenditure is mostly identified with an exogenously growing flow of either consumption or non-capacity creating government expenditure.

In this paper, we draw a sharp distinction between the terms autonomous and exogenous. What defines the autonomous character of expenditure is that it is not determined by (but may have a causal influence on) *short-run* output. To the extent that it is influenced by the stock of wealth accumulation, and by the progress of labour productivity, it is more appropriately labelled 'semi-autonomous'.[1] In what follows, semi-autonomous expenditure occurs in a market economy without government intervention and is supplied by three sources: (i) endogenous R&D activity; (ii) endogenous modernization expenditures carried out by firms producing final output, with the aim of introducing best practice knowledge into production; (iii) an exogenous flow e of money expenditure for consumption, financed by profits. The money wage is fixed and money prices fall with labour productivity, causing the growth of autonomous consumption. Firms, wishing to stay in the market, are forced by competition to carry out modernization expenditures, that are increasing with the rate of technological progress, and with the size of their capital stock. In this way, technological progress is introduced in an aggregate model with fixed capital, thus avoiding the complications of vintage models or of joint production. It may also be worth observing that, since technology in the final output sector is Leontiev, modernization expenditures are not capacity creating, in that the full capacity output at time t is proportional to the capital stock K_t, hence it is independent of labour productivity. To facilitate comparison with contributions (Freitas and Serrano, 2015; Allain, 2015; Lavoie, 2016) in which autonomous demand is exogenous, we provide, first, a preliminary version of the model in which modernization expenditures grow through time as a result of exogenous innovation.

In the more complex, endogenous-growth version of the model, modern-

[1] Other sources of semi-autonomous expenditure are identified by Fiebiger (2018) and Fiebiger and Lavoie (2017) in household investment and durable consumption financed by mortgage and consumer credit.

ization 'software' is supplied by a monopolist, holding a property right on the best practice technology, that results from profit seeking R&D. The existence and stability of the growth path requires in this case that the exogenous flow e is not too small.

In the present framework, the link between innovation and firms' expenditure is married with a second link between innovation and labour demand. The overall effect on aggregate demand dynamics will crucially depend on the way in which the productivity gains are distributed between wages and profits. At the present stage of our work, the state of distribution, hence the wage share, is exogenously fixed. The model gives rise to macro-dynamics in the two dimensional space of the state variables long-term expectations and efficiency-units of capital; dynamics may converge to, or give rise to a limit cycle around, an endogenous growth path. Long-run growth is wage led, in that the growth rate is a decreasing function of the profit share. The intuition is that a higher wage share exerts not only persistent level effects on employment, and on productivity-adjusted output and capital; the additional effect is that a larger capital stock expands the potential market of process innovations, thus providing stronger incentives to R&D, leading to faster labour-saving technological progress. In such conditions, a failure of institutions in preserving a constant wage share would produce self-reinforcing effects, by exerting a downward pressure on the level of employment, and would be associated with slower long-run output growth. Thus the model provides insights into the inter-relations between labour-saving technological progress, distribution and growth. These relations, together with the changing nature of policy action (that lies outside the scope of the present analysis) may contribute to explaining the post-1970s phase of slow growth in Europe and other OECD countries.

The organization of the paper is as follows. Section 2 provides an outline of the main arguments and relates them to the literature on demand-led growth. Section 3 presents the exogenous growth framework. The endogenous growth model is spelled out and discussed in section 4. Section 5 concludes.

2 Relation with the literature

Since the publications of Serrano (1995A, 1995B), the growth literature of Kaleckian and classical-Marxian inspiration has shown a revived interest in the role of aggregate-expenditure components that: (i) are not explained by short-run output; (ii) have a causal influence on it. Exports and government expenditure are two obvious examples, but residential construction, the Due-

senberry (1949) ratchet effect and other forms of consumption financed by consumer credit are also in the list. The hypothesis is further elaborated and discussed in Fiebiger and Lavoie (2017), who label such forms of expenditure 'semi-autonomous'. Recent empirical corroboration is provided by Girardi and Pariboni (2015) and Fiebiger (2018).

This paper builds on the premise that there are flows of expenditure that may be broadly related to innovation and that meet the two conditions (i) and (ii) above. This was also the view often expressed by the late Richard Goodwin, in the footsteps of his master J. Schumpeter. First, R&D is more persistent, compared to other components of firms' expenditure, because firing and re-hiring highly specialized R&D personnel implies a substantial loss of firm-specific human capital (Falk, 2006) that cannot be easily transferred to other activities (Harhoff, 1998). Also, a temporary low capacity utilization, if combined with non-negative long-run prospects, would not discourage the modernization and re-organization expenditures that are induced by process innovation.[2] Semi-autonomous demand related to innovation is rarely mentioned in the literature on demand-led growth. The objective of this paper is to consider this hypothesis and to study its main implication: this is that innovation-driven secular growth, not less than other sources of productivity growth, that remain outside the scope of this paper, is influenced by demand and distribution. Consistently with this limited target, we present a prototype model[3] embedding a neat, if highly stylized, formulation of the link relating effective demand, the accumulation of capital, and the market for innovation.

We are also partly motivated by the diffusion of automation and other labour saving techniques in recent decades. On these grounds, we shall assume that technological progress is labour augmenting. Notice that, to the extent that innovation is the only source of long-run growth in the model, this will also guarantee that the long-term growth path is coherent with the labour supply constraint in the economy.

The role assigned to innovation should not be misleading. As will turn out, short-run output is caused by demand (not vice-versa) and the bulk of investment demand is induced by demand expectations. Thus the model is demand-led and to emphasize this point, we shall first consider the simplified case in which R&D expenditure grows exogenously, much as autonomous expenditure is the *exogenous* driver of growth in Allain (2015), Freitas and

[2] Firms will find comparatively less attractive to undertake modernization and restructuring at times of high activity, when these may interfere with current production.

[3] Such models are closer to an abstract thought experiment than to a realistic description of a complex real-world economy, and cannot be expected to mimic macroeconomic time series (Malinvaud 1977, Lavoie 2017).

Serrano (2015) and Lavoie (2016). In this respect, the similarity of our exogenous-growth framework and theirs (especially Lavoie, 2016) is intentional and is meant to underline the qualitative correspondence of many results. In particular, the stability of the positive steady state is local and is conditional upon a sufficiently slow adaptation of long-term expectations, according to a simple Harrodian rule. On the steady-state path, capacity utilization is at its normal (desired) rate and the growth rate is obviously unaffected by distribution. This is parametrized by the value of the profit share, which is exogenous. Drawing a comparative dynamics across steady states, the profit share has only level effects: a lower profit share is associated with higher levels of employment and higher values of the (productivity adjusted) capital stock and output. These results are now well understood, and come from the large difference in the marginal propensity to consume out of wages and profits, the stabilizing effect of autonomous expenditure, and the slow Harrodian adjustment of long-term expectations.

The specific contribution of this paper comes from the more general version of the model, where the growth rate of labour productivity is endogenous. Firms are forced by competition to adopt best-practice processes, thus providing a market for the innovation goods embodying the new ideas. R&D and firms' modernization outlays feed each-other, and are, in the aggregate, positively related with the size of the capital stock. A persistent demand effect induced by a change in distribution, by affecting capital accumulation, will also affect the market for innovation goods, hence the incentives to R&D.

In addition, the model reveals that the existence and local stability of a positive growth path requires a component of autonomous demand that bears no direct relation with the capital stock. This is provided by a constant money flow e of autonomous consumption financed by profits. Thus, the paper contributes to clarifying the limits within which the stabilizing effects of semi-autonomous demand can survive endogenization, through capital accumulation.

The persistent level effects of a change in distribution are consistent with those of the exogenous-growth framework. But there are in addition, and unlike the previous contributions on autonomous demand, persistent growth effects, in that a lower profit share causes a higher rate of long-run growth.

The persistent growth effects of a change in distribution do not act through a long-run deviation of capacity utilization from its normal desired level. This property differentiates the present framework from the class of models, closely associated with the seminal contributions by Rowthorn (1982), Dutt (1984), Amedeo (1986), Marglin and Bhaduri (1990) and Bhaduri and Marglin (1990), where the opposite holds true (Pariboni 2016). Moreover, there is no labour hoarding in the model, and, for simplicity, full ab-

straction is made from any other direct feedback of output on labour productivity, as is characteristic of the Keynesian growth models adopting some version of Verdoorn's law (see Rezai, 2012 and the references quoted therein).

A crucial implication of the present framework is that output growth is divorced from the growth of employment. Employment levels are preserved, in the long run, only if the real wage grows at least in line with productivity. A failure of institutions in preventing a fall of the wage share would likely exert self-reinforcing effects on employment and the wage share itself.

3 Exogenous technological progress

To clarify exposition, and stress the analogies with similar results in the literature, we introduce first the simple case in which autonomous expenditure grows as a result of exogenous technological progress.

Let us consider a standard aggregate model with gross output Y_t that is either used for induced consumption C_t, semi-autonomous consumption expenditure E_t financed by profit income, gross investment I_t, capital modernization expenditure Z_t, or R&D expenditure R_t.

Net investment is defined by:

$$\dot{K}_t = I_t - \delta K_t \tag{1}$$

The aggregate production function is

$$Y_t = \min(\frac{1}{v} K_t, A_t L_t) \tag{2}$$

where L is labour employment and A is labour productivity. Throughout this paper we shall consider trajectories such that output Y_t is constrained by demand, not by capacity $(1/v)\, K_t$, and the adaptation of output to demand occurs though changes in employment. The actual rate of capacity utilization is $u_t = Y_t / Y_{K,t}$, where $Y_{K,t}$ is full capacity output $(1/v)\, K_t$. The need of promptly meeting unexpected peaks in demand, that may result from accidental shocks or endogenous fluctuations, requires that the desired rate of capacity utilization u_n is less than one. Empirical work suggests that firms may regard as 'normal' a rate of utilization u_n that may be as low as 75%, or 80%.[4]

With output never constrained by capacity, we can write $Y_t = A_t L_t$, hence $L_t = a_t Y_t$, where $a_t = 1/A_t$ is labour input per unit of output.

[4]See Trezzini (2017, f. 33) and the surveys cited therein.

Best practice labour productivity grows as a result of R&D expenditure performed by firms and within bounds that are fixed by historically contingent technological opportunities g_T:

$$\frac{\dot{A}_t}{A_t} = g_T \Psi(r_{A,t}) \tag{3}$$

where $r_{A,t} = R_t/A_t$ is productivity-adjusted R&D and the function $\Psi(r_A)$ has the properties $\Psi' > 0$, $\lim_{r_A \to 0} \Psi(r_{A,t}) = 0$ and $\lim_{r_A \to \infty} \Psi(r_{A,t}) = 1$. Here, $g_T > 0$ is the maximum productivity growth offered by historical technological opportunities and $\Psi(r_{A,t})$ is the fraction of these opportunities that is captured by R&D effort $r_{A,t}$. According to this hypothesis, greater knowledge A_t makes R&D activity more complex and demanding. As a prototype formulation, we take:

$$\Psi(r_{A,t}) = \left(1 - \frac{1}{1 + r_{A,t}}\right) \tag{4}$$

In this section we assume an exogenously fixed and constant $r_{A,t} = r_A > 0$. This amounts to assuming a dynamics of R&D expenditure such that

$$\frac{\dot{R}_t}{R_t} = \frac{\dot{A}_t}{A_t} \tag{5}$$

with initial condition $R_0 = r_A A_0$, where A_0 is pre-determined by history.

For the sake of later reference, we define $r_t = R_t/K_t$ and we observe that

$$r_t = r_A k_t^{-1} \tag{6}$$

where $k_t = K_t/A_t$.

To introduce best practice knowledge into production at time $t + \partial t$, firms carry out modernization expenditures Z_t that depend on the rate of technological progress and the size of their capital stock.

$$Z_t = p_z \frac{\dot{A}_t}{A_t} K_t = p_z g_T \Psi(r_A) K_t \tag{7}$$

where p_z is the price of one update, and $z_t = Z_t/K_t$ is the rate of technological obsolescence, as distinguished from physical depreciation δ.

Equation (7) captures the general hypothesis that firms are forced by competition to the costly implementation of process innovations. In the aggregate, the implementation cost bears a positive relation with the number of innovations per unit of time, and with the stock of equipment. The stylized example we have in mind is that of a technology improvement step,

or update, consisting of an innovation routine produced by R&D. For the sake of simplicity, we assume that the routine is embodied in an intermediate good produced with one unit of output, and that, as in the case of the computer, a unit of capital stock is indivisible with respect to the possibility of being updated by new routines. The total cost of updating increases with the price p_z, with the number $k_t = K_t/A_t$ of efficiency units of capital that require updating and with the number $\dot{A}_t$ of updates. It is worth observing that modernization expenditures are not capacity creating, in that the full capacity output from capital stock K_t, is K_t/v, no matter how high labour productivity A_t may be. This is the simplest way in which non-embodied technological progress is introduced into an aggregate model with fixed capital, thus avoiding the complications of vintage models, or of joint production.

Taking into account the alternative uses of gross output Y_t, market clearing in the good market requires:

$$Y_t = Z_t + R_t + C_t + I_t + E_t \tag{8}$$

Non autonomous consumption C_t comes entirely from the expenditure of the wage bill and we assume for simplicity that workers do not save, while non autonomous consumption out of profit is zero:

$$C_t = w_t L_t = w_t a_t Y_t \tag{9}$$

where w is the real wage. Semi-autonomous consumption expenditure E_t is influenced by the productivity level in the economy, according to $E_t = eA_t$. It may be thought of as resulting from a constant flow of money expenditure financed by profits, that grows in real terms, together with the fall of money prices caused by productivity growth. The term $e = E_t/A_t$ is labelled 'productivity adjusted autonomous consumption' and we assume $e > 1$.

Gross investment demand I_t reflects (i) the need of performing maintenance expenditures δK_t, (ii) the state of long term expectations concerning the average future growth of demand γ_t, (iii) the short-term forecast regarding capacity utilization at time t, namely $u_t^e = vY_t^e/K_t$, together with the will to enforce a gradual reduction of the gap between actual and desired capacity utilization, and (iv) a non-negativity constraint:

$$I_t = \max\left(0, \left[\gamma_t + \gamma_u\left(\frac{vY_t^e}{K_t} - u_n\right) + \delta\right] K_t\right)$$

The adjustment parameter γ_u is sufficiently small to ensure that gross investment is positive, whenever $\gamma_t \geq 0$. This requires the restriction $\delta > \gamma_u u_n$. Following in the footsteps of Keynes' 1937 lecture notes (Keynes, 1973, p.

181), we shall however adopt the standard convention of assuming that short-term expectations are fulfilled, to the effect that $Y_t^e = Y_t$. This leads to:

$$I_t = [\gamma_t + \gamma_u(u_t - u_n) + \delta]K_t \tag{10}$$

so that

$$g_{K,t} = \frac{I_t - \delta K_t}{K_t} = \gamma_t + \gamma_u(u_t - u_n) \tag{11}$$

Substituting for C_t in equation (8) from (9), and dividing throughout by K_t, we obtain the short-term-equilibrium rate of capacity utilization:

$$u_t = \frac{v(z_t + r_t + \delta + \gamma_t + ek_t^{-1} - \gamma_u u_n)}{\pi_t - v\gamma_u} \tag{12}$$

where $\pi_t = 1 - w_t a_t$ is the gross profit share in output, and in the present framework it is also the marginal propensity to save. Throughout this paper, we assume the standard Keynesian short-run-stability condition $\pi - v\gamma_u > 0$, stating that gross saving is more responsive than induced investment to changes in short-run output. This ensures the stable adaptation of short-run output to effective demand.

We are concerned with the study of growth paths supported by an exogenously given state of distribution, that we identify with a given and constant profit share $\pi_t = \pi$. This amounts to introducing the working hypothesis that the real wage is growing at rate $\hat{w}_t = \hat{A}_t$. Any consideration about the plausibility of this working hypothesis, and the implications that may follow from different scenarios of real wage dynamics, are postponed to the final discussion in the concluding section.

Using (6), (7), and (12), we write

$$\gamma_u(u_t - u_n) = \Gamma(\gamma_t, k_t) = x\left(p_z g_T \Psi(r_A) + \frac{r_A}{k_t} + \delta + \frac{e}{k_t} + \gamma_t - \frac{\pi u_n}{v}\right) \tag{13}$$

where

$$x = \frac{v\gamma_u}{\pi - v\gamma_u} > 0 \tag{14}$$

The short-term growth rate $g_{K,t}$ is then:

$$g_{K,t} = \gamma_t + \Gamma(\gamma_t, k_t) \tag{15}$$

Equations (12) and (15) define the short-run equilibrium of our economy, supported by the given state of long-term expectations γ_t and by the predetermined k_t. The full dynamic path of the economy is defined by the law of motion of these two state variables. If obtaining the growth rate of k_t

is straightforward, the law of motion of γ_t depends on speculations about expectations formation. Here we build upon Harrod's firm belief that the dynamics of long term expectations is influenced by the observation of the growth path of the economy. A local approximation to this hypothesis yields[5]:

$$\dot{\gamma}_t = \mu\left(g_{K,t} - \gamma_t\right) = \mu\Gamma(\gamma_t, k_t) \tag{16}$$

$$\dot{k}_t = \left(\gamma_t + \Gamma(\gamma_t, k_t) - g_T\Psi(r_A)\right) k_t \tag{17}$$

The dynamic system (16)-(17) has a unique dynamic equilibrium, namely the constant growth path (γ^*, k^*), such that

$$\gamma^* = g_T\Psi(r_A) = g_K^* \tag{18}$$

$$k^* = \frac{r_A + e}{\pi\frac{u_n}{v} - \delta - g_T\Psi(r_A)(1 + p_z)} \tag{19}$$

Recalling the interpretation of $z = p_z g_T\Psi(r_A)$ as rate of technological obsolescence, the positivity of the steady state is ensured by the plausible restriction that the long-run *net* rate of profit $\pi\frac{u_n}{v} - \delta - z$ is higher than the rate of growth $g_T\Psi(r_A)$.

The dynamic equilibrium (γ^*, k^*) is locally asymptotically stable if the adjustment parameter μ is small enough. To see this, we write the Jacobian matrix of system (16)-(17), evaluated at $(\gamma^*,\ k^*)$

$$J(\gamma^*, k^*) = \begin{bmatrix} \mu x & -\mu x(r_A + e)(k^*)^{-2} \\ k^*\left(1 + x\right) & -x(r_A + e)(k^*)^{-1} \end{bmatrix}$$

with the properties:

$$\det J(\gamma^*, k^*) = (k^*)^{-1}(r_A + e)\mu x$$
$$\operatorname{tr} J(\gamma^*, k^*) = x[\mu - (r_A + e)(k^*)^{-1}]$$

The local asymptotic stability of the dynamic equilibrium (γ^*, k^*) relies on the property $\det J\left(\gamma^*, k^*\right) > 0$ and the condition $\operatorname{tr} J\left(\gamma^*, k^*\right) < 0$. This last condition is fulfilled, if and only if the positive adjustment parameter μ is lower than the following critical value

$$\mu_c = \frac{r_A + e}{xk^*} = \frac{\pi\frac{u_n}{v} - \delta - g_T\Psi(r_A)(1 + p_z)}{x}.$$

Stability is strictly local and, as shown in Fig. 1, for initial conditions outside the basin of attraction of $(\gamma^*,\ k^*)$, trajectories diverge to infinity.[6]

[5] Lavoie (2016) has a multiplicative term γ_t on the right-hand side of (16). This term

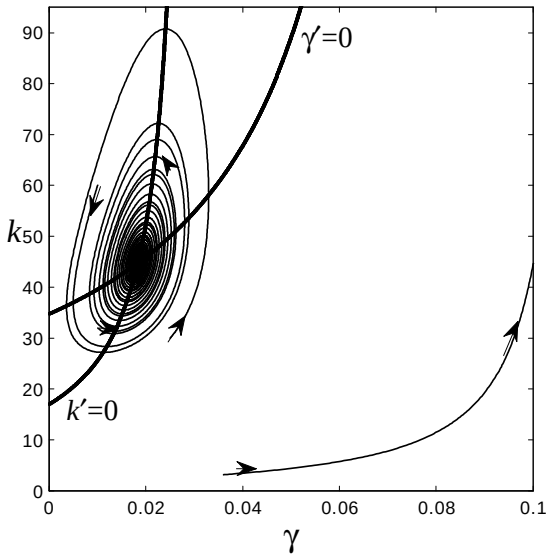

Figure 1: Trajectories in phase space for parameter settings $p_z = 1$, $\mu = 0.05$, $g_T = 0.04$, $\gamma_u = 0.025$, $\pi = 0.34$, $\delta = 0.02$, $u_n = 0.85$, $v = 2.4$, $r_A = 0.85$, $e = 2$ such that $\gamma^* \approx 0.0184$ and $k^* \approx 44.769$. The trajectory on the right is diverging

In the parameter range $0 < \mu < \mu_c$, it is meaningful to consider the persistent effects of a small change in distribution. Since long-term growth is exogenous, the profit share does not have steady-growth effects, but only level effects. A lower profit share causes a persistent demand shock, that will eventually produce a persistently higher productivity adjusted output y^* and capital stock k^*, hence a higher steady-state employment, but has no persistent effect on capacity utilization. This converges, through oscillations, to its desired level u_n, while long term expectations converge to the exogenous growth rate $g_T\Psi(r_A)$. Simulations show that, throughout the convergence path, $0 < u_t < 1$, to the effect that output is constrained by demand, and not by capacity.

The qualitative steady-state properties of system (16)-(17) are in some respects similar to those of other demand-led growth models in which the engine of growth is provided by autonomous expenditure (Allain, 2015; Fre-

is omitted here, to avoid the otherwise necessary restriction $\gamma_t > 0$.

[6]Further analysis of $J(\gamma^*, k^*)$ reveals that, for μ sufficiently close to μ_c, the steady state (γ^*, k^*) is a focus, and the dynamics of (16)-(17) undergoes a Hopf bifurcation as μ goes through its critical value μ_c. We thank one of the referees for drawing our attention to this point. Simulation analysis shows that the local boundedness of solution trajectories is already lost at μ marginally above the critical value.

itas and Serrano, 2015; Lavoie, 2016). A somewhat crucial difference is that in the present framework labour productivity is growing and, provided that the real wage is growing in line with productivity, labour employment would be constant on the steady-state path.

The scenario of rising labour productivity fits well with the assumption that output is never constrained by labour supply, but topics for debate are the plausibility of a rising real wage in the face of a steady level of employment, and the motivation behind the assumed R&D expenditure by firms. The second issue, together with the relation between the profit share and the rate of growth, is addressed in the next section.

4 Endogenous technological progress

In a market economy, the profits from R&D depend on the market-power resulting from innovation, on the ease with which R&D is producing new ideas, and on the market size facing every new idea. In the case of process innovations, this market size bears a positive relation with the stock of equipment. Proceeding with our stylized example, we assume that R&D is carried out by an independent firm, to the end of selling updating tool-kits to firms producing consumption and investment goods. A tool-kit embodies an updating routine and is an intermediate good produced with one unit of output. It has unit price $p_z > 1$ that comes from the intellectual property rights on the routine.[7] We shall abstract from free entry in R&D, for the sake of simplicity. Monopolist's revenue from selling the innovation routine comes from the modernization expenditure Z_t of the other firms in the economy, and affects the net, but not the gross, short-run profit of such firms.[8] With firms' updating expenditure Z_t specified as in (7) above, the profit from selling the updating tool-kits, net of the production and R&D cost, is

$$\Pi_{R,t} = (p_z - 1)K_t g_T \left(1 - \frac{1}{1 + r_A}\right) - R_t \tag{20}$$

For any given $k_t = K_t/A_t$ fixed by past history, the maximization of profit $\Pi_{R,t}$ with respect to R_t yields the productivity adjusted R&D expenditure as a function of k

$$r_A(k) = \begin{cases} 0 & \text{if } 0 < k \leq k_{\min} \\ [g_T(p_z - 1)k]^{1/2} - 1 & \text{if } k > k_{\min} \end{cases} \tag{21}$$

[7] The assumption that the price p_z is fixed and greater than one is justified by the hypothesis that monopoly price is constrained by the potential entry of imitators, who can produce the tool-kit at a constant unit cost $p_z > 1$. See Aghion and Howitt (2009).

[8] As for net profit in general, monopolist's net profit is partly spent on semi-autonomous consumption.

where $k_{\min} = [g_T(p_z - 1)]^{-1} > 0$. In the range $k > k_{\min}$, $r_A(k)$ is an increasing function of k; more precisely,

$$r'_A(k) = \begin{cases} 0 & \text{if } 0 < k \leq k_{\min} \\ \frac{1}{2}[g_T(p_z - 1)]^{1/2} k^{-1/2} & \text{if } k > k_{\min} \end{cases} \tag{22}$$

In what follows, we replace throughout the exogenous value r_A of section 3 with the function $r_A(k)$, to the effect that the rate of productivity growth at time t is now $g_T \Psi(r_A(k_t))$.

The growth rate of the capital stock is

$$g_{K,t} = \gamma_t + F(\gamma_t, k_t), \tag{23}$$

where $F(\gamma_t, k_t)$ is defined by

$$F(\gamma_t, k_t) = x \left[p_z g_T \Psi(r_A(k_t)) + \frac{r_A(k_t)}{k_t} + \delta + \frac{e}{k_t} + \gamma_t - \frac{\pi u_n}{v} \right] \tag{24}$$

The Harrodian adjustment rule (16) for long-term expectations γ_t can now be expressed in compact form as

$$\dot{\gamma}_t = \mu F(\gamma_t, k_t) \tag{25}$$

while the law of motion (17) for k_t becomes:

$$\dot{k}_t = [\gamma_t + F(\gamma_t, k_t) - g_T \Psi(r_A(k_t))]\, k_t \tag{26}$$

As in the previous section, we have a dynamic system in the two state variables γ_t and k_t such that its dynamic equilibria satisfy $\dot{\gamma}_t = \dot{k}_t = 0$. The system admits a positive steady state (γ^*, k^*), where $\gamma^* = \gamma(k^*) = g_T\,[1 - (1 + r_A(k^*))^{-1}]$ and k^* is the positive real solution to $F(\gamma^*(k^*), k^*) = 0$. The properties of the dynamic equilibrium (γ^*, k^*) are discussed below. To this end, let

$$h \equiv 2 \left(\frac{g_T}{p_z - 1} \right)^{1/2} > 0 \tag{27}$$

$$s \equiv \pi u_n / v - \delta - g_T(1 + p_z) \geq 0 \tag{28}$$

Condition (27) relies upon the obvious restriction of a positive monopoly power ($p_z > 1$), while condition (28) requires that when technological progress equals technological opportunity g_T, the net profit rate is not lower than the rate of growth. Appendix $A.1$ shows that, with such restrictions in place, we have:

$$k^* = \left[\frac{2(e - 1)}{h + \Delta^{1/2}} \right]^2 \tag{29}$$

where

$$\Delta = h^2 + 4(e-1)s.$$

Notice from (29) and (27) that steady-state k^* is an increasing function of the expenditure parameter e, and of monopoly power $p_z - 1$, while it is a decreasing function of g_T. A necessary condition for the existence of a positive growth path is that productivity adjusted autonomous consumption meets not only the restriction $e > 1$, but that it is large enough to satisfy the stronger requirement $k^* > k_{\min}$.[9] It may be also worth observing that k^* is negatively related to the value of the profit share, and because γ^* is an increasing function of k^*, we say that growth is wage led in the equilibrium (γ^*, k^*).

To study the local stability of (γ^*, k^*), we write the Jacobian matrix of the first partial derivatives of system (25)-(26), evaluated at (γ^*, k^*).

$$J(\gamma^*, k^*) = \begin{bmatrix} \mu x & \mu F_k(\gamma^*, k^*) \\ (1+x)k^* & k^* F_k(\gamma^*, k^*) - \frac{1}{2} g_T^{1/2} [(p_z - 1)k^*]^{-1/2} \end{bmatrix} \tag{30}$$

This yields:

$$\det(J(\gamma^*, k^*)) = -\mu \left[k^* F_k(\gamma^*, k^*) + x \frac{1}{2} \left(\frac{g_T}{k^*} \right)^{1/2} [(p_z - 1)]^{-1/2} \right]$$

$$\mathrm{tr}(J(\gamma^*, k^*)) = \mu x + k^* F_k(\gamma^*, k^*) - \frac{1}{2} \left(\frac{g_T}{k^*} \right)^{1/2} [(p_z - 1)]^{-1/2}$$

Appendix $A.2$ shows that $F_k(\gamma^*, k^*) < 0$, and its modulus is bounded away from zero $\forall g_T \geq 0$. Recalling that k^* is a decreasing function of g_T, we have: *(i)* $\det(J(\gamma^*, k^*) > 0$ if the value of technological opportunity g_T is not too large; *(ii)* $\mathrm{tr}(J(\gamma^*, k^*)) < 0$ if the positive adjustment parameter μ is lower than a critical value $\bar{\mu}_c$.[10] With both conditions *(i) and (ii)* in place, the steady-state (γ^*, k^*) is locally asymptotically stable.

[9] The reason for this result is that e is now the only source of semi-autonomous expenditure, which is not endogenously depending on capital in efficiency units. Thus, e is the only anchor of the long-run size of k^*. Because the market of process innovations increases with the size of capital, profit from R&D vanishes, if k falls below a lower threshold $k_{\min}$ (see equation 21). To have a positive steady-state supported by purposeful R&D activity, it is necessary that $k^* > k_{\min}$, hence that e is above a minimum threshold. The numerical value of this threshold is related to the profit maximizing solution (21).

[10] Using the property $\mathrm{tr}(J(\gamma^*, k^*)) = 0$ at $\bar{\mu}_c$, and the expression for $F_k(\gamma^*, k^*)$ given in appendix $A.2$, we compute:

$$\bar{\mu}_c = \frac{h + \Delta^{1/2}}{4(e-1)} \left[\left(\frac{g_T}{p_z - 1} \right)^{1/2} \frac{1-x}{x} + h + \Delta^{1/2} \right]$$

Proposition 1 *Assume that the efficiency units e of autonomous consumption are sufficiently larger than 1, and the steady state net rate of profit is not lower than the rate of growth in the full range allowed by technological opportunity g_T. Then, there exists a positive steady state solution (γ^*, k^*) of the dynamic system (25)-(26). (γ^*, k^*) is locally asymptotically stable if the adjustment parameter μ is lower than the critical value $\bar{\mu}_c$.*[11]

The requirement that, in the long-run, the net rate of profit is not lower than the potential growth rate enabled by technological opportunity g_T reflects the long-run feasibility condition that the output resulting from normal capacity utilization u_n, and not spent on induced consumption, or on depreciation and obsolescence allowances, must not be lower than the net investment in capacity implied by steady-state growth. The condition on the flow e of (productivity-adjusted) autonomous consumption is necessary because the flow e sets a positive lower bound to the long-run size of the economic activity, such that the restriction $k^* > k_{\min}$ is fulfilled (see footnote 9).

The steady state is such that: (i) Output and capital stock grow in line with the endogenous growth of labour productivity; (ii) firms are forced by competition to modernize their capital and provide in this way a market to the innovation activity of a profit seeking monopolist; (iii) firms' long-term expectations concerning the growth of demand conform to the endogenous growth rate of the economy; (iv) firms' capacity utilization conforms to its desired level. The local stability of this growth path requires not only that, as in section 3, the adjustment of long-term expectations is sufficiently slow; since R&D expenditure is now endogenous, we have, in addition, the further requirement that the value of technological opportunity g_T is not too large. An illustration of convergence is shown in Fig. 2.

4.1 Comparative analysis

The transitional and steady state effects of a change in distribution on both output and employment are worth considering. In the parameter range in which the local stability of the positive dynamic equilibrium holds, let us contemplate an economy that at time t is fully adjusted to its steady-state position (γ_1^*, k_1^*), corresponding to $\pi = \pi_1$. Labour productivity is A_t and

[11] Not unlike section 3, as μ crosses its critical value, a Hopf-bifurcation occurs, to the effect that there exists locally a family of periodic solutions to system (25)-(26). Simulation analysis shows that the local boundedness of solution trajectories is already lost at μ marginally above the critical value. In sub-section 4.2 we suggest a highly nonlinear adjustment of long-term expectations that extends the parameter range of μ giving rise to bounded, demand-driven economic dynamics.

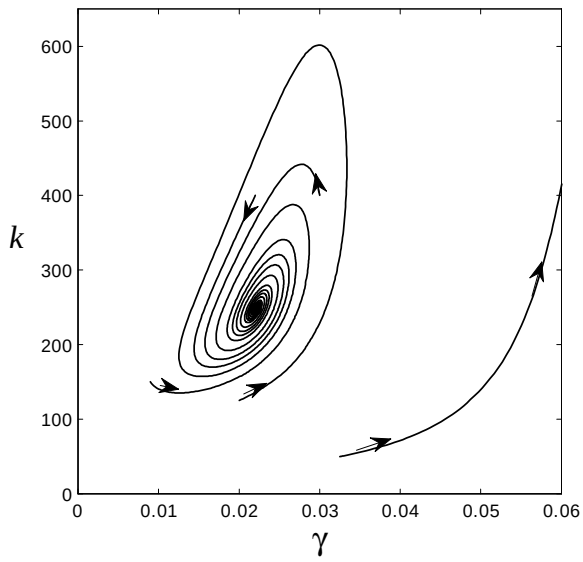

Figure 2: Trajectories in phase space for parameter settings $p_z = 1.5$, $\mu = 0.05$, $g_T = 0.04$, $\gamma_u = 0.025$, $\pi = 0.34$, $\delta = 0.02$, $u_n = 0.85$, $v = 2.4$, $e = 10$ such that $\gamma^* \approx 0.0220$ and $k^* \approx 247.36$. The trajectory on the right is diverging

capacity utilization is $u_t = u_n$; thus, we can write $A_t L_t = u_n K_t$ and $L_t = L_1 = u_n k_1^*$. At time $t + \partial t$ a once and for all small parametric change of the profit share takes place, such that $\Delta\pi = \pi_2 - \pi_1 < 0$. Because k^* is a decreasing function of π, after convergence to the new steady state (γ_2^*, k_2^*), corresponding to π_2, productivity adjusted output is $y_2^* > y_1^*$. The new steady-state level of employment is $L_2^* = u_n k_2^* > L_1^*$. Thus, a once and for all change $\Delta\pi < 0$ of the profit share causes a persistent increase of the growth rate and a persistent rise in employment, but no *persistent* effect on the rate of capacity utilization, that will eventually return to its steady-state normal level u_n. As shown in Fig. 3, capacity utilization u_t is everywhere bounded away from 1 on the transition path ($0.8 < u_t < 0.9$), to the effect that output is determined by demand, and is unconstrained by capacity.

4.2 Limit cycles

As a suggestion for further work, in this section we move some steps towards extending our analysis beyond the parameter range in which the equilibrium (γ^*, k^*) is a local asymptotic attractor of the dynamic system. To this end, we borrow insights from the non-linear adjustment literature (e.g., Goodwin,

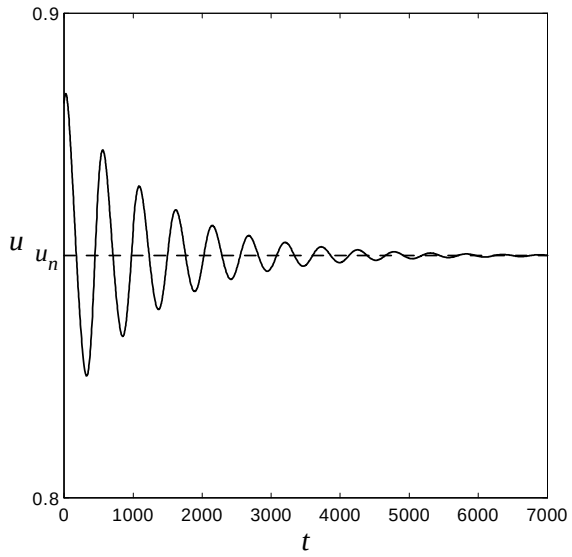

Figure 3: Behaviour in time of the rate of capacity utilization after an exogenous, once and for all change of the profit share $\Delta\pi = -0.01$, with all other parameters as in Fig. 2 and initial condition at the equilibrium (γ^*, k^*)

1951), by imposing that as the gap between the long-term expectation γ_t and the ex-post observation $g_{K,t}$ tends to increase, the adjustment rule of γ_t becomes increasingly conservative. Using (23), we add a non-linear component to the adjustment rule (25), which is replaced with:

$$\dot{\gamma}_t = \mu F(\gamma_t, k_t) - \phi F^3(\gamma_t, k_t) \tag{31}$$

It can be readily observed that (γ^*, k^*) is still an equilibrium of the dynamic system. Moreover, the property $F(\gamma^*, k^*) = 0$ implies that (30) is the Jacobian matrix of (31)-(26) at (γ^*, k^*). From this it follows that the local stability properties of (γ^*, k^*) are unchanged: there exists a critical value $\bar{\mu}_c$ (cf. footnote 10) of the adjustment parameter μ, such that (γ^*, k^*) is locally asymptotically stable if $0 < \mu < \bar{\mu}_c$. In this parameter range, the temporary and persistent qualitative effects of a small change in distribution are those described in sub-section 4.1 above. At $\mu > \bar{\mu}_c$ the dynamic equilibrium (γ^*, k^*) is locally repelling, but if prediction errors induce sufficient caution in the revision of long-term expectations (that is, if μ/ϕ is small enough), the growth trajectories with initial conditions in a neighbourhood of the steady state, converge to a limit cycle around (γ^*, k^*) (Fig. 4). This dynamic behaviour is here illustrated by numerical simulation. A full-fledged mathematical investigation of the highly nonlinear system (31)-(26) requires

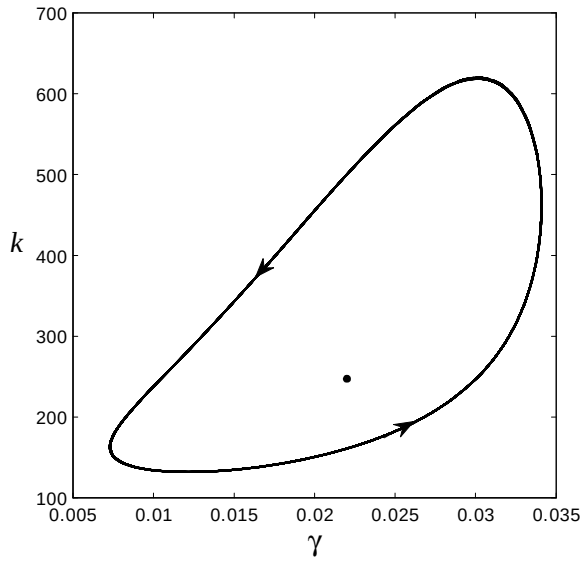

Figure 4: Convergence to a limit cycle in phase space for parameter settings $p_z = 1.5$, $\mu \approx 0.0742$, $g_T = 0.04$, $\gamma_u = 0.025$, $\pi = 0.34$, $\delta = 0.02$, $u_n = 0.85$, $v = 2.4$, $e = 10$, $\phi = 975$

a separate analysis that is left for future work.

The persistent fluctuations around the positive steady state are such that short-run output is never constrained by capacity; moreover, the average rate of capacity utilization over the cycles does not coincide with the steady-state normal value u_n, but is higher (see Fig. 5). This marks a sharp distinction between the long-term time average of a variable and its dynamic equilibrium.[12]

5 Conclusions

This paper builds on the hypothesis that there are forms of R&D and innovation expenditure that, compared to capacity investment, are relatively autonomous with respect to short-run output. If and to the extent that innovations are primarily aimed at reducing the use of the human-labour input in production, while the use of capital inputs per unit of output is fixed, such expenditures do not interfere with expansion investment, as determined by the state of long-term expectations on output growth and by the wish to

[12] Debates over the role and properties of capacity utilization in the analysis of demand-led growth have occasionally overlooked this distinction.

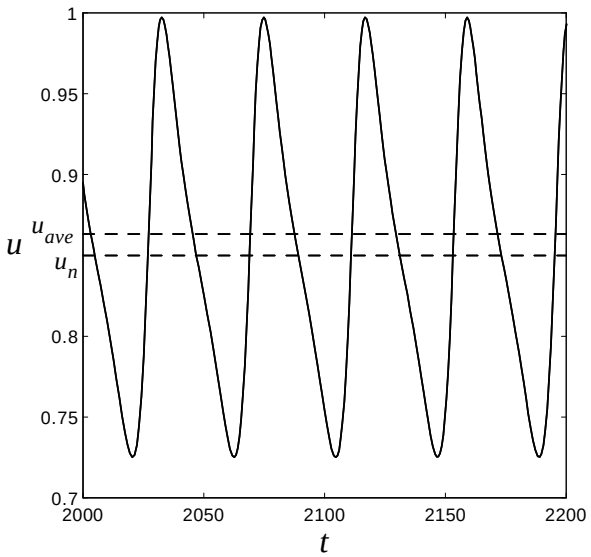

Figure 5: The cyclical behaviour of the rate of capacity utilization over the limit cycle of Fig. 4

bring capacity utilization into line with its desired level. We have explored some implications of these hypotheses in the light of a demand-led endogenous growth model. R&D is carried out by a profit seeking monopolist selling innovation 'software' to firms that are forced by competition to modernize their capital stock. R&D increases with the market for innovation, and with the historically contingent technological opportunities. For the sake of simplicity, we have assumed that the marginal propensity to save out of wages is one and the marginal propensity to save out of profits is zero. In the short-run equilibrium, the average propensity to save depends on the level of autonomous expenditure, and the stability of the short-run equilibrium is ensured by capacity investment reacting more slowly to short-run output than aggregate saving. On the positive growth path aggregate output and capital stock grow in line with the endogenous growth rate of labour productivity, firms' long-term expectations are fulfilled, and capacity utilization conforms to its desired level. The existence of this path requires the feasibility condition that the maximum growth rate g_T enabled by technological opportunity is not higher than the rate of profit, net of depreciation and technical obsolescence. A more subtle condition is that there is a component of autonomous expenditure which, unlike R&D and modernization expenditures, bears no strong direct relation with the size of the capital stock. This flow, interpreted here as autonomous consumption financed by profits, grows

through time with labour productivity.

Provided that the autonomous-consumption flow is not too small, a sufficiently slow adjustment of long term expectations, as parametrized by μ, ensures the local asymptotic stability of the positive growth path. At higher values of μ, the instability of the dynamic equilibrium requires replacing a linear expectation-formation rule, with one entailing that the adaptation of long-term expectations becomes sufficiently conservative, as the gap between observations and predictions tends to increase. If this is the case, the growth trajectories starting in a neighbourhood of the dynamic equilibrium remain bounded and converge to limit cycles. Growth is wage led, both in the sense that long term output growth is inversely related to the profit share, and in the sense that a lower profit share raises the steady state level of productivity adjusted output and employment. Employment is constant on a steady-growth path and the output dynamics tends to be divorced from the employment dynamics. In this framework, any fall in the wage share, whether caused by market forces, or by changes in institutions, tends to produce self-reinforcing effects. Thus the model may shed some light on the association between slower secular growth, falling manufacturing employment and falling wage share, which is a characteristic of the present era in many western countries.

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A Appendix

A.1 Computation of k^*

$$F(\gamma, k) = \gamma_u \left\{ \frac{v[p_z g_T \left(1 - (1 + r_A(k))^{-1}\right) + r_A(k)k^{-1} + \delta + ek^{-1} + \gamma] - \pi u_n}{\pi - v\gamma_u} \right\}$$

Imposing $\gamma = g_T \left(1 - (1 + r_A(k))^{-1}\right)$, the equilibrium restriction $F(\gamma, k) = 0$ yields

$$(1 + p_z)g_T \left(1 - (1 + r_A(k))^{-1}\right) + r_A(k)k^{-1} + ek^{-1} = \frac{\pi u_n}{v} - \delta$$

Substitute for $r_A(k)$ from (21) at $k > k_{\min}$ and rearrange, to obtain

$$k^{-1}(e - 1) - 2k^{-1/2} \left(\frac{g_T}{p_z - 1}\right)^{1/2} = \frac{\pi u_n}{v} - \delta - (1 + p_z)g_T$$

that can be written in compact form as

$$(e-1)y^2 - hy - s = 0$$

where $y = k^{-1/2}$ and $h > 0$, $s \geq 0$ are defined (respectively) by (27) and (28) in the text and by the restrictions spelled out therein.

This leads to

$$y^* = \frac{h + \Delta^{1/2}}{2(e-1)}$$

where $\Delta = h^2 + 4s(e-1)$ and

$$k^* = \left[\frac{2(e-1)}{h + \Delta^{1/2}}\right]^2$$

A.2 Proof that $F_k(\gamma^*, k^*) < 0$

$$F_k(\gamma^*, k^*) = \frac{x}{(k^*)^2}\left\{\frac{1}{2}\left(\frac{g_T k^*}{p_z - 1}\right)^{1/2} + 1 - e\right\}$$

Using (29) the term $\frac{1}{2}\left(g_T k^*\right)^{1/2}$ can be written as

$$\frac{1}{2}\left(g_T k^*\right)^{1/2} = \frac{(e-1)g_T^{1/2}}{h + \Delta^{1/2}} \tag{32}$$

Substituting for h from (27), and using $e > 1$, $p_z > 1$, we have:

$$F_k(\gamma^*, k^*) = \frac{x}{(k^*)^2}\left\{\frac{e-1}{2 + (\Delta/g_T)^{1/2}(p_z - 1)^{1/2}} + 1 - e\right\} < 0$$

References

Aghion, P. and Howitt, P. 2009. *The Economics of Growth*, Cambridge MA, The MIT Press.

Allain, O. 2015. Tackling the instability of growth: A Kaleckian-Harrodian model with an autonomous expenditure component, *Cambridge Journal of Economics*, vol. 39, n. 3, 1351-1371.

Amedeo, E.J. 1986. The role of capacity utilization in long-period analysis, *Political Economy*, vol. 2, n. 2, 147-185.

Bhaduri, A. and Marglin, S.A. 1990. Unemployment and the real wage: the economic basis for contesting political ideologies, *Cambridge Journal of Economics*, vol. 14, n. 4, 375-393.

Cesaratto, S. 2015. Neo-Kaleckian and Sraffian controversies on the theory of accumulation, *Review of Political Economy*, vol. 27, n. 2, 154-182

Duesenberry, J.S. 1949. *Income, Saving and the Theory of Consumer Behavior*, Cambridge MA, Harvard University Press.

Dutt, A.K. 1984. Stagnation, income distribution, and monopoly power, *Cambridge Journal of Economics*, vol. 8, n. 1, 25-40.

Falk, M. 2006. What drives business Research and Development (R&D) intensity across Organisation for Economic Co-operation and Development (OECD) countries?, *Applied Economics*, vol. 38, n. 5, 533-547.

Fiebiger, B. 2018. Semi-autonomous household expenditures as the *causa-causans* of postwar US business cycles: the stability and instability of Luxemburg-type external markets, *Cambridge Journal of Economics*, vol. 42, 155-175.

Fiebiger, B. and Lavoie, M. 2017. Trend and business cycles with *external markets*: Non-capacity generating semi-autonomous expenditures and effective demand, *Metroeconomica*, 00, 1-16.

Freitas, F. and Serrano, F. 2015. Growth rates and level effects, the stability of the adjustment of demand to capacity, and the Sraffian supermultiplier, *Review of Political Economy*, vol. 27, n. 3, 258-281.

Girardi, D. and Pariboni, R. 2015. Autonomous demand and economic growth: some empirical evidence, Working Paper 714, Dipartimento di Economia Politica e Statistica, University of Siena.

Goodwin, R.M. 1951. The nonlinear accelerator and the persistence of business cycles, *Econometrica*, vol. 19, n. 1, 1-17.

Harhoff, D. 1998. Are there financing constraints for R&D and investment in German manufacturing firms?, *Annales d'Économie et de Statistique*, n. 49/50, 421–56.

Hicks, J.R. 1950. *A Contribution to the Theory of the Trade Cycle*, Oxford, The Clarendon Press.

Keynes, J.M. 1973. Ex-post and ex-ante, 1937 lecture notes; reprinted in *The Collected Writings of John Maynard Keynes*, vol 14, London, Macmillan for the Royal Economic Society, pp. 179-183.

Lavoie, M. 2016. Convergence towards the normal rate of capacity utilization in neo-Kaleckian models: the role of non-capacity creating autonomous expenditures, *Metroeconomica*, vol. 67, n. 1, 172-201.

Lavoie, M. 2017. Prototypes, reality and the growth rate of autonomous consumption expenditures: a rejoinder, *Metroeconomica*, vol. 68, n. 1, 194-199.

Malinvaud, E. 1977. *The Theory of Unemployment Reconsidered*, Oxford, Basil Blackwell.

Marglin, S.A. and Bhaduri, A. 1990. Profit squeeze and Keynesian theory, in S.A. Marglin and J.B. Schor (eds), *The Golden Age of Capitalism: Reinterpreting the Postwar Experience*, Oxford, The Clarendon Press, pp. 153–186.

Pariboni, R. 2016. Autonomous demand and the Marglin-Bhaduri model: a critical note, *Review of Keynesian Economics*, vol. 4, n. 4, 409-428.

Rezai, A. 2012. Goodwin cycles, distributional conflict, and productivity growth, *Metroeconomica*, vol. 63, n. 1, 29-39.

Rowthorn, R.E. 1982. Demand, real wages and economic growth, *Studi Economici*, vol. 18, 2-53.

Serrano, F. 1995A. Long-period effective demand, and the Sraffian supermultiplier, *Contributions to Political Economy*, vol. 14, 67-90.

Serrano, F. 1995B. *The Sraffian Supermultiplier*. Unpublished Phd Thesis, University of Cambridge, UK

Serrano, F. and Freitas, F. 2017. The Sraffian supermultiplier as an alternative closure for heterodox growth theory, in *European Journal of Economics and Economic Policies*, vol. 14, 70-91.

Trezzini, A. 2017. Harrodian instability: a misleading concept, Centro Sraffa, Working Papers n. 24

Trezzini A. and Palumbo, A. 2016, The theory of output in the modern classical approach: main principles and controversial issues, *Review of Keynesian Economics*, vol. 4, 503-522.