



Keynes on probability and decision: evidence from the correspondence with Hugh Townshend

This is a pre print version of the following article:

Original:

Zappia, C. (2015). Keynes on probability and decision: evidence from the correspondence with Hugh Townshend. HISTORY OF ECONOMIC IDEAS, 23(2), 145-164 [10.1400/236721].

Availability:

This version is available <http://hdl.handle.net/11365/984573> since 2021-04-05T20:29:18Z

Published:

DOI:10.1400/236721

Terms of use:

Open Access

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. Works made available under a Creative Commons license can be used according to the terms and conditions of said license.

For all terms of use and more information see the publisher's website.

(Article begins on next page)

**Keynes on probability and decision:
evidence from the correspondence with Hugh Townshend.***

1. Introduction

Although not an academic, together with Joan Robinson and Nicholas Kaldor, Hugh Townshend is identified as one of the three «proto-Post Keynesians» (King 2002, 18) who started to develop an interpretative tradition that resolutely denied that Keynes's thinking could be assimilated as a special case of neoclassical thought. Because of its insistence on the revolutionary nature of the *General Theory* (henceforth *GT*), Townshend's note on liquidity preference and the rate of interest published in the *Economic Journal* in March 1937 is amply quoted in the literature on Keynes. Townshend (1937a) took issue with Hicks's attempt to interpret the theory of liquidity preference in terms of the loanable funds theory and argued that, since the market valuation of loans and securities depends on changes of expectations, interests rates may fluctuate even without any exchange of money in the market. Townshend's contribution is praised in particular because of its ability to elucidate what Keynes left implicit in Chapter 17 of the *GT* (Shackle 1967).

But in the literature on Keynes's philosophical ideas it is Townshend's private correspondence with Keynes over the just-published *GT* that has attracted larger attention. The correspondence, printed as part of Volume XXIX of the *Collected Writings* of John M. Keynes, constitutes a relevant piece of evidence in the debate on the continuity of Keynes's thought. Most of the interpretations placing emphasis on the enduring links between the *Treatise on Probability* (henceforth *TP*) and the *GT* make use of excerpts from Keynes's letter to Townshend of December 1938 dealing with probabilities and the weight of argument (O'Donnell 1989, Runde 1994a). Indeed, when discussing the state of long-term expectations in the *GT*, Keynes (1973b [1936], 148) had already suggested that there is a strong analogy between the most probable forecast and the state of confidence, on the one side, and the distinction between probability and weight emphasised in the *TP*, on the other. But it is in his letter to Townshend of December 1938, that Keynes (1979 [1936-1938], 293-294) stressed his point with a reference to the practical relevance of the

* This is a pre-print version of a paper whose revised version has been accepted for publication in *History of Economic Ideas*. Comments on this draft of the paper by Roger Backhouse, Marcello Basili, Mario Cedrini, Jochen Runde and two anonymous referees, and the kind assistance from Jacky Cox, Deputy Keeper of the University Archives at Cambridge University Library, are gratefully acknowledged. Usual disclaimers apply.

distinction:

«I am rather inclined to associate risk premium with probability strictly speaking, and liquidity premium with what in my *Treatise on Probability* I called ‘weight’ ... A liquidity premium ... is a payment, not for the expectation of increased tangible income at the end of the period, but for an increased sense of comfort and confidence during the period».

A different excerpt from the same letter has been quoted by other Keynesian scholars, those who claim that Keynes came to reject classic rational decision criteria, thus focusing on the conventional aspect of the outcomes of markets and on the necessity for economic agents to adopt a different form of rationality, forming expectations on market sentiment, rather than fundamentals (Lawson 1993, Davis 1994). Townshend (1937a) had already made the case that Keynes’s theory of liquidity preference could be applied to all markets, suggesting that valuations dependent on expectations were necessarily conventional. Keynes seems to restate this point in the closing of his December 1938 letter (1979 [1936-1938], 294), when he claims that in making decision it may well happen that:

«To avoid being in the position of Buridan’s ass, we fall back, therefore, and necessarily do so, on motives of another kind, which are not ‘rational’ in the sense of being concerned with the evaluation of consequences, but are decided by habit, instinct, preference, desire, will, etc.»

This paper does not concentrate on the single letter just referred to, but on the whole correspondence and the context in which it unfolded through the years 1936-1938. My aim here is twofold. First, I intend to claim that a comprehensive reading of the exchange provides sounder evidence for the thesis that the analogies that can be envisaged between the *TP* and the *GT* as regards decision-making reveal a continuity of thought which has been pointed out by some interpreters (for instance, see O’Donnell 1989) but questioned by others (for instance see, Bateman 1996).¹ Second, and more importantly, I aim to show that the exchange between Keynes and Townshend followed a discussion thread that is more technical than usually understood. In fact, the correspondence provides evidence for the fact that Keynes still had a keen interest in a problem left unsolved in the *TP*, namely, the definition of an alternative to what we might now call expected utility theory and he termed «normal ethical theory» in the *TP* and identified with «strict mathematical calculation» used by Benthamite calculators in the *GT*.

The paper shows that, late in 1938, the issue of whether a useful decision rule can be devised under uncertainty appears still central in Keynes’s thought, even though, similarly to the

¹ Although a few references to the literature on the continuity issue will be made in what follows, my main aim is not to provide a comprehensive assessment of the controversies on this matter, for which see Gerrard (1992) and the essays collected in Runde and Mizuhara (2003).

conclusions Keynes had already reached in Chapter 26 of the *TP*, a definite solution does not arise from the exchange. In the correspondence, Keynes discusses with Townshend how to distinguish «rational» criteria for decision-making used among «classical» economists from a class of criteria intended to replace them. Asked by Keynes, Townshend examines the contributions of contemporary scholars trying to build up an «expectational economic analysis» on the *GT* in view of the criticism Keynes had raised against mathematical expectation in the *TP*. It can be argued, therefore, that the critique of Benthamite calculus Keynes put forward in Chapter 12 of *GT* and in the 1937 *QJE* defence article was not intended to remain devoid of positive content, even with respect to decision-making. While it is true that the understanding of how conventional judgements may emerge should be considered crucial for an accurate description of actual markets from a Keynesian viewpoint (Runde 2003), it is important to stress that the broader notion of rationality Keynes shows interest in in the *GT* was not confined to social conventions as a basis for judgement. Also, the claim that Keynes's thought mostly relies on the acknowledgement that relevant uncertainty is "irreducible" to probability, as some probabilities are not just unknown but «non-existent» (Skidelsky 2011, 3) seems misplaced. Rather, Keynes provided a balanced picture of the effects of uncertainty on decision-making that, following on his analysis in the *TP*, included suggestions about how alert agents should behave when they deliberately choose not to trust the conventional judgement.

2. The early stages of the correspondence

The correspondence Keynes had with Hugh Townshend that is reproduced in Volume XXIX of Keynes's *Collected Writings* refers to the years of the defence of the *GT*, starting from a letter by Townshend in early March 1936 and ending with a letter from Keynes in December 1938. This last letter, dated December 8th, is the one usually referred to in the critical literature. It should be noted though that Keynes's acquaintance with Townshend originated in the years Townshend spent in Cambridge in the early 1910s while he was taking a First in Mathematics, and that Townshend had been a pupil of Keynes while preparing for Civil Service examinations he took in 1914. Townshend did not enter academics and had a career as civil servant, serving with the Post Office, where his duties included economic forecasting (Chick 1987). We do not know whether Keynes and Townshend remained in contact in the meantime, but the late 1930s correspondence reveals that

Keynes valued Townshend's comments on the *GT* highly.²

Townsend's first letter, dated 6 March 1936, introduces a detailed note commenting on the recently published *GT*. Townshend asks for clarification and suggests corrections on a number of «minor points», almost as if he was acting as a referee, «in the hope—though not without hesitation—that it may be of use to you, in preparing a second edition, to have had your attention called to them» (letter to Keynes, 6 March 1936). On receiving from Townshend such a detailed note, Keynes replies: «It is a great and unusual pleasure for an author to find so close and attentive a reader as you have been! I am very grateful for your corrections, all of which seem to me to be sound». Keynes adds that, as a reprint of his book was in order, «I sent off yesterday to the printer some corrections based on your notes» (letter to Townshend, 11 March 1936).³ A few days later a letter from Townshend appeared in *The Economist*, in which Townshend suggested that the journal's reviewer of the *GT* had misunderstood Keynes's definition of investment. On reading the letter Keynes writes to Townshend to show appreciation for his comments:

«Thank you for your letter in Saturday's *Economist*. I was uneasy about the passage in *The Economist* review to which you refer and am very glad that you have dealt with it. Once more you have shown a complete comprehension of what I am driving at, and I am very grateful» (letter to Townshend 24, March 1936).⁴

In March 1937 Keynes, long-time editor of the journal, publishes Townshend's note on

² In what follows, the scattered information about what part of the correspondence is arguably missing is reported from Donald Moggridge's editorial comments introducing the exchange in Volume XXIX of Keynes's *Collected Writings*. As most of the material collected in this volume, the correspondence with Townshend was retrieved at Keynes's country house in Tilton, Sussex, in 1975, almost thirty years after the original assembly of Keynes's papers (Moggridge 1979, xiii). Townshend had died the year earlier.

³ The *General Theory* was out in print in February 1936. The edition of its first British reprint, dated April 1936, is the one reproduced in Volume VII of the *Collected Writings*. It differs from the original one in that a number of minor corrections were made, among which two are exactly as suggested by Townshend (Moggridge 1973, xix). Townshend's influence on Keynes in the development of his ideas after the publication of the *General Theory* is an issue of interest for its own sake. For instance, Rochon (1996, 12-13) claims that Keynes's (1938) article defending the *GT* in the *Economic Journal* provides a critical analysis of the quantity theory of money that the *GT* does not, and that this was suggested to him by Townshend when claiming that the «quantity theory does not apparently have any positive bearing on monetary policy» (letter to Keynes, 13 April 1936).

⁴ This attitude is reflected also in Keynes's correspondence with Ralph Hawtrey. In order to express dissent from Hawtrey's representation of his analysis as if it implied an identity between saving and investment, Keynes (letter to R. Hawtrey, 24 March 1936) claims: «You will, for example, see this point excellently put in a letter written to last Saturday's *Economist* by Mr. Hugh Townshend» (Keynes, 1973c, 16). It should be noted that the *Economist*'s review was signed E. A. G. R. and can be attributed to Austin Robinson, who was collaborating with Keynes in the editorship of the *Economic Journal*.

«Liquidity-premium and the theory of value» in the *Economic Journal*. As already noted, among Keynesian scholars, Townshend's 1937 essay is attributed a prominent role for its presentation of the subversive character of Chapter 17 of the *GT*. Indeed, on uncertainty issues, it is usually paired with Keynes's own 1937 *QJE* essay (Shackle 1967, 246-248). The correspondence concerning the publication of Townshend's article has not survived, but late in March Townshend writes to Keynes, confirming that he is reviewing Hawtrey's *Capital and Employment* for the *Economic Journal*. On this occasion he sends Keynes a few notes with the aim «to be quite clear as to what the *basic* postulate of classical economics is which you have found to be wrong», with specific regard to «the interpretation of your theory of value in my note in the Journal» (letter to Keynes, 23 March 1937). These notes have not survived, but the following correspondence hinged on the way to characterize «classical» thought, a theme Townshend announces he wants to touch in his review of Hawtrey, and that he finds related more to the dealing with uncertainty than to Say's law.⁵

Townshend reacts warily to Keynes's suggestion that:

«The form of the premise which they [the classical economists] are semi-consciously making is, I fancy, much more like Say's law ... that is to say, [the assumption] that the demand price for aggregate output is always equal to the supply cost, whatever the level of output» (letter to Townshend, 25 March 1937).

In his reply of April 1937, Townshend claims instead that alternative forms of the classical postulate «like Say's law, full employment, etc.» are too general a characterization of the differences between the classical and the newly introduced general theory to be useful. He suggests that:

«we have to deal ... not really with an unstated postulate which does not happen to hold good, but with a logical confusion—an elusive thing which no one states because it cannot strictly be 'stated' at all».

Townshend concludes, first, with a point he already made in his note just appeared in the *Economic Journal*:

«your theory of value [implied in Chapter 17 of the *GT*] in its most general form sh[o]ws that it is not true that the price of *any* durable asset ... is determined by the equations of supply and demand for the others—so that *all* dynamic equational theory must be abandoned (this follows from the effect of expectations in the second-hand market—the exchange for *existing* assets)».

But this is only introductory to a second conclusion, that is, the claim that the «elusive thing» may consist in that:

⁵ Townshend suggests that «*If* [in my notes] I am on the right lines, a brief statement of the point in the review might help to remove the intolerable shoal of read herrings which—so it seems to me—are being thrown, both in Hawtrey's book and elsewhere, across the trail» (letter to Keynes, 23 March 1937). As shown below, significant sections of Townshend's review of Hawtrey represent the outcome of the correspondence with Keynes.

«all the forms of dynamic classical theory which I know seem to me to be trying to describe a world in which *risks exists without uncertainty*. The economic man is supposed both (a) to know the future and (b) not to know it, at the same time. It can surely only be because of (a) that he keeps no inactive balances and because of (b) that he earns profit by risk taking (cf. Knight [*Risk, Uncertainty and Profit*])» (letter to Keynes, 7 April 1937).

This point appears like a summary of Keynes's (1937) essay just published in the February issue of the *Quarterly Journal of Economics*. It does not come as a surprise then that Keynes resolutely accepts Townshend's conclusions:

«I think there is a great deal to be said for your view that the classical theory is not valid on any assumptions, and that you have put your finger on the spot in saying that they [the classical economists] are trying to describe a world in which risks exists without uncertainty» (letter to Townshend, 11 April 1937)

Encouraged by Keynes, Townshend's (1937b) published criticism of Hawtrey stresses both points mentioned in the letter just examined. In his review of the book, Townshend names Hawtrey's theory of value «consistently classical» on two grounds. First, Hawtrey does not admit the possibility that «prices [of bonds, shares or capital goods] may change not only independently of the physical conditions of production, but also independently of the movements of ... money-balances, *merely* because of quasi-unanimous changes in expectations» (Townshend 1937b, 323). Second, he relies on a condition of equilibrium among producers' and consumers' decisions that, «if Mr. Keynes is right», it is undetermined since «the prospect of future returns (whether from enterprise or from the realisation of accumulated assets) is not expressible as a *mathematical* expectation» (Townshend 1937b, 325).

It is also significant that Townshend's own recollection of the meaning of his exchange with Keynes places emphasis on the risk/uncertainty distinction. Late in the 1960s, after George Shackle had praised Townshend's note on liquidity preference in his volume on *The Years of High Theory* (Shackle 1967), the two started corresponding. The Shackle Papers at Cambridge University Library include a 1973 letter from Townshend to Shackle in which Townshend refers to his 1930s correspondence with Keynes. Although he concedes that he is «coming back for a glimpse at economics after 35 years», he finds it «odd, and surprising, that the economic establishment has been so excessively conservative» in elaborating on the lessons of the *GT*. Townshend recalls that one possible approach to decision-making under uncertainty was suggested to him by Keynes, consisting in «a logical analysis of the meaning of probability in real life, as distinct from mathematical probability (calculated "expectations")». He then continues:

«Rather to my surprise, Keynes agreed, without qualification, to a suggestion I had put to him in a letter that what the 'classical' economist were really doing was to try to separate out, in the probabilities of the real life, the indissoluble elements referred to in his *Treatise*

on Probability as ‘risk’ and ‘uncertainty.’ The *Treatise* appeared when I was an undergraduate at Cambridge (about 1912), so I was of course familiar with it; and the ‘risk-uncertainty’ approach, though not taken far by Keynes, had much struck me. So far as I know, it has not been followed up. With this encouragement from Keynes, I tried for sometime to pursue this line of thought in connexion with the economics of the *General Theory*; but I got nowhere». ⁶

3. The *Treatise on Probability* and its relevance for the correspondence.

In 1938, when the correspondence between Keynes and Townshend resumed after Keynes’s recovery from illness, the theoretical question of how to deal with uncertainty became the main focus of discussion.⁷ Keynes asked Townshend to review George Shackle’s volume *Expectations, Investment and Income*. Shackle’s book came out in print in May 1938, while Townshend’s review appeared in the September 1938 issue of the *Economic Journal*. On receiving the draft of Townshend’s review of Shackle, the issue of how to deal with probability in the valuation of decision made in an uncertain environment is dealt with by Keynes as follows:

«The matter you are tackling is a very important and interesting one often in my mind. But the enclosed treatment seems to me still too much half-baked. I fancy one has to tackle it on the basis of ‘equivalent certainties’ ... But a main point to which I would call your attention is that, on my theory of probability, the probabilities themselves, quite apart from their weight or value, are not numerical. So that, even apart from this particular point of weight, the substitution of a numerical measure needs discussion» (letter to Townshend, 27 July 1938)

The letter from Townshend to which Keynes is replying, possibly introducing his review of Shackle’s volume, has not survived. Also, it seems ill-advised to speculate on what Keynes refers to when describing the «enclosed treatment» as «half-baked», since it may relate not to Townshend’s notes, but to a paper sent to him by another correspondent, that he forwarded to Townshend but could not be retrieved (Moggridge 1979, 288 n.).

Still, the use of the term equivalent certainties in quotation marks suggests that the

⁶ Letter from H. Townshend to G. L. S. Shackle, 22 August 1973, Cambridge University Library, Add. MS 7669, 9/3/44. As noted by a referee, the letter also reveals Townshend’s inability to expand upon Keynes’s analysis of probability. Still, Townshend’s interest in the search for a criterion for decision-making under uncertainty he never really pursued – one he thinks Keynes shared with him – is apparent in his following comment to Shackle: «Maybe you will think it is a hopeless attempt to rationalize the irrational – to escape from Plato’s cave. I am not sure».

⁷ As it is well-known, after feeling ill for months, Keynes had an heart attack in May 1937 (Moggridge 1992, 608-609). This made him out of action in varying degrees, including correspondence. As concerns economics and the defence of the *GT*, he resumed corresponding with close colleagues such as Richard Kahn and Dennis Robertson later in summer, but not with others until early 1938. As of September 1939, though, with the outbreak of the Second World War, Keynes’s energies were almost entirely devoted to the problems of war finance and postwar reconstruction, and his correspondence no longer concerned the *GT*.

discussion among the two relates to Shackle's first attempt to deal with investment decisions under uncertainty in his 1938 book. Indeed, in his review of Shackle – after praising the book both for its emphasis on the «inherently expectational character» of most of the concepts treated in it and for devoting an entire chapter to a «*consistently* expectational theory of investment» – Townshend (1938, 523) concludes that «Mr. Shackle's concept of 'equivalent certainties', ingenious and interesting as it is», does not constitute a satisfying solution. Townshend's criticism is that Shackle's decision criterion hinges on a notion, that of numerical probability, that clashes with «the logical objections to this view ... pointed out by Keynes in his *Treatise on Probability*». ⁸ It can be argued then that Townshend refers to the *TP* following on the suggestion formulated by Keynes in his letter of July 1938. ⁹

As noted by Moggridge in his editorial comments, the final part of the exchange with Townshend is peculiar especially because it carries the discussion back to Keynes's earlier concerns, with the *TP* – to which there is no other reference in the whole correspondence about the *GT* reproduced in the *Collected Writings* – mentioned in private correspondence after many years. ¹⁰

⁸ It should be recalled that Shackle's analysis of investment decision-making under uncertainty in *Expectation, Investment and Income* is unrelated to his 1949 proposal in *Expectation in Economics* to replace probability with the notion of potential surprise. Shackle (1938, 64), still endorsing the conventional wisdom, argues that although in practice «the entrepreneur will not have in mind a unique value for the expected yield of the plant at any particular future moment, but a number of values to which he attaches different probabilities ... he will, however, be able to select a single value which, if he could expect it with certainty, would have the same significance for him as the distribution of probability he has actually in mind». Townshend's (1938, 523) suggestion in his review, that he doubts that «the treatment of an economic expectation as uniquely with a value will stand logical analysis without modification», seems to have stricken a chord with Shackle, who took Townshend's comments in great esteem (Harcourt 1981). Just a year later, in an article submitted to Keynes for publication in the *Economic Journal*, Shackle (1939) started thinking in terms of potential surprise instead of probability and of a replacement for mathematical expectation as a decision criterion. In spite of this, Shackle did not take Keynes's *Treatise* into account in the shaping of his theory of decision, missing the fact that Keynes provided a mathematical foundation for his work by constructing interval valued probabilities. In fact, Shackle never really engaged with Keynes's treatment of probabilities until very late (Shackle 1979). On this point see Brady (2013) and [omitted reference to the author's work]

⁹ As suggested by a referee, the discussion with Townshend shows a typical aspect of Keynes's style. Although he was keen to offer suggestions that, if followed-up, would allow the correspondent to figure out what he was thinking about the topic under scrutiny, Keynes was not used to give a correspondent a clear cut answer to a theoretical question.

¹⁰ As a matter of fact, although amply quoted in contemporary statistical literature, the *TP* was rarely mentioned in economics literature. For instance, a search of JSTOR database reveals that, after its 1921 review by Pigou, the *TP* had been mentioned in the *Economic Journal* only once – by Erik Lindahl in a review of the Swedish edition of Myrdal's *Monetary Equilibrium* – before Townshend did it in his review of Shackle.

But what is relevant here is that, through the comments on Shackle's first investigation of an expectational theory of investment decisions, the discussion between Keynes and Townshend shrinks focus on decision-making under uncertainty. While criticizing the conventional way to derive it, Keynes does not deny the usefulness of an appropriate measure apt to identify certainty equivalents for choices entailing uncertain outcomes, and invites Townshend to consider which kind of certainty-equivalent can be used when probabilities are non-numerical.

Moreover his straight comment on the non-numerical character of probabilities is explicitly related to an analysis of human conduct in general:

«the economic problem is of course only a particular department of the general principles of conduct, although particularly striking in this connection because it seems to bring in numerical estimations».

It is not difficult to recognize here the themes Keynes had addressed in Chapter 26 of the *TP* dealing with the «Application of probability to conduct», and indeed Keynes's concluding remarks in his letter to Townshend would serve well as a summary of that chapter:

«One arrives presumably at the numerical estimations by some system of arranging alternative decisions in order of preference, some of which will provide a norm by being numerical. But that still leaves millions of cases over where one cannot even arrange an order of preference. When all is said and done there is an arbitrary element in the situation» (letter to Townshend, 27 July 1938).

In order to clarify what Keynes is referring to when talking of the difficulties to arrive at «numerical estimations», let us then summarize briefly the content of Chapter 26 and its role in the *TP*. As is well-known, the most distinctive feature of the *TP* lies in its conception of probability as a logical relation between propositions. Of these probability relations – epistemic in character, as not derived from frequencies – Keynes offers a formal theory essentially based on comparisons of the form 'at least as probable as'. Contrary to frequency probabilities, Keynes's probability relations do not generally bear numerical values, and yield a partially ordered set of probabilities, allowing only qualitative judgements. Keynes's aim, though, is to offer a theory of the degree of belief that individuals ought to hold even when the available evidence is such that numerical probabilities are not available, and so he does not limit himself to a critique of frequency probability, but develops a theory of «non-numerical», or «numerically undetermined» probabilities. Following on Boole, Keynes (1973a [1921], pp. 134, 176) accounts for a numerical measure of a relation of probability through the method of «numerical approximation», that he describes as «the relating of probabilities, which are not themselves numerical, to probabilities, which are numerical». ¹¹

¹¹ His intuition is that probabilities incapable of numerical measurement, «can be placed nevertheless *between* numerical limits» so as to make «a great number of comparisons or

Moreover, he suggests that when using probability as a guide to action, the degree of completeness of the available evidence must be taken into account. In Keynes's view the use of probabilities should concern both the magnitude of the probability of an argument and the degree of confidence in it. Keynes's claim is that the uncertainty surrounding individuals cannot be represented only through probability, and that the confidence in the probability assessment itself is another relevant dimension in the epistemic state of the individual. In order to exemplify his point, Keynes (1973a [1921], p. 82) compares the probability of drawing a white ball from an urn known to contain black and white balls in equal proportion with the probability of drawing a white ball from an urn containing an unknown proportion of white and black balls. Keynes claims that, while by virtue of the principle of indifference the probability of drawing a white ball from an urn is $\frac{1}{2}$ in both cases, he finds that a greater weight supports the argument that the probability is $\frac{1}{2}$ if draw from the first urn.¹² Other things being equal, Keynes argues, one should prefer «a course of action» whose probability is based on a higher «weight of evidence».

Although the *TP* mainly concerns the logic of probability and related philosophical issues, both the non-numerical character of probability and the weight of argument are used to characterize Keynes's understanding of how to act, as it is apparent from Chapter 26 of the *TP*, that he devoted to explain the practical relevance of his theory. The main aim of Chapter 26 is to justify Keynes's claim that probability is a «guide of life», and that «it is *rational* to be guided by it in action». Keynes's investigation starts off with a critique of current practice. He objects to «normal ethical theory» because its hope of «gradually bringing the moral sciences under the sway of mathematical reasoning» is based on the faulty doctrine of mathematical expectation. Keynes (1973a [1921], 343) maintains that, in order to provide guide to human conduct, «normal ethical theory at the present day makes two assumptions: first, that degrees of goodness are numerically measurable and arithmetically additive, and second, that degrees of probability also are numerically measurable». As a result, ethical theory decides among alternative acts on the basis of their mathematical expectations, which Keynes presented as «a technical expression originally derived from the

appropriate measurements become possible». Thus, Keynes's concern with unmeasurable probabilities can be given operational content, as in the works of Koopman (1940) and Good (1962) that inspired the developments of indeterminate, but interval-valued probabilities (Levi 1980). On these usually disregarded origins of Keynes's discussion of probability see Brady and Arthmar (2012).

¹² This example was later made known in the economics literature by Ellsberg (1961) when introducing ambiguity in decision-making. This is highly significant of the enduring influence of Keynes's insight since current developments of decision theory under uncertainty questioning the subjective expected utility model originate as attempts to account for Ellsberg's ambiguity (Wakker 2008). On this aspect see [omitted reference to the author work].

scientific study of gambling and games of chance» that «stands for the product of the possible gain with the probability of attaining it».

Keynes (1973a [1921], 344-345) disagrees with the use of mathematical expectation as a criterion for action for various reasons, among which two are directly related to his analysis in the previous chapters of the *TP*. First, to assume that «degrees of probability are wholly subject to the law of arithmetic, runs directly counter to the view which has been advocated in part I [of the *TP*]». As just seen, Keynes is referring to his contention that it may then be impossible to identify probability with a single number. This means that in many ordinary cases it may be troublesome to obtain a quantitative measure for making comparisons. The second critique concerns the necessity, when making decisions, to take into account the degree of completeness of information: the use of mathematical expectation is unsound since it «ignores what I have termed the weights of arguments, namely the amount of evidence upon which each probability is founded». Keynes suggests that a low weight may be associated with situations in which the state of confidence worries the decision-maker to the point of feeling unsure whether a single, sharp probability distribution is as reliable as to be used for calculus.¹³

It should be stressed though that Keynes is not criticising the use of mathematics as such. His critique concerns mathematical reasoning only «if we mean, as they [the supporters of normal ethical theory] meant, by mathematics the introduction of precise numerical methods». Mathematical reasoning is an aid in analysis because of its «symbolic», rather than numerical, character. One must admit that «even if we are able to range goods in order of magnitude, and also their probabilities in order of magnitude, yet it does not follow that we can range the products composed of each good and its corresponding probability in this order» (Keynes 1973a [1921], 348). But this is a critique of the «precise» numerical methods used in utilitarian ethics – emerging from Keynes's critique of «precise» numerical probabilities used by the frequency approach – not of mathematical reasoning in itself.¹⁴ **In fact, Keynes aims at positive analysis. On the one hand, non-**

¹³ Keynes deals also with the «risk» represented by the probability of not succeeding in a given gamble, defined as $R = p(1-p) A$, for the gamble which offers A with probability p and 0 with probability $1-p$. It is through risk that Keynes takes utility into account. His critique of the use of mathematical expectation can thus be extended to expected utility theory.

¹⁴ The influence of the early works of Bertrand Russell is apparent here. It is well-known that Keynes intended probability theory as a branch of logic – the logic of uncertainty – inspired by Russell's attempt to found mathematics on the logic of certainty (Gillies 2000). But Keynes followed Russell (1903, 182-183) also in his claim that «those mathematicians who are accustomed to an exclusive emphasis on numbers» are wrong when they argue that «no much can be said with definiteness concerning magnitudes incapable of measurement». Judgements of equality, and of greater and less, «upon which all measurements depend ...are still possible where measurement fails», so that, Russell concludes, «the whole subject of the measurement of

numerical probabilities allow qualitative comparisons that can be attributed mathematical significance by means of «numerical approximation». As noted already, one of Keynes's goal in the *TP* is to show that a logic of non-numerical probabilities can be developed nonetheless. On the other hand, Keynes tries to provide a formal example of how a weight of argument lower than a given maximum would influence decision-making. This is made through an illustration of how probability and weight could be compounded into a coefficient intended to replace standard probabilities in the calculation of expected values (Brady 1993, Gerrard 1995).¹⁵

To sum up on the significance of Chapter 26 of the *TP*. Although Keynes (1973a [1921], 348) concludes that it is doubtful whether an alternative to the notion of mathematical expectations lies «in the discovery of some more complicated function of the probability wherewith to compound the proposed good», the chapter on conduct consists of an investigation of criteria for decision-making. Keynes's criticism of mathematical expectation entails the search for a more general theory. Keynes takes stock of the idea formulated in Part II of the *TP*, that a method of numerical approximation of non-numerical probabilities can be proposed, and provides an extensive discussion of the characteristics to be attributed to a decision rule alternative to the maximization of mathematical expectation. The conclusion Keynes reaches, that «it is not always possible by a mere process of arithmetic to determine which of the alternative ought be chosen» (Keynes 1973a [1921], 344-345), confirms his philosophical claim in Part I of the *TP* that this is the realm of qualitative analysis, where probability measures usually express an incomplete ordering and «the legitimacy of such [probability] comparisons must be a matter for special enquiry in each case» (Keynes 1973a [1921], 40). But this does not mean that Keynes disparaged a formal analysis of this theme.

4. The ultimate meaning of the correspondence

Keynes's analysis of the practical relevance of probability and weight to decision-making provides

quantities is, in fact, one of more practical than theoretical importance».

¹⁵ Keynes introduces coefficient c and argues that, in making a decision concerning a possible «amount of good» A which can be expected with probability p , the standard mathematical expectation $E=pA$ should be replaced by an alternative criterion for choice such as $E'=cA$. Coefficient c depends on both probability and weight as follows: $c=2pw/(1+q)(1+w)$, where p is the probability of an event, $q=1-p$ the probability of its complement, and w is the weight, ranging from 0 to 1. Except for the taking of «risk» into account, which Keynes obtains dividing p by $(1+q)$, when the weight of argument is at its maximum there is no need to modify E as a decision rule. However when lower than 1, a decision rule like E' is worth considering. It is significant that Keynes's decision rule allows for both risk aversion and uncertainty aversion (Brady 1993).

the basis for a theory of decision under uncertainty that, in its critique of mathematical expectations in the *TP*, constitutes the grounds on which Benthamite calculation is deemed to be ill-suited to deal with uncertainty in the *GT*. In his last letter to Townshend, this aspect – a crucial one emphasized by authors claiming that there is a continuity between the *TP* and the *GT* – clearly emerges (Runde 1994a). As already noted, Keynes reminds Townshend that he is inclined to associate «risk premium with probability strictly speaking, and liquidity premium with what in my *Treatise on Probability* I called ‘weight’». Also, the correspondence shows that significant technical aspects of the *TP* survived Ramsey’s critique and Keynes did not endorse the subjective probability viewpoint suggested by Ramsey (1978 [1926]). Had he yielded to Ramsey’s on the possibility to derive point probabilities from action in every instances, Keynes would not refer to non-numerical probabilities as such strong an objection to received analysis of decision-making under uncertainty (Runde 1994b). Keynes’s analogy in his last letter to Townshend, associating the liquidity premium with «an increased sense of comfort and confidence», cannot be accommodated with Ramsey’s subjectivist perspective, in which there is no room for a measure representing the degree of reliance on a probability assessment. So he may have been perplexed, in the assessment of his early beliefs, about the significance of defending the epistemological underpinnings of his theory of probability (Raffaelli 2006, Marchionatti 2010), and even inclined, in the *GT*, to replace explicit treatment of probability with explicit treatment of conventions (Dow 2003). But the correspondence shows that Keynes never stopped thinking of possible uses of his theory of probability.¹⁶

The final part of the exchange with Townshend provides further evidence for this continuity thesis. In fact, it helps reveal to which extent Keynes’s thoughts about uncertainty issues in the years of the defence of the *GT* were still related to his analysis in the *TP*, with specific regard to decision-making and its possible mathematical treatment. This is especially true since in his last letter Townshend submits to Keynes an extended examination of the «alternative lines» on which «those [scholars] who, following on the appearance of the *General Theory*, are trying to develop further an expectational economic analysis, seem to me to be proceeding» (letter to Keynes, 25

¹⁶ Nor he abstained from thinking in terms of certainty-equivalents. This is an element of continuity in Keynes’s thought rarely mentioned in secondary literature, with the notable exception of Gerrard (1994). In the *GT* Keynes (1973b [1936], 24) defines short term expectations as «the amount of proceeds which the entrepreneurs expect to receive from the corresponding output», and he adopts a certainty-equivalence modelling strategy when clarifies that this is «the expectation of proceeds which, if it were held with certainty, would lead to the same behaviour as does the bundle of vague and more various possibilities which actually make up his state of expectation when he reaches his decision». As noted by Gerrard (1994), in conformity with his probability theory, Keynes allows for both the entrepreneur to possess a probability distribution over outcomes and that probability to be indeterminate, since expectation can be held «with varying degrees of probability and definiteness».

November 1938). As noted above, this was suggested by Keynes himself in his letter of July 1938, with his reference to the *TP*.

In the letter of November 25th 1938, Townshend offers Keynes a deep analysis of «methods hitherto used for expectational economic analysis», classifying them into two groups. First, there is Shackle and his attempt to use numerical probabilities to determine equivalent certainties, a line of approach, Townshend reminds Keynes, he already objected to in his 1937 review in the *Economic Journal*. Second, there is the attempt to avoid the difficulties implicit in the use of probabilities by envisaging a time interval short enough to suppose that the investor expects with certainty the values of the assets in which she holds her wealth. This method of analysis, attributed to Hicks and Robertson, is considered illegitimate since it conceals the doubt of investors as to what may happen to the values of their assets «before the end of any period, however short». Townshend argues that neither of the methods is quite valid, but his analysis does not arrive to a definite judgement.¹⁷ He claims that his investigation

«leaves open the question whether, as you suggested in your letter, it may not be possible to develop a logical doctrine of equivalent certainties free from the assumption of numerical probabilities, and perhaps of wider than economic application».

But his main conclusion is that

«the element of arbitrariness in judgements of probability, to which you refer ... really implies a criticism, or at least calls for further analysis, of the basic concept of economic man, defined as *determinately* motivated by (his) judgements of maximum (in some sense) anticipated profitability» (letter to Keynes, 25 November 1938).

It must be stressed then that the subject matter of the last letter from Keynes to Townshend – the one usually quoted in secondary literature – is the classic notion of «economic man» and his reliance on an orthodox criterion for decision-making. In his reply, Keynes considers the problems discussed in the letter as «elusive», but declares himself in accord with Townshend's treatment. Keynes comments on both the specific aspect of the interval to be taken into account when considering expectations and the general issue of rational choice. As to the interval, he agrees with Townshend's scepticism and maintains that «doubt as to what may happen within the interval is a factor of which we must take account». As to the traditional notion of economic calculus, he claims that «all this is not particularly an economic problem, but affects every rational choice concerning conduct where consequences enter into the rational calculation» (letter to Townshend, 7 December

¹⁷ Townshend's tone is pessimistic: «I do not really believe that anything more can be hoped for in economics from the sort of abstract reasoning in which I am indulging (with others!) than the negative result of exposing current fallacies». See also Townshend's recollection in his 1973 letter to Shackle reported above.

1938).

It is from this viewpoint, pertaining to the notion of «rational calculation» in traditional fashion, that Keynes argues that

«generally speaking, in making a decision we have therefore before us a large number of alternatives, none of which is demonstrably more ‘rational’ than the others».

Hence, Keynes is addressing «rational calculation» and the traditional picture of the «economic man» when he concludes with the often-quoted sentence that

«to avoid being in the position of the Buridan’s ass, we fall back ... on motives of another kind, which are not ‘rational’ in the sense of being concerned with the evaluation of consequences, but are decided by habit, instinct, preference, desire, will, etc».

Keynes is still talking of the established view of «economic man» he wants to criticize when concluding in his last remark that «it may well be, as you suggest, that when we remember all this we have to abate somewhat from the traditional picture of the latter» (letter to Townshend, 7 December 1938).¹⁸

Eventually the issue of identifying what alternative theory of decision-making can be put forward remains unsolved. But there appears to be no retreat from the conclusion already reached in Chapter 26 of the *TP*. Among the «motives of another kind» – those that are not «rational» in the classical meaning of the term – Keynes first lists «habit», suggesting the significance of conventional aspects in judgement. But he also lists «instinct», «preference», «desire» and «will», all of which suggest the necessity to look for aspects of behaviour that can help derive a generalised notion of rational behaviour.¹⁹

5. Concluding remarks

¹⁸ Keynes (1973a [1921], 82) had already used the metaphor of Buridan’s ass in the *TP*, in order to clarify the meaning attributed to the principle of indifference by mathematicians such as James Bernoulli and Laplace when considering equi-probable cases. However he does not seem to attribute the metaphor any relevant meaning with respect to decision-making. For an alternative view, see Carabelli (1998).

¹⁹ It is worth noting that this generalised notion of rational behaviour allows for «animal spirits». The list of motives just quoted provides support to a reading of Keynesian «animal spirits» that does not confine them to an exogenously given «element of caprice» representing irrational action. In fact, Keynes seems to refer to an attitude towards uncertainty that may justify taking action in spite of uncertainty, even when classic rational calculation cannot stand. Among Keynesian scholars, Dow and Dow (2011) provide a compelling analysis of why the identification of animal spirits with irrationality tout court should be rejected.

Most of the critical literature on Keynes has insisted that Keynes's rejection of mathematical expectation and probability calculus entails the endorsement of a theory of decision-making that, in the *GT*, is mostly based on conventional valuations. This has generated much controversy over the degree to which Keynes retained his theory of probability in his later work, giving birth to a debate on continuity or change. Certainly, Keynes came to recognise that the purely logical analysis of human behaviour he put forward in the *TP* is inadequate for understanding practical decision-making when individuals face uncertainty, and in Chapter 12 of the *GT* he drew on his understanding of social systems and experience of the actual economy to develop kind of ordinary logic he did not appeal to in the *TP*. But the idea that Keynes came to reject his early beliefs seems to be too an extreme reading of his thought (Gerrard 2003).

Admittedly, for Keynes people in market may fall back on conventions that give them assurance that they are doing the right thing, and, in order to do so, they use a variety of techniques which require concentrating on the «average opinion» (1973b [1936], 156). However, Keynes contrasted his apparently sceptical attitude about the ability of economic agents to take reasonable actions under uncertainty with claims that incorporate normative considerations for agents who reject following conventions. For instance, he suggested that «[t]he social object of skilled investment should be to defeat the dark forces of time and ignorance which envelope our future» (Keynes 1973b, [1936] p. 155), and, after introducing the features of instability in markets related to human nature, he argued that «[w]e should not conclude from this that everything depends on waves of irrational psychology» (Keynes 1973b, [1936] pp. 162-63). Moreover, after summarizing the conventional aspects of the attitude towards uncertainty in his *QJE* summation article, Keynes (1937, p. 215) concluded that «the theory we devise in the study of how we behave in the market place should not itself submit to market-place idols». There exists, then, the need to explain how skilled Keynesian agents, such as expert professional endowed with superior knowledge or long-term investors, should behave in markets when they do not want to follow conventional valuations, for instance because they think these are on the brink of failure.

In view of the assessment of the correspondence with Townshend this paper has provided, the taking into account of these normatively oriented comments seems appropriate. For sure, the appeal to direct judgement and intuition characterizes Keynes's stance, as it also appears from the most famous and often-quoted excerpts of the last letter from Keynes to Townshend. But the understanding of those excerpts we gather from a comprehensive reading of the whole exchange suggests that they should be interpreted as a response to the classical notion of rational calculus, more than to rationality in itself. An all-embracing reading of Keynes's correspondence with Townshend shows that, while defending the *GT*, Keynes still aimed at providing the guidelines for

an alternative approach to decision-making under uncertainty. This is pursued through an examination of what is the role for investment decision-making under uncertainty when standard probabilities cannot be attributed to possible outcomes. The correspondence indicates that Keynes's interest in methods for evaluating decision-making under uncertainty – a relevant matter of discussion in the *TP* – did not fade away even when considering the cases in which Benthamite calculus is inapplicable he discussed in the *GT* and in the 1937 *QJE* essay.

Furthermore, the correspondence shows an interest in the formal content of decision-making that has been usually disregarded in the literature. When authors such as Davidson (2009) and Skidelsky (2011) place emphasis on the crucial importance of uncertainty about the future, they argue that, in Keynes, this implies that uncertainty cannot be model using probability calculus. But this seems to be at odds with other characteristics of Keynes's thought. Indeed, it has been noted that Keynes's discontent with classical orthodoxy did not stop him from framing his argument in mathematical terms and that, in principle, he did not condemn the construction of theories that were more formal than his own (Backhouse 2010). The close reading of the correspondence offered in the present paper provides further evidence for the claim that Keynes's interest in a formal representation for decision-making under uncertainty never really vanished.

REFERENCES

- Backhouse R. E. 2010, «'An Abstruse and Mathematical Argument': the Use of Mathematical Reasoning in the General Theory», in B. Bateman, W. T. Hirai and C. Marcuzzo (eds), *The Return of Keynes: Keynes and Keynesian Policies in the New Millennium*, Cambridge, MA, Harvard University Press.
- Brady M. E. 1993, «J. M. Keynes's theoretical approach to decision making under condition of risk and uncertainty», *The British Journal for the Philosophy of Science*, 44, 357-76.
- Brady, M. E. 2013, «The economic consequences of G. L. S. Shackle's ignorance of Keynes's theory of probability, uncertainty, and decision making», mimeo, available at SSRN: <http://ssrn.com/abstract=2309259>
- Brady M. E. and Arthmar, R. 2012, «Keynes, Boole and the interval approach to probability. *History of Economic Ideas*, 20, 65-84.
- Bateman B. 1996, *Keynes's Uncertain Revolution*, Ann Arbor, MI, University of Michigan Press.
- Carabelli A. M. 1998, «Keynes on Probability, Uncertainty and Tragic Choices», *Cahiers d'économie politique*, 30, 187-226.

- Chick V. 1987, «Townshend, Hugh (1890-1974), in J. Eatwell, M. Milgate and P. Newman (eds), *New Palgrave. A Dictionary of Economics*, London, Macmillan.
- Davidson P. 2009, *The Keynes Solution: The Path to Global Economic Prosperity*, London, Palgrave.
- Davis J. B. 1994, *Keynes's Philosophical Developments*, Cambridge, Cambridge University Press.
- Dow S. C. 2003, «Probability, uncertainty and convention», in J. Runde and S. Mizuhara (eds), *The Philosophy of Keynes's Economics*, London, Routledge.
- Dow A. and Dow S. C. 2011, «Animal spirits revisited», *Capitalism and Society*, 6. Available at SSRN: <http://ssrn.com/abstract=2208040>
- Ellsberg D. 1961, «Risk, ambiguity, and the Savage axioms», *Quarterly Journal of Economics*, 75, 643-69.
- Gerrard B. 1992, «From *A Treatise on Probability* to the *General Theory*: continuity or change in Keynes's thought?» in B. Gerrard and J. Hillard (eds), *The Philosophy and Economics of J.M. Keynes*, Aldershot, Edward Elgar.
- Gerrard B. 1994, «Beyond rational expectations. A constructive interpretation of Keynes's analysis of behaviour under uncertainty», *Economic Journal*, 104, 327-37.
- Gerrard B. 1995, «Probability, uncertainty and behaviour: a Keynesian perspective», in S. C. Dow and J. V. Hillard (eds), *Keynes, Knowledge and Uncertainty*, Aldershot, Edward Elgar.
- Gerrard B. 2003, «Keynesian uncertainty: What do we know?», in J. Runde and S. Mizuhara (eds), *The Philosophy of Keynes's Economics*, London, Routledge.
- Gillies D. 2000, *Philosophical Theories of Probability*, London, Routledge.
- Good I. J. 1962, «Subjective Probability as a Measure of a Non-Measurable Set», in E. Nagel, P. Suppes and A. Tarski (eds), *Logic, Methodology and Philosophy of Science*, Stanford, Stanford University Press.
- Harcourt G. C. 1981, «Notes on an Economic Querist: G. L. S. Shackle», *Journal of Post Keynesian Economics*, 4, 136-144.
- Keynes J. M. 1973a [1921], *A Treatise on Probability. The Collected Writings of John Maynard Keynes, Vol. VIII*, London, Macmillan.
- Keynes J. M. 1973b [1936], *The General Theory of Employment, Interest and Money. The Collected Writings of John Maynard Keynes, Vol. VII*, London, Macmillan.
- Keynes J. M. 1973c, *The General Theory and After: Part II, Defence and Development. The Collected Writings of John Maynard Keynes, Vol. XIV*, London, Macmillan.
- Keynes J. M. 1979 [1936-1938], «Letters to H. Townshend, various dates», in *The General Theory and After: A Supplement. The Collected Writings of John Maynard Keynes, Vol. XXIX*,

London, Macmillan.

- Keynes J. M. 1937, «The general theory of employment», *Quarterly Journal of Economics*, 51, 209-223.
- King J. E. 2002, *A History of Post Keynesian Economics since 1936*, Cheltenham, Edward Elgar.
- Koopman B. O. 1940, «The axioms and algebra of intuitive probability», *Annals of Mathematics*, 41, 269-92.
- Lawson T. 1993, «Keynes and Conventions», *Review of Social Economy*, 51, 174-200
- Levi I. 1980, *The Enterprise of Knowledge. An Essay on Knowledge, Credal Probability and Chance*, Cambridge (MA), The MIT Press.
- Marchionatti R. 2010, «J. M. Keynes, thinker of economic complexity», *History of Economic Ideas*, 18, 115-146.
- Moggridge D. E. 1973, «Editorial Introduction», in *The General Theory of Employment, Interest and Money. The Collected Writings of John Maynard Keynes, Vol. VII*, London, Macmillan.
- Moggridge D. E. 1979, «Editorial Note», in *The General Theory and After: A Supplement. The Collected Writings of John Maynard Keynes, Vol. XXIX*, London, Macmillan.
- Moggridge D. E. 1992, *Maynard Keynes: An Economist's Biography*, London, Routledge.
- O'Donnell R. M. 1989, *Keynes: Philosophy, Economics and Politics*, London, Macmillan.
- Raffaelli T. 2006, «Keynes and philosophers», in R. E. Backhouse and B. W. Bateman (eds), *The Cambridge Companion to Keynes*, Cambridge, Cambridge University Press.
- Ramsey F. P. 1978 [1926], «Truth and probability», in *Foundations: Essays in Philosophy, Logic, Mathematics and Economics*, edited by D. H. Mellor. London, Routledge & Kegan Paul.
- Rochon L-P. 1999, *Credit, Money and Production. An Alternative Post-Keynesian Approach*. Cheltenham, Edward Elgar.
- Runde J. 1994a, «Keynesian uncertainty and liquidity preference», *Cambridge Journal of Economics*, 18, 129-44.
- Runde J. 1994b, «Keynes after Ramsey. In defence of *A Treatise on Probability*», *Studies in History and Philosophy of Science Part A*, 25, 97-121.
- Runde, J. 2003, «On some explicit links between Keynes's *A Treatise on Probability* and *The General Theory*», in J. Runde and S. Mizuhara (eds), *The Philosophy of Keynes's Economics*, London, Routledge.
- Runde J. and Mizuhara S. (eds), 2003, *The Philosophy of Keynes's Economics*, London, Routledge.
- Russell B. 1903, *Principles of Mathematics. Vol. I*, Cambridge, Cambridge University Press.
- Shackle G. L. S. 1938, *Expectations, Investment and Income*, Oxford, Oxford University Press.
- Shackle G. L. S. 1939, «Expectations and employment», *Economic Journal*, 49, 442-52.

- Shackle G. L. S. 1967, *The Years of High Theory*, Cambridge, Cambridge University Press.
- Shackle G. L. S. 1979, *Imagination and the Nature of Choice*, Cambridge, Cambridge University Press.
- Skidelsky R. 2011, «The relevance of Keynes», *Cambridge Journal of Economics*, 35, 1-13.
- Townshend H. 1979 [1936-1938], «Letters to J. M. Keynes, various dates», in *The General Theory and After: A Supplement. The Collected Writings of John Maynard Keynes, Vol. XXIX*, London, Macmillan.
- Townshend H. 1937a, «Liquidity-Premium and the Theory of Value», *The Economic Journal*, 47, 157-169.
- Townshend H. 1937b, «Review of *Capital and Employment* by R. G. Hawtrey», *Economic Journal*, 47, 321-26.
- Townshend H. 1938, «Review of *Expectations, Investment and Income* by G. L. S. Shackle», *Economic Journal*, 48, 520-23.
- Townshend H. 1940, «Review of *Profits, Interest and Investment* by F. A. von Hayek», *Economic Journal*, 50, 99-103.
- Wakker P. (2008), «Uncertainty», in L. Blume and S. N. Durlauf (eds), *The New Palgrave: A Dictionary of Economics*, 2nd Edition, London, Palgrave Macmillan.