Spatial equilibrium in deviations: An application to skill-premium and skill-mix heterogeneity

Spatial Equilibrium in Deviations.

An application to skill-premium and skill-mix heterogeneity.

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Abstract. In this paper, we propose a novel - and general-purpose - modeling approach. We give a linear representation of the spatial general equilibrium, expressed in terms of local percentage deviations from the benchmark case of symmetry, where all the areas in the economy are taken to be initially identical. To illustrate the flexibility of our approach, we revisit the literature on the spatial heterogeneity of local skill premia and local skill mix. We show that our approach is able to encompass a variety of alternative explanations in a simple "unifying framework". Finally, we exploit a graphical version of the model to show how to implement empirical tests.

Keywords: local wage structure, spatial general equilibrium.

JEL Classification: R3, J31.

Short running title: *Spatial equilibrium in deviations*

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1. INTRODUCTION

The main contribution of this paper is to offer a linear representation of the standard Spatial General Equilibrium model derived from Roback (1982), assuming that technology and preferences are Cobb-Douglas functions, possibly the simplest parametrization in this literature (see Glaeser, 2008). We claim that such a representation is flexible and powerful enough to encompass a number of implications that –in the existing literature- require a multiplicity of distinct models. To support our claim, we use our framework to revisit the explanations that have been given in the literature to observed local skill premia, and local skill mix.

The adoption of Cobb-Douglas functions seems to impose major limitations to modelling capacity. For instance, the use of standard Cobb-Douglas production functions is universally abandoned in favor of Constant Elasticity of Substitution (CES) specifications, when skill biased technical change is considered; see, e.g., Acemoglu (2002). However, as shown in Accetturo, Dalmazzo, de Blasio (2014), skill-biasedness in technology can still be represented by a Cobb-Douglas under the "share-altering" hypothesis.[1] On the other hand, Cobb-Douglas preferences are homothetic. This is a substantial limit, since evidence shows that expenditure shares are not constant over income levels: see, e.g., Handbury (2013). Nevertheless, we will show that Cobb-Douglas preferences can still do a good job in mimicking the implications of non-homotheticity, once expenditure shares are allowed to vary across income or education levels. The formal simplicity due to Cobb-Douglas functions enables to obtain a Spatial General Equilibrium model which can be immediately log-linearized. A novelty of our approach is that we can rewrite all the linearized equations of the model in terms of percentage deviations from the case of "symmetry", that is, from an ideal symmetric economy where all places are initially the same. This approach - in the static world we consider - bears some resemblance with the approach followed by the

[1] Accetturo et al. (2014) demonstrate that skill-biased technical change can be represented *even without* CES production functions, which are adopted by the overwhelming majority of the papers on the subject: see, e.g., Beaudry et al. (2010). For a discussion on the limits of the CES, see Acemoglu and Autor (2011).

macroeconomic literature on Dynamic Stochastic General Equilibrium models, where the building equations are log-linearized and re-expressed in terms of deviations from their steady-state values (see Galì, 2008). As argued by Walsh (2017), the linearity of equations where variables are expressed in percentage changes is a main factor of the success of DSGE models.

In order to illustrate the advantages of our modelling strategy, we reconsider the analysis of the determinants of local skill premia, and of the local skill mix. The local skill premium is the wedge between the wages received, respectively, by skilled and unskilled workers in a certain location. On the other hand, the local skill mix is defined as the local ratio between skilled and unskilled individuals. A main point we want to emphasize here is that each existing explanation for skill premium and skill mix heterogeneity across areas is associated with a single specific model. The model we propose here, by contrast, encompasses several explanations in a simple unifying framework.

As emphasized by Glaeser (2008, p.85), the basic Roback model predicts that the local composition of the labor force adapts so as to guarantee the *uniformity* of skill premia across locations, a conclusion that is against the existing evidence[2]. Indeed, the homogeneity result holds only when several assumptions are satisfied. In particular, spatial homogeneity of skill premia requires that:

(i) Skilled and unskilled workers enjoy the same local amenities.

(ii) Skilled and unskilled workers do not bear any "mobility cost" when moving across locations.

(iii) Skilled and unskilled workers buy the same kind of housing services.

(iv) Skilled and unskilled workers have homothetic preferences.

A number of contributions have investigated the consequences of the elimination of each of such assumptions. For instance, Glaeser (2008, p.94,98) considers the case when educated and less

[2] See Glaeser (2008) and references therein. See also Lee (2010), and Beaudry et al. (2010).

educated people enjoy different types of local amenities in a Cobb-Douglas model. However, when analyzing the impact of skill-biased technical change on the local wage structure and skill distribution, Glaeser (2008) switches from Cobb-Douglas to CES production functions. Moretti (2013) investigates "real wage inequality" in a spatial model with mobility costs and local technical change, assuming that households consume one unit of housing and skills are fully segregated across local plants. Starting with a standard Roback's framework, Glaeser (2008, p.89-90) also considers the possibility that the local housing market is segmented, that is, skilled and unskilled individuals consume different types of housing. Finally, Black, Kolesnikova, and Taylor (2009) investigate the consequences on local premia of non-homothetic preferences, such that the elasticity of housing demand to income is different from one. Ganong and Shoag (2017) postulate that skill types are *perfect* substitutes. To model non-homothetic preferences, demand for housing is taken to be inelastic and equal to one unit. Under these assumptions, they show that different expenditure shares for housing across skilled and unskilled individuals may discourage the low-skilled to move towards more productive places, thus reducing income convergence.

A main point to be stressed is that all these contributions build on a variety of different models: one for each story, each one with specific functional forms. Here, we provide a *unifying* framework for the variety of explanations of local skill premia heterogeneity. Our model embeds in a single analytical framework the different factors that explain why skill premia – and the local skill mix - may differ across locations. This is just an application of our general-purpose modeling approach.[3] We consider an economy composed of two locations and, as mentioned, we refer to the benchmark case where such locations start with *identical* features, the "symmetric" case. We then express the linearized model in terms of local *percentage deviations from symmetry*. This methodology, which is novel to the literature and delivers handy linear expressions, allows us to discuss in a straightforward way the comparative static implications of local shocks to amenities, technology,

[3] A similar methodology is exploited in Auricchio et al. (2017) to investigate the impact of local public employment.

etc., on equilibrium outcomes. Finally, from our methodology we derive some suggestions on how the model's testable predictions can be put into effect.

The paper is composed as follows. Section 2 develops the theoretical model, illustrating the methodology and discussing its implications. Section 3 presents a graphical version of the model to show how to implement empirical tests. Section 4 concludes.

2. THE ANALYTICAL FRAMEWORK

We adopt a spatial general equilibrium model which builds on Roback (1982) and allow for idiosyncratic preferences for locations as, for example, in Nakajima and Tabuchi (2011), Moretti (2013) and Diamond (2016). The economy is parted into two regions, $\{a,b\}$. Firms use skilled and unskilled labor to produce an economy-wide *tradable* good. While firms are assumed to be fully mobile across regions, workers are subject to idiosyncratic preference shocks for each location. Such shocks generate "mobility costs" across areas which, in contrast with Roback's original framework, make the local labor supply imperfectly elastic to local real wages. Residential supply in each area is taken as exogenous (depending, possibly, on land availability and building regulation), and landowners are absentee. Both technology and preferences are assumed to be Cobb-Douglas.

We start by characterizing the behavior of firms, and then we consider preferences, both for skilled and unskilled individuals. Last, we analyze the local housing market equilibrium.

All equilibrium conditions will eventually be linearized and expressed in deviations from "symmetry", where places are initially identical.

2.1 Firms.

Denote, respectively, with $\{w_c^s, w_c^u\}$ the local wage level paid to skilled and unskilled workers, and set the price of the tradable good equal to one. In each location $c = \{a, b\}$, firms adopt a Cobb-Douglas constant-returns-to-scale technology which uses skilled and unskilled labor:

$$Y_c = Z_c \cdot (N_c^s)^{\alpha_c} \cdot (N_c^u)^{1-\alpha_c} \tag{1}$$

where $\{N_c^s, N_c^u\}$ are, respectively, the skilled and unskilled labor input employed by a typical firm operating in c, with $\alpha_c \in (0,1)$ and Z_c measuring for local total factor productivity. For brevity, we will define $1 - \alpha_c \equiv \beta_c$. Notice that (1) allows the Cobb-Douglas technology to have different factor-shares across locations. Indeed, as shown in Accetturo, Dalmazzo, de Blasio (2014), different areas can exhibit different degrees of skill biasedness in technology, as measured by the ratio α_c/β_c.[4]

The first-order conditions for the optimal choice of skilled and unskilled labor in region c are, respectively:

[4] Skilled-biased technical change can be represented in different ways. The most common representation works through CES technologies, as in Glaeser (2008, p.82), or Beaudry et al. (2010). In this case, the local production function (1) has the form $Y_c = Z_c \cdot \left[\psi_c \cdot \left(N_c^s\right)^{\sigma} + \left(N_c^u\right)^{\sigma}\right]^{\frac{1}{\sigma}}$, with $\sigma \in (0,1)$, where $\psi_c \geq 1$ is a location-specific productivity parameter that increases skilled productivity in place c. Profit-maximization will imply that $\frac{w_c^s}{w_c^u} = \psi_c \cdot \left(\frac{N_c^u}{N_c^s}\right)^{1-\sigma}$. Thus, given the local skill-mix $\frac{N_c^s}{N_c^u}$, a local skill-biased technological shock tends to raise the local wage-premium. However, similar implications can be reached by assuming "skill-biased share-altering technical change", as in Accetturo et al. (2014), after Seater (2005). Starting from a Cobb-Douglas technology $Y_c = Z_c \cdot (N_c^s)^{\alpha} \cdot (N_c^u)^{\beta}$ with $\beta \equiv 1 - \alpha$, a local share-altering skilled-biased shock $\Delta_c \in [0, \beta)$, such that the local production function becomes $Y_c = Z_c \cdot (N_c^s)^{\alpha+\Delta_c} \cdot (N_c^u)^{\beta-\Delta_c}$, will imply that $\frac{w_c^s}{w_c^u} = \frac{\alpha+\Delta_c}{\beta-\Delta_c} \cdot \left(\frac{N_c^u}{N_c^s}\right)$. Again, given the local skill-mix $\frac{N_c^s}{N_c^u}$, the local skill-biased technical shock $\Delta_c > 0$ tends to raise the local wage-premium.

$$\alpha_c \cdot Z_c \cdot (N_c^s)^{\alpha_c - 1} \cdot (N_c^u)^{1-\alpha_c} - w_c^s = 0 \quad (2)$$

$$\beta_c \cdot Z_c \cdot (N_c^s)^{\alpha_c} \cdot (N_c^u)^{-\alpha_c} - w_c^u = 0 \quad (3)$$

From (2) and (3) we obtain the following expression:

$$\frac{\alpha_c}{\beta_c} \cdot \frac{N_c^u}{N_c^s} = \frac{w_c^s}{w_c^u} \;\; with\ c = \{a, b\} \quad (4)$$

where $\frac{w_c^s}{w_c^u}$ is Glaeser's (2008) measure of the local skill premium, the object of interest here.

From (4), one can immediately derive an expression that relates relative wages across the economy. In particular, it holds that:

$$\frac{\alpha_b}{\alpha_a} \cdot \frac{\beta_a}{\beta_b} \cdot \frac{N_b^u}{N_a^u} \cdot \frac{N_a^s}{N_b^s} = \frac{w_b^s}{w_a^s} \cdot \frac{w_a^u}{w_b^u} \quad (5)$$

By taking logs of (5) and differentiating around the symmetric case, in which the two locations are taken to be initially identical, we obtain the following expression:

$$\frac{dw_b^s - dw_a^s}{w^s} - \frac{dw_b^u - dw_a^u}{w^u} = \frac{d\alpha_b - d\alpha_a}{\alpha} - \frac{d\beta_b - d\beta_a}{\beta} + \frac{dN_b^u - dN_a^u}{N^u} - \frac{dN_b^s - dN_a^s}{N^s}\,. \qquad (6)$$

Notice that for each variable, say x_c, we use the following notation: under "symmetry", it holds that $x_a = x_b = x$. Equation (6) can be seen as a relative "labor demand" equation, where the demand for each type of skill is decreasing in its own relative wage.

2.2 Preferences

We assume that *skilled workers*, located in *c*, have Cobb-Douglas preferences (as, e.g., in Diamond, 2016) given by

$$U_c^s = \ln A_c^s + (1-\mu)\cdot \ln H + \mu \cdot \ln Y + \varepsilon_c^s \qquad (7)$$

where H denotes consumption of housing services, rented at the local price r_c, Y is consumption of the tradable good (sold at an economy-wide price equal to one), A_c^s is a local amenity term which, in principle, may contain amenities which are particularly attractive to the skilled. Finally, ε_c^s denotes an idiosyncratic preference shock for location c, which is i.i.d. and follows a Type I Extreme Value distribution[5] with scale parameter equal to $\phi^s \geq 0$. The parameter ϕ^s governs the strength of individual preferences towards locations and, eventually, determines the degree of labour mobility across areas. The closer ϕ^s gets to zero, the more workers will react to differences in local prices

[5] See, e.g., Nakajima and Tabuchi (2011), Moretti and Kline (2014), Diamond (2016).

and amenities. Indeed, when $\phi^s = 0$, the model degenerates into the case of full mobility, corresponding to the basic Roback framework.

For each alternative location $c = \{a, b\}$, utility (7) is maximized under the budget constraint $w_c^s = r_c \cdot H + Y$, and delivers the following indirect utility function:

$$V_c^s = \ln \eta + \ln A_c^s + \ln w_c^s - (1-\mu) \cdot \ln r_c + \varepsilon_c^s \equiv \ln \eta + v_c^s + \varepsilon_c^s, \qquad with\ c = \{a, b\} \qquad (8)$$

where $\eta \equiv (1-\mu)^{1-\mu}\mu^{\mu}$ is a constant, and $v_c^s \equiv \ln A_c^s + \ln w_c^s - (1-\mu) \cdot \ln r_c$.

A skilled worker will prefer location *a* to location *b* when the following condition holds true:

$$\varepsilon_a^s - \varepsilon_b^s \geq v_b^s - v_a^s\,. \qquad (9)$$

As recalled in Anderson et al. (1992, p.60), the difference $\varepsilon_a^s - \varepsilon_b^s$ between two independent Type I Extreme Value distributions has a Logistic distribution with zero mean and CDF equal to $F(x) = \frac{\exp\left\{\frac{x}{\phi^s}\right\}}{1+\exp\left\{\frac{x}{\phi^s}\right\}}$. Thus, the fraction of people living in location *b*, denoted as $\frac{N_b^s}{N_a^s+N_b^s}$, is equal to $F(v_b^s - v_a^s) = \frac{\exp\left\{\frac{v_b^s - v_a^s}{\phi^s}\right\}}{1+\exp\left\{\frac{v_b^s - v_a^s}{\phi^s}\right\}}$. Re-arranging, one obtains $\frac{N_b^s}{N_a^s} = \exp\left\{\frac{v_b^s - v_a^s}{\phi^s}\right\}$. By taking logs and expressing $v_b^s - v_a^s$ as $(\ln A_b^s - \ln A_a^s) + (\ln w_b^s - \ln w_a^s) - (1-\mu) \cdot (\ln r_b - \ln r_a)$, the following holds:

$$\phi^s \cdot \ln\left(\frac{N_b^s}{N_a^s}\right) = \ln\left(\frac{A_b^s}{A_a^s}\right) + \ln\left(\frac{w_b^s}{w_a^s}\right) - (1-\mu) \cdot \ln\left(\frac{r_b}{r_a}\right). \tag{10}$$

Expression (10) suggests that the relative supply of skilled workers in location b rises when the relative wage paid in that location goes up. On the contrary, when rents are relatively high in location b, local *real* wages will be lower, and a smaller number of skilled individuals will be willing to live there. In other words, equation (10) is a labor-supply relation.

Similarly, *unskilled workers* maximize utility

$$U_c^u = \ln A_c^u + (1-\mu) \cdot \ln H + \mu \cdot \ln Y + \varepsilon_c^u \tag{11}$$

under the constraint $w_c^u = r_c \cdot H + Y$. The local amenity term A_c^u still allows for the possibility that the unskilled enjoy amenities that are different from those enjoyed by the skilled, that is $A_c^u \neq A_c^s$. By assuming again that the location shock ε_c^u follows a Type I Extreme Value distribution with scale parameter equal to $\phi^u \geq 0$, we can solve the unskilled worker's problem with the same procedure adopted for the skilled and obtain:

$$\phi^u \cdot \ln\left(\frac{N_b^u}{N_a^u}\right) = \ln\left(\frac{A_b^u}{A_a^u}\right) + \ln\left(\frac{w_b^u}{w_a^u}\right) - (1-\mu) \cdot \ln\left(\frac{r_b}{r_a}\right). \tag{12}$$

By differentiating (10) and (12) around symmetry, we obtain:

$$\frac{dA_b^s - dA_a^s}{A^s} + \frac{dw_b^s - dw_a^s}{w^s} - (1-\mu)\frac{dr_b - dr_a}{r} \; = \; \phi^s \cdot \frac{dN_b^s - dN_a^s}{N^s} \tag{13}$$

and

$$\frac{dA_b^u - dA_a^u}{A^u} + \frac{dw_b^u - dw_a^u}{w^u} - (1-\mu)\frac{dr_b - dr_a}{r} \; = \; \phi^u \cdot \frac{dN_b^u - dN_a^u}{N^u} \tag{14}$$

where $N^s \equiv \frac{1}{2}(N_a^s + N_b^s)$ and $N^u \equiv \frac{1}{2}(N_a^u + N_b^u)$.[6]

In what follows, we will assume that skilled and unskilled individuals have the *same* measure of mobility costs. That is, we will assume that $\phi^s = \phi^u = \phi$.[7]

2.3 Local housing market equilibrium.

When landowners are absentee, the local housing market equilibrium condition is given by:

$$h_c = (1-\mu) \cdot \left[\frac{w_c^s}{r_c} N_c^s + \frac{w_c^u}{r_c} N_c^u\right]. \tag{15}$$

[6] Notice that if the labour supply of skilled and unskilled workers is given at the aggregate level, then (N^s, N^u) are constants. Then, it holds that $\frac{dN_b^s - dN_a^s}{N^s} = 2\frac{dN_b^s}{N^s}$, and $\frac{dN_b^u - dN_a^u}{N^u} = 2\frac{dN_b^u}{N^u}$.

[7] Auricchio et al. (2017) analyze a similar linear spatial model where the measure of mobility costs is allowed to vary across skill groups. The relevance of mobility costs in the US labor market is discussed in Kemeny and Storper (2012).

The left-hand side is the (exogenously[8] given) local supply for housing, given by h_c. Supply can depend on local land availability, as well as local regulation. The right-hand side is aggregate demand for housing services, recalling that individual demand for housing is given by $(1-\mu) \cdot w/r$.

From (15) we obtain the following expression:

$$\frac{h_b}{h_a} \cdot \frac{r_b}{r_a} = \left[\frac{w_b^s \cdot N_b^s + w_b^u \cdot N_b^u}{w_a^s \cdot N_a^s + w_a^u \cdot N_a^u}\right] = \left[\frac{\alpha_a}{\alpha_b} \cdot \frac{w_b^s \cdot N_b^s}{w_a^s \cdot N_a^s}\right] = \left[\frac{\beta_a}{\beta_b} \cdot \frac{w_b^u \cdot N_b^u}{w_a^u \cdot N_a^u}\right] \tag{16}$$

recalling that, from first-order conditions for profit maximization, it holds that $w_c^s \cdot \frac{N_c^s}{\alpha_c} = w_c^u \cdot \frac{N_c^u}{\beta_c}$. By taking logs of (16) and differentiating around symmetry, we obtain:

$$\frac{dh_b - dh_a}{h} + \frac{dr_b - dr_a}{r} = \frac{dw_b^s - dw_a^s}{w^s} + \frac{dN_b^s - dN_a^s}{N^s} - \frac{d\alpha_b - d\alpha_a}{\alpha} \ . \tag{17}$$

An expression similar to (17) can be obtained when differentiating with respect to $\{w_c^u, N_c^u\}$.

We can now provide the linear representation of the spatial general equilibrium model. Consider equations (6), (13), (14) and (17). Define $\tilde{x}$ such that

[8] The local housing supply can be taken to be increasing in the local rent level r_c without any substantial qualitative difference: see Auricchio et al. (2017).

$$\tilde{x} \equiv \frac{dx_b - dx_a}{x}. \tag{18}$$

Thus, $\tilde{x}$ is the difference between the rate of change of variable x in area b and the rate of change of the same variable in area a. Expressions (13), (14), (6) and (17) can be written, respectively, as:

$$\tilde{A}^s + \widetilde{w}^s - (1-\mu) \cdot \tilde{r} \; = \; \phi \cdot \widetilde{N}^s \tag{19}$$

$$\tilde{A}^u + \widetilde{w}^u - (1-\mu) \cdot \tilde{r} \; = \; \phi \cdot \widetilde{N}^u \tag{20}$$

$$\widetilde{w}^s - \widetilde{w}^u = \left(\tilde{\alpha} - \tilde{\beta}\right) - \widetilde{N}^s + \widetilde{N}^u \tag{21}$$

$$\tilde{h} + \tilde{r} = \widetilde{N}^s + \widetilde{w}^s - \tilde{\alpha} = \widetilde{N}^u + \widetilde{w}^u - \tilde{\beta}. \tag{22}$$

In what follows, by using definition (18) and $\beta_c \equiv 1 - \alpha_c$, we will re-write our measure $\left(\tilde{\alpha} - \tilde{\beta}\right)$ of change in local technology skill-biasedness as $\frac{1}{\beta} \cdot \tilde{\alpha}$.

We are now ready to discuss the main implications of the model.

2.4 Main Results.

Under the assumptions we made, the three equations (19)-(20)-(21) are sufficient to determine our object of interest, the relative skill premium $\widetilde{w}^s - \widetilde{w}^u$ in deviations from symmetry. By subtracting (20) from (19), and using (21) to get rid of the (relative) local skill mix $\widetilde{N}^s - \widetilde{N}^u$, we obtain the following expression:

$$\widetilde{w}^s - \widetilde{w}^u = \frac{-\tilde{A}^s + \tilde{A}^u + \dfrac{\phi}{\beta} \cdot \tilde{\alpha}}{1+\phi} \ . \tag{23}$$

Expression (23) bears several implications:

(i) If the skilled and the unskilled have no mobility costs (ϕ =0) and enjoy the *same amenities*, so that $\tilde{A}^s = \tilde{A}^u = \tilde{A}$, then the skill premium will be constant across areas, as predicted by the basic Roback model (see Glaeser, 2008). Thus, it will hold that $\widetilde{w}^s - \widetilde{w}^u = 0$. In other words, under these conditions, a change in the skilled wage *level* in a region will go together with an equi-proportional change in the unskilled wage level, so to maintain the skill premium constant economy-wide.[9]

(ii) The idea that different skill groups may evaluate local amenities differently is quite common in the spatial equilibrium literature: see, for example, Glaeser et al. (2001), Glaeser (2008), Dalmazzo and de Blasio (2011), Moretti (2013), and Handbury (2013). This case can be immediately considered in our framework. Suppose there are no mobility costs ($\phi = 0$). If the skilled and the

[9] In the absence of mobility costs, it is immediate to show that the skill-premium *level* in every location c of the economy is equal to $\frac{w_c^s}{w_c^u} = \frac{\bar{v}^s}{\bar{v}^u}$, where $\{\bar{v}^s, \bar{v}^u\}$ denote, respectively, the utility levels attained by the skilled and the unskilled, which have to be *constant* across locations. See also Glaeser (2008).

unskilled do *not* enjoy the same amenities, and such amenities are unevenly distributed across space (that is, $\tilde{A}^s \neq \tilde{A}^u$), then it will hold that $\tilde{w}^s - \tilde{w}^u = -\tilde{A}^s + \tilde{A}^u$. In particular, following definition (18), when place b is relatively richer in skilled-biased amenities, it will exhibit a *lower* skill premium. This implication is supported by Adamson et al. (2004) and Lee (2010), who find that high skills are relatively cheaper in cities that offer a greater variety of cultural and consumption amenities. Notice also that, abstracting from spatial heterogeneity in technology (that is, $\tilde{\alpha} = 0$), the presence of mobility costs ($\phi > 0$) reduces the impact of skill-biased amenities on the local premium. Indeed, skilled population movements will react less to differences in local conditions and, thus, exert less pressure on local wages.

(iii) Skilled-biased technical heterogeneity, represented here as share-altering technical heterogeneity (i.e., $\tilde{\alpha} \neq 0$) across areas as in Accetturo et al. (2014), will have an impact on the local skill premium *only if* the local labor supply is not perfectly elastic to wages. Put it in other words, skill-biased technical heterogeneity across locations affects the local premium only when the mobility cost measure ϕ is strictly positive. Indeed, it will hold that $\tilde{w}^s - \tilde{w}^u = \frac{\phi \cdot \tilde{\alpha}}{\beta \cdot (1+\phi)}$. Then, if firms in location b adopt technologies that rely more on skilled labor –such that $\tilde{\alpha} > 0$ holds true- this location will exhibit a *larger* premium. This implication is discussed both in Moretti (2013) and in Beaudry et al. (2010). In particular, Beaudry et al. conclude that the impact of changes in local technology on the local skill premium is only partially curbed by labor mobility.

The results on the skill premium have immediate implications for the local distribution of skills. [10] By substituting the premium expression (23) into (21), we obtain the local skill mix as:

[10] In the Appendix A1, we also derive and discuss the equilibrium expression for $\tilde{r}$, the relative change in local rents.

$$\widetilde{N}^s - \widetilde{N}^u = \frac{\tilde{A}^s - \tilde{A}^u + \frac{1}{\beta} \cdot \tilde{\alpha}}{1 + \phi} \ . \tag{24}$$

For $\phi \geq 0$, both the relative availability of skill-biased amenities and the presence of skill-biased technologies will go together with a higher local skill-mix, consistently with findings in Lee (2010), Beaudry et al. (2010), and Brown and Scott (2012). As hinted in Berry and Glaeser (2005), amenities and technology may also have self-reinforcing effects on the local skill distribution. Glaeser et al. (2009) suggest that places that attract skilled workers are more likely to attract skilled-biased industries.

In what follows, we extend the model to look at two additional factors leading to heterogeneity of skill premia across location. In particular, we will investigate housing market segregation across skills in Sect. 2.5, and heterogeneous expenditure shares across skills in Sect. 2.6.

2.5 Housing market segregation.

Together with ethnicity (see, e.g., Cutler and Glaeser, 1997, Card et al., 2008, Gabriel and Painter, 2012, Ibraimovic and Hess, 2017), income and education play a major role in neighborhood segregation and gentrification, as argued by Rosenthal and Ross (2015).[11] To illustrate the implications of segregation on local skill premia, consider the (extreme) case when the skilled buy housing services from a local market which is perfectly separated from the market which serves the unskilled, as in Glaeser (2008). Now, the skilled and the unskilled will pay different local rents per unit, respectively (r_c^s, r_c^u), for housing in supplies equal to (h_c^s, h_c^u).

[11] See also Bayer et al. (2007), Brasington et al. (2015), and Liu (2017). For a theoretical analysis from a Schelling's perspective, see Zhang (2011).

The analogues of equations (10) and (12) in this case are:

$$\phi \cdot \ln\left(\frac{N_b^s}{N_a^s}\right) = \ln\left(\frac{A_b^s}{A_a^s}\right) + \ln\left(\frac{w_b^s}{w_a^s}\right) - (1-\mu) \cdot \ln\left(\frac{r_b^s}{r_a^s}\right), \tag{25}$$

$$\phi \cdot \ln\left(\frac{N_b^u}{N_a^u}\right) = \ln\left(\frac{A_b^u}{A_a^u}\right) + \ln\left(\frac{w_b^u}{w_a^u}\right) - (1-\mu) \cdot \ln\left(\frac{r_b^u}{r_a^u}\right). \tag{26}$$

We also have two market clearing conditions for the local housing markets:

$$h_c^s = (1-\mu) \cdot \left[\frac{w_c^s}{r_c^s} N_c^s\right] \quad \text{and} \quad h_c^u = (1-\mu) \cdot \left[\frac{w_c^u}{r_c^u} N_c^u\right] \tag{27}$$

which yield

$$\frac{h_b^s}{h_a^s} \cdot \frac{r_b^s}{r_a^s} = \frac{w_b^s \cdot N_b^s}{w_a^s \cdot N_a^s}; \qquad \frac{h_b^u}{h_a^u} \cdot \frac{r_b^u}{r_a^u} = \frac{w_b^u \cdot N_b^u}{w_a^u \cdot N_a^u}. \tag{28}$$

By differentiating around symmetry, expressions (25) and (26) deliver

$$\tilde{A}^s + \widetilde{w}^s - (1-\mu) \cdot \tilde{r}^s \;=\; \phi \cdot \widetilde{N}^s \tag{29}$$

and

$$\tilde{A}^u + \widetilde{w}^u - (1-\mu)\cdot \tilde{r}^u \ = \ \phi \cdot \widetilde{N}^u, \tag{30}$$

while expressions in (28) give:

$$\tilde{h}^s + \tilde{r}^s = \widetilde{N}^s + \widetilde{w}^s \tag{31}$$

and

$$\tilde{h}^u + \tilde{r}^u = \widetilde{N}^u + \widetilde{w}^u \,. \tag{32}$$

By using equations from (29) to (32) together with (21), we obtain the following expression for the skill premium:

$$\widetilde{w}^s - \widetilde{w}^u = \frac{-\tilde{A}^s + \tilde{A}^u + \frac{\phi}{\beta}\cdot \tilde{\alpha} + (1-\mu)\cdot(\tilde{r}^s - \tilde{r}^u)}{1+\phi}\,. \tag{33}$$

Apart from the term in $(\tilde{r}^s - \tilde{r}^u)$, expression (33) coincides with expression (23) above. Housing market segregation predicts that –even after taking care of amenities and technological heterogeneity- a larger differential between the rents paid by skilled and unskilled individuals will be associated with a larger local skill premium. By using (31)-(32), equation (33) can be re-arranged to get rid of the term $(\tilde{r}^s - \tilde{r}^u)$. This delivers the following expression:

$$\widetilde{w}^s - \widetilde{w}^u = \frac{-\tilde{A}^s + \tilde{A}^u - (1-\mu)\cdot\left(\tilde{h}^s - \tilde{h}^u\right) + \frac{1-\mu+\phi}{\beta}\cdot\tilde{\alpha}}{1+\phi} \quad . \tag{34}$$

Notice that the skill premium in area *b* is decreasing in the relative availability of housing services for the skilled in the same location, denoted by $\left(\tilde{h}^s - \tilde{h}^u\right)$. This case can be used to represent the impact of "gentrification", implying greater availability of housing services for the richer at the poorer's expense: see, for example, Ellen and O'Regan (2011), and Guerrieri et al. (2013).

Further, under housing market segregation, technological heterogeneity ($\tilde{\alpha} \neq 0$) is able to affect the local premium *even* in the absence of mobility costs. The intuition for this result is quite straightforward. Skill-biased technical change increases local demand for skilled people, who exert pressure on *their own* housing market. As a consequence, the higher rents paid by the skilled will have to be compensated by higher wages.

The local skill mix under housing-market segregation is given by the following expression:

$$\widetilde{N}^s - \widetilde{N}^u = \frac{\tilde{A}^s - \tilde{A}^u + (1-\mu)\cdot\left(\tilde{h}^s - \tilde{h}^u\right) + \frac{\mu}{\beta}\cdot\tilde{\alpha}}{1+\phi} \quad . \tag{35}$$

Thus, in addition to local skilled-biased amenities and technical change, gentrification - as represented by $(\tilde{h}^s - \tilde{h}^u) > 0$ - further reinforces the rise of skilled workers in the local skill distribution.

2.6 Non-homothetic preferences.

Black, Kolesnikova, Taylor (2009) have emphasised that spatial general equilibrium models are generally built on homothetic preferences, such as Cobb-Douglas utilities, so that - for all goods - income elasticity is equal to one. By contrast, a large empirical literature shows that the income elasticity of housing differs from one (see Black et al., 2009, p.28) and, moreover, there is substantial variation in housing prices across U.S. cities (p.27).[12] As a consequence, non-homotheticity implies that housing prices will affect the size of the local skill-premium. More recently, Ganong and Shoag (2017) have argued that housing prices have disproportionally risen in high-income cities, redirecting low-skill migration away from such places, and slowing income convergence in the U.S.

Since we build on (homothetic) Cobb-Douglas preferences, our framework seems utterly unfit to tackle this issue. Nonetheless, we can "mimic" non-homotheticity even in this environment by assuming that skilled and unskilled individuals exhibit different expenditure shares. As remarked in Black et al. (2008, p.29), if the income elasticity of housing is less than one, the expenditure share on housing will decline with income and vice-versa. Hence, we assume that the original utility of the skilled, given by (7) above, is modified as follows:

$$U_c^s = \ln A_c^s + (1 - \mu + \delta) \cdot \ln H + (\mu - \delta) \cdot \ln Y + \varepsilon_c^s \tag{36}$$

[12] Handbury (2013) considers the implications of non-homotheticity also with regard to the availability of product variety across different cities, an issue which closely related to the idea of "consumer city": see Glaeser et al. (2001).

where $\delta \in (\mu - 1, \mu)$. By contrast, the expenditure shares in the unskilled utility remain the same as in (11). This simple assumption allows us to relate the size of the expenditure shares to the level of skill and, thus, to the level of income. Utility maximization implies that the indirect utility is $V_c^s = \ln \eta' + \ln A_c^s + \ln w_c^s - (1 - \mu + \delta) \cdot \ln r_c + \varepsilon_c^s$, where $\eta' \equiv (1 - \mu + \delta)^{1-\mu+\delta} \cdot (\mu - \delta)^{\mu-\delta}$. The analogue of equation (10) is, in this case,

$$\phi \cdot \ln\left(\frac{N_b^s}{N_a^s}\right) = \ln\left(\frac{A_b^s}{A_a^s}\right) + \ln\left(\frac{w_b^s}{w_a^s}\right) - (1 - \mu + \delta) \cdot \ln\left(\frac{r_b}{r_a}\right). \tag{37}$$

By differentiating (37) we obtain:

$$\tilde{A}^s + \widetilde{w}^s - (1 - \mu + \delta) \cdot \tilde{r} \; = \; \phi \cdot \widetilde{N}^s. \tag{38}$$

Using equation (38) together with (20)-(21), we obtain that –when expenditure shares differ across types- the skill premium is equal to:

$$\widetilde{w}^s - \widetilde{w}^u = \frac{-\tilde{A}^s + \tilde{A}^u + \dfrac{\phi}{\beta} \cdot \tilde{\alpha} + \delta \cdot \tilde{r}}{1 + \phi}. \tag{39}$$

Note that, apart from the term in $\tilde{r}$, expression (39) coincides with expression (23). Expression (39) has a remarkable feature. Differences in expenditure shares between skilled and unskilled workers imply that – after taking care of amenities and technical heterogeneity across places- the local skill premium will also depend on the (average) level of local rents, which is exactly the point made by Black et al. (2008, p.25). The sign of the relationship, though, depends on the sign of parameter δ, which is *negative* when the skilled spend a smaller fraction of their income on housing, relative to the unskilled. This is indeed the case, since income elasticity of demand for housing in the U.S. is estimated to be around 0.7 (see Black et al., p.28). In conclusion, non-homotheticity implies that "the return to education is lower in cities that are more expensive" (Black et al., 2009, p.29).

By getting rid of the (endogenous) value of $\tilde{r}$ in (38), we can write the premium as:[13]

$$\widetilde{w}^s - \widetilde{w}^u = \frac{-f \cdot \tilde{A}^s + g \cdot \tilde{A}^u + k \cdot \tilde{\alpha} - \delta \cdot \phi \cdot \tilde{h} + \delta \cdot (1+\phi) \cdot \tilde{Z}}{(1+\phi)(\phi + 1 - \mu + \delta \cdot \alpha)}, \tag{40}$$

where $f \equiv \phi + 1 - \mu > 0$, $g \equiv \phi + 1 - \mu + \delta > 0$, and $k \equiv \phi \cdot \frac{(1-\mu+\delta\cdot\alpha)\cdot(\phi+1-\mu+\delta\cdot\alpha)+\cdot\delta^2\alpha\cdot\beta}{\beta(1-\mu+\delta\cdot\alpha)} + \delta \cdot (1+\phi) \cdot \alpha \cdot \epsilon$; the constant $\epsilon > 0$ is defined in Appendix A1. The sign of k is –in general- positive, since δ is rather small in absolute value.[14] Still, non-homothetic preferences dampen the impact of skill-biased technical change on the local skill premium.

Expression (40) implies that – when expenditure-shares across skills differ by $\delta < 0$ - the skill premium differential is increasing in relative local housing supply $\tilde{h}$. Another result specific to non-homothetic preferences is that the premium is decreasing in the local TFP, $\tilde{Z}$, since more productive

[13] We use the fact that $\tilde{\beta} = -\frac{\alpha}{\beta} \cdot \tilde{\alpha}$. See the Appendix A2 for additional details.

[14] It is quite straightforward to verify that (40) reduces to expression (23) when it holds that $\delta = 0$.

areas are also more expensive. The same conclusion is reached in Black et al. (2009, p.29). Interestingly, Black et al. (2009) consider the case when skilled and unskilled individuals enjoy the same amenities, that is, $\tilde{A}^s = \tilde{A}^u = \tilde{A}$. This assumption, in our basic model with homothetic preferences, would have a simple implication: local wage premia are unaffected by local amenities (see equation 23). However, when the skilled and unskilled enjoy the same amenities but have different expenditure shares on housing, amenities still matter. From (40), when it holds that $\tilde{A}^s = \tilde{A}^u = \tilde{A}$, we obtain that $-f + g = \delta < 0$. Thus, an increase in local amenities will *decrease* the local wage premium. This conclusion, reached also by Black et al. (2009, p.31-32), has a simple intuition. Higher local amenities, as well as higher local productivity, increase the local price of housing (as shown in Appendix A2). Since the unskilled spend relatively more on housing, they will be happy to stay only if they receive higher wages. As a results, the local wage premium is compressed.

We can now consider the implications of non-homotheticity for the local skill mix. Using (21) together with (39), one obtains:

$$\tilde{N}^s - \tilde{N}^u = \frac{\tilde{A}^s - \tilde{A}^u + \frac{1}{\beta} \cdot \tilde{\alpha} - \delta \cdot \tilde{r}}{1 + \phi} \ . \tag{41}$$

Since the evidence suggests that $\delta < 0$ (skilled people's expenditure share on housing is smaller), whatever generates an increase in the local cost of living (that is, $\tilde{r} > 0$) will generate an increase in the skill mix. This is exactly the idea put forward by the model and the findings of Ganong and Shoag (2017): cities that exhibit high housing costs discourage low-skill immigration. Indeed, unskilled people are less willing to live in cities that are more expensive, since they spend relatively more on housing. A similar conclusion is reached in Liu (2017, p.895).

The skill mix can be rewritten by exploiting (21) and (40), so to obtain:

$$\widetilde{N}^s - \widetilde{N}^u = \frac{f \cdot \tilde{A}^s - g \cdot \tilde{A}^u + m \cdot \tilde{\alpha} + \delta \cdot \phi \cdot \tilde{h} - \delta \cdot (1+\phi) \cdot \tilde{Z}}{(1+\phi)(\phi + 1 - \mu + \delta \cdot \alpha)}, \tag{42}$$

where $f > 0$ and $g > 0$ are defined above, and $m \equiv \frac{(1-\mu+\delta\cdot\alpha)\cdot(1-\mu+\delta\cdot\alpha+\phi)-\delta^2\cdot\alpha\cdot\beta\cdot\phi}{\beta(1-\mu+\delta\cdot\alpha)} - \delta \cdot (1 + \phi) \cdot \alpha \cdot \epsilon > 0$.

Again, as in Ganong and Shoag (2017), stricter regulation reducing relative local land supply, $\tilde{h} < 0$, will increase the skill-mix. Further, if preferences are not homothetic, a rise in local TFP (i.e., $\tilde{Z} > 0$), will also raise the skill mix. Indeed, an increase in local productivity raises the local price of housing, inducing unskilled workers to migrate away.[15]

3. WRAP-UP OF THE MODEL AND SUGGESTIONS FOR EMPIRICAL RESEARCH

The theoretical model has highlighted the presence of different explanations for the existence of skill premia heterogeneity across locations. Testing the empirical relevance of such explanations is beyond the scope of this paper. However, in this section, we will provide some guidance for empirical analysis. First, we will give a simple graphical representation of the model in which we represent our main theoretical implications in the space $\left(\widetilde{N}^s - \widetilde{N}^u, \widetilde{w}^s - \widetilde{w}^u\right)$. Then, we will use

[15] Also notice that when skilled and unskilled enjoy the same amenities, so that $\tilde{A}^s = \tilde{A}^u = \tilde{A}$, an increase in amenities ($\tilde{A} > 0$) will have the same qualitative effect on the skill mix as an increase in local TFP, since it holds that $f - g = -\delta > 0$.

data from the Italian Labour Force Survey to illustrate how it is possible to organize empirical tests of the model.

3.1 The model in a graph.

The basic model is summarized by equations from (19) to (22). By subtracting (20) from (19), we obtain the "labor supply curve" (in deviations):

$$\widetilde{w}^s - \widetilde{w}^u = -(\tilde{A}^s - \tilde{A}^u) + \phi \cdot (\widetilde{N}^s - \widetilde{N}^u). \tag{43}$$

Equation (43) is upward sloping when "mobility costs" are strictly positive ($\phi > 0$). By contrast, if mobility costs are nil, the skill premium will only depend on the differential in amenity endowments of the skilled versus the unskilled. Notice that (43) will shift downwards when $(\tilde{A}^s - \tilde{A}^u)$ increases.

The "labor demand" expression, given by (21), can be re-written as:

$$\widetilde{w}^s - \widetilde{w}^u = \frac{1}{\beta} \cdot \tilde{\alpha} - (\widetilde{N}^s - \widetilde{N}^u). \tag{44}$$

Thus, labor demand is downward sloping, no matter the size of mobility costs. Notice that (44) will shift upwards, if local technology becomes more skill-biased, that is, if $\tilde{\alpha}$ increases.

Figure 1 illustrates the impact of an increase in skill-biased local amenities against a rise in local technology skill-biasedness. A similar identification criterion is adopted in Partridge (2010, p.518-519).

[FIGURE 1 here]

Suppose that locations are, initially, in point A. An increase in the relative endowment of skill-biased amenities in location b, relative to location a, $\left(\tilde{A}^s - \tilde{A}^u > 0\right)$, will shift the supply curve downwards. As point A' is reached, the skill premium will fall, while the skill mix will increase.

On the other hand, when the local technology becomes more skill-biased, that is, when $\tilde{\alpha} > 0$, the demand curve will shift up, and equilibrium will be attained in point A''. In this case, the skill premium will increase together with the local skill mix.

Thus, by looking at the pattern of observed values in the space $\left(\tilde{N}^s - \tilde{N}^u, \tilde{w}^s - \tilde{w}^u\right)$, we can assess whether skill-biased amenities prevail – in this case, the observed pattern is downward sloping – or, on the contrary, skill-biased technical change dominates. In the latter case, the observed pattern is upward sloping.

Next, we briefly discuss how the implications that we have just derived are modified when one takes into accounts (i) segregation and, (ii) non-homothetic preferences.

Consider first housing segregation. While the demand side of the model is still represented by equation (44), the supply side is substantially affected. In case of housing segregation, the labor supply expression becomes:[16]

$$\widetilde{w}^s - \widetilde{w}^u = -\frac{1}{\mu}\left(\tilde{A}^s - \tilde{A}^u\right) - \left(\frac{1-\mu}{\mu}\right) \cdot \left(\tilde{h}^s - \tilde{h}^u\right) + \left(\frac{1-\mu+\phi}{\mu}\right) \cdot \left(\widetilde{N}^s - \widetilde{N}^u\right). \tag{45}$$

The supply curve (45) shares most of the features of (43): it is upward sloping in the skill-mix and it shifts downwards when skill-biased amenities increase. Moreover, an increase in housing available to the skilled, as described by $\left(\tilde{h}^s - \tilde{h}^u\right) > 0$, will decrease the skill premium, since it shifts (45) downward. Indeed, as an increase in housing space available to the skilled reduces the rents they pay, they will be willing to accept lower salaries. In conclusion, when housing segregation is important, a downward sloping pattern between skill premia and skill mix can be explained not only by skilled-biased amenities, but also by "gentrification".

Finally, we consider non-homothetic preferences. Even in this case, the demand curve (44) is unaffected. The supply function, now, is given by:[17]

$$\widetilde{w}^s - \widetilde{w}^u = \frac{-f \cdot \tilde{A}^s + g \cdot \tilde{A}^u}{\phi+1-\mu+\delta \cdot \alpha} + \frac{\delta(1+\phi) \cdot \tilde{Z} - \delta\phi \cdot \tilde{h}}{\phi+1-\mu+\delta \cdot \alpha} + \frac{q}{\phi+1-\mu+\delta \cdot \alpha} \cdot \tilde{\alpha} + \phi \cdot \left(\widetilde{N}^s - \widetilde{N}^u\right) \tag{46}$$

[16] To obtain equation (45), we subtracted (30) from (29), and replaced $(\tilde{r}^s - \tilde{r}^u)$ by exploiting the expression obtained by subtracting (32) from (31).
[17] To derive equation (46), we subtract equation (20) from (38) and substitute away for $\tilde{r}$ by using (A9) in the Appendix.

where $\delta < 0$, the constants $(f, g) > 0$ are defined above, and $q \equiv \delta \cdot \left[(1+\phi)\cdot\alpha\cdot\epsilon + \frac{\delta\cdot\alpha\cdot\phi}{1-\mu+\delta\cdot\alpha}\right]$ has an ambiguous sign. Under non-homothetic preferences, the supply curve (46) is still upward sloping and, as usual, it shifts down when skill-biased amenities increase. Here, however, its position will shift up as local land supply increases $(\tilde{h} > 0)$.[18] Thus, non-homotheticity is such that locations with abundant housing space will tend to be associated with a higher premium and a lower skill-mix. On the other hand, the supply curve shifts down when local TFP increases $(\tilde{Z} > 0)$.[19] As a consequence, if locations which adopt skill-biased technologies also have higher TFP (as suggested, e.g., in Eeckhout et al 2014), non-homothetic preferences will tend to attenuate the impact of technological skill-biasedness on the premium.

Notice finally that increases in local technological skill-bias, as measured by $\tilde{\alpha} > 0$, will have an ambiguous impact on the position of (46). In general, however, such an impact will be rather small.

3.2 Some evidence from Italy.

In what follows, we present some evidence from Italy on the relationship between skill mix and skill premia, to give a first assessment whether amenities or technological factors prevail. We use the Labour Force Survey to compute – for each commuting zone – the university graduate wage premium (skill premium) and the (log) share of local workers with a tertiary degree (skill mix). Details on data construction are provided in the Appendix A3.

Figure 2 shows the results. The left panel presents the relationship for commuting zones with more than 200,000 people.[20] The right panel shows the results for all commuting zones with the linear fit that is weighted according to the size of population. In both cases, the relationship is positive and

[18] An increase in land is more beneficial to unskilled workers, who spend relatively more on housing.
[19] An increase in local TFP raises local rents. Thus, unskilled workers, who have higher expenditure shares on housing, will require relatively higher wages.
[20] This is a standard threshold to define a urban area in Italy, see Accetturo et al. (2018).

statistically significant. The slope of the regression line in the left panel is 0.919 with a standard error of 0.330; the slope in the right panel is 1.577 with a standard error of 0.234.

[FIGURE 2 here]

In a model with only skill-biased amenities or technologies, this evidence might be taken as suggestive that the latter is the dominant source of skill premia heterogeneity. In a richer model, as the one we proposed in this paper, the positive impact of skill-biased technologies on premium and mix can be attenuated by housing segregation and non-homothetic preferences.

In general, one could devise an econometric strategy to check the relative importance of various drivers of skill-premia differences. The idea is based on the Omitted Variable Bias (OVB) formula (Angrist and Pischke, 2008). The empirical counterpart of equations (43)-(46) can be written as:

$$SP_i = A_s + B_s \cdot SM_i + \varepsilon_i^s \tag{47}$$

where SP_i is the skill premium in city *i* and SM_i is the skill mix in the same area. Ordinary Least Squares estimation of equation (47) leads to the following estimate of parameter *B*:

$$\hat{B}_s = B + \gamma \cdot \frac{cov(SM,Z)}{V(SM)}, \tag{48}$$

where *B* is the "true" relationship between skill premia and skill mix and *Z* is an omitted variable. Without frictions (skill biased amenities or technologies, segregation, etc.), *B* should be always equal to zero. However, Italian data actually show that $\hat{B}_s > 0$, thus implying that there is an omitted variable in the estimation of equation (47).[21]

[21] There can be reasons for which the correlation between skill premia and skill mix in actual data could be different from zero; this could occur, for example, in presence of knowledge spillovers across locations. In

In order to circumvent the OVB problem, we can run "longer" regressions by adding additional regressors that are likely to reduce the bias. Suppose we have (exogenous) empirical counterparts for skill-biased amenities or technologies, housing segregation, and non-homothetic preferences. We use them as regressors (Z) in the following equation:

$$SP_i = A_L + B_L \cdot SM_i + \gamma \cdot Z + \varepsilon_i^L \tag{49}$$

If the true model included only skill-biased amenities or technologies, controlling for these sources of spatial misalignment should be sufficient to drive $\hat{B}_L$ to zero. However, if housing segregation and/or non-homothetic preferences matter, only their inclusion will provide an estimate of $\hat{B}_L$ free of OVB.

4. CONCLUDING REMARKS

As its main contribution, the present paper offers a new, general-purpose methodology, to represent the standard spatial general equilibrium model. In particular, the methodology we present delivers a linear framework expressed in percentage changes. Indeed, we argue that our linearized spatial equilibrium model makes the interpretation of the theoretical results both formally neater and more intuitive. In order to demonstrate the flexibility of our approach, we look at the competing explanations of local skill premia and skill distribution, two relevant sources of income inequality at the local level (see, e.g., Beaudry et al., 2010, and Eeckhout et al., 2014). The multiplicity of these explanations generally requires an equal number of specific models: each story, one model. Here, however, we showed that such explanations are easily recomposed in a unifying framework, which neatly emphasizes the factors that contract, or widen, the local skill premium and the local skill mix.

this example, we abstract from these cases (or, equivalently, we assume that we can control for all additional confounding factors).

By using a graphical representation of our model, we also show how the framework we propose can be used to identify the drivers of skill premia and skill mix observed in the data.

Clearly, the generality of our approach goes much beyond the specific example chosen here, as illustrated by Auricchio et al. (2017) with regard to the analysis of the effects of local public employment.

APPENDIX

A1. Derivation of the equilibrium value of $\tilde{r}$ *in the basic model.*

In order to determine the equilibrium value of $\tilde{r}$, one can exploit $\tilde{h}+\tilde{r}=\tilde{N}^s+\tilde{w}^s-\tilde{\alpha}$ from equation (22). By deriving $\tilde{N}^s$ from equation (19) and substituting, one obtains:

$$\phi\cdot\tilde{h}+(\phi+1-\mu)\cdot\tilde{r}=\tilde{A}^s+(1+\phi)\cdot\tilde{w}^s-\phi\cdot\tilde{\alpha}\,. \qquad (A1)$$

To solve for $\tilde{r}$, we have to substitute $\tilde{w}^s$ away. To this purpose, we can exploit (2), the first-order condition that firms follow in order to determine the optimal use of skilled labor. In particular, by taking logs of (2) and differentiating, we obtain:

$$\frac{dw_c^s}{w^s}=\frac{dZ_c}{Z}-\beta\cdot\left(\frac{dN_c^s}{N^s}-\frac{dN_c^u}{N^u}\right)+\left[1+\alpha\cdot ln\left(\frac{N_c^s}{N_c^u}\right)\right]\cdot\frac{d\alpha_c}{\alpha}\,. \qquad (A2)$$

As discussed in Accetturo et al. (2014), the condition for the implementation of a skilled-biased technology (that is, $d\alpha_c>0$) requires that $\frac{N_c^s}{N_c^u}>1$. In area b, we suppose that $\frac{N_b^s}{N_b^u}=1+\epsilon$, with $\epsilon>0$, holds true. Then, it follows that $d\alpha_b>0$. By contrast, in area a, we suppose that $\frac{N_b^s}{N_b^u}=1$, so that $d\alpha_a=0$. Under these conditions, by calculating (A2) for $c=\{a,b\}$, one obtains the following:

$$\widetilde{w}^s = \tilde{Z} - \beta \cdot \left(\widetilde{N}^s - \widetilde{N}^u\right) + (1 + \alpha \cdot \epsilon) \cdot \tilde{\alpha}\,. \qquad (A3)$$

Expression (24) can be substituted into (A3) to obtain the equilibrium expression for $\widetilde{w}^s$:

$$\widetilde{w}^s = \frac{-\beta \cdot \left(\tilde{A}^s - \tilde{A}^u\right) + [\phi + (1 + \phi) \cdot \alpha \cdot \epsilon] \cdot \tilde{\alpha} + (1 + \phi) \cdot \tilde{Z}}{1 + \phi}\,. \qquad (A4)$$

Equation (A4) can then be used to substitute for $\widetilde{w}^s$ into (A1), so to get:

$$\tilde{r} = \frac{\alpha \cdot \tilde{A}^s + \beta \cdot \tilde{A}^u + [(1 + \phi) \cdot \alpha \cdot \epsilon] \cdot \tilde{\alpha} - \phi \cdot \tilde{h} + (1 + \phi) \cdot \tilde{Z}}{\phi + 1 - \mu}\,. \qquad (A5)$$

Thus, the local rent differential $\tilde{r}$ between area b and area a is: (i) increasing in relative amenity endowments and TFP, respectively $\{\tilde{A}^s,\ \tilde{A}^u, \tilde{Z}\}$, and (ii) decreasing in the relative local housing supply $\tilde{h}$ of area b. Moreover, as in Accetturo et al. (2014), local skill-biased technical change – when implementable- will have a positive impact on local rents and local output, as denoted by y_c, since from equation (15) it follows that $\tilde{y} = \tilde{r} + \tilde{h}$.

A2. Derivation of the equilibrium value of $\tilde{r}$ *in the model with heterogeneous expenditure shares.*

In this case, the housing market equilibrium condition is given by

$$h_c = (1-\mu) \cdot \left[\frac{w_c^s}{r_c} N_c^s + \frac{w_c^u}{r_c} N_c^u\right] + \delta \cdot \frac{w_c^s}{r_c} N_c^s = \left[\frac{1-\mu}{\alpha_c} + \delta\right] \cdot \frac{w_c^s}{r_c} N_c^s \qquad (A6)$$

which, after some manipulations, delivers the analogue of (22) for the present case:

$$\tilde{h} + \tilde{r} = \widetilde{N}^s + \widetilde{w}^s - \left(\frac{1-\mu}{1-\mu+\delta \cdot \alpha}\right) \cdot \tilde{\alpha} \, . \qquad (A7)$$

When solving for $\tilde{r}$ with heterogeneous expenditure shares, one gets the analogue of (A1):

$$\phi \cdot \tilde{h} + (\phi + 1 - \mu + \delta) \cdot \tilde{r} = -\frac{\phi \cdot (1-\mu)}{1-\mu+\delta \cdot \alpha} \cdot \tilde{\alpha} + \tilde{A}^s + (1+\phi) \cdot \widetilde{w}^s \, . \qquad (A8)$$

The procedure to obtain the equilibrium expression of $\tilde{r}$ is similar to the one we followed to get (A5) in the basic model. In particular, we use (A3) to get rid of $\widetilde{w}^s$ and, then we subtract (20) from (38) to substitute away for $\left(\widetilde{N}^s - \widetilde{N}^u\right)$ to get:

$$\tilde{r} = \frac{\alpha \cdot \tilde{A}^s + \beta \cdot \tilde{A}^u + \left((1+\phi) \cdot \alpha \cdot \epsilon + \frac{\delta \cdot \alpha \cdot \phi}{1-\mu+\delta \cdot \alpha}\right) \cdot \tilde{\alpha} - \phi \cdot \tilde{h} + (1+\phi) \cdot \tilde{Z}}{(\phi + 1 - \mu + \delta \cdot \alpha)} \, . \qquad (A9)$$

Local skill-biased technical change ($\tilde{\alpha} > 0$) will have an ambiguous effect on local rents. On the one hand, the adoption of skill-biased technologies increases productivity which is capitalized into rents. However, this effect is dampened by the fact that places that become more expensive displace the unskilled, since they spend relatively more on housing.

A3. Details on data construction.

We pool the Italian Labour Force Survey microdata for 12 consecutive waves, from the first quarter in 2009 to the last quarter in 2011. Each wave contains observations on more than 400,000 individuals. Data include – for all employees – wages, education, age, gender, sector of occupation, and city where the individual works.

All data are collapsed over time at Local Labour System (LLS) level.[22]

Wage and employment data are constructed for employees only between 15 and 64 years old. To take into account differences in the sectoral, age, and gender compositions in the labor force across LLS, wage premia are computed using the residuals of a regression (at individual level) of wages on industry dummies (2 digit Nace), 5-years age dummies, and a dummy for females. Population thresholds and weights are calculated at LLS level by using administrative data.

[22] The LLSs are the Italian equivalent for commuting zones. Each LLS is made of several municipalities; LLSs are constructed by using commuting flows from the Italian 2011 census. By construction, in each LLS, at least 75% of the population works and lives. In our data, we are able to collect information on roughly 450 LLSs.

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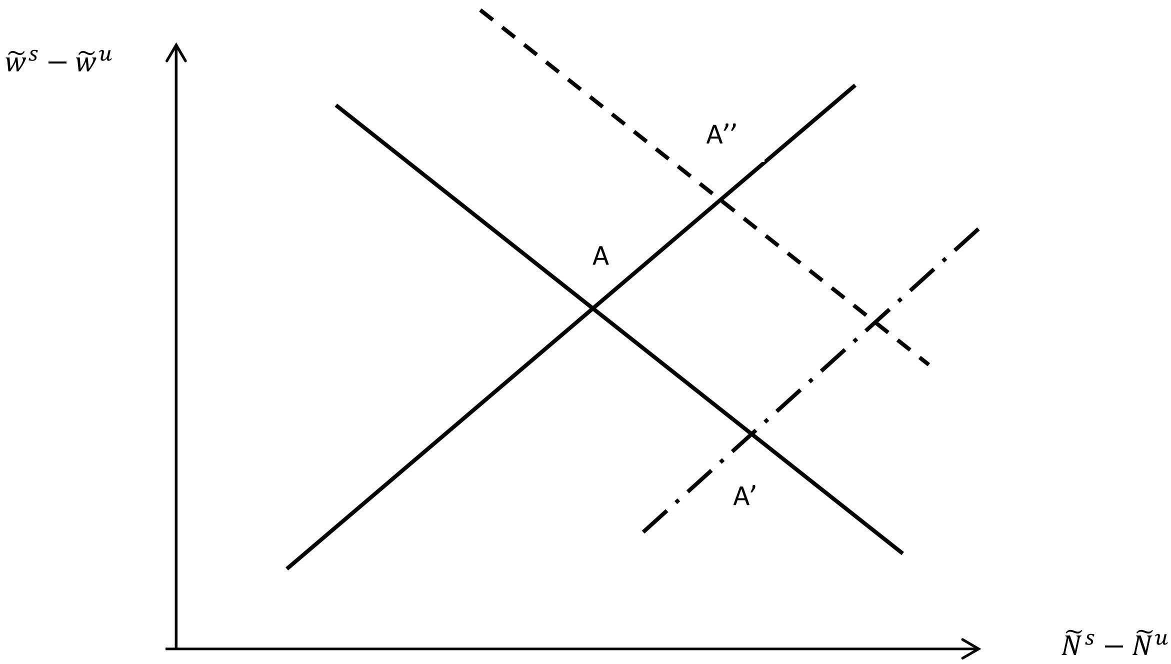

FIGURE 1: The basic model

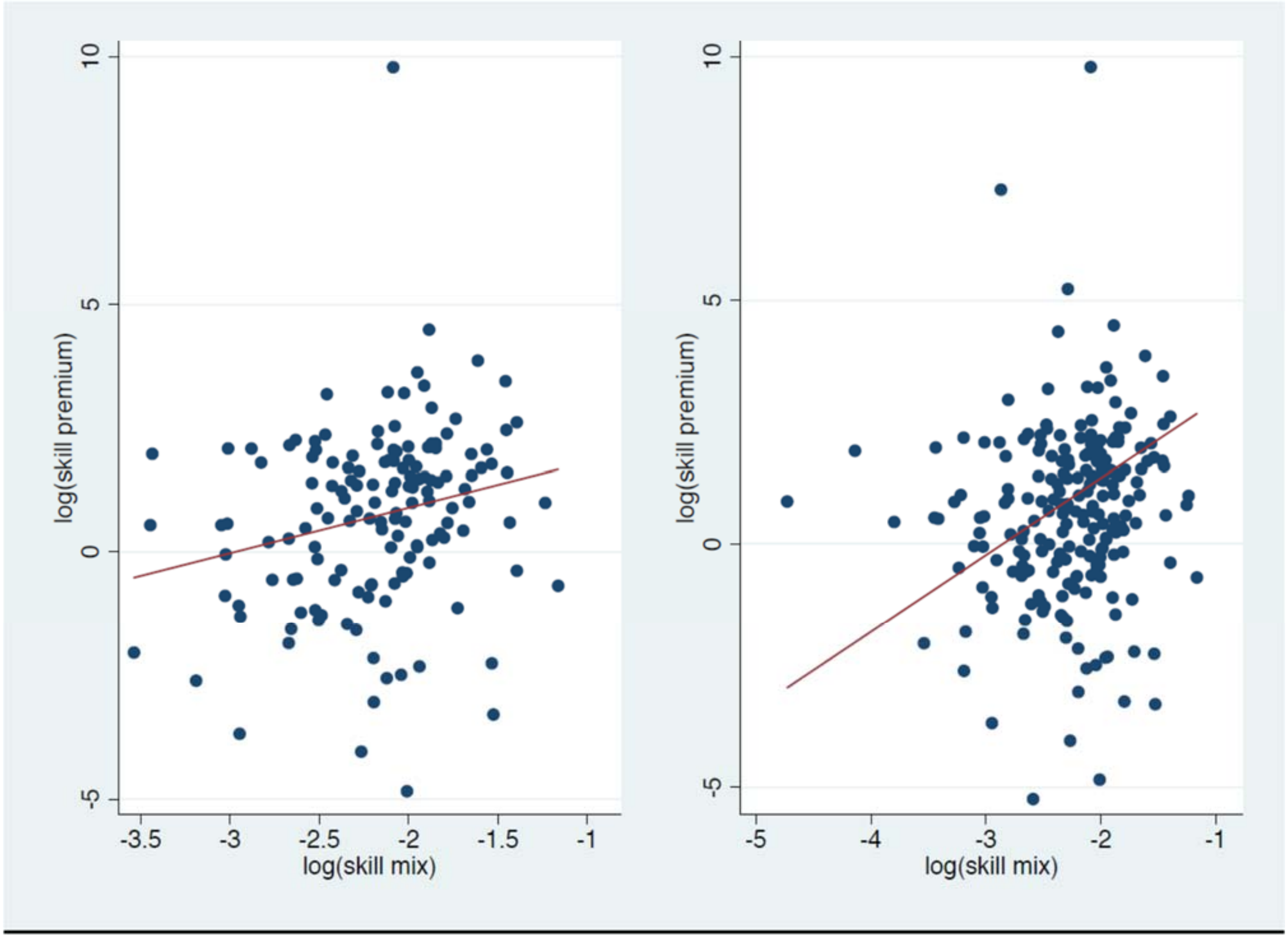

FIGURE 2: The relationship between skill premia and skill mix in Italy

Notes: Authors' calculations on the Italian Labour Force Survey. Unit of observation: commuting zones. On the y-axis: (log) university wage premia; on the x-axis: (log) share of workers with a university degree. The red line represents the linear fit. Left panel: all commuting zones with more than 200,000 inhabitants. Right panel: all commuting zones; linear fit weighted according to the population of each commuting zone.