



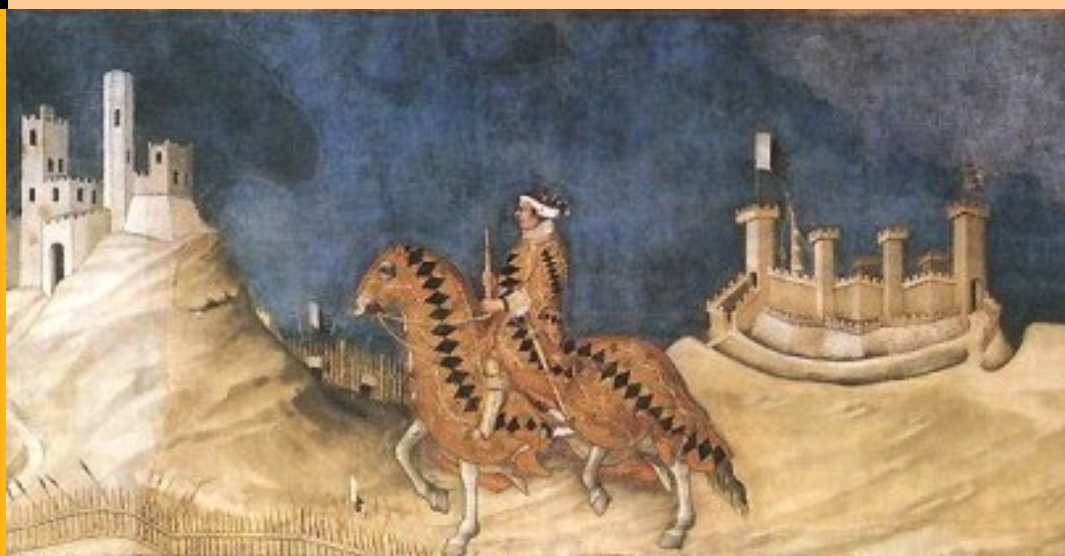
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Comparing small area techniques for estimating poverty measures

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Abstract

The Europe 2020 Strategy has formulated key policy objectives or so-called “headline targets” which the EU as a whole and Member States are individually committed to achieving by 2020. One of the five headline targets is directly related to key quality aspects of life, namely social inclusion; within these targets, the EU-SILC headline indicators at-risk-of-poverty or social exclusion and its components will be included in the budgeting of structural funds, one of the main instruments through which policy targets are attained. For this purpose, DG Regional Policy of the European Commission is aiming to use sub-national/regional level data (NUTS 2). Starting from this, the focus of the present paper is on the “regional dimension” of well-being. In fact, we compare two small area techniques, namely the cumulation and the spatial EBLUP (SEBLUP), on the basis of EU-SILC data from Austria and Spain.

Keywords: small area estimation; poverty; inequality; SILC

JEL Classification: C21, I32, C23

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1. Introduction

In the last two decades there has been increased interest in comparative analysis of poverty and social exclusion in the European Union. The Statistical Office of the European Union (Eurostat) launched the European Community Household Panel study (ECHP, 1994-2001) and later the EU Statistics on Income and Living Conditions (EU-SILC, 2004-to date), in order to create a European standardised data base to generate comparative measures of poverty and social exclusion among the Member States. A comprehensive set of common indicators, termed the Laeken Indicators, has been adopted for countries of the European Union (Atkinson *et al.*, 2002). These indicators are produced on a regular basis at national level, and are mainly based on the EU-SILC. EU-SILC surveys involve a rotational panel design conducted annually in each country. Microdata from the surveys are available to the research community in the form of a Users' Data Base (UDB). The national sample designs and sizes have been determined primarily for the purpose of estimation and reporting of indicators at national level, with limited breakdown by major socio-demographic subgroups of the population.

The Europe 2020 Strategy (European Commission, 2010) has formulated key policy objectives or so-called "headline targets" which the EU as a whole and Member States individually are committed to achieving by 2020. One of the five headline targets is directly related to key quality aspects of life, namely social inclusion; within these targets, the EU-SILC headline indicators at-risk-of-poverty or social exclusion (AROPe, which is also known as Head Count Ratio (HCR) and FGT(0) in the family of Foster, Greer and Thorbecke, 1984) and its components will be included in the budgeting of structural funds, one of the main instruments through which policy targets are attained.

For this purpose, DG Regional Policy of the European Commission is aiming to use sub-national/ regional level data (NUTS 2¹, and exceptionally NUTS 1 for a couple of big countries) for the social headline indicators, in order to complement GDP per capita, in defining regions that can apply for funding directed to the Convergence Objective. As a first step in this direction, for the funding period 2014-2020 these indicators will be used for benchmarking and assessing the efficiency of regional policies and programmes. Therefore, there is an urgent policy need for regional values of social policy indicators. The focus should be on accurately and correctly identifying regions with the highest proportion of people being poor or socially excluded, in order to target policy measures accordingly.

¹ NUTS is an abbreviation for Nomenclature of Statistical Territorial Units. This is Eurostat's hierarchical classification of regions, from Member States (NUTS 0) down to smaller areas.

For these reasons, the focus of the present paper is on the “regional dimension” of well-being. While the above-mentioned EU-wide comparative data sets, namely the ECHP and the EU-SILC, can serve as unique sources for generating comparative indicators of well-being, or rather of lack of welfare manifest such as poverty and deprivation, these sources are designed primarily to serve at national level, and appropriate methodologies are required to extend their use to the level of sub-national regions: such methodologies are known as small area estimation (SAE) techniques.

There is a wide variety of techniques available (SAE) in the literature, and the field is rapidly expanding. The suitability and efficiency of a particular technique depends on the specific situation and on the nature of the statistical data available for the purpose. Standard reference on small area estimation methodology are Handerson (1950), Gosh and Rao (1994) and, above all, Rao (2003); Betti *et al.* (2012) focus on small area estimation methods for poverty and inequality measures.

One class of techniques aims at making the best use of available data from national sample surveys, such as by cumulating and consolidating the information to obtain more robust measures which permit greater spatial disaggregation; this class is described in Section 2, where the particular method of cumulating three-years of the EU-SILC survey is described and applied.

Another class of techniques is based on small area models; in the literature these are classified as: (i) area level random effect models (Fay and Herriot, 1979), which are used when auxiliary information is available only at area level (such as the prevailing unemployment rate); (ii) nested error unit level regression model, used if unit specific covariates (such as the individual’s or the household’s employment situation) are available at unit level (Battese *et al.*, 1988).

In Section 3 one technique of class (i) is taken into account, namely the Empirical Best Linear Unbiased Predictor (EBLUP), and its developments in a spatial environment. One well known methodology of class (ii) is often undertaken by the World Bank, namely the Poverty Mapping (Elbers, Lanjouw and Lanjouw, 2003, ELL); however, it requires direct access to census data, which is not usually available for university researchers.

Finally, in Section 4 we compare the results obtained by the cumulation method and the spatial EBLUP (SEBLUP) method, based on Austria and Spain; some concluding remarks are also reported at the end of the paper.

Both methodologies applied in Sections 2 and 3 are based on the SILC, which is the major source of comparative statistics on income and living conditions in Europe. EU-SILC covers data and data sources of various types: cross-sectional and longitudinal; household-level and person-level; on income and social conditions; and from registers and interview surveys depending on the country. A standard integrated design has been adopted by nearly all EU countries. It involves a rotational panel in which a new sample

of households and persons is introduced each year to replace one quarter of the existing sample. Persons enumerated in each new sample are followed-up in the survey for four years. The design yields each year a cross-sectional sample, as well as longitudinal samples of various durations.

2. Cumulative measures of poverty

This section focuses on pooling of different sources pertaining to the same population or largely overlapping and similar populations. In particular, the interest is in pooling over survey waves in a national survey in order to increase the precision of regional estimates. Estimates from samples from the same population are most efficiently pooled with weights in proportion to their variances (meaning, with similar designs, in direct proportion to their sample sizes). Alternatively, the samples may be pooled at the micro level, with unit weights inversely proportional to their probabilities of appearing in any of the samples. This latter procedure may be more efficient (e.g., O’Muircheartaigh and Pedlow, 2002), but may be impossible to apply as it requires information, for every unit in the pooled sample, on its probability of selection into each of the samples irrespective of whether or not the unit actually appears in the particular sample (Wells, 1998). Another serious difficulty in pooling samples is that, in the presence of complex sampling designs, the structure of the resulting pooled sample can become too complex or even unknown to permit proper variance estimation. In any case, different waves of a survey like EU-SILC do not correspond to exactly the same population. The problem is akin to that of combining samples selected from multiple frames, for which it has been noted that micro level pooling is generally not the most efficient method (Lohr and Rao, 1996). For the above reasons, pooling of wave-specific estimates rather than of micro data sets is generally the more appropriate approach to aggregation over time from surveys such as EU-SILC.

2.1 Gain in precision from cumulation over survey waves

Consider that for each wave of a survey like EU-SILC, a person’s poverty status (poor or non-poor) is determined from his/her income within the income distribution of that wave, independently for each EU-SILC year, and then the proportion of poor at each wave is computed. These proportions are then averaged over a number of consecutive waves.

The issue is to quantify the gain in sampling precision from such pooling, compared to results based on a single wave.

The quantification of efficiency gains from averaging across multiple years is not straightforward in surveys, such as EU-SILC, that are based on rotational panel, given that data from different waves of a rotational panel are highly correlated.

A large proportion of the individuals are common in the different cross-sections. However, a certain proportion of individuals are different from one wave to the other. The cross-sectional samples are thus not independent, resulting in correlation between measures from different waves.

Apart from correlations at the individual level, we have to deal also with additional correlation that arises because of the common structure (stratification and clustering) of the waves of a panel. Such correlation would exist in, for instance, samples coming from the same clusters even if there is no overlap in terms of individual households.

In order to quantify the gain in precision from averaging over waves of a rotational panel, we provide the following simplified procedure that could be of help in better clarifying the point. It illustrates the statistical mechanism of how the gain is achieved. Indicating by p_j and p'_j the (1, 0) indicators of poverty of individual j over the two adjacent waves, we have the following result for the population variances:

$$\text{var}(p_j) = \sum (p_j - p)^2 = p \cdot (1 - p) = V;$$

similarly,

$$\text{var}(p'_j) = p' \cdot (1 - p') = V',$$

$$\text{cov}(p_j, p'_j) = \sum (p_j - p) \cdot (p'_j - p') = a - p \cdot p' = c_1, \text{ say,}$$

where ' a ' is the persistent poverty rate over the two adjacent years.

Under the two waves model and in the extreme case of a completely full sample overlap and $p' = p$, the variance V_A of the average over two waves of the concerned poverty measure can be estimated as:

$$V_A = \frac{V}{2} \cdot (1 + \rho) \tag{1}$$

where ρ represents the correlation between the two waves that in our simplified case can be quantified by:

$$\rho = \left(\frac{c_1}{V} \right) = \left(\frac{a - p^2}{p - p^2} \right).$$

Alternatively, if the overlap between the two waves is only partial like in the EU-SILC survey, and cross-sectional variances are not necessarily equal, it is necessary to allow for variations in cross-sectional sample sizes and partial overlaps:

$$V_A = \frac{1}{2} \cdot \left(\frac{V_1 + V_2}{2} \right) \cdot \left(1 + \rho \cdot \left(\frac{n}{n_H} \right) \right) \tag{2}$$

where V_1 and V_2 are the variances in each of the two waves, n is the sample overlap, n_H is the harmonic mean of different wave sizes, ρ as above (Verma *et al.*, 2013).

A replication method for variance estimation

The Jackknife Repeated Replication (JRR) is one of the classes of practical methods for variance estimation in complex samples based on measures of observed variability among replications of the full sample.

All replicated variance estimation procedures are based on comparisons among replications generated through repeated re-sampling of the same parent sample. Once the set of replications has been appropriately defined for any complex design, the same variance estimation algorithm can be applied to a statistic of any complexity.

The basic requirement is that the full sample is composed of a number of subsamples or replications, each with the same design and reflecting complexity of the full sample, enumerated using the same procedures. A replication differs from the full sample only in size. But its own size should be large enough for it to reflect the structure of the full sample, and for any estimate based on a single replication to be close to the corresponding estimate based on the full sample.

At the same time, the number of replications available should be large enough for the comparison among replications to give a stable estimate of the sampling variability in practice.

JRR provides a versatile and straightforward technique for variance estimation in situations like the ones we are concerned with.

Briefly, the standard JRR involves the following.

Let z be a full-sample estimate of any complexity. We use the subscript i to indicate a sample primary sampling unit (PSU) and h to indicate its stratum; $a_h \geq 2$ is the number of PSUs in stratum h . Let $z_{(hi)}$ be the estimate produced using the same procedure after eliminating primary unit i in stratum h and increasing the weight of the remaining $(a_h - 1)$ units in the stratum by an appropriate factor g_h (see below). Let $z_{(h)}$ be the simple average of the $z_{(hi)}$ over the a_h sample units in h . The variance of z is then estimated as:

$$\text{var}(z) = \sum_h \left[(1 - f_h) \cdot g_h \cdot \sum_i \left(z_{(hi)} - z_{(h)} \right)^2 \right] \quad (3)$$

$(1 - f_h)$ is the finite population correction and it is usually ~ 1 for samples in typical social surveys.

While one may take factor g_h as $g_h = a_h / (a_h - 1)$, it is more appropriate to use $g_h = w_h / (w_h - w_{hi})$, where $w_h = \sum_i w_{hi}$, with $w_{hi} = \sum_j w_{hij}$ as the sum of sample weights of ultimate units j in primary selection units i . This means that in each replication (hi) , the weights for individual units are redefined and rescaled as follows: i) for unit j not in stratum h : $w'_{hij} = w_{hij}$; ii) for unit j in stratum h but not in PSU i : $w'_{hij} = g_h \cdot w_{hij}$; iii) unit j in stratum h and in PSU i : $w'_{hij} = 0$. The second form for g_h retains the total weight of the

included sample cases unchanged across the replications created, so as to have the same total as the one for the full sample. With the sample weights scaled in such a way that their sum is equal (or proportional) to some external more reliable population total, population aggregates from the sample can be estimated more efficiently, often with the same precision as proportions or means (Verma and Betti, 2011).

2.2 Quantifying the gain in sampling precision using EU-SILC survey

The formulae presented in Section 2.1 have been applied to the EU-SILC *cross-sectional* datasets in order to obtain averaged measures over waves.

When complete information on sample structure is available and, more specifically, when identifiers are provided to *link* strata and PSUs throughout different EU-SILC *cross-sectional* datasets, it is possible to cumulate waves and quantify the gain in sampling precision achieved with this methodology.

When the above requirement is met, that is when full information on sample structure is available, the gain in sampling precision can be easily quantified by applying the standard JRR methodology presented above on the basis of the following considerations.

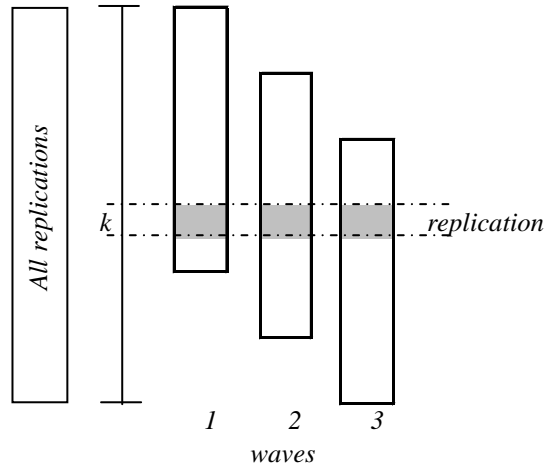
The total sample of interest is formed by the union of all the cross-sectional samples being compared or aggregated. Using the common structure of this total sample as a basis, a set of JRR replications is defined in the usual way.

Each replication is formed in such a way that when a unit is to be excluded in its construction, it is excluded simultaneously from every wave where the unit appears.

For each replication, the required measure is constructed for each of the cross-sectional samples involved, and these measures are used to obtain the required averaged measure for the replication. Variance of the statistic of interest is then estimated from the replication estimates in the usual way.

Let us clarify this procedure, presenting an empirical example. Consider that we have the cross-sectional dataset of the EU-SILC survey for three consecutive years and want to estimate the average of a given poverty measure over the three years. We proceed as follows. We first construct a common structure of strata and PSUs from the union of the three cross-sectional datasets; that is, we keep the list of all the strata and PSUs of each of the three datasets and construct a new list that is the result of the union of the three samples. Then we will create the replications from this common structure.

In the standard JRR methodology, replications are created by eliminating one PSU at a time, a replication being identified by the particular PSU (say k) eliminated in constructing it. In the combined dataset, the concerned PSU, if present, is eliminated from all the three cross-sectional datasets to obtain a 'combined' replication.



Next, we assign new weights to this common structure which are equal to the average of the weights of the three years:

$$w^{(t)\text{Common}} = (w)^{\text{Average}} = (w^{(1)} + w^{(2)} + w^{(3)})/3 \quad (4)$$

For each year (t) and for each replication (k), we can estimate $y_k^{(t)}$ where $t=1, 2, 3$ and from this, the required statistic

$$y_k^{\text{Average}} = \sum_t a_t y_k^{(t)} ; \quad (5)$$

that in our example on three waves is:

$$y_k^{\text{Average}} = (y_k^{(1)} + y_k^{(2)} + y_k^{(3)})/3 . \quad (6)$$

The variance estimate of this measure can be easily estimated applying the usual JRR for variance estimation procedure using the 'combined' replications as defined above, as if the statistic were a common cross sectional measure.

It is necessary to underline again that such procedures can be applied only if *full information on the sample structure is available*.

We have developed an alternative procedure for dealing with a situation in which full information on the sample structure is lacking (Verma *et al.*, 2010).

2.3 Empirical results

We have applied the methodologies described above to calculate the average measures for three years (2009, 2010 and 2011) to EU-SILC data for Austria (AT) and Spain (ES).

The empirical analysis has been performed only on these two countries for the following reasons. In the public version of the EU-SILC data, the so called UDB, the variables necessary for constructing the structure of the sample (namely, the PSUs 'DB060' and

the strata 'DB050') are not present and no link is possible across cross-sectional dataset either at micro (unit) level or at macro (structure) level. This problem is reflected also in Section 3.

Thanks to a project with the OECD for Spain we had access to all the necessary information on the sample structure and the linkage of the cross-sectional datasets.

For Austria, all necessary information (linkage of the structure for the 3 cross sectional data sets) was not available to us, but, given that the Austria sample structure could be assimilated to a simple random sampling, we used for the computation the indirect procedure, mentioned above.

Results at the national level for Austria and Spain are shown in Table 1, and results at regional NUTS 2 level in Austria and Spain in Table 2 and 3.

Table 1 Average over three years, Austria and Spain.

	(a)	(b)	(c)	(d)
AUSTRIA				
HCR 60% national p.l.	13.8	0.608	0.426	0.700
S80/S20	4.0	0.084	0.066	0.786
SPAIN				
HCR 60% national p.l.	22.0	0.478	0.311	0.650
S80/S20	6.5	0.154	0.110	0.718

(a) Estimate 2011

(b) s.e. 2011

(c) s.e. 3-years average

(d) ratio s.e. 3-years average over s.e. single year

The results at national level show a sensible reduction of the standard error (s.e.) using the three years average with the two measures concerned. The reduction of the standard errors that we get using the three years averages compared to the estimate for a single year (column (d)), ranges from 12% for S80/S20 index for Austria, up to 35% for HCR for Spain. In general the two methodologies (direct and indirect) for the estimation of standard errors of averaged measures over three years perform well and give similar results both at national and regional level, as we have already shown in our past work. The comparison of standard errors between one-year and three-year estimates is more complex at regional NUTS 2 level, given the instability of the one-year estimates because of small samples. This problem is particularly evident for regions with a small number of PSUs. The cumulated estimates in fact have been chosen to overcome to the high instability of the single year estimates.

Generally, also in this case we can appreciate a reduction of the standard error, both in mean and median, for the two measures. The reduction can be better appreciated considering the median, which is not affected by extreme values that are present in the results given the instability of the estimates for single years.

Table 2. Average over three years, Austria regional NUTS 2 level.

HCR 60%, national p.l.	(a)	(b)	(c)
Burgenland	3.721	2.438	0.655
Niederösterreich	1.065	0.783	0.735
Wien	1.859	1.352	0.727
Kärnten	3.347	2.240	0.669
Steiermark	1.297	1.100	0.848
Oberösterreich	1.119	0.718	0.642
Salzburg	1.877	1.323	0.705
Tirol	1.914	1.146	0.599
Voralberg	1.989	1.595	0.802
	Mean		0.709
	Median		0.705
S80/S20			
Burgenland	0.477	0.323	0.677
Niederösterreich	0.177	0.138	0.780
Wien	0.225	0.169	0.751
Kärnten	0.315	0.238	0.754
Steiermark	0.218	0.163	0.749
Oberösterreich	0.181	0.140	0.773
Salzburg	0.361	0.257	0.712
Tirol	0.303	0.217	0.715
Voralberg	0.425	0.405	0.951
	Mean		0.763
	Median		0.751

(a) s.e. 2011

(b) s.e. 3-years average

(c) ratio s.e. 3-years average over s.e. single year

Table 3a. Average over three years, Spain regional NUTS 2 level, HCR.

HCR 60%, national p.l.	(a)	(b)	(c)
Galicia	1.167	0.828	0.710
Principado de Asturias	1.035	0.708	0.684
Cantabria	0.948	2.084	2.199
País Vasco	0.840	0.475	0.565
Comunidad Foral de Navarra	0.831	0.602	0.725
La Rioja	1.502	0.980	0.653
Aragón	1.521	2.720	1.788
Comunidad de Madrid	1.908	0.931	0.488
Castilla y León	1.388	1.327	0.956
Castilla-La Mancha	1.735	1.183	0.682
Extremadura	2.045	2.326	1.137
Cataluña	1.033	0.549	0.531
Comunidad Valenciana	1.007	1.040	1.033
Illes Balears	1.127	1.845	1.637
Andalucía	1.175	0.944	0.804
Región de Murcia	1.563	1.313	0.840
Ciudad Autónoma de Ceuta	1.340	2.460	1.837
Ciudad Autónoma de Melilla	2.154	1.514	0.703
Canarias	1.187	1.071	0.902
	<i>Mean</i>		0.993
	<i>Median</i>		0.804

The results are very stable across regions in Austria. Furthermore, the results for mean and median measures are nearly the same, showing a reduction in variance of about 25-30% with pooling over 3 years.

For Spain the largest reductions in this case are in S80/S20, where, in median, we have a decrease of 38%; for HCR the decrease in median is 20%.

Table 3b. Average over three years, Spain regional NUTS 2 level, S80/S20.

S80/S20			
Galicia	0.359	0.180	0.503
Principado de Asturias	0.423	0.221	0.521
Cantabria	0.286	0.222	0.777
País Vasco	0.284	0.167	0.587
Comunidad Foral de Navarra	0.479	0.193	0.402
La Rioja	0.539	0.276	0.511
Aragón	0.358	0.247	0.690
Comunidad de Madrid	0.249	0.206	0.827
Castilla y León	1.644	0.597	0.363
Castilla-La Mancha	0.968	0.466	0.482
Extremadura	0.586	0.363	0.619
Cataluña	0.286	0.165	0.579
Comunidad Valenciana	0.414	0.271	0.655
Illes Balears	0.476	0.427	0.897
Andalucía	0.515	0.313	0.607
Región de Murcia	0.354	0.384	1.084
Ciudad Autónoma de Ceuta	0.392	0.846	2.156
Ciudad Autónoma de Melilla	0.933	0.639	0.684
Canarias	0.523	0.840	1.606
	Mean		0.766
	Median		0.619

(a) s.e. 2011

(b) s.e. 3-years average

(c) ratio s.e. 3-years average over s.e. single year

3. Model based small area estimation

In this section we present the main features of some model-based techniques for small area estimation, namely, the EBLUP estimator based on the model by Fay and Herriot (1979) and the EBLUP estimator based on spatially correlated random effects (Pratesi and Salvati, 2007). The first is an essential tool in dealing with small area estimation when only aggregated auxiliary data at the area level are available, the latter allows for spatial dependence of area level random effects by assuming a Simultaneously Autoregressive Process (SAR). We have applied these estimators to a pair of poverty and inequality measures, namely Head Count Ratio and the S80/S20 index.

3.1 Empirical Best Linear Unbiased Predictor

We are interested in obtaining an estimate of a domain specific parameter θ_i ($i = 1, 2, \dots, m$). In this work it can be either HCR or S80/S20. As true values θ_i are unknown it is assumed that a design-based unbiased estimator of the parameter is available such that $\hat{\theta}_i = \theta_i + e_i$, where e_i ($i = 1, 2, \dots, m$), are the sampling errors for each area, independent of each other and distributed with mean 0 and variance ψ_i . This is known as *sampling model*. It is assumed that the sampling variances ψ_i are known, but in practice it is rarely the case, so they are replaced with estimates $\hat{\psi}_i$ obtained by following a JRR procedure (Verma, 2004).

It is further assumed that true values θ_i are linearly related to a vector of p -area specific auxiliary variables $\mathbf{x}_i = (x_{1i}, x_{2i}, \dots, x_{pi})^T$:

$$\theta_i = \mathbf{x}_i' \boldsymbol{\beta} + z_i v_i$$

where $v_i \sim (0, \sigma_v^2)$ are independent area-level random effects. Normality of random effects can be assumed to obtain Maximum Likelihood (ML) or Restricted Maximum Likelihood (REML) estimates $\hat{\sigma}_v^2$. This model is known as *linking model*.

By combining the results above the Fay and Herriot (1979) model is obtained:

$$\hat{\theta}_i = \mathbf{x}_i' \boldsymbol{\beta} + z_i v_i + e_i \quad (7)$$

where v_i are independent of e_i . Under the model we have $E[\hat{\theta}_i] = \mathbf{x}_i' \boldsymbol{\beta}$ and $V[\hat{\theta}_i] = \psi_i + z_i^2 \sigma_v^2$. The Best Linear Unbiased Predictor of θ_i can be easily obtained by applying the general results of linear mixed effects models and it is equal to:

$$\tilde{\theta}_i^H(\sigma_v^2) = \gamma_i \hat{\theta}_i + (1 - \gamma_i) \mathbf{x}_i' \tilde{\boldsymbol{\beta}} \quad (8)$$

where factor $\gamma_i = z_i^2 \sigma_v^2 / (\psi_i + z_i^2 \sigma_v^2)$ is known as *shrinkage factor* and $\tilde{\boldsymbol{\beta}}$ is the BLUE estimator of $\boldsymbol{\beta}$. The expression above shows that the BLUP estimator is an average of the direct estimator $\hat{\theta}_i$ and the synthetic estimator $\mathbf{x}_i' \tilde{\boldsymbol{\beta}}$. It can be noted that the lower the sampling variance is the more the weight is attached to the direct estimator $\hat{\theta}_i$, in fact when $\psi_i \rightarrow 0$ then $\gamma_i \rightarrow 1$ meaning that $\tilde{\theta}_i^H \rightarrow \hat{\theta}_i$. We can say that the BLUP estimator is *design consistent*. In matrix notation the model can be expressed as:

$$\hat{\boldsymbol{\theta}} = \mathbf{X} \boldsymbol{\beta} + \mathbf{Z} \mathbf{v} + \mathbf{e} \quad (9)$$

The BLUP estimator is unknown as it depends on random effects variance σ_v^2 . By substituting it with a consistent estimator $\hat{\sigma}_v^2$ we obtain a two stage estimator which can be referred to as Empirical BLUP (EBLUP) and it is indicated as $\hat{\theta}_i^H = \tilde{\theta}_i^H(\hat{\sigma}_v^2) = \hat{\gamma}_i \hat{\theta}_i + (1 - \hat{\gamma}_i) \mathbf{x}_i' \tilde{\boldsymbol{\beta}}$. Under the model (7) the MSE of the BLUP estimator is:

$$\text{MSE}[\tilde{\theta}_i^H] = E[\tilde{\theta}_i^H - \theta_i] = g_{1i}(\sigma_v^2) + g_{2i}(\sigma_v^2)$$

Where:

$$g_{1i}(\sigma_v^2) = \psi_i [1 - \gamma_i]$$

$$g_{2i} = \gamma_i^2 + \mathbf{x}_i' \left\{ \sum_{i=1}^m (\sigma_v^2 + \psi_i)^{-1} \mathbf{x}_i \mathbf{x}_i' \right\}^{-1} \mathbf{x}_i$$

where $g_{1i}(\sigma_v^2)$ is due to the prediction of the random effect v_i and is $O(1)$ for large m , g_{2i} is due to the estimation of $\boldsymbol{\beta}$ and is $O(m^{-1})$. This means that a large reduction in MSE over $MSE(\hat{\theta}_i) = \psi_i$ can be obtained when $1 - \gamma_i$ is large.

Under normality of random effects and errors, the MSE of the EBLUP is:

$$MSE(\hat{\theta}_i^H) = MSE[\tilde{\theta}_i^H] + E[\hat{\theta}_i^H - \tilde{\theta}_i^H] = g_{1i}(\sigma_v^2) + g_{2i}(\sigma_v^2) + g_{3i}(\sigma_v^2) + o(m^{-1})$$

where $g_{3i}(\sigma_v^2) = \gamma_i^2(\sigma_v^2 + \psi_i)^{-1} \bar{V}(\hat{\sigma}_v^2)$, and \bar{V} is the asymptotic variance of the estimator $\hat{\sigma}_v^2$. The classic Fay and Herriot model (7) can be extended by considering that the vector \boldsymbol{v} follows a Simultaneously Autoregressive Process (SAR) with spatial autoregressive coefficient ρ and proximity matrix \boldsymbol{W} (Cressie, 1993).

$$\boldsymbol{v} = \rho \boldsymbol{W} \boldsymbol{v} + \boldsymbol{u} \rightarrow \boldsymbol{v} = (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{u} \quad (10)$$

where $\boldsymbol{u} \sim N(\mathbf{0}, \sigma_u^2 \boldsymbol{I})$ which implies that \boldsymbol{v} has mean vector $\mathbf{0}$ and variance-covariance matrix

$$\boldsymbol{G} = \sigma_u^2 [(\boldsymbol{I} - \rho \boldsymbol{W})(\boldsymbol{I} - \rho \boldsymbol{W}^T)]^{-1}.$$

Combining equations (9)-(10) the model with spatially correlated random effects is:

$$\hat{\boldsymbol{\theta}} = \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{Z} (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{u} + \boldsymbol{e} \quad (11)$$

Under the model above the covariance matrix of the estimator $\hat{\boldsymbol{\theta}}$ is $\boldsymbol{V} = \boldsymbol{R} + \boldsymbol{Z} \boldsymbol{G} \boldsymbol{Z}^T$, where $\boldsymbol{R} = \text{diag}(\psi_i)$ is the covariance matrix of the error term \boldsymbol{e} .

Matrix \boldsymbol{W} describe the spatial contiguity among areas while $\rho \in [-1, 1]$ describes the strength of spatial relationship among the random effects associated with neighbouring areas.

Under the model (11) the Spatial Best Linear Unbiased Predictor is:

$$\tilde{\boldsymbol{\theta}}_i^S(\sigma_u^2, \rho) = \boldsymbol{x}_i \tilde{\boldsymbol{\beta}} + \boldsymbol{b}_i^T \{ \sigma_u^2 [(\boldsymbol{I} - \rho \boldsymbol{W})(\boldsymbol{I} - \rho \boldsymbol{W}^T)]^{-1} \} \boldsymbol{Z}^T \{ \text{diag}(\boldsymbol{\psi}_i) + \boldsymbol{Z} \sigma_u^2 [(\boldsymbol{I} - \rho \boldsymbol{W})(\boldsymbol{I} - \rho \boldsymbol{W}^T)]^{-1} \boldsymbol{Z}^T \}^{-1} (\hat{\boldsymbol{\theta}} - \boldsymbol{X} \tilde{\boldsymbol{\beta}}) \quad (12)$$

which is equal to the classic BLUP if $\rho = 0$.

The estimator is unknown because it depends on the unknown parameters σ_u^2 and ρ . By substituting them with a consistent estimator (ML or REML) $\hat{\sigma}_u^2$ and $\hat{\rho}$ a two stage estimator $\hat{\boldsymbol{\theta}}_i^S = \tilde{\boldsymbol{\theta}}_i^S(\hat{\sigma}_u^2, \hat{\rho})$ is obtained which can be referred to as a Spatial EBLUP (Pratesi and Salvati, 2007).

The spatial weight matrix reflects the neighbouring structure of the small areas. In the next applications the structure has been specified by following an approach based on contiguity and on distance threshold (see Cliff and Ord, 1981, for further details).

The former specifies the spatial dependence between two areas by assigning spatial weight $w_{ij} = 1$ if area i and j are adjacent and zero otherwise. The latter assigns spatial weight $w_{ij} = 1$ if the distance between the centroid of area i and j is less than a distance threshold τ . Generally, the matrix \boldsymbol{W} is row-standardized, so it is row-stochastic and ρ is called a spatial autocorrelation parameter (Banerjee *et.al*, 2004).

Under the model (12) the Mean Square Error of the Spatial BLUP estimator depending on two variance components (ρ, σ_v^2) is given by:

$$\text{MSE}[\tilde{\theta}_i^S(\rho, \sigma_u^2)] = g_{1i}(\rho, \sigma_u^2) + g_{2i}(\rho, \sigma_u^2)$$

where the first term is due to the estimation of random effects and it is of order $O(1)$, while the second is due to the estimation of β and it is of order $O(m^{-1})$.

For the Spatial EBLUP, given the normality of random effects, we have:

$$\text{MSE}[\tilde{\theta}_i^S(\hat{\rho}, \hat{\sigma}_u^2)] = \text{MSE}[\tilde{\theta}_i^S(\rho, \sigma_u^2)] + E[\tilde{\theta}_i^S(\hat{\rho}, \hat{\sigma}_u^2) - \tilde{\theta}_i^S(\rho, \sigma_u^2)]$$

The last term is intractable and therefore it needs to be approximated. For full details on the MSE of the spatial EBLUP and its estimation procedure, see Pratesi and Salvati (2007).

3.2 Applications

In this section we apply the above estimators to EU-SILC data available for Austria and Spain at NUTS 2 level. We have 16 NUTS 2 for Spain and 9 NUTS 2 for Austria at our disposal. Following Bivand, Rubio, and Pebesma (2008) there are basically three different types of spatial data: 1) Spatial Point Processes 2) Geostatistical data 3) Areal Data. In this paper we focus on areal data which means that when it is collected data regard a particular region, country, small area of any kind. The aim is to investigate how and to what extent data observed in one region is influenced by what has been observed in other regions. Tables 4 and 5 show direct, EBLUP and SEBLUP estimates. The gain in efficiency is quantified on the following lines: for each area we calculate the ratio between the estimate of the standard error obtained by the model-based estimator and the estimate of the standard error of the direct estimate. Then, these values are averaged to get the gain in efficiency.

Table 4 shows that the Spatial EBLUP based on a distance approach ($\hat{\theta}_i^{Sd}$) leads to the highest gain in efficiency. The gain is quite small, this is due to the fact that the direct estimates of HCR already have an appropriate level of accuracy. Proof of this can be had by observing the estimates of the shrinkage factors which are all close to 1. This means that, in traditional EBLUP case, the weight is attached mostly to the direct estimates rather than to regression estimate meaning that sampling variances $\hat{\psi}_i$ are small with respect to the total model-variance.

Regarding the S80/S20 Index, the results in Table 5 show that the highest gain in efficiency is obtained with the traditional EBLUP. This is not surprising because the estimated spatial autocorrelation coefficient is lower than the one estimated in the HCR case. It is interesting to note that when the coefficient is almost zero, the estimates obtained with EBLUP and SEBLUP are nearly the same. In fact, when $\rho = 0$ the EBLUP and SEBLUP estimators are equal (Bivand, Rubio and Pebesma, 2008).

Table 4. HCR Estimates. Spain 2011

NUTS	Direct Estimates	SE	EBLUP	gamma	se	SEBLUP (contiguity)	se	SEBLUP (distance)	se
Galicia	0.184	0.012	0.188	0.945	0.011	0.184	0.011	0.183	0.011
Asturias	0.103	0.010	0.109	0.956	0.010	0.109	0.010	0.109	0.010
Cantabria	0.199	0.009	0.199	0.963	0.009	0.197	0.009	0.197	0.009
País Vasco	0.110	0.008	0.109	0.971	0.008	0.108	0.008	0.108	0.008
Navarra	0.095	0.008	0.096	0.971	0.008	0.098	0.008	0.098	0.008
Rioja	0.250	0.015	0.248	0.912	0.015	0.245	0.015	0.247	0.014
Aragón	0.179	0.015	0.182	0.910	0.015	0.182	0.015	0.182	0.015
Madrid	0.164	0.019	0.159	0.866	0.018	0.162	0.018	0.166	0.018
Castilla y León	0.230	0.014	0.232	0.924	0.014	0.232	0.013	0.231	0.013
Castilla - La Mancha	0.318	0.017	0.310	0.886	0.017	0.310	0.016	0.306	0.016
Extremadura	0.352	0.020	0.343	0.849	0.019	0.347	0.019	0.347	0.019
Cataluña	0.171	0.010	0.171	0.957	0.010	0.170	0.010	0.171	0.010
Comunitat Valenciana	0.201	0.010	0.202	0.959	0.010	0.202	0.010	0.203	0.010
Balears	0.186	0.011	0.186	0.949	0.011	0.187	0.011	0.187	0.011
Andalucía	0.320	0.012	0.316	0.945	0.012	0.317	0.011	0.317	0.011
Murcia	0.247	0.016	0.248	0.906	0.015	0.252	0.015	0.249	0.015
Ceuta	0.235	0.013	0.234	0.929	0.013	0.233	0.013	0.234	0.013
Melilla	0.318	0.022	0.319	0.835	0.021	0.319	0.020	0.322	0.020
Canarias	0.320	0.012	0.319	0.943	0.012	0.319	0.012	0.319	0.012
mean	0.220	0.013	0.219	0.925	0.013	0.220	0.013	0.220	0.013
gain					2.43%		2.80%		3.01%
						$\rho = 0.64$		$\rho=0.57$	

Table 5. S80/S20 Index. Spain 2011

NUTS	Direct Estimates	se	gamma	EBLUP	se	SEBLUP (contiguity)	se	SEBLUP (distance)	se
Galicia	5.581	0.359	0.883	5.704	0.344	5.666	0.348	5.711	0.349
Asturias	4.740	0.423	0.844	5.003	0.399	4.980	0.410	5.010	0.413
Cantabria	6.081	0.286	0.922	6.079	0.279	6.066	0.281	6.080	0.280
País Vasco	5.142	0.284	0.923	5.090	0.281	5.094	0.283	5.091	0.282
Navarra	4.757	0.479	0.809	4.869	0.448	4.933	0.455	4.865	0.455
Rioja	7.644	0.539	0.770	7.354	0.490	7.355	0.503	7.350	0.499
Aragón	6.227	0.358	0.883	6.221	0.343	6.211	0.349	6.223	0.354
Madrid	5.989	0.249	0.940	5.940	0.246	5.944	0.246	5.939	0.247
Castilla y León	6.332	1.644	0.264	6.562	0.897	6.585	0.970	6.553	0.949
Castilla - La Mancha	7.358	0.968	0.509	7.040	0.733	7.080	0.776	7.057	0.778
Extremadura	7.266	0.586	0.739	7.268	0.531	7.371	0.539	7.246	0.548
Cataluña	5.363	0.286	0.922	5.397	0.279	5.384	0.281	5.397	0.281
Comunitat Valenciana	5.426	0.414	0.850	5.583	0.392	5.552	0.400	5.586	0.398
Balears	6.748	0.476	0.811	6.603	0.442	6.610	0.445	6.603	0.444
Andalucía	8.565	0.515	0.785	8.166	0.473	8.120	0.483	8.171	0.489
Murcia	5.401	0.354	0.886	5.557	0.341	5.588	0.345	5.558	0.344
Ceuta	5.319	0.392	0.863	5.475	0.373	5.487	0.375	5.458	0.382
Melilla	10.275	0.933	0.527	9.076	0.752	9.093	0.759	9.093	0.802
Canarias	7.603	0.523	0.780	7.542	0.484	7.556	0.487	7.540	0.486
mean	6.411	0.530	0.785	6.344	0.449	6.351	0.460	6.344	0.462
gain					9.38%		7.80%		7.38%
						$\rho=0.31$		$\rho=-0.03$	

Figure 1 and 2 show the geographical distribution of the Spatial EBLUP estimates based on a distance approach for both HCR and S80/S20 indices. It can be observed that the southern regions of Spain show the highest percentage of poor individuals. In the left corner of the image the islands Ceuta, Melilla and Canarias are reported widely.

Figure 1. HCR spatial EBLUP estimates. Spain 2011

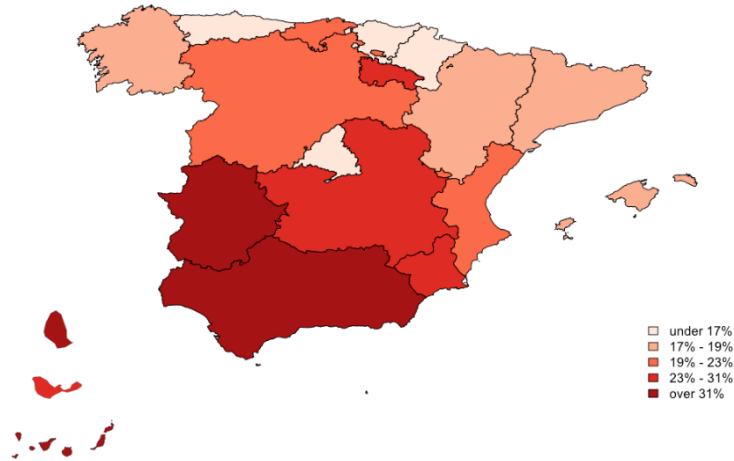
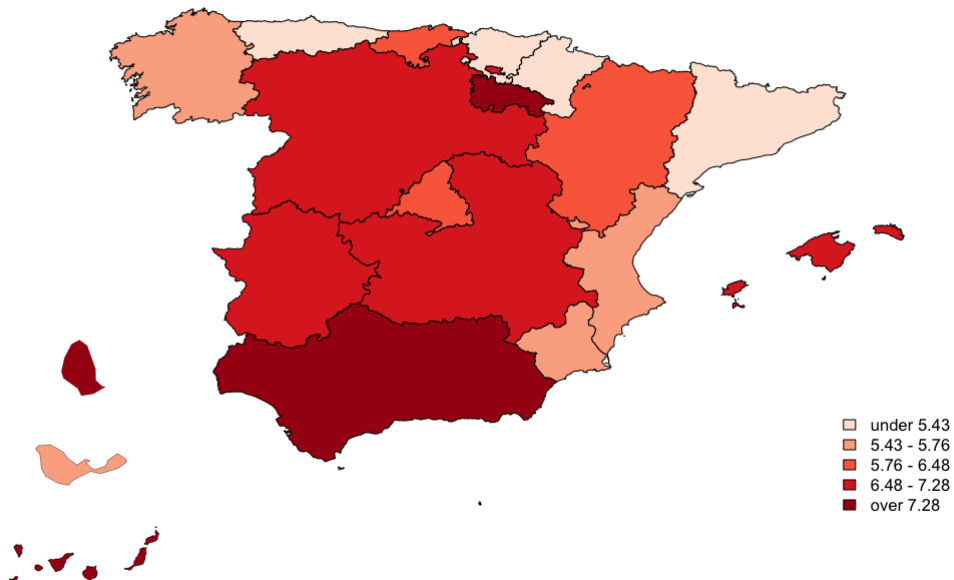


Figure 2. S80/S20 spatial EBLUP estimates. Spain 2011



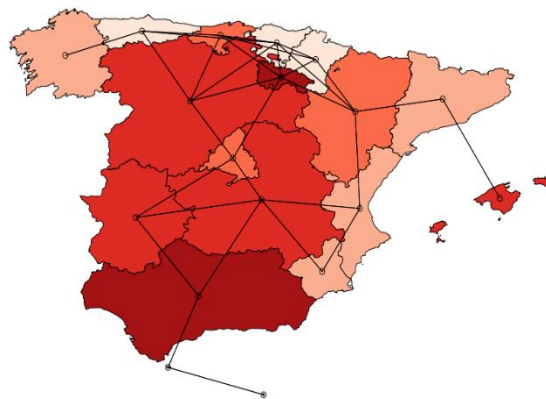
Comparing Figure 1 with Figure 2, we can observe that, generally, those regions showing high values of HCR also present high values of S80/S20 index; this confirms that relative

poverty and inequality are generally correlated. On one hand, Madrid is an exception, since a low HCR is accompanied by a high level of inequality. On the other hand Melilla is a region with a high value of poverty and very little inequality.

By looking at the estimates spatial autoregressive coefficients it can be observed that, in both cases, this is lower when considering a distance threshold approach. In fact, the distance threshold is taken to guarantee at least one linkage for each area apart from the Canarias Islands. By doing this, most linkages between areas are found in northern Spain where we find rich (unequal) and poor (equal) regions. This can be understood by looking at Figure 3.

Each area is linked to its neighbours by means of black lines visible in the figure. Obviously the neighbouring structure is influenced by the approach followed by the researcher. In the figure we follow a distance based approach, that is two areas are called neighbours if the distance from each other is less than a threshold (taking as reference the centroid of each area as already explained in Section 3.1).

Figure 3. Distance threshold neighbours



Tables 6 and 7 show instead direct, EBLUP and Spatial EBLUP estimates. It is interesting to compare here the performance of the model based estimators proposed with respect to the available sample size for each small area. It is expected that the gain in efficiency by adopting the EBLUP (or SEBLUP) estimator will be higher for those areas where the sample size is lower or where the estimated relative standard error is high. Table 6

shows the results for the HCR Index. In the states of Burgenland and Carinthia the shrinkage factors are much lower than the others, meaning that most weight is attached to the regression synthetic estimator. It is not surprising that these areas show few sample sizes as well as a high estimated relative standard error, respectively 26% and 18%. Here model-based estimates are much lower than direct ones. The estimates of the spatial autoregression coefficients are moderate and the highest gain in efficiency is obtained with EBLUP. This suggests that if the spatial dependence is weak then it is better to use the traditional EBLUP. The results for S80/S20 Index show that for the areas of Burgenland and Vorarlberg the most weight is attached to the regression estimator while for Lower Austria the most weight is given to direct estimate. This is consistent with the expected results as the first two areas show low sample size and an estimate of the relative standard error of 14% and 11% respectively. On the contrary, high sample size is found in Lower Austria apart from a very low estimate of the relative standard error (4%). The Spatial EBLUP estimator leads to a very reduced gain in efficiency. This may be due to the fact that the estimated coefficient of spatial autoregression is moderate and negative, in the HCR case as well.

Table 6. HCR Estimates. Austria 2011

NUTS	Direct Estimates	se	EBLUP	gamma	se	SEBLUP (contiguity)	se	SEBLUP (distance)	se
Burgenland	0.143	0.037	0.082	0.110	0.019	0.081	0.021	0.086	0.022
Carinthia	0.184	0.033	0.119	0.132	0.016	0.117	0.015	0.124	0.019
Lower Austria	0.099	0.011	0.099	0.600	0.011	0.100	0.014	0.099	0.012
Upper Austria	0.089	0.011	0.094	0.576	0.012	0.094	0.013	0.093	0.013
Salzburg	0.099	0.019	0.120	0.326	0.015	0.120	0.016	0.118	0.017
Styria	0.109	0.013	0.110	0.503	0.013	0.110	0.014	0.110	0.014
Tyrol	0.126	0.019	0.122	0.317	0.015	0.122	0.015	0.126	0.018
Vorarlberg	0.085	0.020	0.104	0.301	0.015	0.105	0.011	0.102	0.016
Vienna	0.192	0.019	0.178	0.411	0.019	0.178	0.021	0.178	0.020
mean	0.125	0.020	0.114	0.364	0.015	0.114	0.016	0.115	0.017
gain					17.87%		12.70%		8.82%
					$\rho=0.07$		$\rho=-0.39$		

Table 7. S80/S20 Index. Austria 2011

NUTS	Direct Estimates	se	EBLUP	gamma	se	SEBLUP (contiguity)	se	SEBLUP (distance)	se
Burgenland	3.967	0.477	3.375	0.246	0.332	3.383	0.380	3.327	0.362
Carinthia	3.712	0.315	3.712	0.428	0.258	3.702	0.280	3.747	0.284
Lower Austria	3.556	0.177	3.536	0.703	0.181	3.531	0.218	3.532	0.200
Upper Austria	3.330	0.181	3.467	0.693	0.181	3.469	0.197	3.473	0.201
Salzburg	4.310	0.361	4.252	0.363	0.279	4.267	0.317	4.260	0.322
Styria	3.669	0.218	3.743	0.611	0.208	3.744	0.230	3.737	0.233
Tyrol	4.163	0.303	3.924	0.447	0.253	3.950	0.280	3.960	0.266
Vorarlberg	3.197	0.425	3.652	0.291	0.282	3.615	0.295	3.584	0.305
Vienna	5.001	0.225	4.900	0.595	0.241	4.897	0.270	4.893	0.256
Mean	3.878	0.298	3.840	0.486	0.246	3.840	0.274	3.835	0.270
Gain					13.07%		2.71%		4.58%
					$\rho=-0.19$		$\rho=-0.29$		

Figures 4 and 5 show Spatial EBLUP estimates based on a contiguity approach for HCR and S80/S20 in Austria. It can be observed that here areas with high values of HCR show also high values of S80/S20 and viceversa.

Figure 4. Spatial EBLUP estimates. S80/S20 Index. Austria

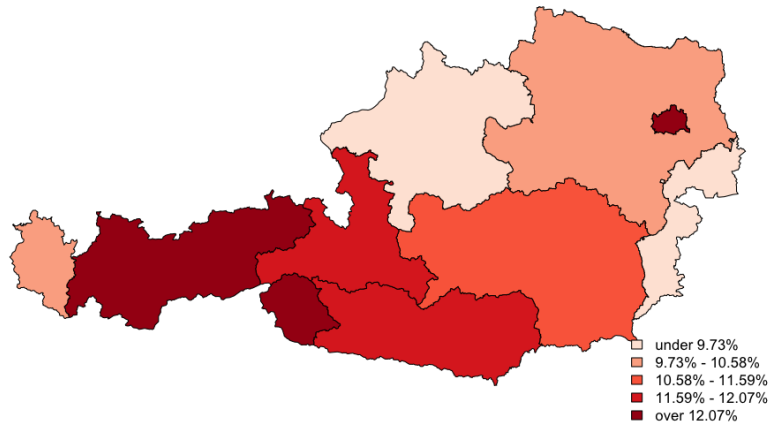
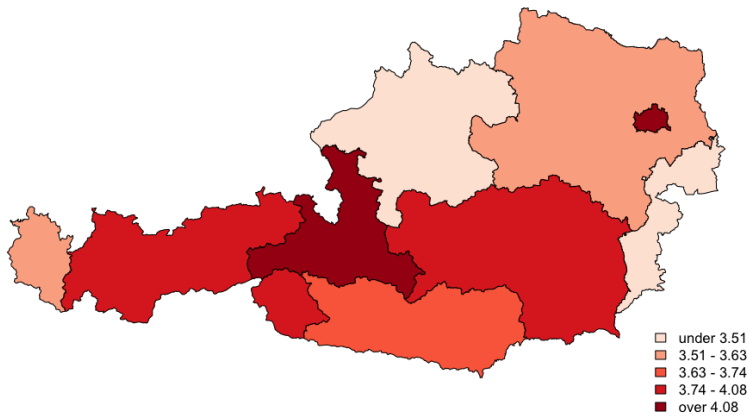


Figure 5. Spatial EBLUP estimates- S80/S20 Index. Austria



4. Discussion and concluding remarks

In this paper we have addressed the problem of estimating measures of well-being on their “regional dimension”; in fact, we have presented and compared two small area techniques, namely the cumulation and the spatial EBLUP (SEBLUP), on the basis of EU-SILC data from Austria and Spain. Both methodologies have been analysed observing both advantages and drawbacks.

In general, estimates computed with the cumulation method show standard errors which are smaller than those computed with EBLUP or SEBLUP. The gain of pooling SILC data over three years is therefore relevant, and may allow researchers to prefer this method. However, we would like to emphasise a point of great practical concern. Assessment of sampling precision of the estimates, taking into account the actual structure of the SILC sample, on which the data are based, has an essential requirement: *provision of codes describing the sample in the survey micro data itself, along with accompanying documentation describing the design and the code*. Inadequate (or sometimes even absence of) information on sample structure in survey data files is a long-standing and persistent problem in estimation from sample surveys. Unfortunately, even outstanding and highly standardised multi-country surveys such as EU-SILC have this sort of shortcomings, as underlined in this paper. A second drawback of the cumulation approach consists in the loss of the reference period to which the estimated measures coming from the data pooling are anchored. For example, in our exercise which is the reference year? 2010, which is the middle year, or 2011, which is the last available year (and comparable with SEBLUP estimates)? The debate on this issue is still open, and the present paper would intend to be a new starting point in this debate.

On the other hand, when considering techniques such EBLUP and SEBLUP, some features of the areas for which new estimates are needed should be properly taken into account; first of all, the presence of islands or other types of geographical barriers; may be that some computational procedures would fail in the presence of such a situation. In such cases, the problem needs to be addressed adequately. The analysis may be restricted to those areas having at least one linkage with another and at the same time leaving the remaining as separate cases (this is usually done in the US with Alaska and Hawaii). If the spatial dependence is not an essential feature of data, meaning an estimated spatial autocorrelation coefficient nearly equal to zero, then a possible solution could be that of adopting the traditional EBLUP estimator. This estimator, by assuming the independence of the area-level random effects, does not suffer from spatial boundaries and, consequently, it is not sensitive to whether a region is an island or not. Obviously, this kind of problem can be easily overcome by following a design-based approach to small area estimation as in the case with the cumulation of estimates.

On the other hand, the fact that in the presented results the cumulation method performs better than EBLUP and SEBLUP should be judged taking into account an additional issue: when choosing the set of regressors in the EBLUP or SEBLUP, in general researchers do not have full access to information (regressors) present at area level. From this point of view, National Statistical Offices could in general perform better, having the possibility to access a large set of regressors.

Finally, we want to highlight that in the paper the estimation of the MSE of the SEBLUP estimator has been carried out by following a procedure which considers the analytical approximation of the MSE itself. Other estimators based on bootstrap procedures have been developed (see for instance Molina, Pratesi and Salvati, 2009). We have tried to apply these procedures; however, results have been unsatisfactory and some computational issues have been raised. Again, this is another aspect which future research should be focused on.

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