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Strength of Justification
The Rational Degree of Certainty Approach

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In this paper, I present the fundamental ideas of a new theory of justification strength. This theory is based on the epistemological approach to argumentation. Even the thesis of a valid justification can be false for various reasons. The theory outlined here identifies such possible errors. Justification strength is equated with the degree to which such possible errors are excluded. The natural expression of this kind of justification strength is the (rational) degree of certainty of the belief in the thesis.

KEYWORDS: argument strength, Bayesian updating, certainty of belief, dimensions of justification strength, epistemological approach to argumentation, metatheoretical certainty, preciseness, probabilism, strength of justification, validity

1. THE SENSE OF A THEORY OF JUSTIFICATION STRENGTH AND THE AIM OF THIS ARTICLE

Amongst the founding ideas of the new argumentation theory was that there are arguments besides deductive arguments, which, however, are not certain and, therefore, are defeasible. Defeasibility in particular implies that two good (i.e., argumentatively valid and adequate [1]) arguments (or non-argumentative justifications), *a* and *b*, may be in logical conflict because their theses, t_a and t_b, contradict each other. Sometimes it is possible to combine the data from conflicting arguments

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[1] '*Argumentative validity*' here means, roughly, that an argument's premises are true and the inferential relation is correct; and '*adequacy*' means that both the premises and the inferential relation are epistemically accessible to the argument's addressee (Lumer, 2005a, pp. 220; 225-231; 234-236).

into a total datum and an overall argument. For example, if two testimonies contradict each other, one can try to hypothetically explain how the contradiction arose, and assign probabilities to these hypothetical explanations. Forming such a comprehensive overall argument amounts to dissolving the epistemic conflict by *coherentising the competing justifications*. If such a dissolution of the contradiction by means of coherentisation in an encompassing argument is possible, normally it should be undertaken; so coherentisation takes precedence over the cases to be considered in what follows. However, such an integrating coherentisation is often *not* possible, so that the conflict remains. A standard approach to resolving such epistemic and argumentative conflicts is to decide according to the strength of justification of the conflicting arguments or theses, and then to reject the weaker justified. This presupposes, however, a normative theory of justification strength, providing the criteria for making such comparisons.

There are several approaches to a theory of justification strength, e.g., Pollock (2001; 2002; 2010), Hahn & Oaksford (2006), Betz (2010; 2012), Gordon & Walton (2011), Godden & Zenker (2016). Generally, however, the subject is not given much attention. Although I am unsatisfied with the theories aforementioned, for reasons of space, I cannot here expand upon my discontent in detail. Nevertheless, in the next section I will briefly criticize the probabilistic approach, according to which justification strength is identical to the probability assigned to a thesis by the argument: in cases of probabilistic theses, the probability belongs to the content of the thesis and does not indicate the degree of its justification. Although the probability calculus, with its quantitative precision and its flexibility, constitutes a formal paragon for the theory of justification strength to be developed here, it is not the desired theory of justification strength.

My main purpose in this paper is to develop the fundamental ideas of a normative, epistemologically-oriented theory of justification strength. The epistemological approach (overview: Lumer, 2005b) holds, inter alia, foundationalism about the formation of opinions: rational belief is justified belief; it results from an analytical process of verifying compliance with criteria, the fulfilment of which guarantees the truth, probability, or acceptability of the thesis. Heeding this conception, then, in the present context analyticity is of central importance, i.e., that the fulfilment of the single conditions of a criterion is examined, whereby the criterion itself is epistemologically justified. If this verification procedure is carried out correctly and leads to a positive result (sc. the conditions of the criterion are met), then the

thesis is true, probably true, or acceptable. The restriction 'true, probably true, or acceptable' arises from the fact that not all justification procedures can guarantee the thesis' truth; for example, probabilistic or practical justifications e.g. cannot. In this respect the justification procedure is, therefore, uncertain. The strength of justification of a thesis, then, is an index that tries to estimate such uncertainties, or, more precisely: by estimating such uncertainties it attempts to determine how well the thesis justified in this way approaches the truth, and thereby how much one can rely on it. In addition to the uncertainty of the justification procedure, there are a number of other factors on account of which truth can be missed: The data or premises on which the procedure is based are uncertain or not sufficiently yielding; the process is accurate only to a limited degree; mistakes were made in the application of the procedure; the validity of the method is metatheoretically uncertain, etc.. Several of these components of uncertainty can be expressed as probabilities, others cannot. The resulting total uncertainty, or the degree of certainty of the substantiated thesis, however, does *not* reside in the content of this thesis, i.e., in the substantiated judgment, e.g., as a probability indicated there. Rather, the degree of certainty, in rational subjects, is represented by the (rational) degree of certainty of the belief in the thesis. Therefore, the central claim of the approach developed here is that strength of justification is identical with the rational degree of certainty of belief in the thesis. In determining the strength of justification, however, the approach presented does not rely on the subjective feeling of the subject, but rather tries to establish the degree of certainty analytically, viz. by way of the different dimensions of possible uncertainty. The degree of certainty of the belief of rational subjects should, then, correspond to the degree of certainty established analytically.

This essay sketches essential elements of an elaborated normative, epistemological theory of justification strength, a series of details, and the way for further development. It does not, however, present the complete theory. The latter requires much more detail, especially with regard to the dimensions of uncertainty and their (comparative) measurement for individual justification types.[2]

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[2] Overview of argumentative justification types from an epistemological viewpoint: Lumer 2011a.

2. AGAINST PROBABILISM: STRENGTH OF JUSTIFICATION IS NOT IDENTICAL WITH PROBABILITY

If one could equate justification strength with the probability of propositions, then this kind of probabilism would be the simplest and most elegant theory of justification strength. However, this is not possible for a number of reasons. This impossibility holds equally for various interpretations of probability: frequentist probability (according to which probabilities are determined on the basis of relative frequencies), natural/objective probability (probability as propensity or interpreted frequentistically), and probability understood as subjective degree of belief (in particular subjectivist Bayesianism). However, the epistemological approach advocated here cannot accommodate a purely subjectivist interpretation of probability, understood as subjective degree of belief. This is because the degree of belief would be a mere feeling and would thereby no longer be grounded.[3]

The first problem for probabilistic approaches to justification strength is technical: where is probability to be located? There are two possible interpretations: 1. The first is that probability resides within the proposition. Then we have to consider several types of propositions. There are probabilistic propositions, e.g. '$P(h)=0.9$'; for these propositions, the probability specification belongs to the proposition itself, hence it does not give the justification strength of *this* probabilistic proposition. Therefore, one could assume that the given probability (0.9) expresses the probability and justification strength of the skeleton proposition remaining after the removal of the probability specification (in the example: 'h'). In this case non-probabilistic propositions would have a probability of 1 as well as (according to the idea just discussed) a justification strength of 1. This latter implication, however, is extremely implausible since many weakly or uncertainly founded propositions are not probabilised, e.g., propositions regarding the existence of God, moral principles, the value of science, responsibility for the last state crisis. Moreover, what would be the

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[3] Some subjectivists add a probabilistic coherence requirement to the subjective degree of belief interpretation of probability. But this does not resolve the original problem: in case of incoherent probabilities, some of one's probabilities have to be changed, but even a (merely) coherentist version of subjectivism says nothing about which of the various ways to arrive at a coherent probability attribution ought to be chosen, thus leaving the choice to the subject's arbitrariness. Moreover, it leads to a new problem: Since nobody's probabilities are coherent, we could not attribute subjective probabilities to anybody.

justification strength of probabilistic propositions that do not simply assign a probability to a skeleton proposition, e.g., conditional probabilities like '$P(h|e)=0.8$'? The skeleton remaining after removing the probability assignment, i.e. '$(h|e)$', is no longer a proposition, hence nothing that can be believed, justified and possess justification strength. Would such probabilistic propositions—e.g., '$P(h|e)=0.8$'—also have the probability of 1? This too is extremely implausible. It would, for example, exclude the possibility of learning with respect to relative frequencies. 2. An alternative interpretation holds that probability resides outside of the proposition, viz. in the (rational or subjective) degree of belief, which, again, should follow the probability calculus. On this interpretation, however, if the believed proposition is itself probabilistic we would encounter double probabilities: one that is internal to the proposition and another that is external, pertaining to the degree of belief. However, it is unclear how such probabilities of probabilities are to be interpreted. If the inner probability (given in the proposition) is a natural, objective probability—such as the probability of a uranium atom decaying within a certain time period—this might make sense. But in many cases, since inner probability is a point value, which one would rarely ever hit, the external probability should be equal, or close, to zero—contrary to our expectations regarding the strength of justification of statements about natural probabilities. And the situation becomes even more problematic if the inner probability itself is only a subjective probability. Would this mean that one is unsure about one's own degree of belief? This, however, would be a question other than that of the probability of the skeleton proposition. – The purely technical problem underlying these interpretative difficulties is that *all* substantiated or believed theses have a degree of justification, while only part of the propositions contain a separable degree of probability.

Another problem with equating the justification strength of a proposition with its probability is this: usually assumed degrees of probability neither contain nor reflect much of the information relevant to the degree of justification and to the rational certainty of belief: 1. the *diligence*, and therefore the probability of error, in the execution of the reasoning (this holds for statements justified with certainty, e.g., deductively justified beliefs, as well as, e.g., for frequentistically justified probability statements); 2. for statistically justified probabilities, the *different strengths of the statistical procedures* (e.g., differently sized data bases, more or less sophisticated evaluation methods); 3. the *level of detail and thoroughness of the justification and the amount of the material used* (in practical arguments, for example, the number of

consequences and alternatives considered); 4. *uncertainty regarding the correct application of the reasoning procedure* (for example, in the case of difficult deductive justifications of necessary truths, which, if executed correctly, have a probability of 1, the error probability can nevertheless be greater than 0, the reasoning strength thus smaller than 1); 5. *metatheoretical uncertainty* regarding the value and rational permissibility of the type of reasoning itself. Now, different justifications of the same thesis may ascribe to it the same probability in spite of considerable differences in one or more of these five factors. This shows that these factors are not taken into account in probability assignments, though they considerably influence the strength of justification.

A further problem for the probabilistic approach to justification strength is this: if we measure quantitative empirical values (e.g., the present temperature at this measuring station, the mean income of the population, the hardness of a metal), the probability that the measured value corresponds to the actual value is often equal or close to zero, because the measured value is only more or less point-accurate. In such cases, the measured value should be regarded as the centre of a more or less broad interval in which lies the correct value. However, it usually makes more sense to determine an estimation function over the possible values, e.g., a normal distribution indicating how likely the actual value, according to the measurement, lies within certain intervals around the measured value. This estimation function expresses the precision of the measurement, i.e., how precisely the measured value represents reality. This function, in particular its standard deviation, is thus a measure of the justification strength of the measured value: the lower the standard deviation, the better the estimate and the justification strength. However, such an estimation function and its standard deviation are not probabilities.

Amongst many other problems for the probabilistic interpretation of justification strength is this: according to those interpretations of probability that are not completely subjectivist, necessary truths have a probability of 1. This is true, however, independently of the justification, owing to the semantic content of the proposition alone. The corresponding probability term is therefore one-adic. The concept of justification, however, is at least two-adic; that is, it refers to both a thesis and a corresponding justification, measuring how strongly the thesis is justified *by the justification* (see Pollock, 2010, p. 11). Probability therefore measures something besides the strength of justification.

3. STRENGTH OF JUSTIFICATION AS RATIONAL DEGREE OF CERTAINTY – FUNDAMENTAL ELEMENTS OF THE THEORY

The basic idea of an epistemological theory of justification strength is this: the strength of justification must cover everything that could cause a (putatively) validly justified thesis to nevertheless fall short of the truth. More precisely, justification strength is an index representing how well possible sources of error in the (supposedly) valid justification of a thesis were avoided, and hence how much one can trust the result, viz. the justified thesis; so, justification strength represents the thesis' estimated truthfulness. The natural expression of this kind of justification strength and trust in the thesis is the degree of certainty of the belief in the thesis. However, according to the epistemological approach, the subjective feeling of certainty cannot simply be taken as the measure of justification strength; instead it must be *rational* certainty, corresponding to an analytically determined degree of error exclusion. Otherwise, the most stupid and naïve person would have the most strongly justified beliefs and theses, while the rational sceptic, whose opinions are carefully examined and justified, would have the most weakly justified beliefs. Above all, justification strength should represent the actual degree of error exclusion. To this end, it must also be objective—otherwise different people, despite operating with the same justification and thesis, could have different degrees of certainty, which would also render the strength of justification subjectivist in the wrong way. To elaborate, *objectivity* here means, inter alia:

i. *Independence of persons:* The strength of justification must be based on objective factors, which are independent of the person, which can be replicated and which has only to do with the justification procedures used.
ii. *Objective factors:* It is necessary to specify—as far as possible—on what grounds the certainty is based. The objective elements that contribute to the strength of the justification must be specified, e.g., the nature of the procedures or of the data, etc...
iii. *Analyticity:* The strength of justification and subjective certainty is based on several factors such as diligence, the reasoning procedure. These factors, components or dimensions of subjective certainty must be assessed individually.
iv. *Foundation:* The specified factors contributing to the strength of justification must explain why the presumed certainty follows from them, or vice versa. That is, they must explain why, owing to their non-compliance or

diminished compliance, the risk of the justified thesis
falling short of the truth increases; so, they must justify
the degree of certainty.

The various ways in which (supposedly) valid justifications can
miss the truth of the thesis, or, vice versa, the ways in which valid
justifications can lead still better, more reliably or closer, to the truth
are objective factors influencing justification strength, and which can be
analytically differentiated. The *basic technical idea* of the theory of
justification strength developed here is: (i) to differentiate several basic
and mutually independent sources of errors (or, vice versa, to
differentiate possibilities of bringing closer to the truth) as dimensions
of justification strength, (ii) to determine the strength in the individual
dimensions, and (iii) to aggregate these individual strengths into a joint
index of justification strength. Such dimensions comprise:

    1. justification strength of the premises or the data used,
    2. truthfulness of the justification procedure,
    3. examination intensity and extensity,
    4. the yieldingness of the foundational material,
    5. correctness, fault-freeness in the application of the
    justification procedure,
    6. metatheoretical certainty about the justification procedure.

The justification strengths of several of the foregoing dimensions can be
determined by frequentist-probabilistic methods; other components of
justification strength, however, represent something quite different
from the frequency of failure, e.g., the degree of precision (a
subdimension of dimension 2, truthfulness of the justification
procedure); for others still, such as metatheoretical certainty (6), only
an estimate can be derived from consensus, theoretical penetration, and
historical stability. In analogy to other indices, which represent a
spectrum from total absence to completeness, the strengths of
justification in the individual dimensions should be expressed with
values from the interval [0; 1]. The total strength of justification then
corresponds to the product of the components' justification strengths.

Next, I give an overview of the aforementioned dimensions of
justification strength; in the following section some of them will be
explained in greater detail.

*1. Justification strength of the premises (only in case of inferential
justification):* Many justification procedures are inferential, sc. they
proceed from premises to a thesis. In particular, all argumentative
justification procedures are inferential. Observations, on the other hand,

are not inferential. The strength or weakness of the premises necessary for the justification is also reflected in the justification strength of the conclusion. To avoid running into vicious circles when determining justification strengths, one must begin with non-inferential justifications and only then proceed onto the justifications which use the non-inferentially justified propositions as premises etc.

2. Truthfulness of the justification procedure: A reliable justification procedure such as deduction, if applied correctly, leads from certain premises to a certain conclusion; the deductive justification procedure therefore has a truthfulness of 1. Uncertain justification procedures cannot guarantee this complete truthfulness.

3. Examination intensity and extensity: Some reasoning procedures can be applied more or less intensively, resulting in more or less consolidated statements. For example, observations (with or without instrumental assistance) can be more or less thorough, thereby influencing the probability of finding sought-after traits; in practical justifications different lengths of time can be devoted to identifying alternatives or possible consequences, thus the chances of finding relevant alternatives and consequences are increased and decreased, respectively. In the case of deductions, however, this differentiation does not exist.

4. Yieldingness of the justification material (for the thesis): In some justification procedures, the quality of the material known thus far and used as the starting point, or the material initially captured during the application of the cognizing procedure determines the strength or weakness of conclusions that can be drawn from it: Since the material—with respect to its (further) use in the justification procedures—is (not) particularly yielding, the result of the justification procedure's application is (not) particularly certain. Examples of different yieldingness of the justification material include: an observer could hardly decide whether object *a* was brighter than *b*; the difference in desirability between the best and second best options is so great that the optimality judgment is very certain; on the basis of the sonogram of the foetus, it is difficult to decide whether it is a girl or a boy.

5. Correctness (freedom from error) in the application of the reasoning procedure: Have all the steps of cognizing, as required by the justification procedure, been carried out carefully and correctly? Even a very good cognizing procedure can be applied superficially or sloppily: For example somebody does not observe exactly or in the right order; or "trivia" such as the exact place or time are ignored and only estimated; calculations or inferences are made incorrectly, etc.. The justification method can be completely correct, but if it is not applied correctly, the

result does not say much. The more complex the reasoning procedure, the more likely application errors are to occur. This, of course, also applies to certain justifications such as deduction.

6. Metatheoretical certainty about the justification procedure: Justification procedures are instruments whose reliability we estimate in dimension 2 (truthfulness of the procedure) and attempt to guarantee by metatheoretical justifications. But these justifications could also be wrong: Is the justification procedure all right? Is its reliability not lower than assumed? Often this metatheoretical degree of certainty of the justification procedure cannot be determined precisely. But there are obvious differences in metatheoretical certainty, for example between deduction and theoretically more controversial procedures, such as the use of significance levels or Bayesian updating as the standard foundation process.

What has just been described can be formulated more formally as follows. We need to define the quantitative term 'the justification strength of thesis t in virtue of justification j': $JS(t,j)$—where j is an inferential, e.g., argumentative, or a non-inferential cognitive justification process, such as an observation or an intuitive estimate. Let $JS_i(t,j)$ be the justification strength of t by the justification j in dimension i—with function values ranging from 0 to 1. The justification strength of thesis t in virtue of the justification j is then defined as the product of the justification strengths in all dimensions:

$$JS(t,j) := \Pi_i\, JS_i(t,j).$$

Determining justification strength according to the method just described is complicated and costly. In addition, individual components of reasoning strength are often difficult to determine, or can at best be estimated. For the intended application of the theory of justification, this is for the most part not a problem, since only a comparative determination of justification strengths of a pair of justifications with contradicting results must be undertaken. In such a pair comparison, one can ignore all the components of justification strength that are (about) equally strong in both justifications, and concentrate instead on the essential differences. Thus, one may arrive at useful practical results, even though information gaps exist and there are uncertainties with respect to considerable parts of the justification strength. This is also a general consolation: the theory of justification strength provided here is neither elegant, simple, nor beautiful. However, in practical cases of conflict between justifications, it can still be helpful.

How do we apply the foregoing conception of justification strength to the problem of justified theses contradicting one another—where one thesis is, therefore, a defeater of the other? Several cases have to be distinguished:

1. One of the justifications is not (argumentatively) valid or not adequate: If one of the justifications (especially arguments) is not (argumentatively) valid or situationally inadequate (on this terminology see footnote 1), whereas the other fulfils these conditions, then the valid and adequate justification always takes precedence.

2. An integration of the justifications to a comprehensive overall justification is possible: Sometimes (valid and adequate) justifications of competing theses can be integrated into a comprehensive overall justification. This is, as I mentioned in the introduction, e.g., often the case with regard to conflicting testimony, or if one's own observations contradict the statements of an expert, or if practical arguments are in contrast. The integration is then undertaken, e.g., through interpretive arguments (Lumer, 1990, pp. 224-246; Lumer, 1992; Lumer, 2010), where comprehensive hypothetical explanations for the contradictory statements are sought, and subsequently the probabilities of these hypothetical explanations are determined. The integration of competing practical arguments (Lumer, 2014) with (partly) different sets of options or consequences can be undertaken by constructing a third, comprehensive argument that takes up all of the options and all of the consequences considered in at least one of the two original arguments. (Examination intensity and extensity (dimension 3) are higher for the third argument than for the first two arguments.) Comparing justification strengths in order to eliminate the weaker justified thesis should only be made if such an integration into a comprehensive overall justification is not possible.

3. The justification strengths of the competing theses are clearly different: If the justifications are valid and adequate and cannot be integrated into an encompassing justification, and, moreover, if the justification strengths of the competing theses are clearly different, then the weaker justified thesis is to be rejected and the better justified thesis accepted.

4. The justification strengths of the competing theses are at least approximately equal: If the justifications are valid and adequate and cannot be integrated into an overall justification, and the justification strengths of the competing theses are equal or approximately equal, then, provided the topic is a are purely theoretical question, belief in the theses should be (temporarily) suspended. If, on the other hand, the questions are, or get to be, of practical importance for a practical

decision, then they should be treated like decisions under uncertainty. The rule most frequently used in decisions under uncertainty is the assignment of Pascal probabilities, viz. assigning the same probability to all of the mutually exclusive and together complete alternatives, hence in cases of two alternative theses assigning to each a probability of 0.5. This Pascalian solution is rationally required if you need the information for predictions in practical deliberations. To instead leave open the critical question in the decision, that is, to exclude it from the decision-theoretic calculus, often is tantamount to implicitly—and, of course, unjustifiedly—affirming one of the alternative theses. For example, if somebody plans a picnic and has equally good justifications for the predictions that it will and will not be raining at the scheduled time, then simply ignoring the question of rain means—unjustifiedly—deciding as if it will not rain.

4. SOME DIMENSIONS OF THE STRENGTH OF JUSTIFICATION

For reasons of space, here I can illustrate details of only some of the dimensions of justification strength.

2. Truthfulness of the justification process: Observation and *deduction* are procedures that, with correct application (and, in deduction, additionally with true premises) lead to true results. They are each completely truthful justification procedures. I assume that *probabilistic inferences* also have a validity of 1: They transfer with certainty the (limited) probability of the premises to the conclusion. Let us consider the following inference: '$P_0(h|e)=0.8$; $P_1(e)=1 \Rightarrow P_1(h)=0.8$'. This inference, unlike statistical projections, is not really ampliative, i.e., the informational content of the conclusion is no greater than that of the premises. Instead, correctly understood, the thesis only reports the tendency of the present information, contained in the premises, concerning the truth of *h*. (For this interpretation of probability, cf.: Lumer, 2011b.) The inference is certain on the condition that the data base contains no better information about the conclusion (see ibid.): the uncertain informational content of the premises is transferred to the conclusion. – Many other justification procedures, instead, are uncertain, and truthful only within limits; notwithstanding correct application and a certain data base, they do not always lead to correct results. A subdimension of truthfulness for, e.g., *inferentially statistical methods* is statistical validity, which could also be referred to as "projective-inferential validity" or "projective inference strength". It concerns whether the data obtained allow for a sufficient degree of certainty to make an inference to the population. One measure of

projective inference strength is, for example, the significance level *p* (or more precisely: the value 1-*p*). (By the way, this shows that the individual dimensions of justification strength may be captured by measures quite different from probabilities.) – A subdimension of truthfulness is *precision*. A measurement's degree of precision can be expressed as an interval around the measured value in which lies the actual value. A more precise specification of the precision degree of measurements is, however, the estimated variance of this type of measurement. Examples of precision in the argumentation types commonly discussed by argumentation theorists include the following phenomena: for all *probabilistic arguments*, the given probability values are only more or less precise; they must be interpreted as intervals. For *practical arguments* (Lumer, 2014), the precision of the assessment itself depends very much on the set and number of consequences taken into account. It also depends on the precision of the evaluation of these consequences, for example, whether, for the evaluation, the chains of consequences are followed up to intrinsically relevant consequences (to which clear criteria for intrinsic desirability can then be applied); or whether the whole chain's desirability is only estimated holistically. Appraisal of the consequences itself in practical arguments is already a premise (hence regards dimension 1 of justification strength), but how far and how ramified the examination of consequences is pursued is a matter of evaluation precision. In practical arguments which attempt to determine the best option the precision of the optimality judgment depends also on the number and quality of the options considered.

4. Yieldingness of the justification material (for the thesis): Several examples of the different yieldingness of the justification material for the justification of the thesis have already been mentioned above: fuzzy, distorted observations, disturbance by noise or unclean instrumental data. Some further examples include: *Evidence* as the basis of an interpretative argument may be more or less dense, i.e. inter alia more or less detailed. As the evidence becomes denser, the conclusion becomes more certain. *Radical uncertainty* in *practical arguments* means that one does not even know which relevant effects could occur (e.g., non-terrestrial bacteria or viruses, future changes in one's life philosophy); empirical knowledge, providing one basis for the evaluation, is therefore quite poor to this end.

5. EXAMPLES OF THE APPLICATION OF THE THEORY AND A CRITIQUE OF BAYESIANISM

Let us consider some examples to illustrate how the theory works.

Example 1: Contradiction between statements of experts of different quality: Jones, expert e_1, a local general practitioner, says:

> a_1: "u_1 is a very good urologist who, according to what I know from his doctor's letters and from my patients, recognizes and optimally treats almost all (98%) diseases that can be recognized and treated today; u_2 instead is a bad urologist who both fails to see and misdiagnoses many diseases (probability of success 80%)".

Smith, e_2, a friend, who is inclined towards alternative medicine, however, says:

> a_2: "u_1 is a bad urologist, who could not help many of my acquaintances (frequency of success 70%), many of whom were later completely satisfied with u_2 (frequency of success 90%)".

Both experts, e_1 and e_2, base their claims—'u_1 is clearly better than u_2' and 'u_1 is clearly inferior to u_2'—on empirical experience with u_1 and u_2. A reconstruction with an overall explanation regarding how the experts arrived at the indicated underlying relative frequencies is scarcely possible in this case: Could u_1 not heal only those patients for whom orthodox medicine so far had nothing to offer? Did the alternative-medicine-oriented patients not get along with u_1, while u_2 had the aura and suggestive power appealing to this patient group? Is e_1 perhaps friends with u_1 or e_2 with u_2, so that e_1 or e_2 misrepresents the truth? Is u_2 perhaps really a better doctor, but unpopular with the school doctors? These and many other possibilities can hardly be combined into a small set of useful general explanations, which can then also be assigned plausible a priori probabilities. In this case, however, one can compare the justification strengths of the arguments from expert opinion, namely the methodical genesis of the expert judgements about the success rates. e_1 relies on a relatively randomly-distributed sample, viz. his patients and medical letters obtained from u_1 and u_2; e_1 can also check success through the physicians' letters, which have the diagnoses listed and may be of quite different quality, the direct observation of his patients' healing processes, or his patients' reports thereof. e_2, on the other hand, builds upon a sample of presumably partially-biased subjects: his acquaintances, who are themselves likely to lean towards alternative medicine as well. Moreover, the sample of patients who have visited u_2 appears to consist of those who were thus dissatisfied with u_1—u_1 could not heal them because the chemistry between them and u_1

was not right, or because they expected and hoped for the use of different healing methods. Furthermore, the empirical material on which e_2's judgement is based seems primarily to be satisfaction judgments, rather than information about actual healing success. These characteristics of the justification of e_2's judgment lead to problems and a decrease in justification strength, at the least, on the dimension 'truthfulness', more precisely: on the (sub)dimension of validity: with respect to construct validity (is health actually captured or only general satisfaction?), internal validity (are the observed subjects neutral or perhaps biased?), and statistical validity (the sample of e_2 appears to be much smaller than that of e_1). The justification a_1 of e_1 is indeed not a methodically assured investigation, but a collection of experiences. Nevertheless, in the aforementioned respects it achieves a notably higher validity. According to the criterion proposed here, then, in this case the judgment of e_1 is better (more strongly) justified than that of e_2. Accordingly, the expert statement and justification of e_1 should be accepted, and those of e_2 rejected.

Example 2: Contradiction between two possibilities of probabilistic reasoning: Bayesianism against frequentism: In the second detailed example two probabilistic arguments from witness testimony are compared, in which the probability of the witnessed event is determined once frequentistically, and once following the Bayesian method. The testimony report and premise 1 of both arguments is: P1: 'An acquaintance, s, tells me, "Yesterday I saw a serious accident on the Autopalio (i.e. the four-lane highway between Siena and Florence) with casualties, ambulances, etc., at the exit to Siena"'. The following abbreviations are used below:

h is the hypothesis: 'Yesterday there was a serious accident on the Autopalio at the exit to Siena.'
e is the event described in P1, that is, s's witness statement that she observed h.

Basic probability establishing argument from testimony: a_1:
'P1: s says she has observed h.
P2: s is a reliable person whose statements about the lifeworld are true in most cases (90%).
P3: h is about the lifeworld.
(*P4:* No better information (NBI): I have no better information about s and h as expressed in premises P1 to P3.)
(*P5:* Foundation principle for basic probabilities: If x% of the Fs are E, and some y is F, and the data base d contains no

better information about y's possibly being E, then the probability that y is E on the data base d is x%.)
Therefore:
T1: $P(h)=0.9$.'

Bayesian argument from testimony: a_2: The following probabilities are taken from a typical response among responses given by my acquaintances and students on a test:
'*P1: e*: s says she has observed h.
P6: The a priori probability of h before e is 0.1. ($P_0(h)=0.1$.).
P7: The probability that s tells me she had seen a serious accident on the Autopalio at the exit to Siena, when there really was this accident is 0.9. ($P_0(e|h)=0.9$.)
P8: The probability that s tells me she had seen a serious accident on the Autopalio at the exit to Siena, although there was in fact no such accident is 0.1. ($P_0(e|\neg h)=0.1$).
(*P9:* Bayes' rule: $P(h|e) = (P(e|h){\cdot}P(h))$ / $(P(e|h){\cdot}P(h)+P(e|\neg h){\cdot}P(\neg h))$.)
From this it follows by insertion:
L: $P_0(h|e) = 0.9{\cdot}0.1$ / $(0.9{\cdot}0.1+0.1{\cdot}(1-0.1)) = 0.5$.
Because of P1 with $P_1(e) = 1$ it follows via Bayesian updating:
T2: $P_1(h)=0.5$.'

The arguments are (for the time being) argumentatively valid. However, while they proceed from the same assertion of the witness, they arrive at contradictory theses. It is not possible to integrate the two arguments into a comprehensive overall argument because the probability assumptions are incoherent: to arrive at T2 with an argument such as a_1, the reliability assumption in P2 would have to drop to 50%. Therefore, in order to arrive at a dissolution of the argumentative contradiction, the justification strengths of the two arguments must be compared. The uncertainties of both arguments in dimensions 2 to 6 are roughly comparable. However, the justification strengths of the premises are very different. P2, P6, P7, and P8 are based on estimates—all other premises are without problem. Beliefs about P2 (the premise about the reliability of s) can emerge as impressions during encounters with s; who has a functioning sense for numbers can estimate this reliability with useful preciseness, perhaps in a confidence interval of 0.2. By contrast, P6, P7 and P8 (premises of a_2) presuppose, for a useful estimate, theoretical considerations and an abundance of further assumptions, in part extremely complicated assumptions. As a result, the error probability here is very high and precision extremely low. Correspondingly, all probability estimates in P6, P7, and P8 are grotesquely incorrect. *Ad P6:* An expressway exit where a major

accident occurred in the course of a random day with a probability of 0.1 (i.e., in 10 days with a probability of 0.6126) would be closed immediately in a civilized country. A useful probability value can be calculated as follows: according to a rather high estimate, a severe accident happens on the Autopalio perhaps every ten days; that one happened yesterday therefore has a probability of 1/10; that it happened exactly at the exit to Siena (the total length of the Autopalio being 60 km but with increased risks at the exits) may therefore have a probability of, maximally, 1/100. Thus one arrives at an overall probability estimate of 1/1000, i.e., $P_0'(h)=0.001$. The estimation error in P6 with $P_0(h)=0.1$ is thereby in two orders of magnitude. With similar, but considerably longer, considerations (that I cannot expound here for reasons of space) one arrives at the conclusion that P7 perhaps exceeds the true value by a factor of 200, and P8 by a factor of 200,000! Thus, according to these considerations, the precision of the premises of argumentation a_2 is extremely low. Indeed, it is considerably lower than that of the premises of argument a_1. Given the same justification strengths of a_1 and a_2 in all other dimensions, the premise precision and therefore the justification strength of the premises is decisive. The conclusion is: In the present example, according to the approach proposed here, from the two competing arguments a_1 and a_2, the Bayesian argument from witness testimony, a_2, with its thesis, T2, should be rejected, and the statistical argument from witness testimony, a_1, with its thesis, T1, should be accepted.

This last example illustrates a general weakness of Bayesianism. Of course, the Bayesian calculus as such is correct: Bayes' rule follows from the axioms of probability theory. But the fundamental idea of Bayesianism is to justify probabilities exclusively by Bayesian updating. One is thereby compelled to reason from specific kinds of premises, viz. the a priori probability of the hypothesis h ($P_0(h)$), and the conditional probabilities $P_0(e|h)$ and, e.g., $P_0(e|\neg h)$. If one has no opinion at all regarding these premises or, if one has an opinion, but does not have a justification for it, then, owing to Bayesian updating one is obliged to draw on respective estimates—that are often quite crude and quantitatively absurd. Apart from the fact that Bayesianism does not require Bayesian updating for the justification of an observation e itself—and thus recognizes the existence of other kinds of justifications—, the constraint to use particular premises and to always update in the Bayesian way is in contrast, first, to the variety of forms of rational justifications, and, second, to a fundamental tenet of the epistemological approach to argumentation theory: to build on rationally-founded premises. Moreover, in everyday life, of course, there

exists the notorious problem that the Bayesian calculations are too complicated.

6. CONCLUSION

In this article, I have presented some basic principles of a new theory of justification strength. The basic idea of this theory originates in the epistemological approach to argumentation theory: Rational justification consists in examining the truth or acceptability of a thesis by means of truth criteria or—epistemologically justified—acceptability criteria for this thesis. Such justifications can fail on account of various errors and problems. Strength of justification is an index that is intended to measure the exclusion of such errors, and thereby approximation to the truth. In the lifeworld, this corresponds to the—rational—degree of certainty about one's opinions. The systematically next steps in developing the theory consisted in: identifying and illuminating such error possibilities, which correspond to the dimensions of justification strength; presenting the first approaches to determine justification strength in these individual dimensions; and, lastly, defining an index which aggregates the justification strengths in the foregoing dimensions to the total justification strength. The examples given at the end of this article have shown that this theory can be fruitfully applied. What is still missing is a further elaboration of this theory.

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