UNIVERSITÀ DI SIENA
1240

QUADERNI DEL DIPARTIMENTO DI ECONOMIA POLITICA E STATISTICA

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n. 737 – Ottobre 2016

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October 21, 2016

Abstract

This paper complements the Cournot collaboration game outlined in Goyal and Joshi (2003, sect. 4), with the hypothesis that pairwise R&D alliance is constrained by knowledge distance. Potential asymmetry of distance between two knowledge sets is formalized through a quasi-metric in knowldge space. If the knowledge constraints to collaboration are weak enough, the paper replicates the result by Goyal and Joshi (2003, sect. 4), that a firm is either isolated, or is connected to every other non-isolated firm in the industry. If absoprtion of ideas from one's potential partner requires sufficiently high knowledge proximity, the stable R&D networks in Cournot oligopoly are shown to display the clustering property, that is characteristic of real-world industry networks, and of social networks more generally.

JEL classification: D85, L13, O30

Keywords: Cournot collaboration game, directed knowledge distance, R&D networks, degree assortativity, clustering

1 Introduction

R&D collaboration between firms has been growing dramatically in the last decades of the twentieth century. A large body of empirical evidence documents the diffusion of R&D alliances, especially in the high-tech industries, (pharmaceuticals, ICT, aerospace and defence) where product innovation bears close roots in abstract and codifiable scientific knowledge (Powell et al. 2005, Roijakkers and Hagedoorn 2006). Diffusion of R&D networks has been also significant in a number of medium-tech sectors, like instrumentation and medical equipment, chemicals, automotive, consumer electronics (Hagedoorn 1982,

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Schilling and Phelps 2007, Tomasello et al. 2013).[1] The target of R&D alliances was not circumscribed to product innovation. To the extent that the development of cost effective processes was often a key to business success, process innovation could not be disregarded, even where product innovation seemed to play a prominent role (Pisano 1997). The partial decay of R&D networks after 1998 (Gulati et al. 2012) marks a new phase of what Gilsing and Noteboom (2006) interpret as a cyclical evolution, and posits yet another challenge to economic understanding.

This paper bridges different strands of the theoretical literature on R&D network formation. The knowledge portfolio foundation of R&D collaboration and knowledge spillovers (Cowan and Jordan 2009, Baum et al. 2010, Caminati 2016) is married with the hypothesis that firms, after cooperating in R&D, compete in a homogeneous-product market. As in d'Aspremont and Jacquemin (1988), Salant and Shaffer (1998), Kamien and Zang (2000), Goyal and Joshi (2003), and many others, it will be assumed that firms engage in Cournot competition. On the assumption that the potential gains from an R&D partnership are only backed by a fixed collaboration cost, a variety of stable network topologies may be produced, depending on the knowledge constraints to collaboration. In this perspective, Goyal and Joshi's (2003) dominant group architecture, in which every non-isolated firm in the network cooperates with every other similar firm, is a particular outcome, supported by sufficiently weak knowledge requirements. The paper identifies conditions such that the stable network topologies, in spite of their variety, share a feature, differentiating social networks from other types of networks (Newman and Park 2003).

If a large common understanding is necessary to absorb ideas form an R&D partner, two R&D partners of the same firm will be most likley closer in knowledge space, than if they were picked up at random from the set of non isolated firms in the industry. This produces a higher local clustering of R&D links, than it would be observed in a random network of a corresponding size and average degree.

The rest of the paper is organised as follows. In section 2, the theoretical background of the analysis is discussed, in the light of the relevant contributions to the literature. Section 3 introduces the formal definition of directed distance in knowledge space, which is then used to provide a foundation to the notion that the potential knowldge gain from R&D-collaboration may be asymmetric. The collaboration game played by oligopolistic firms is introduced in section 4, together with the main propositions. Results are discussed in section 5, and section 6 concludes.

[1] Around 1980, the fraction of R&D alliances in high-tech and medium-tech industries were not dissimilar. After 1980, the latter declined to the advantage of the former (Hagedoorn, 2002, p. 482).

2 Theoretical background

A common backbone of diverse models of R&D-network formation, is that the sharing of heterogeneous, complementary R&D inputs between direct partners produces a potential pay-off, through the effects on product or process innovation. Firm profit is then functionally related to the firm's alliance network. A plausible rationality requirement states that (i) a bilateral R&D partnership is agreed upon only if the alternative course of action does not increase a subscriber's profit-flow; (ii) two firms do not abstain from joining a partnership, if their profits are both higher after link formation. The two conditions above are embodied in the notion of pairwise network stability (Jackson and Wolinsky 1996); this is strengthened by Goyal and Joshi (2003) with the further requirement that a firm cannot increase its profit by severing all its links at once.

A branch of the literature assumes that firms produce a homogeneous product, and R&D aims at process, cost-reducing, innovations. Following a tradition initiated with d'Aspremont and Jacquemin (1988), a number of papers restricts the analysis to a Cournot duopoly, in which a firm's marginal cost is a function of the variable R&D investments carried out by the firm, and its potential partner[2]. Goyal and Joshi (2003) extend the analysis to a oligopolistc industry with a variable number n of firms, on the simplifying assumption that the marginal cost reduction from an R&D alliance is fixed and uniform across all potential partners, and is backed by a fixed collaboration cost. A firm's choice to form (or confirm) an allaince depends on the way in which the state of competition in the market for output is related to the possibly complex topology of the collaboration links within the network. Goyal and Joshi (2003) show that if the firms in the industry engage in Cournot competition, a form of 'increasing returns' to neighbor selection applies. On the assumption that a firm i in the industry produces its Cournot output $q_i(g)$, conditional on the network g of alliances, i's profit gain, from adding a link ih to its alliances in g, is lower than the gain from adding a second link ij to its alliances in $(g + ih)$[3]. With costly collaboration, this form of increasing returns to link formation implies that isolated and non-isolated firms may be simultaneously present in a stable network of the industry; but a firm is either isolated, or has an R&D link with any other non-isolated firm in the network. In other words, a stable network has at most one non-trivial component, and the firms in this components are fully connected among them. We may therefore conclude that quantity competition, in the framework of Gayal and Joshi (2003), implies that every highy-degree firm[4], if there is any, is invariably linked to other similar firms. Notice that this is a peculiar form of positive assortativity by degree.

In a second class of models (Baum et al. 2010, Cowan and Jonard 2009, Eg-

[2] d'Aspremont and Jacquemin (1988) focus on Cournot equilibria with cooperative symmetric R&D investment of the two potential partners. Other contributions in the same vein are reviewed in De Bondt (1996). Salant and Shaffer (1998) introduce the possibility of asymmetric R&D investment by the partners.

[3] $(g + ih)$ is the network obtained by adding the link ih, joining firms i and h, to the set of alliances in g.

[4] The degree of a network member is its number of links in the network.

betokun and Savin 2014, Caminati 2016) firms produce a heterogeneous product, and competition in the market for output is abstracted from. The resources shared between R&D partners bear more explicitly, in this case, upon the cognitive competences grounded in the firms knowledge portfolios. The relevance of this interpretation finds empirical corroboration in Mowery et al. (1998).

The foundations of the knowledge-portfolio approach are traceable in the seminal intuitions elaborated in Nooteboom (2000), Nooteboom et al. (2007), Gilsing et al. (2008). They relate the effectiveness of firm's interaction with a potential R&D partner to the novel ideas potentially accessible in the partner's knowledge base, and the capability to absorb (Cohen and Levinthal 1990)) the novelty in question. On the one hand, absorptive capacity is conditional upon a background of common understanding, hence it is frustrated when the sharing of ideas between partners is too ephimeral (novelty is too large). On the other, interaction does not lead to any substantial gain in competence or creativity, if cognitive distance, hence novelty, is too low. Building upon this broad intuition, Nooteboom (2000) goes as far as suggesting the notion of an 'optimal cognitive distance', stemming from a relation between effectiveness of knowledge interaction and cognitive distance, that is described by an inverted U curve[5].

The topology of pairwise stable networks produced by the knowledge portfolio models are bound to depend on the assumed distribution of knowledge. There is however agreement on the finding that R&D alliances tend to organize into clusters, broadly corresponding to knowledge communities, and, on average, the number of pairwise alliances in the set of a firm's direct partners is larger than it would be expected, if the wiring of connections was random[6] (Baum et al. 2010, Caminati 2016).

A third class of models (Kamien and Zang 2000, D'Agata and Santangelo 2003) assumes Cournot competition in the market for output, but contrary to Goyal and Joshi (2003) and d'Aspremont and Jacquemin (1988), the marginal-cost reduction from alliance formation is not independent of the partners specialized knowledge. The idea is that the knowledge spillovers between two R&D partners depend on the congruence of their R&D approaches (Kamien and Zang 2000), or the distance between their respective technological profiles[7] (D'Agata and Santangelo 2003). In these models, the R&D approach, or the technology profiles, are not inherited from history, but are endogenous choice variables, that are fixed before alliance formation. D'Agata and Santangelo (2003) introduce Granstrand's (1994) (symmetric) distance on the σ algebra of the subsets of a knowledge space. Relying upon Nooteboom (2000), they provide a foundation for symmetric knowledge spillovers between two R&D partners, as a non-linear

[5]Wuyts et al. (2006) find evidence of a inverted U relationship between firm innovation success and cognitive distance with respect to R&D partners. Some empirical proxy-measures of cognitive distance (Nooteboom 2000, p. 301, Nooteboom et al. 2007) are closer to a notion of knowledge overlap.

[6]In other words, the clustering coefficient is larger than in a random network of a corresponding size and average degree. Further restrictions (concerning degree assortativity) are produced by sufficient variation in the size of network clusters (Caminati 2016).

[7]A technological profile defines the area of a firm's knowledge specialization.

function of the cognitive distance between them.

The presents paper bridges the above strands of the literature, while departing in one, or more, respects from each. In the short run, a firm's initial knowledge base is inherited from history, hence, it is exogenously fixed by a given cross-firm knowledge distribution $A = \{A_1, ..., A_n\}$. The pairwise interaction between potential R&D partners yields asymmetric spillover opportunities, that depend on the directed (non-symmetric) cognitive distances between the partners. This requires introducing a quasi-metric in knowledge space. Collaboration will materialize only if the spillover opportunities of both partners are non-negligible.

As in Goyal and Joshi (2003, section 4), we assume that the n firms in a oligopolistic industry play a two-stage Cournot-collaboration game[8]. In the first stage, an R&D network g is formed, giving rise to the knowledge spillovers that are elicited by the exogenous knowledge distribution A. In the second stage, conditional on the alliances in g, the n firms compete over quantities in the homogeneous product market. In this framework, network stability is a temporary-equilbrium notion, conditional on a given A distribution at time t. An A-stable network g meets the two conditions (i) and (ii) defining Jackson and Wolinsky (1996) pairwise-stable networks, and the further restriction that a firm does not have a strict incentive to sever all its links at once (Goyal and Joshi 2003).

The results below show that a pairwise R&D alliance will form, only if the directed distances between the partners are not too large or small. This implies that, contrary to Goyal and Joshi (2003), a non-empty A-stable network may embed more than one non-trivial component. In addition, an empirical property of social networks is recovered in this framework. If a sufficiently large common understanding is necessary to make an R&D collaboration viable, the clustering coefficient in a stable network is larger than it would be if the wiring of connections between the firms in the network was random. To the extent that a large common understanding signals membership in a knowledge community, the result is reminiscent of the argument by Newman and Park (2003), explaining the high local clusering of social networks, on the ground that human agents are typically organized into social communities.

In the long-run, the knowledge distribution among the n firms in the industry is endogenous, as a result of the idea spillovers between partners, and of the innovations produced by R&D. It is argued that the focus on asymmetric directed knowledge distances bears implications for network evolution. Brief euristic remarks on this point are offered in section 5, as a suggestion for further work.

[8] In Goyal and Joshi (2003) other forms of imperfect competition are also considered.

3 Cognitive distance and knowledge spillovers

3.1 Directed distance in knowledge space

What follows avoids taking a theoretical stance on the issues concerning what human knowledge is and how it works.[9] Following in the footsteps of Granstrand (1994), our analysis is built on the main parsimonious assumptions that: (*i*) there is a meaningful set-theoretical description of a knowledge base as a subset of a knowledge universe Ω, and (*ii*) the description obeys to the mathematical operations of set union, intersection and complementation. On this premise, we introduce a quasi-metric on a σ-algebra of subsets of Ω, that defines the *directed distance* $d(A, B)$ of a knowledge base A relative to a knowledge base B, with the property $d(A, B) \neq d(B, A)$.

Unlike the knowledge representation most prevalent in game theory[10], the items composing the knowledge universe Ω are not possible occurrences, that may or may not obtain. It will be assumed that the items in Ω are all 'true', in that each of them identifies an idea accepted by (a subset of) the scientific and egineering community at the given date t. It is worth observing in this respect that scientific ideas are often accepted as useful items of knowledge, long before a proof is available of their consistency with, and position in the hierarchy of, the other idas in Ω[11]. With this motivation, we resort to a knowledge representation avoiding any committment concerning the hierarchy/consistency of the ideas in Ω.

Knowledge heterogeneity among the firms in the same industry may reflect the diversity of their research programs, of their category representations[12] of the same real-world objects, or, more generally, of their histories. The sources of knowledge heterogeneity are neglected, in what follows. The initial knowledge distribution results from the random assignment of any idea in the industry knowledge base Ω to the firms in the industry. Contrary to Olsson (2000), Ω is defined at a given date t and does not include the ideas that have yet to be discovered at t.

Let Ψ be a σ-algebra of subsets of Ω, that is, a collection of subsets of Ω, containing the empty-set $\varnothing$, and closed with respect to the formation of complements and of countable unions and intersections. In particular, we have that $\Omega \in \Psi$. The couple (Ω, Ψ) defines a measurable space and we assume the existence of a finite measure μ on (Ω, Ψ)[13]. As a trivial example, consider the

[9] Nooteboom (2000) argues that such issues are highly consequential to the theory of organizational learning.

[10] The information of an individual about real world occurrences is there described as a partition on the set of the possible states, or histories, of the world (Aumann 1999).

[11] The history of the reduction of heath theory to statistical mechanics is a classic example (Nagel 1979, pp. 338-345).

[12] Representations may be more abstract or concrete (Boland et al 2001).

[13] μ is a non-negative real valued set function $\mu : \Psi \to [0, \infty)$ such that $\mu(\varnothing) = 0$, $\mu(\Omega) < \infty$, and for $E_k \in \Psi$, any countable disjoint collection $\{E_k\}_{k=1}^{\infty}$ satisfies the condition

$$\mu\left(\cup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mu(E_k).$$

case in which Ω is a finite set, and for any $E \in \Psi$, define $\mu(E) = \#E$.

A metric on the set Ψ is a non-negative distance function $d : \Psi \times \Psi \to [0, \infty)$ such that for any triplet of subsets $A, B, C \in \Psi$ the following properties hold:

1. $d(A, A) = 0$ identity recognition
2. $d(A, B) = 0$ only if $A = B$ identity of indiscernibles
3. $d(A, B) = d(B, A)$ symmetry
4. $d(A, C) \leq d(A, B) + d(B, C)$ triangle inequality

If the non-negative distance function d meets 1, 3, 4, but 2 may fail, it is called a pseudo-metric on Ψ. If it meets 1, 2, 4, but the symmetry condition 3 may fail, it is a quasi-metric (Wilson 1931). Finally, the function d is a pseudo-quasi-metric on Ψ, if it meets 1 and 4 above[14].

Let $N = \{1, ..., n\}$ be a finite set of firms; $A = \{A_i\}_{i \in N}$ is a knowledge distribution on N; A_i is the knowledge base of firm $i \in N$, and $\cup_i A_i = \Omega$ is the knowledge base of the industry. The distance between subsets of Ω is the metric $d : \Psi \times \Psi \to [0, 1]$ defined by (Granstrand 1994):

$$d(A_i, A_j) = \frac{\mu(A_i \cup A_j) - \mu(A_i \cap A_j)}{\mu(A_i \cup A_j)} \text{ for } A_i \neq A_j,$$

and $d(A_i, A_j) = 0$ otherwise.

The undirected cognitive distance between firms i, $j \in N$ is the pseudo-distance $dk : \mathcal{N} \times \mathcal{N} \to [0, 1]$ defined by $dk(i, j) = d(A_i, A_j)$.

For the sake of later reference we introduce the following simplifying notation. The measure of the set of ideas that are known to i, but not to j, is the novelty $n_{ij} = \mu(A_i - A_i \cap A_j)$. The knowledge overlap between i and j is $\lambda_{ij} = \mu(A_i \cap A_j)$.

Definition 1 The *directed distance* of a knowledge set A_i relative to a knowledge set A_j is the quasi-metric $dd : \Psi \times \Psi \to [0, 1]$ defined by:

$$dd(A_i, A_j) = \frac{\mu(A_i - A_i \cap A_j)}{\mu(A_i \cup A_j)}. \tag{1}$$

Because dd trivially meets identity recognition and identity of indiscernibles, only the triangle inequality remains to be ascertained. This is proved in the appendix.

Notice that $dd(A_i, A_j) + dd(A_j, A_i) = d(A_i, A_j)$ is a metric on Ψ. As a striking motivation for introducing a directed distance in the σ-algebra of subsets of the universal knowledge set Ω, consider the case in which $A_i \subset A_j \neq \varnothing$, where

Cf. Royden and Fitzpatrik (2010), pp. 337-340.

[14] See Künzi (1992, 2001). A less than fully settled nomenclature may produce occasional differences in definition. See, for instance, Basili and Vannucci (2013).

$\subset$ is strict set inclusion. In this case, $A_i \cap A_j = A_i$, and $dd(A_i, A_j) = 0$, whereas $dd(A_j, A_i) > 0$.

Definition 2 The directed cognitive distance of firm i relative to firm j is the pseudo-quasi-metric $\delta : N \times N \to [0,1]$, defined by $\delta(i,j) = dd(A_i, A_j) = \delta_{ij}$.

Straightforward computation reveals that for i, $j \in N$, $\delta_{ii} = 0$, and $\delta_{ij} + \delta_{ji}$ is the pseudo cognitive distance between i and j.

$$0 \leq \delta_{ij} + \delta_{ji} = dk(i,j) \leq 1 \tag{2}$$

3.2 Knowledge spillovers

Two firms i, $j \in N$ are prepared to share their knowledge if and only if they form an R&D alliance. The R&D relation between them is described by the binary variable $ij \in \{0,1\}$, with the interpretation that $ij = 1$ if the firms are linked by an R&D partnership, and $ij = 0$ otherwise. The set of pairwise relationships between firms is represented by the network $g = \{(ij)_{i,\ j \in N}\}$, $ii(g) = 0$, $\forall i \in g$. With some abuse of language, we say that the firms i, $j \in N$ are nodes of g. In this section, a given distribution of knowledge $A = \{A_i\}_{i \in N}$ is fixed, to define the opportunity S_{ij} of a knowledge spillover from firm j to firm i. Spillover opportunity S_{ij} is defined by

$$S_{ij} = \delta_{ji} \cdot [\max(0, \theta - \delta_{ij} - \delta_{ji})].$$

Recalling that

$$(\delta_{ij} + \delta_{ji}) = dk(i,j) = 1 - \frac{\lambda_{ij}}{\mu(A_i \cup A_j)},$$

firm i has a positive opportunity to draw a knowledge gain from its partner j, only if novelty $n_{ji} > 0$, and the ratio of knowledge overlap λ_{ij} to joint knowledge $\mu(A_i \cup A_j)$ is larger than $1 - \theta > 0$. Here $\theta \in (0,1)$ is a maximum threshold beyond which excessive undirected knowledge distance (low knowledge proximity) frustrates knowledge interaction. [15] The value of θ reflects the nature of the R&D projects shared by firms, collaboration on more incremental projects requiring higher proximity of the competence bases, hence a lower θ.

In this paper we focus on the qualitative nature of the conditions enabling, or forbidding, effective knwledge interaction between partners, avoiding any more demanding discussion on spillover size.[16] To this end, we introduce the simplifying assumption that, provided the conditions for effective knowledge absorption from an R&D partner are fulfilled, the directed flow of ideas materializing in a

[15] The statement is reminiscent of, but weaker than, Nooteboom's (2000) notion of an inverse-U shaped relation between cognitive distance and effectiveness of knowledge interaction. Moreover, we do nat have here a 'optimal cognitive distance' in Nooteboom's sense.

[16] A discussion of this kind would itsef require a deeper understanding and discussion of knowledge measurement. D'Agata and Santangelo (2003) define the undireted knowledge spillover between two R&D partners i and j, in terms of the undirected knowledge distance $dk(i,j)$ between them.

unit time interval is fixed. Given a network g, and knowledge distribution A, firm i can absorb ideas from firm j if and only if $ij(g) = 1$, and the spillover opportuinity S_{ij} is larger than a minimum threshold τ. The size of τ is related to the indivisibility od ideas. More formally, we assume that the directed spillover $s_{ij}(g)$ from j to i is a binary variable $s_{ij}(g) \in \{0, 1\}$:

$$\forall i, j \in N, \;\; s_{ij}(g) = 1, \text{ if } \; ij(g) = 1, \text{ and } S_{ij} > \tau, \tag{3}$$

$$s_{ij}(g) = 0 \text{ otherwise} \tag{4}$$

It is worth stressing that the knowledge incentives to collaboration may well be asymmetric, if network g is arbitrary, as it may be the case that $s_{ij}(g) = 1$, but $s_{ij}(g) = 0$, or vice versa. The knowledge-portfolio literature mostly overlooks the distinction between directed and undirected knowledge distance, and the potential asymmetry of the knowledge incentives to collaboration (Cowan and Jonard 2009, Egbetokun and Savin 2014, D'Agata and Santangelo 2003. An exception is Caminati 2016). The distinction is redundant only if the knowledge endowments are of equal measure, $\mu(A_i) = \mu(A_j)$, $i, j \in N$, with the implication that the directed knowledge distances, and collaboration incentives, are symmetric.

4 Collaboration in Cournot oligopoly

The foundation of knowledge interaction on directed cognitive distance is used in this section to build a collaboration game played by oligopolistic firms. The game is otherwise reminishent of Gojal and Joshi (2003). Any two firms in the set $N = \{1, ..., n\}$ have the opportunity to collaborate on a shared R&D project. For every research link a firm incurs a fixed collaboration cost $f > 0$, and obtains the potential access to the partner's private knowledge, which is relevant to the project in question. After collaboration, if any, the n firms engage in Cournot competition in the market for the homogeneous product.

To the reader's convenience, we recall some standard definitions on networks (Gojal and Joshi 2003), and the related notation used in the sequel. The *empty network* g^e is defined by $ij(g^e) = 0, \forall i, j \in N$. The undirected network $g + ij$ is obtained by replacing $ij(g) = 0$ with $ij(g) = 1$. $g - ij$ is the undirected network obtained by replacing $ij(g) = 1$ with $ij(g) = 0$. g_{-i} is the network obtained by severing i's links in g, replacing any $ij(g) = 1$ with $ij(g) = 0$. If i is a isolated node of g, $g_{-i} = g$.

A *path* of length m in g, connecting firms i and j, is a set of m distinct firms $\{i_1, ..., i_m\}$ such that $ii_1 = i_1 i_2 = ... i_{m-1} i_m = i_m j = 1$. Two *neighbors* i and j are connected by a path of length zero, that is, $ij = 1$. A network $g' \subseteq g$ is a *component* of g if: (i) for all $i, j \in g'$, with $i \neq j$, there is a path in g' connecting i and j; (ii) for all $i \in g'$ and $j \in g$, $ij(g) = 1$ implies $j \in g'$. A network g is *connected* if it has a unique component g', and $g' = g$. The complete network g^c is defined by $ij(g^c) = 1, \forall i, j \in N$.

In this paper we restrict our attention to costly collaboration. On the assumption that $f > 0$, firms i and j are not prepared to form an R&D alliance unless they both have the opportunity to earn a positive spillover from their partner; if this is the case, we say that the link $ij(g) = 1$ is viable.

Definition 3. For any network g, and firms i, $j \in N$, a link $ij(g) = 1$ is viable if and only if $S_{ij} > \tau$, and $S_{ji} > \tau$, hence ony if $(\delta_{ij} + \delta_{ji}) \cdot [\theta - (\delta_{ij} + \delta_{ji})] > 2\tau$. Using (3) above, any viable link $ij(g) = 1$ is such that $s_{ij}(g) = ij(g) = s_{ji}(g)$.

Firm i process innovations resuts from the number of its viable collaborations with the other firms; this is

$$s_i(g) = \sum_{j \neq i} s_{ij}(g). \tag{5}$$

Firms face a constant returns to scale production technology with marginal production cost

$$c_i(g) = \gamma_0 - \gamma s_i(g) \tag{6}$$

where $\gamma > 0$, and γ_0 is the marginal cost of any isolated firm in g. In particular, $c_i(g^e) = \gamma_0$. The total collaboration cost borne by firm i is $f \cdot \eta_i(g)$, where $\eta_i(g) = \sum_{j \neq i} ij(g)$ is the number of i's network partners. Notice that $\eta_i(g) \geq s_i(g)$, and strict inequality holds if, and only if, there are i's links in g that are not viable.

Assumption A1. The knowledge distribution $A = \{A_i\}_{i \in N}$ is obtained from the random assignment to each $i \in N$ of any idea $a \in \Omega$, with uniform probability $p(a) = 0.5$. On the understanding that $\mu(\Omega)$ and n are large enough, a representative A distribution is assumed such that each pair of firms in N have a non-vanishing knowledge overlap, and each pair member has some knowledge that is unknown to the other:

$$\delta_{ij} > 0, \text{ and } \quad dk(i,j) < 1, \forall i, j \in N, i \neq j. \tag{7}$$

The expected directed and undirected knowledge distance between two firms in the industry are defined by:

$$E(\delta_{ij})_{i \neq j} = \frac{1}{n} \sum_i \left(\frac{1}{n-1} \sum_j \delta_{ij} \right); \qquad E(dk(i,j))_{i \neq j} = E(\delta_{ij} + \delta_{ji})_{i \neq j} = 2E(\delta_{ij})_{i \neq j}. \tag{8}$$

Given a knowledge distribution A, the n firms form collaborations in the first stage of the game and engage in quantity Cournot competition on the homogeneous-product market, in the second stage. The linear inverse market-demand function is

$$p = \alpha - \sum_{i \in N} q_i, \quad \alpha > 0$$

The gross profit of firm i induced by g is

$$\pi_i(g) = p \cdot q_i(g) - c_i(g)q_i(g), \tag{9}$$

and net profit is $\Pi_i(g) = \pi_i(g) - f \cdot \eta_i(g)$. The standard solution procedure starts from the second stage, and finds the unique product-market equilibrium resulting from g, and the induced marginal cost vector $c(g) = \{c_1(g), ..., c_n(g)\}$.

The incentives to form new links, and to delate or preserve existing ones, depend on the knowledge spillovers triggered by the directed knowledge distances between partners, and their interaction with Cournot output. A network g is A-stable if the stability condition of Jackson and Wolinsky (1996), and the additional global-check condition of Goyal and Joshi (2003, p. 73) are satisfied: the former requires that any two partners do not have a strict incentive to sever their link, and any two firms, that are not members of a partnership, do not have a strict incentive to form one; the latter requires that a firm cannot increase its net profit by severing all its links at once. More formally:

Definition 4. Given a knowledge allocation A, an A-stable network g meets the following conditions.
(i) If $g_{ij} = g_{ji} = 1$, then: $\pi_i(g) - \pi_i(g - ij) \geq f$, and $\pi_j(g) - \pi_j(g - ji) \geq f$.
(ii) If $g_{ij} = g_{ji} = 0$, then $\pi_i(g+ij) - \pi_i(g) > f$ implies $\pi_j(g+ji) - \pi_j(g) < f$.
(iii) $\pi_i(g) - f \cdot \eta_i(g) \geq \pi_i(g_{-i}), \forall i \in N$.

In the second-stage game each Cournot ologopolist i formulates a prediction on $q_{j \neq i}$ and maximizes profit (9), subject to the prediction. Given a network g, the Cournot equibrium output of the game is

$$q_i(g) = \frac{\alpha - \gamma_0 + n\gamma \cdot s_i(g) - \gamma \sum_{j \neq i} s_j(g)}{n+1}. \tag{10}$$

In particular, the Cournot output in the empty network is uniform, $q_i(g^e) = (\alpha - \gamma_0)/(n+1)$. We assume that γ is sufficiently low relative to $(\alpha - \gamma_0)$ that, for any network g, and any $i \in g$, the Cournot output $q_i(g)$ is strictly positive. The Cournot-equilibrium gross profit is:

$$\pi_i(g) = q_i^2(g) \tag{11}$$

Remark 1. *Given a network g, and firms i, $j \in N$, such that $ij(g) = 0$, the change of i's gross profit, following upon the formation of a link with firm j, obeys to:*

$$\pi_i(g + ij) - \pi_i(g) = (q_i(g + ij) - q_i(g)) \cdot [2q_i(g) + (q_i(g + ij) - q_i(g))] \tag{12}$$

The following properties hold:

$$sign\ \ [\pi_i(g + ij) - \pi_i(g)] = sign\ \ [q_i(g + ij) - q_i(g)] \tag{13}$$

$$\pi_i(g + ij) - \pi_i(g) > f \quad iff \quad q_i(g + ij) - q_i(g) > x(q_i(g), f) \tag{14}$$

where $x(q_i(g), f)$ is the minimum change in i's output making firm i willing to add link ij to its connections in g, and is defned by $x(q, f) \equiv [(q^2 + f)^{1/2} - q]$. *The function $x(q, f)$ is strictly increasing with f, decreasing with q, and* $\lim_{f \to 0} x(q, f) = 0$.

We can check from equation (10) that, if a link ij is not viable, then, $q_i(g + ij) - q_i(g) = 0$. In this case, because the function $x(q_i(g), f)$ is strictly positive at $f > 0$, we have $q_i(g + ij) - q_i(g) < x(q_i(g), f)$. This shows that a necessary, but not sufficient, condition to the formation of any link $ij = 1$, is that the link in question is viable. When this is the case, $s_{ij}(g + ij) = s_{ji}(g + ij) = 1$, and

$$q_i(g + ij) - q_i(g) = \frac{\gamma(n-1)}{n+1}. \tag{15}$$

Proposition 1 *Assume that g is A-stable, and $ij(g) = 1$. The following necessary twin conditions apply:*

$$\gamma \frac{n-1}{n+1} \geq x(q_i(g - ij), f) \tag{16}$$

$$\gamma \frac{n-1}{n+1} \geq x(q_j(g - ij), f) \tag{17}$$

It is worth observing that a non-negligible collaboration cost $f > 0$, gives rise to a mild form of increasing returns to link formation. To see this, we use remark 1 to state that, if firm i is willing to add the link $ij = 1$ to its collaborations within a network g, then, it must be the case that corresponding change in i's Cournot output satisfies $q_i(g + ij) - q_i(g) \geq x(q_i(g), f) > 0$. Assuming that the network $(g + ij)$ is formed, we observe that the change in Cournot output making firm i willing to form also a second link $ih = 1$ is at least as large as $q_i(g + ij + ih) - q_i(g + ij) \geq x(q_i(g + ij), f)$. Because the function $x(q, f)$ is strictly decreasing in q, we have:

$$0 < x(q_i(g + ij), f) < x(q_i(g), f)$$

Obviously enough, the second link $ih = 1$ will not form, if it is not viable, in which case $q_z(g + ij + ih) - q_z(g + ij) = 0$, for $z = i$, $z = h$, or both.

A stronger form of increasing returns to link formation is enforced, if for any network g, and $\forall i, j \in N$, the equality $s_{ij}(g) = ij(g)$ holds true.[17] Within such a fremework, Goyal and Joshi (2003, lemma 4.1) prove that the change in i's gross profit from adding any marginal link to the network $(g+ij)$, is larger than the change in i's gross profit from adding the link ij to the network g. More formally, $\pi_i(g + ij + ih) - \pi_i(g + ij) > \pi_i(g + ij) - \pi_i(g)$. In this paper, the statement may not hold, simply because the link ih may not be viable. Contrary

[17] This happens in the present paper, if knowledge distances do not matter, because all links are viable (see below).

to Goyal and Joshi (2003), it may here be the case that firm i is not prepared to jointly sever all its links at once, but is prepared to sever one, or more, of its links separately, if they are not viable. Condition (iii), of the stability definition 3, is therefore no stronger than condition (i), and the former does not generally imply the latter.

Proposition 2 *Assume that the network g is A-stable. If $i, j \in N(g)$, and $ij(g) = 0$, the link ij is not viable.*

Proof. As one may easily check from equation (10), the equality between the degree $\eta_i(g)$ and the total spillover $s_i(g)$, earned by members of an A-stable network g, induces a perfect correlation between degree $\eta_i(g)$ and Cournot output $q_i(g)$, across network participants. To the extent that q_i is perfectly correlated with degree, and the function $x(q_i(g), f)$ is decreasing in $q_i(g)$, i's constraints to form a new link with any other firm $j \in N$ are weaker, if i's degree $\eta_i(g)$ is higher. The assumption $i, j \in N(g)$ can be written $\eta_i(g) \geq 1$, $\eta_j(g) \geq 1$. By proposition 1, there is $h \in N(g)$, $z \in N(g)$ such that

$$\gamma \frac{n-1}{n+1} \geq x(q_i(g - ih), f)$$

$$\gamma \frac{n-1}{n+1} \geq x(q_j(g - jz), f)$$

Now suppose proposition 2 is false, hence the link ij is viable. Because $q_i(g) > q_i(g - ih)$, and $q_j(g) > q_j(g - jz)$, it must be the case that:

$$q_i(g + ij) - q_i(g) = \frac{\gamma(n-1)}{n+1} > x(q_i(g), f)$$

$$q_j(g + ij) - q_j(g) = \frac{\gamma(n-1)}{n+1} > x(q_j(g), f)$$

This contradicts the assumption that g is A-stable, and completes the proof.

4.1 Viable links

To fix a reference point, it is worth considering the twin restrictions $\tau = 0$, $\theta = 1$, that make the Cournot collaboration game of this paper a replica of the corresponding game in Goyal and Joshi (2003, section 4). With such restrictions in place, all links are viable, hence knowledge distances do not matter. This leads to $s_{ij}(g) = ij(g)$, and $s_i(g) = \eta_i(g)$, $\forall i, j \in N$.[18] Using (ii) of the stability definition 4, if g is A-stable, and $ij(g) = 0$, then, i and j cannot both have a strict incentive to form the link $ij = 1$. Equivalently, it cannot be the case that, for both $h = i$, and $h = j$, the following inequality holds:

$$\gamma \frac{n-1}{n+1} > x(q_h(g), f) \tag{18}$$

[18] Notice that lemma 4.1 of Goyal and Joshi (2003) applies in this special case.

By continuity of $x(q,f)$, in its domain of definition, there exists a critical value $f_{Min} > 0$, such that If the collaboration cost $f < f_{Min}$ the right-hand side of (18) is strictly lower than the left-hand side, hence, the inequality (18) is necessarily verified. This means that at $0 < f < f_{Min}$ the unique A-stable network g is the complete network $g = g^c$. If the collaboration cost f is larger than a critical value f_{Max}, an A-stable network g is empty.[19] At $0 \leq f \leq f_{Max}$ there exist a non-empty set $G(f)$ of A-stable networks, such that, if $g \in G(f)$, the set $N(g)$ of non-isolated firms in g has size $n(g)$, with $2 \leq n(g) \leq n$.

Because all links are viable, proposition 2 implies that if a non-empty network g is A-stable, and $i,\ j \in N(g)$, then $ij(g) = 1$. Equivalently, at $0 \leq f \leq f_{Max}$, a non-empty A-stable network g has the dominant-group architecture, $g = g^k$, with $k = n(g)$ (Goyal and Joshi 2003, sect. 4). Any firm $i \in N(g)$ has uniform degree $\eta_i(g) = k-1$, and uniform Cournot output[20]

$$q_i(g) = \frac{\alpha - \gamma_0 + \gamma \cdot (k-1)(n+1-k)}{n+1}.$$

A isolated firm $i \in \{N - N(g)\}$ has Cournot output

$$q_i(g) = \frac{\alpha - \gamma_0 - \gamma k(k-1)}{n+1}.$$

The subnetwork formed by the connected component of g has global clustering coefficient[21] $C_N = 1$; this is no larger than it would be in a random network of equal size and average degree.

4.2 Viable and non-viable links: clustering in an A-stable network

This section introduces tighter conditions for link viability. In particular, we assume $\theta < 1$ and $\tau > 0$, such that each firm i in the industry has a viable link with some, but not all other firms in the industry. More formally, the following restrictions of assumption $A.1$ hold.

$$\forall i \in N,\ \exists\ j \in N, \text{ such that } [\min(\delta_{ij}, \delta_{ji})] \cdot [\theta - (\delta_{ij} + \delta_{ji})] < \tau \qquad (19)$$

$$\forall i \in N,\ \exists\ h \in N, \text{ such that } [\min(\delta_{ih}, \delta_{hi})] \cdot [\theta - (\delta_{ij} + \delta_{ji}) > \tau. \qquad (20)$$

Restriction (19) implies that link viability will strictly bind the formation of some alliances, no matter how small is the collaboration cost f. Contrary to

[19] f_{Min} and f_{Max} correspond to F_0 and F_3 of proposition 4.1, Goyal and Joshi (2003).

[20] The correspondence between the collaboration cost f and the interval $\{k_{Min}(f)$, $k_{Min}(f)+1$, ..., $k_{Max}(f)\}$ spanning the (non-trivial) component size k of a A-stable network $g = g^k$, is described by proposition 4.1 of Goyal and Joshi (2003).

[21] The global clustering coefficient of a network g is

$$C(g) = \frac{\text{number of closed triangles in } g}{\text{number of triangles in } g}.$$

the results in the previous section, at $0 < f < f_{Min}$ the complete network $g = g^c$ is no longer A-stable, and a non-empty A-stable network g does not have the dominant group architecture. By (20) every firm $i \in N$ has a viable link with some other firm; thus, every firm has a strict incentive to form a collaboration with some other firm, if the collaboration cost is low enough; when this is the case, an A-stable network g has a number of non isolated nodes $n(g) = n$, has average degree

$$\bar{\eta}(g) = \frac{1}{n} \sum_{i \in N} \eta_i(g) < n - 1,$$

and may have any number K of components, $1 \leq K \leq n/2$.

We are now ready to prove the result:

Proposition 3 *Assume (7), (8), (19), (20). If the maximum undirected knowledge distance consistent with link formation is low enough, an A-stable network g is such that (global) clustering coefficient $C(g)$ is larger than it would be expected, if the wiring among the non-isolated nodes of g was random, and the number of links among them was unchanged.*

Proof. Assume g is A-stable. For the sake of the argument, let us first consider the restriction $0 < f < f_{Min}$, with the implication that the set of non isolated firms in g is $N(g) = N$. Firms i and j can be R&D partners in g, only if $dk(i,j) < \theta$. A corresponding constraint $dk(i,h) < \theta$ applies, if the same firm i is linked to a second firm h. Moreover, the triangle inequality implies $dk(j,h) \leq dk(i,j) + dk(i,h) < 2\theta$. Recalling (8), on the assumption that θ is lower than $E(\delta_{ij})_{i \neq j}$, we conclude that $dk(j,h) < E(dk(i,j))_{i \neq j} = 2E(\delta_{ij})_{i \neq j}$. In words, the undirected distance between any two firms j, $h \in N(g)$, that are linked to an identical third partner, is strictly lower than it would be expected, if two firms were picked-up at random from the set $N(g) = N$. If we now consider the full range $0 < f < f_{Max}$, it may well be the case that $N(g) \subset N$. Stronger partner selection induced by a higher collaboration cost, only requires that a sufficiently high Cournot output is necessary to the membership in the set $N(g)$; it does not however impinge upon the *expected* undirected knowledge distance between two random members of $N(g)$. It is still the case that, if θ is low enough, the undirected knowledge distance between two firms, that are linked to an identical third partner in $N(g)$, is strictly lower than it would be expected, if two firms were picked-up at random from $N(g)$. Because link formation is constrained by viability, we conclude that the frequency with which two neighbors of the same partner are direct neighbors in $N(g)$ is higher than the average frequency of collaboration in $N(g)$, or equivalently, it is larger than it would be expected, if the wiring of the connections among the firms in $N(g)$ was random, and the number of links among them was unchanged. This proves the proposition.

5 Discussion

Variety in the size and composition of the knowledge endowments, induced by a cross-firm knowledge distribution A, affects the competence-based opportunities of R&D alliance. In a oligopolistic industry with quantity competition, the topology of an A-stable R&D network is produced by the way these opportunities interact with the distribution of Cournot output across the n firms in the industry.

When the knowledge constraints to cooperation are not biting, it is as if the knowledge spillovers between partners fall 'like manna from haven'. In this case all potential links are knowledge-viable, and the game played by firms produces the dominant-group architecture $g = g^k$, such that the non isolated nodes of g form a complete subnetwork (Goyal Joshi 2003, sect. 4). It is worth observing that in a g^k architecture, the fraction of one's neighbors that are neighbors is maximal, but is no larger than it would be, if the wiring of connctions among the non-isolated firms in g was random. The obvious reason is that, when each non-isolated node is linked to any other, there is no scope for randomness in the wiring of connections.

Different properties are enforced by the assumption that knowledge spillovers between partners do not fall from haven. Although a perfect correlation remains between firm degree and output, and a higher Cournot output makes collaboration less vulnerable to a rise of collaboration costs, tight knowledge constraints to cooperation may prevent R&D alliance between firms, no matter how large their output may be.[22]

Tight knowledge proximity constraints to cooperation produce the further property that, in an A-stable network g, the fraction of one's neighbors that are neighbors is larger than it would be, if the wiring of connctions among the non-isolated firms in g was random. High local clustering of R&D links is a charachteristic prediction of knowledge-based models of R&D network formation (Baum et al. 2010). The remarks above show that the prediction may not be lost, when knowledge based R&D collaboration is followed by quantity competition in the market for output.[23].

It may be worth adding that, if the A distribution produces a modular cross-firm distribution of the knowledge distances[24], the local clustering of R&D links leads to a multiplicity of clusters, such that the frequency of connection within a cluster is higher than the frequency of connection between the clusters. In view of the knowledge-proximity constraints to cooperation, a cluster may then be interpreted as a knowledge community. On the hypothesis that there is sufficient variation in cluster size, clustering is associated to the further property that a disproportionately large fraction of high-degree nodes are linked to other high-

[22] The possiblity of a multiplicity of network components cannot be ruled out.

[23] If the local clusters of nodes are connected by sparse clique-spanning ties, the A-stable network g is a small-world (Watts and Strogatz 1998, Watts 1999).

[24] This means that the set N of firms in the industry can be partitioned into a multiplicity of groups, such that average undirected knowledge distance within a group is lower than average undirected knowledge distance between groups.

degree nodes. In other words, assortativity by degree is positive, as is typically the case in social networks (Newman and Park 2003, Caminati 2016).[25]

This paper claims that the formation of R&D alliances may be affected by the potential asymmetry of the pairwise knowledge distances between firms. Other implications of this asymmetry bear on the persistency, rather than the formation, of R&D networks. This is clarified by the following remark.

Innovation and knowledge spillovers make the knowledge distribution endogenous, with potential convergence or divergence in the size and composition of the knowledge endowments, that depend crucially on the topology of the R&D network. In one specific instance, the wildly complex influence of network topology can be thamed by simple euristic arguments leading to cogent conclusions. If a firm $i \in N$ is the terminal node of a path in an A-stable network g (that is, there is $i \in N$, such that $\eta_i(g) = 1$), the firm i in question lacks sources of knowledge, outside the collaboration with its unique R&D partner, say j. If the knowledge produced by the alliance ij is shared between the partners, the directed knowledge distance δ_{ij} is bound to converge to zero, in the long run. As a result, j's pay-off to collaborate with i will eventually vanish, bringing the collaboration to an end. This suggests that some forms of network topology, like the star network, though possibly stable in the short.run, are inherently unpersistent in the long run.

6 Conclusions

This paper complements the Cournot collaboration game outlined in Goyal and Joshi (2003, sect. 4), with the knowledge portfolio approach to R&D colaboration (Cowan and Jonard 2009, Baum et al. 2010, Caminati 2016). In particular, the paper assumes that a firm can receive a knowledge spillover from another, only if the latter has some novel ideas to offer, and the two firms in question share a sufficiently large mutual understanding. It is agued that short-run R&D alliance formation is constrained by the directed knowledge distances between the potential partners, reflecting the given initial cross-firm knowledge distribution.

Our results show that, unless the knowledge constraints to collaboration are weak enough, a stable network will not generally display the dominat group architecture. In this sense, the result is a generalization of Goyal and Joshi (2003, sect. 4). Although a perfect correlation between firm's degree and Cournot

[25] The empirical relevance of this conclusion is limited by the following remarks. In a narrowly defined industry, modularity in knowledge distribution is less plausible. In fact, Tomasello et al. (2013) find that in most (SIC three digits) sector networks, assortativity by degree is very close to zero or even negative. It is positive in multisector networks.

The assortativity coefficient $r(g)$ is defined by:

$$r(g) = \frac{\sum_{ij} \left(ij(g) - \eta_i(g) \cdot \eta_j(g)/T\right) \eta_i(g) \cdot \eta_j(g)}{\sum_{ij} \left(\eta_i(g)\Delta(i,j) - \eta_i(g) \cdot \eta_j(g)/T\right) \eta_i(g) \cdot \eta_j(g)}$$

where $\Delta(i,j) = 1$, if $i = j$, and $\Delta(i,j) = 0$ otherwise; $T = \sum_i \eta_i(g)$.

output still applies, and a higher Cournot output makes collaboration less vulnerable to collaboration costs, R&D cooperation between two positive-degree firms may be prevented by tight knowledge constraints.

It is shown that sufficiently tight knowledge-proximity constraints to R&D collaboration produce stable networks with a (global) clustering coefficient, that is higher than it would be expected, if the wiring among the non-isolated firms in the network was random, while the number of links among them was unchanged.

High local clustering is a characteristic of many real-world social networks, and differentaiates them from other types of networks (Newman and Park 2003).

It is finally suggested that the constraints to alliance formation induced by asymmetric knowledge distances between the partners may bear strong implications for network evolution, and the interpretation of network decay.

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Appendix

A. Directed distance in knowledge space

Let A, B, C any triplet of subsets in Ψ. The *directed-distance* function $dd : \Psi \times \Psi \to [0,1]$ defined by

$$dd(A,B) = \frac{\mu(A - A \cap B)}{\mu(A \cup B)} \tag{21}$$

is a quasi-metric on Ψ.

Because dd trivially meets identity recognition and identity of indiscernibles, only the triangle inequality remains to be ascertained, that is:

$$dd(A,B) + dd(B,C) \geq dd(A,C) \tag{22}$$

This is proved as follows. First, we observe that the following fact holds[26]:

$$(A - A \cap B) \cup (B - B \cap C) \supseteq (A - A \cap C) \tag{23}$$

Statement (23) implies:

$$A \cup B \cup C = (A \cap B \cap C) \cup (A - A \cap B) \cup (B - B \cap C) \cup (C - C \cap A) \tag{24}$$

and, the more stringent equality

$$\mu(A \cup B \cup C) = \mu(A \cap B \cap C) + \mu(A - A \cap B) + \mu(B - B \cap C) + \mu(C - C \cap A). \tag{25}$$

Now use (25) and the obviuos fact

$$\mu(A \cup C) = \mu(A \cap C) + \mu(A - A \cap C) + \mu(C - C \cap A)$$

to obtain:

$$\frac{\mu(A - A \cap B)}{\mu(A \cup B)} + \frac{\mu(B - B \cap C)}{\mu(B \cup C)} \geq \frac{\mu(A - A \cap B) + \mu(B - B \cap C)}{\mu(A \cup B \cup C)} =$$

[26] If $a \in (A - A \cap C)$, then either $a \in (A - A \cap B)$, or $a \in A \cap B$, hence $a \in (B - B \cap C)$. This proves (23).

$$= \frac{\mu(A \cup B \cup C) - \mu(A \cap B \cap C) - \mu(C - C \cap A)}{\mu(A \cup B \cup C)} \geq 1 - \frac{\mu(A \cap B \cap C)}{\mu(A \cup B \cup C)} - \frac{\mu(C - C \cap A)}{\mu(A \cup C)} =$$

$$-\frac{\mu(A \cap B \cap C)}{\mu(A \cup B \cup C)} + \frac{\mu(A \cap C)}{\mu(A \cup C)} + \frac{\mu(A - A \cap C)}{\mu(A \cup C)} \geq \frac{\mu(A - A \cap C)}{\mu(A \cup C)}$$

This proves the triangle inequality (22).

B. Proof of Remark 1

Statements (12) and (13) are straightforward. The condition $\pi_i(g + ij) - \pi_i(g) > f$ can be written:

$$y_i^2(g + ij) + 2q_i(g) \cdot y_i(g + ij) > f$$

where: $y_i(g + ij) = q_i(g + ij) - q_i(g)$. We conclude that $\pi_i(g + ij) - \pi_i(g) > f$ if and only if:

$$y_i(g + ij) > [(q_i^2(g) + f)^{1/2} - q_i(g)] \equiv x(q_i(g), f)$$

This proves statement (14).

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