Conditional loyalty and its implications for pricing

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Abstract

Bertrand-Edgeworth competition has recently been analyzed under imperfect buyer mobility, as a game in which, once prices are chosen, a static buyer subgame (BS) is played where the buyers choose which seller to visit (see, e.g., Burdett et al, 2001). Our paper considers a symmetric duopoly where two buyers play a two-stage BS of imperfect information after price setting. With prices sufficiently close, an equilibrium of the BS is characterized in which the buyers keep loyal if previously served. Conditional loyalty is proved to increase the firms' market power: at the corresponding subgame perfect equilibrium of the entire game, the price is higher than that corresponding to the equilibrium of the BS in which the buyers are persistently randomizing.

Keywords: Bertrand-Edgeworth competition, matching, imperfect buyer mobility, conditional loyalty, assessment equilibrium.

JEL Classification Codes: D430, L130.

1 Introduction

Some recents contributions have addressed Bertrand-Edgeworth competition under imperfect buyer mobility, inasmuch as each buyer is allowed to visit only one seller after pricing decisions (Peters, 1984; Deneckere and Peck, 1995; Burdett et al., 2001; Geromichalos, 2014). Due to multiplicity of pure strategy equilibria (PSEs) of the buyer subgame (BS), price determination has been analyzed subject to the mixed strategy equilibrium (MSE) of the BS, where "mismatchings" - firms selling below capacity along with more expensive rivals receiving positive demand - may arise.

Unlike most of this literature on pricing and "directed" search, Shi (2016) develops a multistage model where, in each stage, the firms announce prices as well as any service priority they might offer to loyal buyers and the buyers, based on this information and the history of previous matchings at the various firms, choose the probability of visiting any seller. However, even over a period in which prices remain fixed, buyers frequently seem to be able to move to another seller; furthermore, they might be uncertain as to recent capacity utilization at the various sellers. To take account of these features in the simplest way, our paper incorporates imperfect mobility into a model where two firms, whose total capacity equals an inelastic total demand, compete in prices and next two buyers play a two-stage BS of imperfect information, each time choosing which firm to visit. With prices sufficiently close to each other, two alternative equilibria of the two-stage BS are characterized. In one equilibrium, the MSE of the static BS is played repeatedly. But another equilibrium exists in which, if served, buyers keep loyal to the seller previously chosen: on the corresponding equilibrium path, both buyers are served in the second stage of the BS. Most importantly, the corresponding (subgame-perfect) equilibrium price is higher than under constant randomization.

The rest of the paper is organized as follows. To prepare the ground for our positive contribution, Section 2 analyzes duopolistic price determination when the two buyers play a static BS.[1] Section 3 develops our model of price determination in which the two buyers are playing a two-stage BS of imperfect information, in each stage choosing which seller to visit. Section 4 briefly concludes.

2 Pricing with a static buyer subgame

Two risk-neutral firms, A and B, sell a homogeneous commodity to two risk-neutral buyers, h and k. (Below, notation will most often be introduced in terms of buyer h.) Each firm $i \in \{A, B\}$ independently sets price p_i; then a BS is played. In the static BS, any h demands one unit if $\min\{p_A, p_B\} \leq 1$, 1 being each buyer's reservation price, and chooses υ_h, the probability of visiting A (we conveniently denote h's strategy by υ_h rather than $\sigma_h = (\upsilon_h, 1 - \upsilon_h)$); then each i costlessly produces y_i, the minimum between its forthcoming demand and capacity $\overline{y}_i = 1$. Hereafter, we take as given that

$$\max\{p_A, p_B\} \leq 1. \tag{1}$$

The set of possible events buyer h may experience is $\mathcal{E}_h = \{e_h\} = \{As_h, Ar_h, Bs_h, Br_h\}$: is_h (ir_h) stands for h visiting $i \in \{A, B]$ and being served (rationed). With both buyers at i, each is served with equal probability. Denote by $\pi(e_h)_{(\upsilon_h,\upsilon_k)}$, $u_h(\upsilon_h, \upsilon_k)$, $(Ey_i)_{(\upsilon_h,\upsilon_k)}$, and $(E\Pi_i)_{(\upsilon_h,\upsilon_k)} = p_i(Ey_i)_{(\upsilon_h,\upsilon_k)}$, respectively, the probability of e_h, h's payoff (expected surplus), i's expected output and profit under strategy profile (υ_h, υ_k). Given υ_k, h's service probability and payoff are $\pi(As_h)_{(1,\upsilon_k)} = \frac{\upsilon_k}{2} + 1 - \upsilon_k$ and $u_h(1, \upsilon_k) = (1 - p_A)\pi(As_h)_{(1,\upsilon_k)}$, respectively, if visiting A, and $\pi(Bs_h)_{(0,\upsilon_k)} = \upsilon_k + \frac{1-\upsilon_k}{2}$ and $u_h(0, \upsilon_k) = (1 - p_B)\pi(Bs_h)_{(0,\upsilon_k)}$ if visiting B. With p_A and p_B meeting system

$$2p_B - 1 \leq p_A \leq \frac{1 + p_B}{2}, \tag{2}$$

the BS has a symmetric equilibrium, $(\upsilon_h, \upsilon_k) = (\widetilde{\upsilon}, \widetilde{\upsilon})$, where buyers are indifferent between the firms: $\widetilde{\upsilon}$, the solution of

$$(1 - p_A)\pi(As_h)_{(1,\upsilon)} = (1 - p_B)\pi(Bs_h)_{(0,\upsilon)}, \tag{3}$$

equals

$$\widetilde{\upsilon} = \widetilde{\upsilon}(p_A, p_B) = \frac{1 - 2p_A + p_B}{2 - p_A - p_B}. \tag{4}$$

[1] The basic result is already in Burdett et al. (2001, pp. 1062-1067) where the two-seller two-buyer case is presented before the m-seller n-buyer one.

Note that $p_B < 1$ ($p_B = 1$) is equivalent to $2p_B - 1 < \frac{1+p_B}{2} < 1$ ($2p_B - 1 = \frac{1+p_B}{2} = 1$); then, holding system (2), $p_A < 1$ ($p_A = 1$) too. Clearly, $\widetilde{\upsilon}(p,p) = 1/2$ (any $p < 1$); for definiteness, we also let $\widetilde{\upsilon}(1,1) = 1/2$. With $p_B < 1$, $\widetilde{\upsilon} \in (0,1)$ if and only if

$$2p_B - 1 < p_A < \frac{1+p_B}{2}. \tag{2'}$$

Then two asymmetric pure strategy equilibria (PSEs) also exist, $(\upsilon_h, \upsilon_k) = (\upsilon_k, \upsilon_h) = (1,0)$: they Pareto dominate the mixed strategy equilibrium (MSE) $(\widetilde{\upsilon}, \widetilde{\upsilon})$ since $u_h(\widetilde{\upsilon}, \widetilde{\upsilon}) < \min\{1 - p_A, 1 - p_B\}$. Yet, absent preplay communication, the MSE seems compelling given the likely mismatchings between demands and capacities.[2] Finally, strategy $\upsilon_h = 0$ ($\upsilon_h = 1$) is strictly dominant if $p_A > (1+p_B)/2$ ($p_A < 2p_B - 1$). Based on the above, we rely on equilibrium $(\upsilon_h, \upsilon_k) = (\upsilon^*, \upsilon^*)$ of the BS, where[3]

$$\upsilon^* = \upsilon^*(p_A, p_B) = \begin{cases} 1 & \text{if} \quad p_A < 2p_B - 1 \\ \widetilde{\upsilon} \in [0,1] & \text{if} \quad 2p_B - 1 \le p_A \le \frac{1+p_B}{2} \\ 0 & \text{if} \quad p_A > \frac{1+p_B}{2}. \end{cases} \tag{5}$$

Then $Ey_A = (Ey_A)_{(\upsilon^*,\upsilon^*)} = \upsilon^{*2} + 2\upsilon^*(1-\upsilon^*)$ and $E\Pi_A = p_A(Ey_A)_{(\upsilon^*,\upsilon^*)}$, respectively. It follows from Eq. (5) that $p_i \in (0,1)$ since, no matter $p_B \in [0,1]$, $p_A = 1$ and $p_A = 0$ are never best responses.[4] This stands in stark contrast with the case of perfect mobility, where $p_i = 1$ is strictly dominant (no matter p_B, A sells capacity for any $p_A \le 1$).[5] We have $\frac{\partial E\Pi_A}{\partial p_A} = (Ey_A)_{(\upsilon^*,\upsilon^*)} + p_A \frac{d(Ey_A)_{(\upsilon,\upsilon)}}{d\upsilon}\frac{\partial \upsilon^*}{\partial p_A}$; holding system (2′), $\upsilon^* = \widetilde{\upsilon}$, $\frac{\partial \upsilon^*}{\partial p_A} = \frac{\partial \widetilde{\upsilon}}{\partial p_A} = \frac{-3(1-p_B)}{(2-p_A-p_B)^2}$, $\frac{\partial^2 \upsilon^*}{\partial p_A^2} = \frac{\partial^2 \widetilde{\upsilon}}{\partial p_A^2} = \frac{-6(1-p_B)}{(2-p_A-p_B)^3}$, and $\frac{\partial^2 E\Pi_A}{\partial p_A^2} = 2\frac{d(Ey_A)_{(\upsilon,\upsilon)}}{d\upsilon}\frac{\partial \widetilde{\upsilon}}{\partial p_A} + p_A \frac{d^2(Ey_A)_{(\upsilon,\upsilon)}}{d\upsilon^2}\left(\frac{\partial \widetilde{\upsilon}}{\partial p_A}\right)^2 + p_A \frac{d(Ey_A)_{(\upsilon,\upsilon)}}{d\upsilon}\frac{\partial^2 \widetilde{\upsilon}}{\partial p_A^2} < 0$. At a symmetric equilibrium, $\widetilde{\upsilon} = 1/2$ and $p_A = p_B = p$: hence $\frac{\partial E\Pi_A}{\partial p_A} = 0$ yields $p = 1/2$.

3 Pricing with a dynamic buyer subgame

In this section, a two-stage BS of imperfect information is played after price announcements in $t = 0$: in stage $t = 1, 2$, h demands one unit if $\min\{p_A, p_B\} \le 1$ and chooses $\upsilon_{h,t}$, the probability of visiting A; each $i \in \{A, B\}$ produces the minimum between its forthcoming demand and capacity $\overline{y}_i = 1$. We let h maximize his expected total surplus and i maximize its expected total profits, $\sum_{t=1}^{2} E\Pi_{i,t} = p_i \sum_{t=1}^{2} Ey_{i,t}$. $\mathcal{E}_{h,t} = \{e_{h,t}\} = \{As_{h,t}, Ar_{h,t}, Bs_{h,t}, Br_{h,t}\}$ stands for the set of stage-t possible events regarding h, $\pi(e_{h,t})_{(\upsilon_{h,t},\upsilon_{k,t})}$ for the probability of $e_{h,t}$ under $(\upsilon_{h,t}, \upsilon_{k,t})$. With both buyers at i in $t = 2$, each is served with equal probability,

[2] Of course, coordination would be increasingly problematic the larger the number of buyers n. For instance, with $\overline{y}_A = \overline{y}_B = n/2$ and $p_A = p_B$, there are $\binom{n}{n/2}$ PSEs.

[3] With either $\widetilde{\upsilon} = 1$ (i.e., $p_A = 2p_B - 1$) or $\widetilde{\upsilon} = 0$ (i.e., $p_A = (1+p_B)/2$), $(\upsilon_h, \upsilon_k) = (\widetilde{\upsilon}, \widetilde{\upsilon})$ is an equilibrium in weakly dominant strategies in the continuum of equilibria $(\upsilon_h, \upsilon_k) = (\widetilde{\upsilon}, \upsilon_k)$.

[4] If $p_B \in [0,1)$, then, for $p_A = 1$, $\upsilon^* = E\Pi_A = 0$, whereas, for $p_A \in \left(0, \frac{1+p_B}{2}\right)$, $E\Pi_A > 0$; if $p_B = 1$, then, for $p_A = 1$, $E\Pi_A = 3/4$ (since $\upsilon^*(1,1) = \widetilde{\upsilon}(1,1) = 1/2$), whereas, for p_A negligibly less than 1, $\upsilon^* = E\Pi_A = 1$.

[5] With perfect mobility and $p_B < p_A \le 1$, buyers try B first and then the rationed buyer moves to A.

regardless of whom i served in $t=1$. When choosing $\upsilon_{h,2}$, h recalls $e_{h,1}$ and makes an inference on $e_{k,1} \in \mathcal{E}_{k,1} = \{As_{k,1}, Ar_{k,1}, Bs_{k,1}, Br_{k,1}\}$ from his information set $I_h = (e_{h,1}, (p_A, p_B))$ (henceforth, $I_h = (e_{h,1}, \cdot)$, for brevity) and his conjecture on k's previous move.

A behavioral strategy, Θ_h, is a pair of functions $(\upsilon_{h,1}(p_A, p_B), \upsilon_{h,2}(I_h))$. $\Theta_{h|2}$ stands for a strategy prescribing $\upsilon_{h,2} = \upsilon_{h,2}(I_h)$, precisely as Θ_h. In any BS, h's (ex-ante) payoff with strategy profile (Θ_h, Θ_k) is written $U_h(\Theta_h, \Theta_k) = \sum_{t=1}^{2} u_{h,t}(\Theta_h, \Theta_k)$, where $u_{h,t}(\Theta_h, \Theta_k)$ $(t = 1, 2)$ is h's (ex-ante) stage-t payoff; $u_{h,t}(\upsilon_{h,t}, \Theta_k)$ stands for h's (ex-ante) stage-t payoff, if playing $\upsilon_{h,t}$ and with k adhering to Θ_k.

In BS*s* where system ($2'$) holds, there are equilibria in which, already in $t = 1$, one of the two asymmetric PSE*s* of the static BS is played. Absent preplay communication, though, we look at alternative equilibria. In one such equilibrium there is constant randomization, under system ($2'$).

Proposition 1 *Let* $\Theta^* = (\upsilon^*_{h,1}, \upsilon^*_{h,2}) = (\upsilon^*, \upsilon^*)$. *Strategy profile* $(\Theta_h, \Theta_k) = (\Theta^*, \Theta^*)$ *induces a Nash equilibrium in each BS.*

Proof. $u_{h,t}(\upsilon_{h,t}, \Theta^*) = \upsilon_{h,t}\pi(As_{h,t})_{(1,\upsilon^*)}(1-p_A) + (1-(\upsilon_{h,t}))\pi(Bs_{h,t})_{(0,\upsilon^*)}(1-p_B)$. Under (2), $u_{h,t}(\upsilon_{h,t}, \Theta^*) = \pi(As_{h,t})_{(1,\widetilde{\upsilon})}(1-p_A) = \pi(Bs_{h,t})_{(0,\widetilde{\upsilon})}(1-p_B) = u_h(\widetilde{\upsilon}, \widetilde{\upsilon}) = u_{h,t}(\Theta^*, \Theta^*)$ no matter $\upsilon_{h,t}$ (see Eq. (5) and (3)). If $p_A > \frac{1+p_B}{2}$, $\Theta_h \neq \Theta^*$ implies $\upsilon_{h,1}(p_A, p_B) > 0$ and/or $\upsilon_{h,2}(I_h) > 0$ (some I_h). Thus, at any deviating stage t, $u_{h,t}(\Theta_h, \Theta^*)$ is a convex linear combination of $(1-p_A)$ and $\frac{1-p_B}{2}$, less than $u_{h,t}(\Theta^*, \Theta^*) = \frac{1-p_B}{2}$ for any positive weight upon $1-p_A$. A similar argument holds if $p_A < 2p_B - 1$. ■

However, another equilibrium exists where, with prices sufficiently close to each other, the following norm of "conditional loyalty" (CL) is observed.

Definition 1. According to CL, if previouly served, a buyer visits in $t=2$ the same seller as in $t=1$, while visiting the other seller if rationed. □

CL has straightforward implications.

Proposition 2 *Under CL, both buyers are served in t=2; a unilateral deviation from CL results in each buyer being rationed with positive probability.*

Proof. In $t=1$, let h be served and k be rationed by A or h and k be served by A and B, respectively. Under CL, $(\upsilon_{h,2}, \upsilon_{k,2}) = (1, 0)$ and both are served in $t=2$. If h deviates, $\upsilon_{h,2} < 1$ and service probability is $\upsilon_{h,2} + \frac{1-\upsilon_{h,2}}{2} < 1$ for each; if k deviates, $\upsilon_{k,2} > 0$, implying service probability $\frac{\upsilon_{k,2}}{2} + 1 - \upsilon_{k,2} < 1$ for each. ■

The following strategy incorporates CL.

Definition 2. $\Theta^{**} = (\upsilon^{**}_{h,1}, \upsilon^{**}_{h,2})$, where

$$\upsilon^{**}_{h,1} = \upsilon^{**}_{h,1}(p_A, p_B) = \begin{cases} 1 & \text{if} \quad p_A < \frac{3p_B-1}{2}, \\ \tilde{\tilde{\upsilon}} \in [0,1] & \text{if} \quad \frac{3p_B-1}{2} \leq p_A \leq \frac{1+2p_B}{3}, \\ 0 & \text{if} \quad p_A > \frac{1+2p_B}{3} \end{cases}, \tag{6}$$

$$\upsilon^{**}_{h,2} = \upsilon^{**}_{h,2}(I_h) = \begin{cases} 1 \quad \text{if } e_{h,1} \in \{As_{h,1}, Br_{h,1}\} \text{ and } 2p_B - 1 < p_A < \frac{1+p_B}{2}, \\ \qquad \text{if } p_A \leq 2p_B - 1 < 1, \text{ or if } p_A < p_B = 1, \\ 0 \quad \text{if } e_{h,1} \in \{Bs_{h,1}, Ar_{h,1}\} \text{ and } 2p_B - 1 < p_A < \frac{1+p_B}{2}, \\ \qquad \text{if } \frac{1+p_B}{2} \leq p_A < 1, \text{ or if } p_B < p_A = 1, \\ \frac{1}{2} \quad \text{if} \quad p_A = p_B = 1, \end{cases} \tag{7}$$

and

$$\tilde{\tilde{\upsilon}} = \tilde{\tilde{\upsilon}}(p_A, p_B) = \begin{cases} \frac{1+2p_B-3p_A}{2-p_A-p_B}, & \text{if } \frac{3p_B-1}{2} < p_A \leq \frac{1+2p_B}{3} \text{ or } \frac{3p_B-1}{2} \leq p_A < \frac{1+2p_B}{3}, \\ \frac{1}{2} & \text{if} \quad p_A = p_B = 1. \end{cases} \tag{8}$$

□

Remarks. Before establishing $(\Theta^{**}, \Theta^{**})$ as an equilibrium, some remarks on Θ^{**} are in order.

1. $p_B < 1$ ($p_B = 1$) is equivalent to $2p_B - 1 < \frac{3p_B-1}{2} < \frac{1+2p_B}{3} < \frac{1+p_B}{2} < 1$ ($2p_B - 1 = \frac{3p_B-1}{2} = \frac{1+2p_B}{3} = \frac{1+p_B}{2} = 1$). Consequently: with $p_B < 1$, system
$$\frac{3p_B - 1}{2} \leq p_A \leq \frac{1 + 2p_B}{3} \tag{9}$$
is stricter than system (2′) and implies $p_A < 1$; with $p_B = 1$, system (9) is equivalent to system (2) and implies $p_A = 1$.

2. According to Eq. (6), CL applies if system (2′) holds; if (2′) doesn't hold, then in $t = 2$ the cheapest firm is visited if $p_A \neq p_B$ while $\upsilon^{**}_{h,2} = 1/2$ if $p_A = p_B = 1$.

3. Holding system (9), $\upsilon^{**}_{h,1} = \tilde{\tilde{\upsilon}}$. More specifically, according to Eq. (8): with at least one of inequalities (9) strict, $\tilde{\tilde{\upsilon}}$ is the unique solution of equation
$$\begin{aligned} &\pi(As_{k,1})_{(\upsilon_{h,1},1)}(1 - p_A)2 + \left(1 - \pi(As_{k,1})_{(\upsilon_{h,1},1)}\right)(1 - p_B) \\ &= \pi(Bs_{k,1})_{(\upsilon_{h,1},0)}(1 - p_B)2 + \left(1 - \pi(Bs_{k,1})_{(\upsilon_{h,1},0)}\right)(1 - p_A), \end{aligned} \tag{10}$$
making k's payoff (under CL by h and k) be independent of $\upsilon_{k,1}$;[6] if instead $p_A = p_B = 1$, $\tilde{\tilde{\upsilon}} = 1/2$ (for definiteness). If system (9) does not hold, the cheapest firm is visited in $t = 1$. □

Proposition 3 *Strategy profile* $(\Theta_h, \Theta_k) = (\Theta^{**}, \Theta^{**})$ *induces a Nash equilibrium in each BS.*

[6] By Proposition 2, the LHS (RHS) of Eq. (10) is k's payoff if visiting A (B) in $t = 1$, conditional on $\upsilon_{h,1}$ and CL by h and k. (By the way, $\pi(As_{k,1})_{(\upsilon_{h,1},1)} = \frac{\upsilon_{h,1}}{2} + 1 - \upsilon_{h,1}$ and $\pi(Bs_{k,1})_{(\upsilon_{h,1},0)} = \upsilon_{h,1} + \frac{1-\upsilon_{h,1}}{2}$.)

Proof. In the Appendix. ■

Since any dynamic BS has no proper subgames, a Nash equilibrium of it is subgame perfect. Hence one should check that the equilibrium strategy prescribes a best response at information sets off the equilibrium path. It is obviously so for the equilibrium (Θ^*, Θ^*), since $e_{h,1}$ does not affect h's prediction on k's move in $t = 2$. Proposition 4 will show that, along with a proper belief system μ, the equilibrium $(\Theta^{**}, \Theta^{**})$ represents an "assessment equilibrium",[7] in the specific meaning to be specified shortly. At any I_h, buyer h holds a belief $\mu(\mathcal{E}_{k,1} \mid I_h)$, an (ex-post) probability distribution over $\mathcal{E}_{k,1}$, which allows him to compute $u_{h,2}(\upsilon_{h,2}, \Theta^{**}_{|2}; \mu(\mathcal{E}_{k,1} \mid I_h) \mid I_h)$, his stage-2 payoff conditional on I_h, when playing $\upsilon_{h,2}$ and with k adhering to Θ^{**} in $t = 2$. The assessment $(\Theta^{**}, \Theta^{**}; \mu)$ is "sequentially rational": at any I_h, $\upsilon_{h,2} = \upsilon^{**}_{h,2}(I_h)$ maximizes $u_{h,2}(\upsilon_{h,2}, \Theta^{**}_{|2}; \mu(\mathcal{E}_{k,1} \mid I_h) \mid I_h)$. It is also "structurally consistent": at any I_h, the belief $\mu(\mathcal{E}_{k,1} \mid I_h)$ is derived by Bayes' rule and the strategy k is conjectured to have followed in $t = 1$.

Proposition 4 *Any assessment $(\Theta^{**}, \Theta^{**}; \mu)$ - μ being any structurally consistent belief system - meets sequential rationality in any BS.*

Proof. Sequential rationality is trivial if $p_A = p_B = 1$; it is also immediate if $\frac{1+p_B}{2} \leq p_A < 1$ or $p_B < p_A = 1$ ($p_A \leq 2p_B - 1 < 1$ or $p_A < p_B = 1$) since then $\upsilon_{h,2} = 0$ ($\upsilon_{h,2} = 1$) is obviously a best response to $\upsilon^{**}_{k,2} = 0$ ($\upsilon^{**}_{k,2} = 1$). Then consider BS*s* where system (2′) holds. At $I_h = (Ar_{h,1}, \cdot)$, h obviously infers that k was served by A in $t = 1$ and k's loyalty is thus predicted: $u_{h,2}(\upsilon_{h,2}, \Theta^{**}_{|2}; \mu(\mathcal{E}_{k,1} \mid I_h) \mid I_h) = \upsilon_{h,2}(1 - p_A)/2 + (1 - \upsilon_{h,2})(1 - p_B)$, which is maximal for $\upsilon_{h,2} = \upsilon^{**}_{h,2}(Ar_{h,1}, \cdot) = 0$ since $p_A > 2p_B - 1$. At $I_h = (As_{h,1}, \cdot)$, any structurally consistent belief is such that $\mu(Ar_{k,1} \mid I_h) + \mu(Bs_{k,1} \mid I_h) = 1$.[8] Therefore, k is expected to visit B in $t = 2$: $u_{h,2}(\upsilon_{h,2}, \Theta^{**}_{|2}; \mu(\mathcal{E}_{k,1} \mid I_h) \mid I_h) = \upsilon_{h,2}(1 - p_A) + (1 - \upsilon_{h,2})(1 - p_B)/2$, which is maximal for $\upsilon_{h,2} = \upsilon^{**}_{h,2}(As_{h,1}, \cdot) = 1$ since $p_A < \frac{1+p_B}{2}$. Similar arguments hold for $I_h = (Bs_{h,1}, \cdot)$) and $I_h = (Br_{h,1}, \cdot)$. ■

Based on Propositions 1 and 3, we now solve for the entire game.

Proposition 5 *(i)* $((p_A, p_B), (\Theta_h, \Theta_k)) = ((p^*, p^*), (\Theta^*, \Theta^*))$, *with* $p^* = 1/2$, *is a subgame perfect equilibrium (SPE) of the entire game.*

(ii) $((p_A, p_B), (\Theta_h, \Theta_k)) = ((p^{**}, p^{**}), (\Theta^{**}, \Theta^{**}))$, *with* $p^{**} = 7/12$, *is another SPE.*

Proof. (i) If $(\Theta_h, \Theta_k) = (\Theta^*, \Theta^*)$, $\sum_{t=1}^{2} E\Pi_{A,t}(p_A, p_B) = 2p_A \left(Ey_A\right)_{(\upsilon^*, \upsilon^*)}$. Holding system (2), $\partial \sum_{t=1}^{2} E\Pi_{A,t}(p_A, p_B)/\partial p_A = 2\left((Ey_A)_{(\upsilon,\upsilon)} + p_A \frac{d(Ey_A)_{(\upsilon,\upsilon)}}{d\upsilon}\frac{\partial \widetilde{\upsilon}}{\partial p_A}\right)$. At a symmetric equilibrium, $\upsilon = 1/2$ and $p_A = p_B = p$: hence $\partial \sum_{t=1}^{2} E\Pi_{A,t}(p_A, p_B)/\partial p_A = 0$ yields $p = 1/2$, as with the static BS.

[7] By this terminology Binmore (1992, pp. 536-540) refers to a weakened version of Kreps and Wilson's (1982) "sequential equilibrium."

[8] For instance, if k is conjectured to have obeyed Θ^{**} in $t = 1$, $\mu(Ar_{k,1} \mid I_h) = \frac{\widetilde{\widetilde{\upsilon}}}{2-\widetilde{\widetilde{\upsilon}}}$ and $\mu(Bs_{k,1} \mid I_h) = \frac{2-2\widetilde{\widetilde{\upsilon}}}{2-\widetilde{\widetilde{\upsilon}}}$ if $\frac{3p_B-1}{2} \leq p_A \leq \frac{1+2p_B}{3}$, $\mu(Bs_{k,1} \mid I_h) = 1$ if $(1 + 2p_B)/3 < p_A$, and $\mu(Ar_{k,1} \mid I_h) = 1$ if $p_A < (3p_B - 1)/2$.

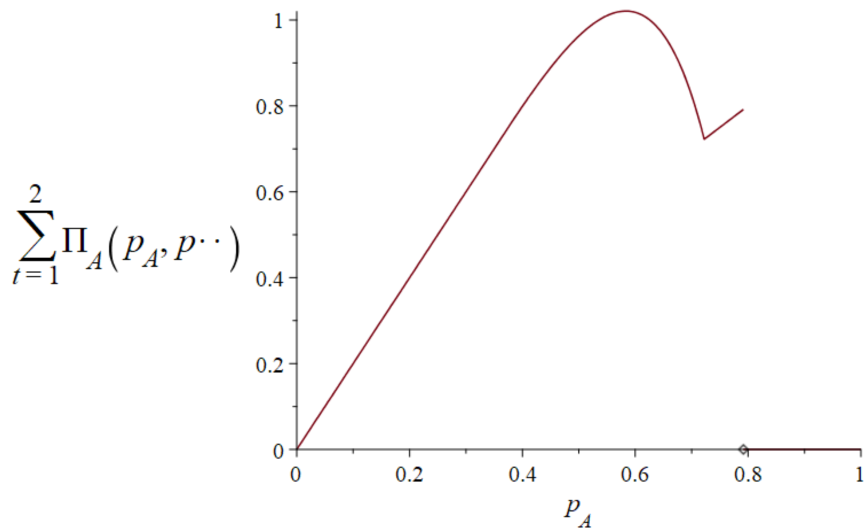

Figure 1: *The curve shows A's payoff function for* $p_B = p^{**} = 7/12 = 0.58\overline{3}$*, conditional on the equilibrium* $(\Theta^{**}, \Theta^{**})$ *of the BS.*

(ii) For any $p_B \in (0;1)$,[9] with $(\Theta_h, \Theta_k) = (\Theta^{**}, \Theta^{**})$ A's payoff is

$$\sum_{t=1}^{2} E\Pi_{A,t}(p_A, p_B) = \begin{cases} 2p_A & \textit{if} \quad 0 \le p_A < \frac{3p_B-1}{2} \\ p_A \left(Ey_A\right)_{(\widetilde{\widetilde{v}},\widetilde{\widetilde{v}})} + p_A & \textit{if} \quad \max\left\{0, \frac{3p_B-1}{2}\right\} \le p_A \le \frac{1+2p_B}{3}, \\ p_A & \textit{if} \quad \frac{1+2p_B}{3} < p_A < \frac{1+p_B}{2} \\ 0 & \textit{if} \quad p_A \ge \frac{1+p_B}{2}, \end{cases} \tag{11}$$

a continuous function of p_A for $p_A \in \left[0, \frac{1+p_B}{2}\right)$. Over the range $\left(\max\left\{0, \frac{3p_B-1}{2}\right\}, \frac{1+2p_B}{3}\right)$, $\frac{\partial \sum_{t=1}^{2} E\Pi_{A,t}(p_A,\cdot)}{\partial p_A} = (Ey_A)_{(v,v)} + p_A \frac{d(Ey_A)_{(v,v)}}{dv} \frac{\partial \widetilde{\widetilde{v}}}{\partial p_A} + 1$, which is positive on a right neighbourhood of $\max\left\{0, \frac{3p_B-1}{2}\right\}$ and negative on a left neighbourhood of $\frac{1+2p_B}{3}$, while $\frac{\partial^2 \sum_{t=1}^{2} E\Pi_{A,t}(p_A,\cdot)}{\partial p_A^2} < 0$:[10] thus, in that range, $\sum_{t=1}^{2} E\Pi_{A,t}(p_A, \cdot)$ has a unique, internal maximum. $\sum_{t=1}^{2} E\Pi_{A,t}(p_A, \cdot)$ is kinked at $p_A = \frac{1+2p_B}{3}$ [11] and increasing for $p_A \in \left(\frac{1+2p_B}{3}, \frac{1+p_B}{2}\right)$. At a symmetric equilibrium, $(p_A, p_B) = (p^{**}, p^{**})$ $(p^{**} = 7/12)$. (In fact, $p^{**} = \arg\max \sum_{t=1}^{2} E\Pi_{A,t}(p_A, p^{**})$ since, while $\partial \sum_{t=1}^{2} E\Pi_{A,t}(p_A, p^{**})/\partial p_A > 0$ for $p_A \in \left(\frac{1+2p^{**}}{3}, \frac{1+p^{**}}{2}\right)$, $\sum_{t=1}^{2} E\Pi_{A,t}(p^{**}, p^{**}) = 49/48 > \lim_{p_A \to \frac{1+p^{**}}{2}^-} \sum_{t=1}^{2} E\Pi_{A,t}(p_A, p^{**}) = \frac{1+p^{**}}{2} = 38/48$ (see Figure 1). ■

To see why CL raises the firm's market power, note that $\frac{\partial \sum_{t=1}^{2} E\Pi_{A,t}(p_A,p_B)}{\partial p_A} |_{(p_A,p_B)=(p,p)}$ reads

[9] One can easily prove that, with $(\Theta_h, \Theta_k) = (\Theta^{**}, \Theta^{**})$, $p_i = 1$ (as well as $p_i = 0$) is never a best response.

[10] $\frac{\partial^2 \sum_{t=1}^{2} E\Pi_{A,t}(p_A,\cdot)}{\partial p_A^2} = 2\frac{d(Ey_A)_{(v,v)}}{dv}\frac{\partial \widetilde{\widetilde{v}}}{\partial p_A} + p_A \frac{d^2(Ey_A)_{(v,v)}}{dv^2}\left(\frac{\partial \widetilde{\widetilde{v}}}{\partial p_A}\right)^2 + p_A \frac{d(Ey_A)_{(v,v)}}{dv}\frac{\partial^2 \widetilde{\widetilde{v}}}{\partial p_A^2}$, with $\frac{\partial \widetilde{\widetilde{v}}}{\partial p_A} = -\frac{5(1-p_B)}{(2-p_A-p_B)^2} < 0$ and $\frac{\partial^2 \widetilde{\widetilde{v}}}{\partial p_A^2} = \frac{-10(1-p_B)}{(2-p_A-p_B)^3} < 0$.

[11] $\lim_{p_A \to \left(\frac{1+2p_B}{3}\right)^-} \frac{\partial \sum_{t=1}^{2} E\Pi_A(\cdot)}{\partial p_A} = \frac{-1-17p_B}{5(1-p_B)} < 0 < \lim_{p_A \to \left(\frac{1+2p_B}{3}\right)^+} \frac{\partial \sum_{t=1}^{2} E\Pi_A(\cdot)}{\partial p_A} = 1$.

$2\left(Ey_A\right)_{(1/2,1/2)} + 2p\frac{d(Ey_A)_{(\upsilon,\upsilon)}}{d\upsilon}\mid_{\upsilon=\frac{1}{2}} \times \frac{\partial\widetilde{\upsilon}}{\partial p_A}\mid_{(p_A,p_B)=(p,p)} = \left(2\times\frac{3}{4}\right) + \left(-\frac{3}{2}\frac{p}{1-p}\right)$ under equilibrium (Θ^*,Θ^*) and $\left(Ey_A\right)_{(1/2,1/2)} + 1 + p\frac{d(Ey_A)_{(\upsilon,\upsilon)}}{d\upsilon}\mid_{\upsilon=\frac{1}{2}} \times \frac{\partial\widetilde{\widetilde{\upsilon}}}{\partial p_A}\mid_{(p_A,p_B)=(p,p)} = \left(\frac{3}{4}+1\right) + \left(-\frac{5}{4}\frac{p}{1-p}\right)$ under equilibrium $(\Theta^{**},\Theta^{**})$. Now, $\frac{3}{4}+1 > 2\times\frac{3}{4}$ and $-\frac{5}{4}\frac{p}{1-p} > -\frac{3}{2}\frac{p}{1-p}$: either inequality follows since $(\Theta^{**},\Theta^{**})$ implies full capacity utilization in $t=2$. Thus, the incentive to unilaterally increasing p_A is higher under equilibrium $(\Theta^{**},\Theta^{**})$ and, as a consequence, $\frac{\partial\sum_{t=1}^{2}E\Pi_{A,t}(p_A,p_B)}{\partial p_A} > 0$ at $(p_A,p_B)=(p^*,p^*)$.

4 Conclusion

Our duopolistic price game with a two-buyer dynamic BS provides two main insights. First, even with product homogeneity, repeat purchasing decisions over the time period in which prices are fixed creates an incentive for conditional loyalty. Quite remarkably, this incentive arises even with no service priority to loyal customers and with imperfect information on other buyers' previous moves. Second, the equilibrium of the BS exhibiting conditional loyalty does affect the firm's market power. Whether similar results arise in more general models is an issue that we leave to future research. One might check whether a (properly defined) strategy incorporating conditional loyalty is again part of an "assessment equilibrium" of the BS when n buyers are playing a multistage BS in the face of m sellers;[12] furthermore, one might explore how such an equilibrium would affect pricing as well as entry and capacity decisions.

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APPENDIX

Proof of Prop. 3. In BS*s* where system (9) holds, $U_h(\Theta^{**},\Theta^{**}) = \sum_{t=1}^{2} u_{h,t}(\Theta^{**},\Theta^{**})$, where

$$u_{h,1}(\Theta^{**},\Theta^{**}) = \left(\frac{\widetilde{\widetilde{\upsilon}}^2}{2} + \widetilde{\widetilde{\upsilon}}(1-\widetilde{\widetilde{\upsilon}})\right)(1-p_A) + \left(\frac{(1-\widetilde{\widetilde{\upsilon}})^2}{2} + (1-\widetilde{\widetilde{\upsilon}})\widetilde{\widetilde{\upsilon}}\right)(1-p_B), \tag{12}$$

and[13]

$$\begin{aligned}
u_{h,2}(\Theta^{**},\Theta^{**}) = \sum_{e_{h,1}\in\mathcal{E}_{h,1}} \pi\left(e_{h,1}\right)_{(\upsilon^{**}_{h,1},\upsilon^{**}_{k,1})} u_{h,2}(\Theta^{**},\Theta^{**} \mid (e_{h,1},\cdot)) = \\
\pi\left(As_{h,1}\right)_{(\upsilon^{**}_{h,1},\upsilon^{**}_{k,1})} u_{h,2}(\Theta^{**},\Theta^{**} \mid (As_{h,1},\cdot)) + \\
\pi\left(Ar_{h,1}\right)_{(\upsilon^{**}_{h,1},\upsilon^{**}_{k,1})} u_{h,2}(\Theta^{**},\Theta^{**} \mid (Ar_{h,1},\cdot)) + \\
\pi\left(Bs_{h,1}\right)_{(\upsilon^{**}_{h,1},\upsilon^{**}_{k,1})} u_{h,2}(\Theta^{**},\Theta^{**} \mid (Bs_{h,1},\cdot)) + \\
\pi(Br_{h,1})_{(\upsilon^{**}_{h,1},\upsilon^{**}_{k,1})} u_{h,2}(\Theta^{**},\Theta^{**} \mid (Br_{h,1},\cdot)) = \\
\left(\frac{\widetilde{\widetilde{\upsilon}}^2}{2} + \widetilde{\widetilde{\upsilon}}(1-\widetilde{\widetilde{\upsilon}})\right)[1-p_A] + \frac{\widetilde{\widetilde{\upsilon}}^2}{2}[1-p_B] + \\
\left(\frac{(1-\widetilde{\widetilde{\upsilon}})^2}{2} + (1-\widetilde{\widetilde{\upsilon}})\widetilde{\widetilde{\upsilon}}\right)[1-p_B] + \frac{(1-\widetilde{\widetilde{\upsilon}})^2}{2}[1-p_A].
\end{aligned} \tag{13}$$

If Θ_h involves a deviation only in $t = 1$ (i.e., $\upsilon_{h,1} \neq \widetilde{\widetilde{\upsilon}}$), $U_h(\Theta_h,\Theta^{**}) = U_h(\Theta^{**},\Theta^{**})$ (see Remark 3). If Θ_h involves a deviation only in $t = 2$, some of the following hold:

[13] For instance, in Eq. (13), $\pi\left(As_{h,1}\right)_{(\upsilon^{**}_{h,1},\upsilon^{**}_{k,1})} = \left(\frac{\widetilde{\widetilde{\upsilon}}^2}{2} + \widetilde{\widetilde{\upsilon}}(1-\widetilde{\widetilde{\upsilon}})\right)$ since $As_{h,1} = (As_{h,1} \cap Ar_{k,1}) \cup (As_{h,1} \cap Bs_{k,1})$; also, $u_{h,2}(\Theta^{**},\Theta^{**} \mid (As_{h,1},\cdot)) = 1-p_A$ is buyer h's stage-2 payoff, under strategy profile $(\Theta^{**}\Theta^{**})$, conditional on $(As_{h,1},\cdot)$.

$\upsilon_{h,2}(As_{h,1},\cdot)<1$, $\upsilon_{h,2}(Ar_{h,1},\cdot)>0$, $\upsilon_{h,2}(Bs_{h,1},\cdot)>0$, $\upsilon_{h,2}(Br_{h,1},\cdot)<1$. Then:

$$u_{h,2}(\Theta_h,\Theta^{**})=\sum_{e_{h,1}\in\mathcal{E}_{h,1}}\pi\left(e_{h,1}\right)_{(\upsilon^{**}_{h,1},\upsilon^{**}_{k,1})}u_{h,2}(\Theta_h,\Theta^{**}\mid(e_{h,1},\cdot))=$$

$$\left(\frac{\widetilde{\widetilde{\upsilon}}^2}{2}+\widetilde{\widetilde{\upsilon}}(1-\widetilde{\widetilde{\upsilon}})\right)\left[\upsilon_{h,2}(As_{h,1},\cdot)(1-p_A)+(1-\upsilon_{h,2}(As_{h,1},\cdot))\,\frac{1-p_B}{2}\right]+ \tag{14}$$

$$\frac{\widetilde{\widetilde{\upsilon}}^2}{2}\left[\upsilon_{h,2}(Ar_{h,1},\cdot)\frac{1-p_A}{2}+(1-\upsilon_{h,2}(Ar_{h,1},\cdot))(1-p_B)\right]+$$

$$\left(\frac{(1-\widetilde{\widetilde{\upsilon}})^2}{2}+(1-\widetilde{\widetilde{\upsilon}})\widetilde{\widetilde{\upsilon}}\right)\left[\upsilon_{h,2}(Bs_{h,1},\cdot)\frac{1-p_A}{2}+(1-\upsilon_{h,2}(Bs_{h,1},\cdot))(1-p_B)\right]+$$

$$\frac{(1-\widetilde{\widetilde{\upsilon}})^2}{2}\left[\upsilon_{h,2}(Br_{h,1},\cdot)(1-p_A)+(1-\upsilon_{h,2}(Br_{h,1},\cdot))\,\frac{1-p_B}{2}\right], \tag{15}$$

Since system (9) holds, system (2′) a fortiori holds (see Remark 1). As a consequence, $u_{h,2}(\Theta_h,\Theta^{**}\mid(e_{h,1},\cdot))<u_{h,2}(\Theta^{**},\Theta^{**}\mid(e_{h,1},\cdot))$ at any $(e_{h,1},\cdot)$ where Θ_h deviates from Θ^{**}. Let, for instance, $\upsilon_{h,2}(As_{h,1},\cdot)<1$. Then $u_{h,2}(\Theta_h,\Theta^{**}\mid(As_{h,1},\cdot))=\upsilon_{h,2}(As_{h,1},\cdot)(1-p_A)+(1-\upsilon_{h,2}(As_{h,1},\cdot))\,\frac{1-p_B}{2}<u_{h,2}(\Theta^{**},\Theta^{**}\mid(As_{h,1},\cdot))=1-p_A$, since $p_A<\frac{1+p_B}{2}$.

Next, let system (2′), but not system (9), hold:[14] for instance, $\frac{1+2p_B}{3}<p_A<\frac{1+p_B}{2}$, so that Θ^{**} prescribes $\upsilon^*_{h,1}=0$ and CL. The argument remains essentially unaltered if Θ_h entails a one-stage deviation in $t=2$. With a one-stage deviation in $t=1$, $\sum_{t=1}^{2}u_{h,t}(\Theta_h,\Theta^{**})=\left[\upsilon_{h,1}(1-p_A)+(1-\upsilon_{h,1})\frac{1-p_B}{2}\right]+\left[\upsilon_{h,1}(1-p_A)+(1-\upsilon_{h,1})\left(\frac{1}{2}(1-p_B)+\frac{1}{2}(1-p_A)\right)\right]$, less than $\sum_{t=1}^{2}u_{h,t}(\Theta^{**},\Theta^{**})=\frac{1-p_B}{2}+\left[\frac{1}{2}(1-p_B)+\frac{1}{2}(1-p_A)\right]$ since $\frac{1+2p_B}{3}<p_A$.

Finally, one can easily check that a two-stage deviation from Θ^{**} is not rewarding (the "one-stage deviation property" holds). ■

[14] In BSs where system (2') does not hold, Θ^{**} makes the same prescriptions as Θ^*.