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The measurement of a single-mode thermal field with a microwave cavity parametric amplifier

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Abstract. In this paper, we present the experimental study of a single-mode thermal field carried out using a microwave parametric amplifier tuned at 1.5 GHz and working at room temperature. The parametric amplifier is based on a variable capacitance diode placed inside a microwave resonant cavity. The measured distribution of the thermal photons inside the resonator follows the expected Bose–Einstein distribution probability.

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Contents

1. Introduction

In our effort to study the dynamical Casimir effect [1, 2], we have built a parametric amplifier based on a microwave resonant cavity whose proper frequency can be modulated by means of a variable capacitance diode (varicap). By a proper tuning of the varicap, an extremely large gain can be obtained, thus allowing one to measure the very small amounts of thermal energy contained in a very narrow frequency band.

The properties of thermal radiation are generally described by applying the laws of statistical mechanics to the radiation in equilibrium at temperature T [3]. The radiation consists of a continuum spectrum of modes whose energy density is described by Planck's formula. When analyzing radiation inside a high-Q resonator, we have to consider that only a single mode of the field is possible, i.e. only photons with energy E around $h\nu_r$, with ν_r the resonance frequency of the cavity, are present. When the mode is populated by n photons, the energy of the mode is $E_n = (n + 1/2)h\nu_r$. The probability $P_\nu(n)$ of finding n photons in the cavity is then

$$P_\nu(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n, \qquad (1)$$

where $\bar{n}$ is the mean photon number

$$\bar{n} = [\exp(h\nu_r/k_B T) - 1]^{-1}. \qquad (2)$$

Equation (1) is the Bose–Einstein distribution, having the peculiar characteristic of being super-Poissonian, with variance $(\Delta n)^2 = \bar{n}(1+\bar{n})$.

The Planck spectrum has been measured several times with extremely high accuracy, the most remarkable measurement being the cosmic microwave background [4]. This is not the case for the single-mode photon distribution, described by (1), since when selecting a very narrow frequency band the number of photons is reduced below detectability. On the other hand, enlarging the measurement band will soon spoil the single mode, reducing its variance as $(\Delta n)^2 = \bar{n}\,(1+\bar{n}/\mu)$, with μ the number of modes. These are the reasons why only a few measurements of the Bose–Einstein distribution have been made [5, 6], but with an average number of photons $\bar{n}{\sim}1$ and with apparatuses that can work only in a very limited temperature range.

In this paper, we present our experimental study of a super-Poissonian single-mode thermal field with $\bar{n}{\sim}10^3$. The measured distribution of the thermal photons inside the resonator follows the distribution probability (1). This measurement was possible due to a novel type of parametric amplifier having an extremely large amplitude gain and high frequency selectivity.

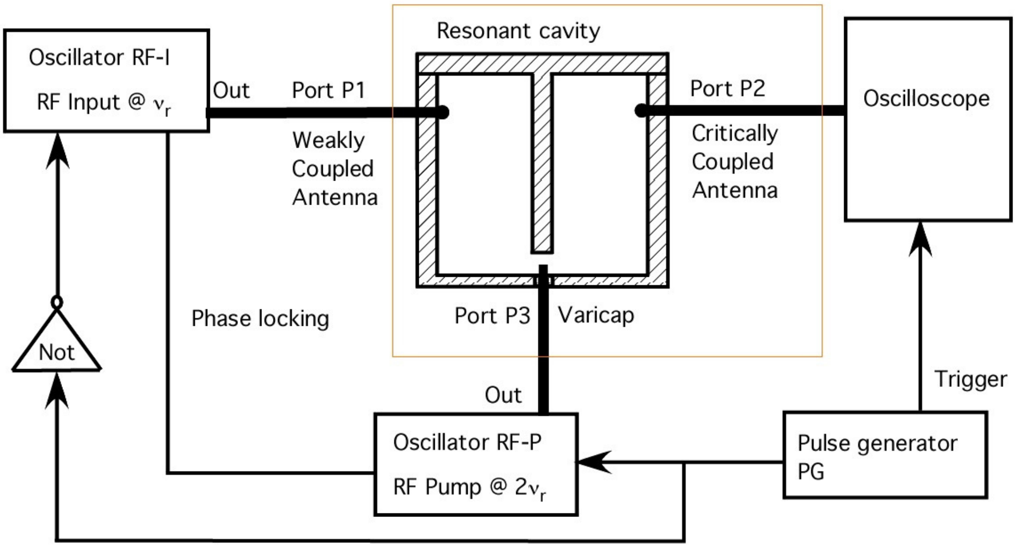

Figure 1. The principle scheme of the microwave parametric amplifier. The copper resonant cavity is kept inside a cryogenic system that can be used both at liquid nitrogen and at liquid helium. The varicap (Macom Ma 46470-91) is a cylinder with about 2.4 mm height and 1 mm diameter.

2. The experimental apparatus

The principle experimental scheme of the apparatus is shown in figure 1. It is a much improved version of the waveguide parametric amplifiers built mainly in the 1960s [7]. A detailed description will be given elsewhere [8]. It is based on a reentrant geometry microwave resonant cavity with circular section, with a diameter of 42 mm and a height of 50 mm. Its resonance frequency ν_r is about 1.5 GHz, with a quality factor $Q = \pi \nu_r \tau_c \approx 200$ (τ_c is the field decay time). The reentrant cylinder, having a diameter of 4 mm, leaves a 5.2 mm gap in front of one of the cavity ends. This gap almost entirely determines the capacitance C_r of the equivalent circuit of the resonator. In front of the reentrant cylinder, a varicap is mounted, with a zero bias capacity of 1.5 pF. The varicap can be seen as a capacitance added in series to C_r to give a total capacitance C'_r and hence a different value of the resonance frequency. By driving the varicap with an ac voltage it is thus possible to periodically modulate the resonance frequency of the system, and the modulation depth strongly depends on the position of the varicap itself inside the resonator.

By using the three ports P1, P2 and P3, it is possible to charge the cavity, to measure the stored energy and to drive the capacitance of the varicap, respectively. Port P1 holds a weakly coupled antenna with a coupling $k_1 \sim -37$ dB. The antenna is connected to the oscillator RF-I used to charge up the microwave cavity in the calibration measurements. Port P2 holds a second antenna whose position can be varied to obtain critical coupling. This antenna is connected to the 50 Ω input of a 6 GHz bandwidth oscilloscope. If the energy in the cavity is constant, the antenna delivers a stationary signal onto the oscilloscope. An ac signal applied by the oscillator RF-P through port P3 is used to drive the varicap. The two oscillators RF-I and RF-P are gated by the pulse generator (PG); the relative phase of their outputs is controlled by a phase locking system, permitting a complete characterization of the parametric amplification process also at very large gain.

By driving the varicap at a frequency $\approx 2\nu_r$, a parametric amplification of the energy stored in the resonator can be realized. In this case, the signal on the scope has an exponential-like growth. The growth rate depends on the efficiency of the parametric process, and deviates toward saturation in a short time. As we do not work in a saturation regime, the oscillator RF-P is activated only for a limited amount of time. We study the time behavior of the amplitude V_a at frequency ν_r of the antenna signal on port P2 when the pump RF-P is present. The output power is then $P_{out} = V_a^2/(2R_g)$, for a corresponding energy in the cavity

$$E_{cavity} = \frac{1}{k_2} P_{out} \tau_c = \frac{1}{k_2} \frac{V_a^2 \tau_c}{2R_g}. \qquad (3)$$

For a critically coupled antenna the coefficient k_2 is equal to unity. Analogously, for the input port, the energy loaded in the cavity is $E_{cavity} = P_{in}\tau_c/k_1$, with P_{in} the power delivered by the oscillator RF-I.

The general trigger for a measurement is the leading edge of a square pulse of the PG; this is a TTL signal with a duration of about 10 μs that enables the output of the pump oscillator RF-P and starts the acquisition by the oscilloscope. When performing calibrations with a known input on port P1, the oscillator RF-I is switched off at the start of the pump signal in order not to have a stationary input in the cavity during amplification. The acquisition system of the oscilloscope limits the repetition rate of the measurements to about 10 Hz, which ensures also that the cavity is again in a stationary state next time the pump is switched on.

3. The amplification process

Let us consider a resonant microwave cavity with unperturbed resonance frequency $\nu_r = \omega_r/2\pi$, quality factor Q, and field decay time $\tau_c = 1/\lambda$. To produce a parametric amplification process, we modulate the resonance frequency following the relation

$$\nu_r(t) = \nu_r[1 + \eta \cos 2(\omega_r t + \theta)], \qquad (4)$$

where η is the modulation depth, $t = 0$ is the time at which the modulation begins and 2θ the pump initial phase. In the absence of dissipation, the evolution of the electromagnetic field amplitude inside the resonator will exhibit an exponential growth of the type e^{st} [9], with rate $s = \pi\eta\nu_r$. In the presence of dissipation, the rate will be diminished by an amount λ. The power output measured by the antenna on port P2 can be described approximately by the following equation:

$$\begin{aligned} P_{out}(t) &\approx P_{out}(0) e^{t/\tau_p} g(\theta - \phi) \\ &\approx P_{out}(0) e^{2(s-\lambda)t} g(\theta - \phi) \quad \text{for } t > 0, \end{aligned} \qquad (5)$$

where ϕ is the phase of the cavity field at $t = 0$ and the factor two in the exponent is due to the change from field to power. In the following, we present evidence that the phase function $g(\theta - \phi)$ can be written simply as $\cos^2(\theta - \phi + \text{constant})$. The time constant $\tau_p = [2(s - \lambda)]^{-1}$ is the characteristic time of the amplification process. The exponential growth in equation (5) cannot last for ever, and the amplitude of the field saturates to a value related to the physical characteristics of the electrical components. For our purposes, the saturation part of $P_{out}(t)$ is not used.

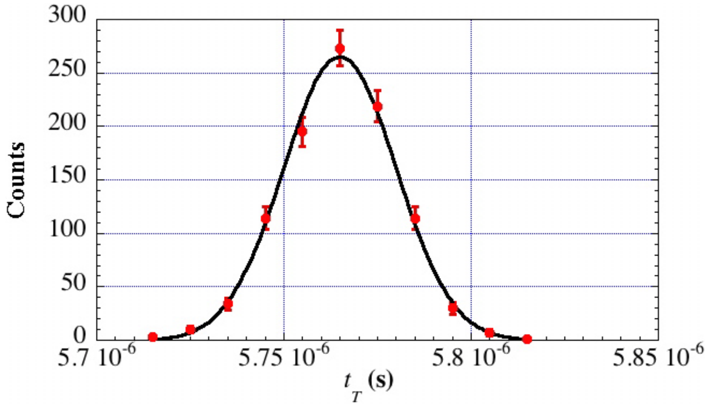

Figure 2. Histogram of 1000 values of t_T for a fixed value of P_{in}, with $P_T = 25\,\mu\text{W}$, and having maximized the phase function $g(\theta - \phi)$. The fitting curve is a Gaussian function, showing a standard deviation of 15 ns with an average value of 5.765 μs.

Measurements are made as follows: an arbitrary value P_T is chosen so that the saturation part of $P_{\text{out}}(t)$ begins well after this value is reached. The time t_T, at which $P_{\text{out}}(t_T) = P_T$, determines the amplifier gain, and in principle it is possible to deduce the initial amplitude $P_{\text{out}}(0)$ by just reversing equation (5). Unfortunately, due to the presence of the phase function $g(\theta - \phi)$, this is not always feasible: while the initial phase θ can be chosen at will, the phase ϕ can only be determined when the cavity field is driven using the oscillator RF-I on port P1, thanks to the phase locking between the input signal and the pump. In this case, the phase ϕ can be easily tuned and the function $g(\theta - \phi)$ maximized: the value of P_{out} is then uniquely determined by P_{in} through relation (3). The time t_T is

$$t_T = \tau_{\text{p}} \left[\ln \frac{P_T}{P_{\text{out}}(0)} - \ln g(\theta - \phi) \right]. \tag{6}$$

In figure 2 a histogram of 1000 values of t_T is shown. These measurements are made with a constant input P_{in}, having maximized the phase function $g(\theta - \phi)$, and with $P_T = 25\,\mu\text{W}$. Making several measurements like those of figure 2 at different values of P_{in} allows for a precise determination of the characteristic time τ_{p} of the parametric amplification process. The results of such measurements are shown in figure 3, fitted using (6). The resulting value is $\tau_{\text{p}} = (13.9 \pm 0.2)$ ns. The parameter τ_{p} gives the temporal difference in t_T between two power inputs whose ratio is e; its value does not depend on the choice of the experimental parameters P_T, k_1 and k_2.

As can be seen from figure 3, the time t_T is stable for input power below a certain value: this indicates that inside the resonator there is a source that cannot be switched off. This source has to be identified with the thermal bath. We will see below that it is possible to determine the energy spectrum and phase distribution of the signal assuming a thermal source as the input. Before doing this, we have to study the phase function $g(\theta - \phi)$. In fact, when doing calibrations, the phase is adjusted to give a maximum of the phase function, but when the parametric process acts upon a signal with random phase, the effect of the function $g(\theta - \phi)$ must be taken into account. In order to resolve this point, measurements of t_T have been made keeping P_{in} fixed and varying the phase ϕ of the oscillator RF-I. These results are shown in figure 4. The experimental points have been fitted through equation (6) using $g(\theta - \phi) = \cos^2(\theta - \phi)$,

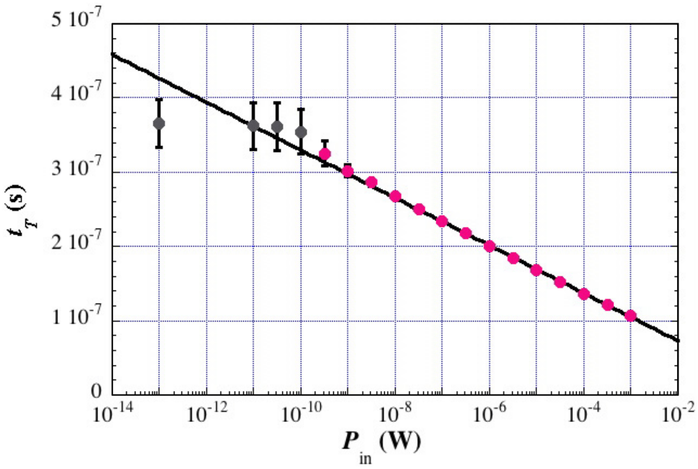

Figure 3. Calibration of the gain parameter τ_p. For all the data $P_T = 25\,\mu$W. The four points on the left were not used in the fit. We find that $\tau_p = (13.9 \pm 0.2)$ ns.

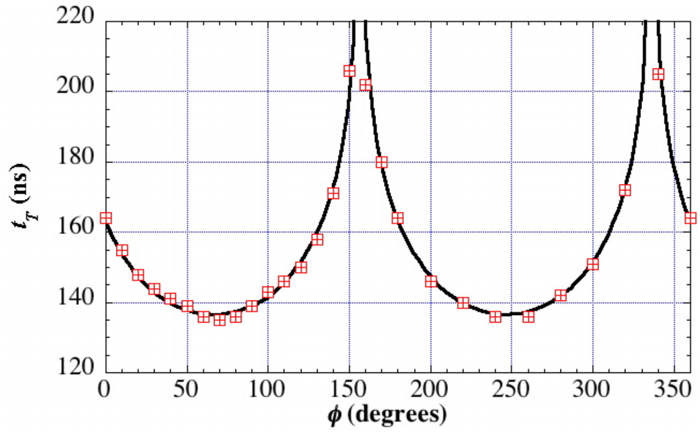

Figure 4. Determination of the phase function. Experimental points are fitted using the function $\tau_p \ln \cos^2(\theta - \phi) + \text{constant}$. A value $\tau_p = (14.3 \pm 1.1)$ ns is found.

leaving θ as a free parameter. The result of the fit is good, with a reduced $\chi^2 = 0.1$, and the gain parameter obtained, $\tau_p = (14.3 \pm 1.1)$ ns, is in agreement with that of the calibration (see figure 3). The results prove the validity of the approximation used in equation (5), i.e. the parametric process is approximated well by an exponential growth. The total gain can reach very large values: for example, in the leftmost data point in figure 3, the initial output power is about $P_{\text{out}}(0) = 1 \times 10^{-13}$ W for a total gain of about 2.5×10^8.

From the measured value of τ_p, we can obtain the modulation depth η. Knowing that $\tau_c = 40$ ns, we obtain $s \approx 6 \times 10^7$ Hz and hence $\eta \approx 1.2 \times 10^{-2}$. This value was obtained with a 13 dBm output level on the oscillator RF-P. Within our setup it is not possible to measure the exact value of the ac component V_{pump} of the RF driving voltage of the varicap, essentially due to the difficulty in obtaining correct impedance matching between the varicap and the feeding line, with the result that most of the driving power is actually reflected. The linearity of η on the pump was roughly verified for small variations of V_{pump}. We also verified that the

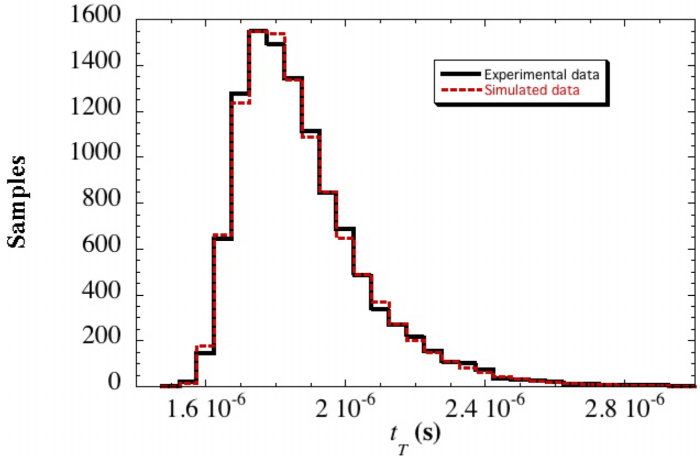

Figure 5. (Continuous line) 11 000 measurements of t_T, taken at room temperature, with no input on cavity port P1 and $P_T = 100\ \mu$W. (Dashed line) MC simulation of the data assuming a single-mode thermal spectrum in the cavity at $t = 0$. In the simulation the only free parameter is a common time t_0 added to all the generated values.

parametric process is present only when the driving frequency has values in a narrow interval centered on $2\nu_r$.

4. The thermal photon bath

As seen above, the parametric amplification process takes place and reaches saturation even when the input power P_{in} is reduced to zero. In this case, the distribution of the time t_T is no longer Gaussian and has greater variance. In figure 5 the continuous line is a histogram of 11 000 values of t_T measured at room temperature with the oscillator RF-I kept constantly off; for these data the characteristic time obtained with the calibration procedure is $\tau_p = (84.0 \pm 1.2)$ ns, with a cavity decay time $\tau_c = 18$ ns (cavity Q is reduced in this measurement owing to a change of the antenna geometry to optimize coupling). After the oscillator RF-P is turned on, the system reaches the value P_T in a time dependent on the energy present in the cavity and on the phase ϕ of the internal field. We will show that the spectrum reflects the presence of the single-mode thermal photon distribution described by relation (1). To this end, we performed a Monte Carlo (MC) simulation of the parametric process: we generated 1.1×10^6 values of t_T assuming that the energy present in the cavity at the starting trigger has the probability distribution given by equation (1) and that the phase ϕ has a flat distribution in the interval $[0 - 2\pi]$. The generated values are arranged in a histogram as the measured data, rescaling for the total number of samples (dashed line in figure 5). One can say that the simulation is done without free parameters: τ_p is measured in the calibration procedure, τ_c comes from the cavity characterization, k_2 is measured with a vector analyzer, P_T is a common choice both for calibration and thermal data, and finally $\bar{n} = 4164$ is determined from equation (2) with $T = 300$ K. By adding a value t_0 to all the generated times, it is possible to superimpose the simulated and the experimental data. It is clear that adding a constant value to all the points does not affect the shape of the distribution, which reproduces the experimental data quite satisfactorily. One reason for the presence of the constant t_0 can be found in the uncertainty on the parameter k_2. An error in this parameter results exactly in a constant added to t_T (cf (3) and (6)). A χ^2 test for the simulated and the real data gives a total $\chi^2 = 27$ for the first 29 bins of the histogram, thus indicating that the chosen hypotheses are consistent with the experimental data.

In order to check that the distribution is due to a single-mode thermal field, we repeated the analysis assuming for the initial field a thermal distribution with a number of modes equal to μ [10]:

$$P_\nu(n) = \frac{(n+\mu-1)!}{(\mu-1)!n!}\frac{1}{[1+\bar{n}/\mu]^{\mu}[1+\mu/\bar{n}]^{n}}. \quad (7)$$

Already with $\mu = 2$, i.e. supposing to have two field modes resonating in the cavity, it was not possible to obtain a good description of the experimental data, with the χ^2 in excess of 100. This proves that no standard distribution but the Bose–Einstein can be used to fit the data.

The width of the curve of figure 5 depends strongly on the type of energy distribution in the cavity and, given that, mainly on the gain parameter τ_p. In the case of the distribution of equation (1) such a width changes very slightly with temperature. This is related to the fact that the Bose–Einstein distribution can always be approximated by an exponential curve if the average photon number is much larger than 1. This very slight dependence of the shape on temperature and the presence of t_0 seem to put a limit on the determination of the absolute temperature of the thermal photon bath inside the resonator from measured values of t_T, at least for temperatures well above absolute zero.

The system has also been tested at cryogenic temperatures, with the cavity placed in a cryostat filled with either liquid nitrogen ($T \approx 77$ K) or liquid helium ($T \approx 5$ K). The operation of the varicap is not prevented by low temperature and the measured spectra are fitted well by following the same procedure used for the room temperature one. However, due to the uncertainties present in the determination of the temperature of the photon bath, it was not possible to verify whether the measured spectra were due to photons in thermal equilibrium with the cavity wall or to some heat leakage into the cavity, for example through the critically coupled antenna. We are currently working to solve this problem.

5. Conclusions

In conclusion, we have built a parametric amplifier based on a microwave resonant cavity with the resonance frequency driven by a varicap. With this amplifier, we have studied the thermal field inside the cavity. By using MC simulation, we have reproduced the experimental result with the hypothesis of a single-mode thermal field inside the resonator, described by a Bose–Einstein-type probability distribution. The description of the energy spectrum inside the cavity in terms of the Bose–Einstein distribution is a proof of the quantum behavior of thermal radiation. Moreover, the energy spectrum is measured at room temperature, i.e. with a large number of photons ($\sim 10^3$). Other proofs of the quantum character of thermal radiation at high temperature were obtained by measuring the thermal corrections of the Casimir [11] and Casimir–Polder forces [12].

The system presented in this paper proves to be a powerful and versatile tool for the measurement of very small signals. Our result could pave the way for the measurement of the dynamical Casimir effect [1, 2, 13, 14], finding also possible applications in the studies of the statistics of photons emitted by conductors [15] and of squeezed thermal radiation [16].

Acknowledgments

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