UNIVERSITÀ DEGLI STUDI DI SIENA

QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA E STATISTICA

Mauro Caminati

Self sustaining R&D networks

n. 653 – Ottobre 2012

Abstract - This paper contributes to the knowledge-incentive based explanation of R&D networks. It argues that knowledge overlap and novelty are complementary inputs of any R&D alliance, and the complementarity coefficients depend on the incremental or radical nature of the research activity. The relation between the specialization of knowledge endowments and the structural properties (clustering, components, asymmetry of link distribution) of the resulting incremental, radical and mixed research networks are investigated. The paper addresses the structural conditions enabling the persistence of a research network through time.

Jell Classification: D85,O30

Keywords: R&D collaboration, specialized knowledge endowment, complementarity between novelty and overlap, pairwise equilibrium, self-sustaining network.

Mauro Caminati, Dipartimento di Economia Politica e Statistica, Università di Siena

1 Introduction

This paper suggests a unified, knowledge-based explanation of inter-organizational R&D collaboration, and its network properties, which are revealed by a growing body of evidence especially in fields of activity where the pace of technological progress is faster (Hagedoorn 2002 [29], Powell et al. 2005 [47], Hagedoorn and Roijakkers 2006 [30]). Our explanation of short-run network formation is a generalization of Cowan and Jonard 2009 [12], in that collaboration incentives depend on the pairwise matchings, and mis-matchings, between the knowledge repertoires of the organization concerned. A dynamic extension is oultined, which identifies necessary conditions for network persistence.

The model recovers stylized properties of R&D networks, which are recurrent in the field studies[1] carried out at a large scale of analyis which can be a industry, a set of industries, a group of related disciplinary fields or application domains.

The network is organized around 1 giant connected component. The architecture of links within the giant component is not uniform, but consists of more densly connected communities of nodes, which are related by sparse between-community ties. This produces a short average and maximum relational distance[2] between the nodes. On average, the fraction of a node's neighbours that are each-other neighbours (average clustering coefficient), is high, at least compared to random networks. The degree distribution, showing how the fraction of nodes relates to their number of links, is asymmetric. Most nodes have a smaller than average degree, and a small, but non negligible fraction of nodes has a very large number of connections. This makes the degree distribution positively (right) skewed[3]. Such properties are often synthetically referred to with the statement that R&D networks conform to the small world structure (Watts 1999 [58], Jackson and Rogers 2005 [23]).[4]

[1] A number of studies are most relevant in the present framework. Schilling and Phelps (2007) [51] emphasize some large-scale properties of 11 different industry-level alliance networks in the U.S.; Powell et al. (2005) [47] study nework structure and dynamics in the life sciences; Gisling and Duysters (2008) [15] examine trends of network evolution in the biotechnology and multimedia industry in the Netherlands; Hanaki, Nakaijma, and Ogura (2010) [31] analyse network evolution in the IT industry in the US.

[2] The relational distance between two nodes is the minimum number of links separating the nodes. Maximum relational distance (diameter) is computed on the set of node pairs.

[3] In the study of Powell et al. (2005) [47] the degree distribution decays almost linearly on a log-log scale and approximates a power-law.

[4] It is notewothy that the properties above, or at least some of them, are also descriptive of research collaboration in different domains of analysis. The map of research collaborations between scholars in a scientific discipline is recovered from easily accessible data on coauthorship and reveals that the near-small-world property is recurrent in different disciplines (Newman 2004 [37], Goyal, van der Leij, and Moraga-González 2006 [21]). M. E. Newman and M. Girvan (2002) [40] apply their method of community-structure identification to the co-authorship network of the scientists attached to a trans-disciplinary research organization, the Santa Fe Institute. They find that the network can be partitioned into more densly linked community clusters, broadly corresponding to scientific fields and sub-fields. Knowledge based collaboration within large business organizations ad its network properties are emphasized by Hansen (2002) [25] and Cohendet (2005) [11], among others. The insight is that a knowledge based explanation of R&D networks, must build upon general design principles, which hold across a variety of domains.

The paper is organized as follows. Section 2 presents the main line of argument and relates it to the literature on R&D network formation. Section 3 builds upon the distinction between exploitation and exploration, to introduce different levels of exploration into the analysis. Sections 4 formalizes and elaborates upon the notions of ideas, knowledge proximity (understanding) and novelty. The complementary role of novelty and understanding in innovation alliance is discussed in section 5. Section 6 introduces the definition of a disciplinary field in the the knowledge space, and motivates the assumptions on specialized knowledge endowments. The model of conditional collaboration within and across fields is presented in section 7, and its main implcations are discussed in section 8. Section 9 elaboraes upon the notion of self-sustaining network. Section 10 concludes.

2 The line of argument and relations with the literature

This paper suggests a knowledge-based explanation of R&D collaboration, which rests on the trade off between the costly participation in a two-sided research agreement and the benefit enabled by the access to novel items of knowledge. Like other theoretical studies of R&D alliance (Goyal and Joshi 2003 [18] 2006 [19], Cowan and Jonard 2009 [12], Wesbrock 2010 [60], König et al. 2011 [27] 2012 [28])[5], we assume that collaboration cost is exogenous[6]. The nodes, or units, in the networks to be considered are mostly business firms, but may be also private or public R&D laboratories, and university research centres (Saviotti 2009 [50]). Such a differentiated set of actors is characterized by different motivations and incentives. On the ground that this paper is focused on knowledge-based incentives, that the incentives in question are of a general nature, and that the qualitative properties to be explained are generic (independent of the specific domain of enquiry), ontological differences between the different actors will be disregarded.

To make novelty productive, collaborators i and j need a common language and understanding. This is closely related to B. Nooteboom's notion of an optimal distance (Nooteboom 1992 [42], 2000 [44], 2004 [45]) of i's knowledge relative to j's (and vice versa). Cowan and Jeonard (2009) [12] make optimal cognitive distance operational by assuming that the necessary and sufficient condition which makes collaboration between any couple of firms i and j attractive, is that their symmetric knowledge overlap meets an optimal standard, which is uniform across firms. This paper argues that 'cognitive distance' is not captured by knowledge overlap, and is generally a non-symmetric notion. Conditional on the agents heterogeneous knowledge repertoires, the symmetric cognitive overlap between i and j yields non symmetric measures of novelty n_{ij}

[5] Some of these studies (Goyal and Moraga-González 2001 [20], Wesbrock 2010 [60], König et al. 2012 [28]) address also welfare issues that are outside the scope of this paper.

[6] In Goyal and Moraga-González 2001 [20] the cost is endogenized through the market effects of collaboration, which result from firm rivalry in the product market.

(of j's repertoire, relative to i's repertoire) and n_{ji}. Our first and quite obvious point is that novelty matters, and this makes i's incentive to enter collaboration (i, j) generally different from j's.

Our second point is that overlap and novelty are *complementary*, hence necessary ingredients of any productive R&D collaboration. Without novelty, mutual understanding makes collaboration uninteresting. Without understanding, difference of knowledge repertoires can not foster, and may even slow down, innovation. We argue that the existence of a collaboration cost gives rise to a minimum overlap threshold $\tilde{m}$ and novelty threshold $\tilde{n}$. Such thresholds vary with the type of exploration activity under way. March's (1991) [34] seminal distinction between exploitation and exploration is generalized to envisage different levels of exploration, such that exploration at the higher level broadens the framework, and cuts loose from some of the constraints which are held fixed in exploration at the lower level (see also Bogenrieder and Nooteboom 2004 [5]). The paper considers two levels of exploration. At the lower level, 'incremental' exploration is a search for new useful ideas within a knowledge space of an unchanging dimension; it is the search for ever fitter variants of a family of ideas and is broadly related to the notion of a technological trajectory (Dosi 1982 [13]). At the higher level, 'radical exploration' is a search for new and potentially useful families of ideas, which are located in a knowledge space of ever larger dimension. We make the hypothesis that radical exploration requires more creativity, hence a higher ratio between novelty and understanding, than is required by incremental exploration.

We study the qualitative, structural implications of the theoretical framework above, under the weak requirement that networks are *pairwise stable equilibria* (Jackson and Wolinsky, 1996 [24]), a notion which is is drawn from the game theoretic, incentive based explanation (as opposed to the statistical explanation) of networks (surveyes are Vega-Redondo 2007 [56], Goyal 2007 [17], Jackson 2008 [22]). In this respect, the paper outlines the main differences between 'incremental-exploration' (I.E.) and 'radical-exploration' (R.E.) networks. We proceed on the bold assumption that an agent's knowledge endowment, though random, belongs in one competence field. A field was produced by some radical discovery in the past, which added one or more new dimensions to the knowledge space. On this ground, fields span different regions of the knowledge space $\mathbf{\Gamma}$, but they are not disjoint. To give a precise formalization to the notions of I.E. and R.E activity, knowledge endowments, knowledge fields and their articulation within the knowledge space, we draw upon a standard representation of adaptive search in knowledge spaces (Auerswald, Kauffman, Lobo and Shell 2000 [2]), and on related results concerning modularity and the division of labour in the production of ideas (Simon, 1962 [52], 1973 [53], 2005 [54], Marengo, Pasquali and Valente 2005 [35], Watson 2006 [57]).

On the premise that I.E. collaborations build upon a necessary, but possibly limited amount of novelty, complemented by a solid background of shared mutual understanding, I.E. collaborations are most likely formed by collaborators from the same field. I.R. collaborations require a larger ratio between novelty and mutual understanding; as a result, they are more likely to be agreed upon by firms with competence repertoires in different, though partly overlapping fields. The set of all pairwise colaborations in the economy defines a mixed I.R.E. (incremental and radical exploration) network. We study the structural properties of the equilibrium networks wich are produced by a random, but field specific, allocation of endowments across firms. It is shown that the average clustering coefficient is positively tied to the overlap threshold $\tilde{m}$ and is expected to be large in I.E. networks (campared to random networks), and larger in I.E. than in R.E. networks. The existence of minimum thresholds $\tilde{m}$ and $\tilde{n}$ makes the degree distribution asymmetric and right skewed. Moreover, in a relevant range of the thresholds, a mixed I.R.E. network typically contains two groups of components. One coicides with a giant component; the other consists of isolated nodes. This equilibrium structure differs from the dominant group architecture[7], in that the giant component, though having a small diameter, is not completely connected. Burt (1992 [6]) argues that agents have a pay-off incentive to act as knowledge brokers, which is the case if some of their links connect otherwise disconnected parts of the network. An optimal link strategy avoids too much redundancy (excessive clustering is a waste), and obtains only if there are 'structural holes' in the egonetwork, a feature also revealed by empirical studies (Ahuja 2000 [1]). Short average relational distance and small diameter are a consequence. In our framework, R.E. collaborations link otherwise disconnected field-specific I.E. networks, which may also host structural holes.

Like Cowan and Jonard (2009) [12], our explanation of the properties above is based on a short-run pairwise analysis of collaboration incentives, which follow from the size and composition of the knowledge stocks of potential collaborators. Unlike König et al. (2011 [27] 2012 [28]) we avoid far sighted agents evaluating intertemporal research-collaboration incentives; in our model, short-run collaboration incentives are independent of network topology. This marks a sharp difference also with respect to the preferential attachment mechanism (Barabasi and Albert 1999) [3][8]. Topology matters only in the long run (section 9), that is, when knowledge endowments are made endogenous.

Research networks, as described above, are *short run equilibria*, conditional on a given distribution of knowledge across firms. It is natural to ask if such equilibrium structures can persist, when the knowledge distribution evolves endogenously.

In I.E networks, the very weak hypothesis on knowledge spillovers, that (ceteris paribus) the knowledge overlap between any couple of firms a and b is an increasing function of their past collaboration episodes, leads to the conclusion that novelty n_{ab} and n_{ba} will eventually be too low to make collaboration at-

[7] A dominant group architecture is defined by Goyal and Joshi (2006) [19] as one in which there are two groups of players. One group consists of players that are mutually completely linked. The other group consists of isolated players.

[8] The authors show that a right skewed, scale free distribution of links is produced by a self-reinforcing mechanism during the growth of a random network. Self-reinforcing is produced by the hypothesis that new nodes preferentially attach to nodes with a high degree. (see also Barabasi and others [4]). Preferential attachment is not particularly well suited to interpret collaboration choices entailing an intentional pairwise agreement, supported by a weighing of benefits against costs.

tractive. Ultimately, this is due to the defining characteristics of I.E. activities that they search in a fixed and finite space.

The implication is that I.E. networks can persist only in a larger environment, in which knowledge diversity is endogenously re-produced. The paper considers one such environment, a mixed I.R.E. network, and defines necessary conditions enabling a mixed network to be a *self sustaining pairwise equilibrium.* In such persistent equilibria, field specific clusters of nodes are linked by R.E. collaborations, connecting heterogeneously specialized units. This pattern broadly conforms to observed empirical properties of R&D networks[9], but any more accurate empirical correlate to a self-sustaining equilibrium can only be accessed through a long-run filtering of the data, on the hypothesis that dynamic evolution in historical time oscillates around (or converges to) a self-sustaining trajectory. It is noteworthy, in this respect, that a theoretical and empirical literature suggests a cyclical evolution of R&D network structures between phases in which novelty and varity abund, and phases in which they become more scarce (Callon 2002 [7], Nooteboom 2000 [44], Gilsing and Nooteboom 2006 [16]).

3 Different levels of exploration

J. March (1991) [34] introduced the seminal distinction between exploration and exploitation. He argued that there is an inescapable trade off between (exploitation) activities resting on accumulated competence and producing further refinement thereof, and (exploration) activities putting the accumulated knowledge stock into question, and extending the possible courses of action (action set). Exploration increases variety and, compared to exploitation, causes a higher variance of the performance distribution, possibly at the loss of the short-run mean performance. Exploration may be nevertheless most rewarding, because a lower short run mean performance is the price to pay for the opportunity to select, within the wider action set, a course of action possibly leading to a higher future mean performance.

Contrary to a common understanding, what is classified under exploration or exploitation is strictly dependent upon the object of analysis. In other words, exploitation and exploration do not define a dichotomic bipartition of activities; rather, there seem to be multiple levels of exploitation and exploration, ranging from the mechanical repetition of a 'production routine' to 'basic research' cutting loose from the received truths. Even in basic science (a locus of exploration, from a technology wide perspective), individual units may occasionally shift their effort from the development a prevailing theoretical hypothesis (exploitation!) to the exploration of alternative or complementary hypotheses. When invention and innovation activities are at stake, it seems most appropriate to abandon the exploitation-exploration dichotomy, and to refer to multiple levels of exploration, such that exploration at the higher level broadens the framework, and cuts loose from some of the constraints which are held fixed in

[9] Shilling and Phelps, 2007 [51]; Gilsing and Nooteboom, 2006 [15]; Gilsing and Duysters, 2008 [15].

exploration at the lower level (see also Bogenrieder and Nooteboom 2004 [5]). The paper considers two levels of exploration. At the lower level, 'incremental' exploration is a search for new useful ideas within a knowledge space of an unchanging dimension; it is the search for ever fitter variants of a family of ideas and is broadly related to the notion of a technological trajectory. At the higher level, 'radical exploration' is a search for new and potentially useful families of ideas, which are located in a knowledge space of ever larger dimension.

4 Ideas, knowledge proximity, and novelty

This paper makes the bold assumption that ideas are codified information strings. Ideas are embodied in human agents' brains, and by extension, in the organizations in which such human agents operate. The statement that $\mathbf{a}$ is an element in the set $\mathbf{A}_i$ of i's ideas, means that i has the capability to carry out a set of operations on a knowledge space, which use the information $\mathbf{a}$ as input[10]. We assume that an organization's knowledge is covered by secrecy and can only be accessed, and partially absorbed, joining a direct collaboration agreement with it.

In what follows we use the term agent to refer to any form of unit organization (firm, public or private R&D laboratory, etc.) performing R&D. Our argument is made more precise by a generalization of the standard binary-string representation of ideas in knowledge spaces. The burden of generalization is carried by the formal definition of a family (type) of ideas, and by the distinction between search within a family of ideas (I.E. activity) and search for new families of ideas (R.E. activity)[11].

Ideas and types: An idea is codified knowledge defined by a string $\mathbf{a} \in \{0,1,s\}^N$ of N elements. $N = N(t)$ is the knowledge horizon as of time t, indicating the number of known dimensions of the knowledge space. We avoid the time script in what follows, if unnecessary. An element a_n is identified by its location $n \in N = \{1, ..., N\}$ on the string. A location n is silent (uneffective) for idea a if and only if $a_n = s$. The set of non silent locations of a is $\mathbf{NS}(\mathbf{a}) = \{n \in \{1, ..., N\}$ s.t. $a_n \neq s\}$. A family, or type, of useful ideas (for instance, the family of the different versions of 'the steam engine') is the subspace $\mathbf{F} \subseteq \{0,1,s\}^N$ of ideas with identical silent and non silent locations: if $\mathbf{a} \in \mathbf{F}$ and $\mathbf{a}' \in \mathbf{F}$, then $a_n = s$ if and only if $a'_n = s$. A type $\mathbf{F}$ is uniquely identified by the set $\mathbf{NS}(\mathbf{F})$. An idea is a configuration of a family type.

Fitness in knowledge spaces: A type $\mathbf{F}$ is known at t, if at least one useful idea in $\mathbf{F}$ has been discovered at t. It is a subspace of the knowledge space $\mathbf{\Omega}_t \equiv \{0,1,s\}^{N(t)}$. $\mathbf{\Gamma}_t$ is the set of known types at t, and $\mu(t) \equiv \#\mathbf{\Gamma}_t$ is the number of such types. We fix a labelling of types in $\mathbf{\Gamma}$, such that $\mathbf{\Gamma} =$

[10] We may refer here to a standard example, which marks the difference between having a mathematics handbook at one's disposal, and having a full grasp of the proofs and potential applications of the mathematical proposition printed in the text.

[11] To the best of our knowledge, family types, and their formalization, were first introduced in an unpublished working paper of the author (Caminati 2009 [9]).

$\{\mathbf{F}_1, \mathbf{F}_2, ..., \mathbf{F}_{\boldsymbol{\mu}(\mathbf{t})}\}$. A configuration of $\mathbf{\Gamma}$, or knowledge configuration, is a list $\boldsymbol{\tau} = \{\mathbf{a}(\mathbf{F}_1), \mathbf{a}(\mathbf{F}_2), ..., \mathbf{a}(\mathbf{F}_{\boldsymbol{\mu}(\mathbf{t})})\}$ specifying one idea configuration $\mathbf{a}_f = \mathbf{a}(\mathbf{F}_f)$ for each family $\mathbf{F}_f$ in $\mathbf{\Gamma}$. $\boldsymbol{\tau} = \{\mathbf{a}_f \cup \mathbf{a}_{-f}\}$, where $\mathbf{a}_{-f}$ is the configuration of the families other than $\mathbf{F}_f$. In general, only a vanishing small fraction of the possible configurations of each idea type is 'useful'. The usefulness of an idea configuration $\mathbf{a}_f$ is, like fitness in biology, a relative, not an absolute concept, and can only be evaluated in the context of the given concomitant knowledge configuration $\mathbf{a}_{-f}$. This is because usefulness is affected by the positive or negative complementarities between ideas. The relative fitness of two ideas $\mathbf{a}_f$ and $\mathbf{a}'_f$ belonging to the same type $\mathbf{F}_f$ is evaluated by a fitness ratio $V_f(\mathbf{a}_f, \mathbf{a}_{-f})/V_f(\mathbf{a}'_f, \mathbf{a}_{-f})$, where $V_f()$ is a real function[12] $V : \mathbf{\Gamma} \rightarrow R_+$.

I.E. versus R.E. activity: At any date t, I.E. activity aims at selecting and evaluating alternative relevant configurations $(\mathbf{a}_f, \mathbf{a}_{-f})$, $(\mathbf{a}'_f, \mathbf{a}_{-f})$ within a given space $\mathbf{\Gamma}_t$. Evaluation takes place through the local 'computation' of $V()$ carried out through laboratory experiments. In contrast, R.E. activity at time t aims at discovering new types, giving rise to a set $\mathbf{\Gamma}_{t+1} \supset \mathbf{\Gamma}_t$. New types may result from recombinations of pre-existing known dimensions of a knowledge space with fixed horizon N_t, or from the discovery of new dimensions $N_t + 1, ..., N_{t+1}$.

Individual endowments: $\mathbf{A}_i$ is the set of ideas that are known by unit i, that is, i's knowledge endowment. Every $\mathbf{a} \in \mathbf{A}_i$ is a configuration of some corresponding type $\mathbf{F}(\mathbf{a}) \in \mathbf{\Gamma}_i \subset \mathbf{\Gamma}$. $\mathbf{\Gamma}_i$ is the set of types that are known by i, and can be interpreted as i's knowledge domain. The active dimensions of the knowledge space that are contemplated in $\mathbf{A}_i$ is $\mathbf{NS}_i = \cup_{\mathbf{F} \in \mathbf{\Gamma}_i} \mathbf{NS}(\mathbf{F})$, and the number of such dimensions is the cardinality measure $\sharp\mathbf{NS}_i \leq N$. The *endowment size* K_i is the number of ideas in i's endowment: $K_i = \sharp\mathbf{A}_i$; the number of ideas which agent i can potentially reach, through individual I.E. activity is $3^{\sharp\mathbf{NS}_i} - 1 \geq K_i$.[13] The maximum number of ideas which any set of agents, or any collaboration of agents, can reach through I.E. activity, within the knowledge horizon $N(t)$, is $3^{N(t)} - 1$.

Lane, Malerba, Maxfield, and Orsenigo (1996) [33] discuss at length the resons why relationships between heterogeneous agents are 'generative', that is, are a potential source of innovation. Within the restricted domain imposed by ruling out tacit knowledge, we draw upon the compositional (also called 'recombinant') approach to knowledge production (Holland 1992 [32], Reiter 2001 [49], Weitzman 1998[59], Watson 2006 [57]), to motivate the assumption that research collaboration (i, j) is attractive for both i and j, *only if* each collaborator brings a net contribution of ideas to the joint knowledge $\mathbf{A}_i \cup \mathbf{A}_j$ of the collaboration (i, j). Research collaboration between units i and j endowed with heterogeneous knowledge bases, forces such units to activate a communication

[12] Agents do not 'know' V_f, but carry out search experiments, which disclose local evaluations of the fitness function.

[13] The maximum number of ideas, which i's I.E. activity (carried out in isolation) can potentially reach, is the number of possible configurations of $\{0, 1, s\}^{\sharp\mathbf{NS}_i}$, except for the configuration with $\sharp\mathbf{NS}_i$ elements in state s, which does not correspond to any idea.

repertoire, based on a commom language and a mutual understanding, which is here measured by the overlap λ_{ij}.

$$\lambda_{ij} = \sharp\{\mathbf{A}_i \cap \mathbf{A}_j\} \quad (1)$$

. The number of new ideas which j can contribute to (i, j) is the novelty:

$$n_{ij} = K_j - \lambda_{ij} \quad (2)$$

Nooteboom (1992 [42], 1999 [43]) introduces the notion of a cognitive distance (CD) of one human agent with respect to another, which he relates to the differences and similarities between the 'cognitive repertoires' with which the agents concerned interpret, understand and evaluate the world. He likewise relates the CD of one firm with respect to another, to the differences and similarities between the interpretation systems (systems of shared meanings) ruling within the former and the latter. In what follows, we abstract from the otherwise important distinction between knowledge in individuals and in organizations. According to Nooteboom, there exists some optimal CD, in the weak sense that, if CD is too low, interaction does not lead to any substantial gain in competence or creativity (novelty is too low), if it is too large, the potential gain is inhibited by the lack of understanding. This implies than an organization faces an inverse U-shaped relation between collaboration pay-off, and its CD with respect to potential collaborators. A recent attempt at empirical corroboration of optimal CD is Wuyts et al. (2006) [61][14]. Some 'empirical proxy-measures' of CD (Nooteboom 2000 [44], p. 301, Nooteboom et al. 2007 [46]) are closer to a notion of 'knowledge proximity'. In our restricted framework, knowledge proximity between i and j is a symmetric notion correlated to the overlap λ_{ij}. Expressions (1) and (2) make it clear that overlap does not yield unambiguous information on novelty, and in general $n_{ij} \neq n_{ji}$, that is, novelty is non-symmetric. Overlap yields information on novelty only after the knowledge repertoires are appropriately specified. This means that any symmetric measure of CD cannot simultaneously account for the competence differences and similarities[15], which matter in R&D collaboration. Cowan and Jonard (2009) [12] assume that the necessary and sufficient condition to the R&D collaboration between i and j is the existence of an optimal overlap λ_{ij} (or optimal 'overlap range'). In their model, optimal overlap is *uniform* across all firms, whatever the differences in the size of their knowledge endowments.

[14] The study tests the hypothesis of an inverse U-shaped relation between firm innovation success, and firm CD with respect to R&D partners.

[15] A non-symmetric measure may have the required properties, if it depends on both novelty and overlap. $CD_{ij} = n_{ij}/\lambda_{ij}$ is a measure of the relative CD of j with respect to i.

5 Novelty and understanding as complementary inputs

In this section, we replace the problematic notion of a uniform optimal cognitive overlap, with the idea that novelty and overlap are complementary inputs of any research alliance. I.E. activity by collaboration (i, j) is interpreted in this paper as the search for fitter configurations of the unchanging knowledge domain $\mathbf{\Gamma}_i \cup \mathbf{\Gamma}_j \subset \mathbf{\Gamma}$.

Herbert Simon seminal contributions (1962[52] 1973 [53] 2005 [54]) on the role of near decomposability in natural and human evolution, and related arguments, made under the heading of 'compositional evolution' (Watson, 2006 [57]) show the benefits and limits of modularity and specialization in R&D. A suitable reduction in the dimension of a search space is available if the space can be partitioned into subspaces, with a limited number of fitness interactions between their components. Knowledge of these interfaces is conducive to a decomposition of the original problems into sub-problems, that can be eventually recomposed, after a simplification of the problem space has been achieved. Evaluation of the R&D benefits from problem decomposition is further strengthened by the adoption of an evolutionary framework. When the fitness loss caused by the fixing of interfaces is of a lower order, problem decomposition may be conducive to faster progress. Frenken, Marengo and Valente (1999) [14] show that a decomposition yielding a satisficing trade off between simplification of the problem space and accurate problem representation offers a selective advantage.

The above ideas motivate the assumption that, provided i and j have a sufficiently large mutual understanding of the (fitness) *relevant* interactions between $\mathbf{\Gamma}_i$ and $\mathbf{\Gamma}_j$, I.E. exploration on $\mathbf{\Gamma}_i \cup \mathbf{\Gamma}_j$ can benefit from division of labour in research. This means that i's specialized knowledge over $\mathbf{\Gamma}_i$ and j's specialized knowledge over $\mathbf{\Gamma}_j$ can be recomposed, reducing the number of search experiments leading to a profitable innovation. We assume that novelty n_{ij} (n_{ji}) is beneficial to I.E. exploration, provided that it is supported by a sufficiently large mutual understanding λ_{ij}. To this end, specialization n_{ij}/λ_{ij} (n_{ji}/λ_{ij}) must not exceed a relevant threshold. We formalize this intuition by assuming complementarity between novelty and understanding. Conditional on the fact that j is prepared to collaborate with i, i's net pay off from I.E. activity within collaboration $(i, j)^I$, rather than alone, is:

$$\Pi^I_{ij} = \min(\alpha^I \lambda_{ij}, \beta^I n_{ij}) - F \tag{3}$$

The upper script I refers to incremental exploration. F is a collaboration cost[16]. Without loss of generaity we assume $F = 1$. α and β are complementarity parameters. Expression (3) assumes a form of free disposal, which rules

[16] The collaboration cost has the nature of an R&D investment. Its size is broadly related to the complexity of the knowledge space. On this ground, it can be assumed that, in the very long run, F grows through time, together with knowledge. Similar considerations on this point are in König et al. (2011) [27].

out the possibility that excess novelty, or understanding, may come at a cost. A more general formulation is spelled out in the attached footnote [17].

The hypothesis that radical innovations are new families of ideas produced by creative recombinations of knowledge, gives ground to believe that a comparatively higher creativity, hence greater novelty, and a comparatively lower mutual understanding (overlap), are necessary in R.E. collaborations than in I.E. collaborations. Conditional on the fact that j is prepared to collaborate with i, i's net pay off from R.E. activity within collaboration $(i,j)^R$ (rather than alone) is[18]:

$$\Pi_{ij}^{R} = \min(\alpha^R \lambda_{ij}, \beta^R n_{ij}) - F \tag{4}$$

We introduce the assumption:

$$\frac{n_{ij}^R}{\lambda_{ij}^R} \geq \frac{n_{ij}^I}{\lambda_{ij}^I}$$

The collaboration cost $F = 1$, implies that there are *minimum* thresholds $\tilde{n} = Z(1/\beta)$ and $\tilde{m} = Z(1/\alpha)$ below which research collaboration is unrewarding. The function $Z() : R_+ \to Z_+$ maps a positive real x to the lowest integer $Z(x) \geq x$. The threshold values are positive and finite, as a result of the fact that both novelty and understanding (overlap) are necessary inputs to R&D collaboration[19], that is, $0 < \alpha \leq 1$, and $0 < \beta \leq 1$. $\tilde{n}$ and $\tilde{m}$ are generally different for I.E. and R.E. activities, and we assume:

$$\tilde{n}^R = Z(1/\beta^R) \geq \tilde{n}^I = (1/\beta^I) \qquad \tilde{m}^R = (1/\alpha^R) \leq \tilde{m}^I = (1/\alpha^I) \tag{5}$$

i is prepared to enter collaboration $(i,j)^I$ if and only if $\Pi_{ij}^I \geq 0$, and is prepared to enter collaboration $(i,j)^R$ if and only if $\Pi_{ij}^R \geq 0$.

[17] If there are *maximum* threshold values $\hat{n}$ and $\hat{\lambda}$ beyond which novelty and understanding may be harmful to incremental innovation, expression 3 generalizes to:

$$\Pi_{ij}^{I} = \min(\alpha^I \lambda_{ij}, \beta^I n_{ij}) - F - \theta(\max(0, n_{ij} - \hat{n}^I) - \phi(\max(0, \lambda_{ij} - \hat{\lambda}^I)$$

where the parameters θ and ϕ measure the cost effect (if any) of excess novelty, and understanding, respectively.

[18] In the absence of free disposal over 'excess' novelty, or understanding (see previous footnote), the expression below generalizes to:

$$\Pi_{ij}^{R} = \min(\alpha^R \lambda_{ij}, \beta^R n_{ij}) - F - \theta(\max(0, n_{ij} - \hat{n}^R) - \phi(\max(0, \lambda_{ij} - \hat{\lambda}^R)$$

[19] This marks a main difference from Cowan and Jonard (2009) [12]. In their model knowledge overlap fully explains research alliance, no matter what the novelty contribution may be: Collaboration between i and j is conditional on the event that the overlap is 'optimal' ($\lambda_{ij} = \lambda^*$), or that it falls in the 'optimal' overlap range ($\lambda_{ij} \in [\lambda^* - \rho, \lambda^* + \rho]$). Our generalized pay-off functions (footnotes 19 and 20 above) can be forced to reproduce that model; in particular, the minimum novelty threshold $\tilde{n}$ tends to vanish, if $\beta \to +\infty$.

6 Knowledge specialization

We assume that there are W disciplinary fields in the knowledge space $\mathbf{\Omega}$. Each field $\mathbf{X}_w$, $w \in \{1, ..., W\} \equiv \mathbf{W}$ defines a restricted sub-domain of $\mathbf{\Omega}$, that is, a subset $\mathbf{NS}(\mathbf{X}_w) \subset \{1, ..., N\}$ of non silent locations. The statements $\mathbf{F} \subseteq \mathbf{X}_w$ and $\mathbf{NS}(\mathbf{F}) \subseteq \mathbf{NS}(\mathbf{X}_w)$ are equivalent. Disciplinary fields are specialzed, but they may not be disjoint: for every $w \in \{1, ..., W\}$ there is $z \in \{1, ..., W\}$, $z \neq w$, such that $\mathbf{NS}(\mathbf{X}_w) \cap \mathbf{NS}(\mathbf{X}_z) \neq \varnothing$. In the absence of any better hypothesis, and for the sake of simplicity, we impose symmetry assumptions on field structure.

- The size of a knowledge field, that is, the number of its active locations, is uniform across fields: for any $w \in \{1, ..., W\}$, *field size* is defined by $d_w \equiv \sharp \mathbf{NS}(\mathbf{X}_w) = d < N$. For every any $w \in \{1, ..., W\}$ the number of ideas defined on $\mathbf{X}_w$ is $\delta = 3^d - 1$.[20]

- The field-neighbor structure is such that, any couple of fields (w, w') shares a positive number $\chi(w, w') \equiv \sharp\{\mathbf{NS}(\mathbf{X}_w) \cap \mathbf{NS}(\mathbf{X}_{w'})\}$ of active dimensions. In this sense, a field is a neighbor of any other. In addition, there is a unique ordering of the labels $\{1, ..., W\}$, such that $0 < \chi(w, w') \leq d - s$, if $|w - w'| = s$. Fields w and w' are adjacent if and only if $|w - w'| = 1$.

- For the sake of simplicity, it is assumed that the dimensional overlap $\chi(w, w')$ is a constant χ, for any couple (w, w') of adjacent fields. That is, for any field $w \neq W$ the dimensional overlap $\chi(w, w + 1)$ is a constant $\chi \leq d - 1$. The number of ideas defined in the intersection $\mathbf{X}_w \cap \mathbf{X}_{w'}$ between two fields w and w' is $\gamma(w, w')$. Notice that $\gamma(w, w + 1) = \gamma \equiv 3^\chi - 1 \leq 3^{d-1} - 1$; in words, the number of ideas, which two adjacent fields have in common, is less than 1/3 the number of ideas defined on their respective domains. The number $\gamma(w, w')$ falls dramatically, if w and w' are non adjacent.

Our assumptions imply that any two neighboring fields do not agree on at least one knowledge dimension. We interpret the restriction as a result of the fact that a new field born at $t + 1$ is the outcome of correlated events of radical innovation and specialization; the innovation adds $\Delta \geq 1$ new dimensions (locations) to the pre-existing number of dimensions $N(t)$ and produces a growth factor $(3^{N(t)+\Delta} - 1)/(3^{N(t)} - 1) \approx 3^\Delta$ of the number of ideas in the knowledge space. We may notice, in passing, that there are radical innovations which do not give rise to the birth of a new field.

Every agent $h \in \{1, ..., H\} \equiv \mathbf{H}$ has a randomly assigned initial knowledge endowment, which is defined on a given and unique field. The subset of agents with endowment derfined on field $\mathbf{X}_w$ is the knowledge community $\mathbf{H}_w$, and the collection of communities $\{\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_W\}$ is a partition of $\mathbf{H}$. For the sake of simplicity, we assume that the number of agents in each community $\mathbf{H}_w$ is H/W for every $w \in \mathbf{W}$.

[20] $3^d - 1$ is the number of different ideas which can be defined on $\{0, 1, s\}^d$.

We assume that every $h \in \mathbf{H}_w$ is randomly assigned any idea $\mathbf{a} \in \mathbf{X}_w$ with uniform probability $p = 0.5$. The random assignment of knowledge endowments and the assumed structure of disciplinary fields, taken together, imply that $h \in \mathbf{H}_w$ and $j \in \mathbf{H}_z$ face a positive, but possibly small probability to have some ideas in common.

The detailed formalization of ideas, knowledge fields and specialized endowments serves the purpose of introducing the distinction between radical and incremental exploration, and weakest restrictions on endowments, for agents operating in different fields. The analysis to follow will be considerably eased by a simplified representation of endowments consistent with the framework above. We adopt a conventional re-ordering $(\mathbf{a}_1^w, \mathbf{a}_2^w, ..., \mathbf{a}_\delta^w)$ of the δ ideas defined on a field $\mathbf{X}_w$, $w = 1, ..., W$. We then represent the endowment $\mathbf{A}_i$ assigned to $i \in \mathbf{H}_w$ by means of a binary string $\mathbf{K}_i^w$ of length δ, such that the jth element in the string is 1 if the idea $\mathbf{a}_j^w$ is in the endowment, and it is 0 otherwise.

7 Conditional collaboration within and across fields

The difference between two initial endowments $\mathbf{A}_i$ and $\mathbf{A}_j$, defined on the same field, is the outcome of random events. Moreover, our symmetry assumptions imply that for any $h \in \mathbf{H}$, the expected number of ideas in $\mathbf{A}_h$ is uniform. Recalling that the probability of $K_h = s$ is $\binom{\delta}{s}(1/2)^\delta$, the expected size of h's endowment is:

$$E(K_h) = \sum_{s=1}^{s=\delta}[s\binom{\delta}{s}(1/2)^\delta] \tag{6}$$

7.1 R&D collaboration within the same field

If i and j belong to the same field, the conditional probability that i and j have m ideas in common, given the size K_i and K_j of their endowments, is $P(\lambda_{ij} = m|K_i, K_j)$, which can be expressed in terms of the binomial coefficients, using the argument made in Cowan and Jonard (2009) [12].

Proposition 1 *(Proof in appendix) Assume that i and j belong to the same community $\mathbf{H}_w$ of specialists in field $\mathbf{X}_w$. Given the size K_i and K_j of i's and j's endowments, the conditional probability that they have m ideas in common is:*

$$\begin{aligned} P(\lambda_{ij} &= m|K_i, K_j) = \\ &= \binom{\max(K_i, K_j)}{m}\binom{\delta - \max(K_i, K_j)}{\min(K_i, K_j) - m}\left[\binom{\delta}{\min(K_i, K_j)}\right]^{-1} \end{aligned} \tag{7}$$

where the following conventions hold throughout: $\binom{z}{m} = 0$, if $m > z$; $\binom{z}{0} = 1$, all $z \in Z_+$ (the set of non negative integers).

For the sake of simplcity we follow the standard practice of assuming that the number of alliances of any organization is unconstrained by its physical capital endowment, or by the number of its employees[21].

A collaboration strategy of firm i is a choice $\mathbf{S}_i = \{e_{i1}, ..., e_{iH}\}$, such that $e_{ij} = 1$ if i makes a collaboration offer to j, and $e_{ij} = 0$ otherwise. A strategy $\mathbf{S}_i$ is a best reply to $\mathbf{S}_{-i}$ (the profile of srategies chosen by firms other than i), *if and only if*, for any $j \neq i$, $e_{ij} = 1$ implies: $e_{ji} = 1$, and $\Pi_{ij} \geq 0$.

A pairwise equilibrium (Jackson and Wolinsky, 1996 [24]) is a strategy profile $\mathbf{S}^* = \{\mathbf{S}_1^*, ..., \mathbf{S}_H^*\}$, such that $\mathbf{S}^*$ is a Nash equilibrium, and is robust to the formation of two-agents coalitions. For every i and j in $\mathbf{H}$, such that $0 = e_{ij} \in \mathbf{S}_i^*$, $0 = e_{ji} \in \mathbf{S}_j^*$, there is no strategy pair $(\mathbf{S}_i', \mathbf{S}_j')$ such that $1 = e_{ij} \in \mathbf{S}_i'$, $1 = e_{ji} \in \mathbf{S}_j'$, and both i and j prefer the strategy profile $\mathbf{S}' = (\mathbf{S}_{-(i+j)}^*, \mathbf{S}_i', \mathbf{S}_j')$[22] to the the profile $\mathbf{S}^*$.

We define the type $V = I, R$ undirected network $\mathbf{G}^V = \{\mathbf{H}, \mathbf{L}^V\}$, where $\mathbf{H}$ is the set of firms, $\mathbf{L}^V$ is the set of undirected links between them, with the property that, for any couple $(i,j) \in \mathbf{H} \times \mathbf{H}$, $(i,j)^V \in \mathbf{L}^V$, if and only if $e_{ij} = 1$ and $e_{ji} = 1$ are mutual best replies.

The complementarity between novelty and understanding brings with it the implication that the minimum thresholds $\tilde{m}^V$ and $\tilde{n}^V$, imposed by the fixed collaboration cost $F = 1$, must be simultaneously met. A necessary condition for $(i,j)^V \in \mathbf{L}^V$ is therefore $K_i \geq \tilde{m}^V + \tilde{n}^V \leq K_j$. The implcation is that the values of the overlap λ_{ij} enabling collaboration $(i,j)^V$ are those weakly exceeding $\tilde{m}^V$, and which are not too high, so that they leave enough scope for novelty: $n_{ij} = K_j - \lambda_{ij} \geq \tilde{n}^V \leq K_i - \lambda_{ij} = n_{ji}$. Summing over all the probability values defined by (7), which meet this twin condition on λ_{ij}, we obtain:

Proposition 2 *Assume that i and j belong to the same community $\mathbf{H}_w$ of specialists in field $\mathbf{X}_w$. For $V = I, R$ the conditional probability that i and j form a collaboration $(i,j)^V$, given the size K_i and K_j of their endowments, is*

$$\begin{aligned} P_w((i,j)^V &\in \mathbf{L}^V | K_i, K_j) = \\ &= \sum\nolimits_{\lambda_{ij}=\lambda^V.\min}^{\lambda_{ij}=\lambda^V.\max} \binom{\max(K_i,K_j)}{\lambda_{ij}} \binom{\delta - \max(K_i,K_j)}{\min(K_i,K_j) - \lambda_{ij}} \left[\binom{\delta}{\min(K_i,K_j)} \right]^{-1} \end{aligned} \quad (8)$$

where $\lambda.\min$, $\lambda.\max$ are the lowest and highest admissible values of λ_{ij}, given the restrictions $\lambda_{ij} \geq \tilde{m}^V$, $n_{ij} \geq \tilde{n}^V$, $n_{ji} \geq \tilde{n}^V$, and the feasibility constraints $\lambda_{ij} = K_i - n_{ji} = K_j - n_{ij}$, $\lambda_{ij} \geq K_i + K_j - \delta$. This yields $\lambda^V.\min = \max(\tilde{m}^V, K_i + K_j - \delta)$; $\lambda^V.\max = \max(0, \min(K_i,K_j) - \tilde{n}^V)$.

[21] Only the organization knowledge repertoire matters, and an implicit side hypothesis is that the unit time interval is sufficiently long that this repertoire can be absorbed by new employees.

[22] With some abuse of notation, the profile $(\mathbf{S}_{-(i+j)}^*, \mathbf{S}_i', \mathbf{S}_j')$ is obtained by replacing strategies $\mathbf{S}_i^*$ and $\mathbf{S}_j^*$ in $\mathbf{S}^*$ with the strategies $\mathbf{S}_i'$ and $\mathbf{S}_j'$, respectively.

7.2 Cross field collaboration

If $i \in \mathbf{H}_w$ and $j \in \mathbf{H}_{w+1}$, so that i and j are specialists in adjacent fields $\mathbf{X}_w$, $\mathbf{X}_{w+1}$, the ideas they may possibly have in common correspond to $\gamma \leq 3^{d-1} - 1$ different configurations of the intersection $\mathbf{X}_w \cap \mathbf{X}_{w+1}$.

Proposition 3 *(Proof in appendix) Assume i and j are specialists in adjacent fields $\mathbf{X}_w$, $\mathbf{X}_{w+1}$, respectively, and define $\bar{K}_i$, and $\bar{K}_j$ the number of i's and j's ideas (respectively), belonging in the intersection $\mathbf{X}_w \cap \mathbf{X}_{w+1}$. The conditional probabilites $P(\bar{K}_i|\mathbf{X}_w, \mathbf{X}_{w+1}, K_i)$ and $P(\bar{K}_j|\mathbf{X}_w, \mathbf{X}_{w+1}, K_j)$ are:*

$$P_{w,w+1}(\bar{K}_h|K_h) = \binom{\gamma}{\bar{K}_h}\binom{\delta-\gamma}{K_h-\bar{K}_h}\left[\binom{\delta}{K_h}\right]^{-1} \qquad h = i, j \tag{9}$$

Proposition 4 *(Proof in appendix) Assume i and j are specialists in adjacent fields $\mathbf{X}_w$ and $\mathbf{X}_{w+1}$, respectively. Given K_i and K_j, the conditional probability that they form a collaboration $(i,j)^V$ is*

$$P_{w,w+1}((i,j)^V \in \mathbf{L}^V|K_i, K_j) = \sum_{\bar{K}_i=\tilde{m}}^{\bar{K}_i=K_i}\sum_{\bar{K}_j=\tilde{m}}^{\bar{K}_j=K_j}\sum\nolimits_{\lambda_{ij}=\lambda.\min}^{\lambda_{ij}=\lambda.\max}\binom{\max(\bar{K}_i,\bar{K}_j)}{\lambda_{ij}} \cdot \binom{\gamma-\max(\bar{K}_i,\bar{K}_j)}{\min(\bar{K}_i,\bar{K}_j)-\lambda_{ij}}\left[\binom{\gamma}{\min(\bar{K}_i,\bar{K}_j)}\right]^{-1} P_{w,w+1}(\bar{K}_i|K_i)\cdot P_{w,w+1}(\bar{K}_j|K_j) \tag{10}$$

where $\lambda^V.\min = \max(\tilde{m}^V, \bar{K}_i+\bar{K}_j-\gamma)$, and $\lambda^V.\max = \max[0, \min(\bar{K}_i, \bar{K}_j, K_i - \tilde{n}^V, K_j - \tilde{n}^V)]$.

7.3 Asymmetry of degree distribution and clustering

The existence of minimum thresholds $\tilde{m}^V$ and $\tilde{n}^V$, which are necessary to enter a type $V = I, R$ collaboration, and the random assignment of initial endowments, have direct bearings on *(i)* the distribution of the average fraction of nodes, with a given expected degree, and *(ii)* the average clustering coefficient.

Information on the average distribution of the expected degree, resulting from the rules above, is recovered in two steps. In step 1 the probability of link formation is used to compute the expected type-V degree $L^V(s)$ of a node $i \in \mathbf{H}_W$, conditional on endowment size $K_i = s$ (see appendix). For mixed I.R.E. networks, the conditional expected degree is $L^{IR}(s) = L^I(s) + L^R(s)$. Step 2 uses the above information to express the expected fraction $H(L^V)$ of units with expected type-V degree L^V, $V = I, R, IR$, $0 \leq L^V \leq H - 1$,

$$H(L^V) = \sum\nolimits_{\{s\in\{0,1,\ldots,\delta\}|L^V(s)=L^V\}} \binom{\delta}{s}(1/2)^\delta, \tag{11}$$

where $\binom{\delta}{s}(1/2)^\delta$ is the probability that an agent is assigned endowment of size s. We obtain the following intuitive result.

Proposition 5 *If the minimum endowment size $\tilde{k}^V$ is large enough, the average distribution of expected degree is asymmetric and right skewed. In particular, if $\tilde{k}^V$ is weakly larger than, or sufficiently close to, the modal value of endowment size, the frequency distribution is strictly decreasing with the expected degree.*

In sample experiments, with large $\tilde{k}^V$, the degree distribution declines almost lineraly on a log-log scale. These findings match observed empirical properties (Powell et. al. 2005).

The probability that i and j form a collaboration, conditional on the event that they both collaborate with the same agent h, is considerably higher than the corresponding unconditonal probability. On average, this clustering effect (difference between the above conditional and unconditional probabilities) is positively affected by an increase of $\tilde{m}^V$, at given $\tilde{n}^V$. To see this, notice that the event $[(i,h) \in \mathbf{L}^V, (j,h) \in \mathbf{L}^V]$ implies, in the first place, $K_i \geq \tilde{k}^V$, $K_j \geq \tilde{k}^V$, $K_h \geq \tilde{k}^V$; in the second place, it implies that any admissible value of K_h fixes a lower bound $\lambda_{ij} \geq 2\tilde{m}^V - K_h$ to the conditional overlap between i and j[23]. The conditional expected value of this lower bound is $2\tilde{m}^V - \left[E(K_h)|\tilde{k}^V\right]$, where the conditional expectation is taken on the support $\tilde{k}^V \leq K_h \leq \delta$. Restricting (6) to this support, $2\tilde{m}^V - E(K_h) = 2\tilde{m}^V - \sum_{s=\tilde{k}^V}^{s=\delta} [s \binom{\delta}{s} (1/2)^{\delta - \tilde{k}^V+1}]$, which grows as a result of a discrete increase of $\tilde{m}^V$, at given $\tilde{k}^V$, or at given $\tilde{n}^V$[24].

Proposition 6 *Consider the clustering coefficient of the R.E. and I.E. networks that are generated by a random assignment of endowments, according to the rules specified in the text. Over a sufficiently large number of random assignments,the average clustering coefficient is 'large' in I.E. networks (compared to random networks), and larger in I.E. than in R.E. networks, because the former are characterized by relatively higher values of $\tilde{m}/\tilde{n}$.*

8 From disciplinary fields, to the structure of I.E. and R.E. networks

Mild assumptions on the way the active dimensions of disciplinary fields are structured within a knowledge space, impose strong limitations on the largest number γ of ideas that specialists in adjacent fields may potentally share. If $i \in \mathbf{H}_w$ and $j \in \mathbf{H}_{w+1}$, any idea $\mathbf{a} \in \{\mathbf{X}_w \cap \mathbf{X}_{w+1}\}$, is assigned with probability

[23] Conditioning is made with respect to the event that both i and j collaborate with h. The number of ideas h shares with both i and j fixes a lower bound to the overlap λ_{ij}. The *maximum* number of ideas h does not share with any collaborator z is $K_h - \tilde{m}^V \geq n_{hz} \geq \tilde{n}^V$. It follows that the *minimum* number of ideas h shares with both i and j is $\tilde{m}^V - K_h - \tilde{m}^V = 2\tilde{m}^V - K_h$.

[24] The second result is explained by the fact that, at given $\tilde{n}^V$, $\Delta\tilde{k}^V = \Delta\tilde{m}^V$, and a change $\Delta\tilde{k}^V$ produces a corresponding change of the lower bound of the support $\tilde{k}^V \leq K_h \leq \delta$. Because the probablity of any endowment K_h in the support is strictly positive, the implication is $\Delta\left[E(K_h)|\tilde{k}^V\right] \leq \Delta\tilde{k}^V$.

1/4 to i and not to j, with probability 1/4 to j and not to i, and with probability 1/4 to both i and j. If instead $\mathbf{a} \in \{\mathbf{X}_w - \mathbf{X}_w \cap \mathbf{X}_{w+1}\}$, the idea is assigned with probability 1/2 to i and not to j, and with probability zero to j and not to i, or to both. [25] Under the mild restriction $\chi \equiv \chi(w, w+1) = d-1$, the ratio between the number of ideas defined on $\{\mathbf{X}_w \cap \mathbf{X}_{w+1}\}$ and $\{\mathbf{X}_w - \mathbf{X}_w \cap \mathbf{X}_{w+1}\}$ is:

$$\frac{\gamma}{\delta - \gamma} = \frac{3^{d-1} - 1}{3^d - 1 - (3^{d-1} - 1)} = \frac{3^{d-1} - 1}{3^{d-1} \cdot 2} \approx \frac{1}{2}$$

As a result, if i and j are specialists in adjacent fields, the unconditional expected novelty $E(n_{ij}) = E(n_{ji})$ is considerably larger than the unconditional expected overlap $E(\lambda_{ij})$. If instead if i and j belong in the same field community, unconditional expected novelty and overlap are the same. The ratio between unconditional expected overlao and novelty falls dramatically when i and j are specialists in neighboring, but non adjacent fields w and w' ($|w - w'| > 1$), to the effect that the probability of collaboration (i, j) is vanishingly small (or zero) if the minimum overlap threshold $\tilde{m}$ is sufficiently close to the largest number $\gamma(w, w')$ of ideas, i and j may potentially share. Recalling that $|w - w'| > 1$ implies $\gamma(w, w') = 3^{\chi(w,w')} - 1 \leq 3^{d-2} - 1$, we obtain the following:

Proposition 7 *If the restrictions imposed by participation in the I.E. collaboration agreements are such that $\tilde{m}^I$ is sufficiently large, and $\tilde{n}^I$ is sufficiently small, 'then, on average, most I.E. collaborations $(i, j)^I$ (if any) are such that $i \in \mathbf{H}_w$ and $j \in \mathbf{H}_w$, that is, i and j are specialists in the same field.*

Proposition 8 *Conversely, if the restrictions imposed by participation in the R.E. collaborations $(i, j)^R$ are such that $\tilde{n}^R/\tilde{m}^R$ is large enough, and $\tilde{m}^R$is sufficiently close to 3^{d-2}, then (if there are R.E. collaborations at all), collaboration between specialists in non adjacent fields is unfrequent relative to collaboration between specialists in adjacent fields.*

8.1 Mixed networks

Some organizations in $\mathbf{H}$ are engaged only in I.E. or R.E. collaborations, others may be engaged in both. The latter are knowledge-rich organizations, because their endowments meet $K_i \geq \tilde{m}^I + \tilde{n}^R$. We define the mixed I.R.E. (incremental and radical exploration) undirected network $\mathbf{G} = \{\mathbf{H}, \mathbf{L}\}$, where $\mathbf{H}$ is the set of nodes, $\mathbf{L} = \mathbf{L}^I \cup \mathbf{L}^R$ is the set of undirected links between them, that is, $(i, j) \in \mathbf{L}$, if and only if: $(i, j)^I \in \mathbf{L}^I$, or $(i, j)^R \in \mathbf{L}^R$, or both. For appropriate values of the complementarity parameters, the mixed network $\mathbf{G}$ has one giant component, which consists of dense community clusters, broadly corresponding to fields and sub fields. Clusters belonging to different fields are typically connected by R.E. links, joining knowledge rich firms. In our simulations, there is typically one and only one component in a mixed network.

[25] The case $\mathbf{a} \in \{\mathbf{X}_{w+1} - \mathbf{X}_w \cap \mathbf{X}_{w+1}\}$ is symmetric.

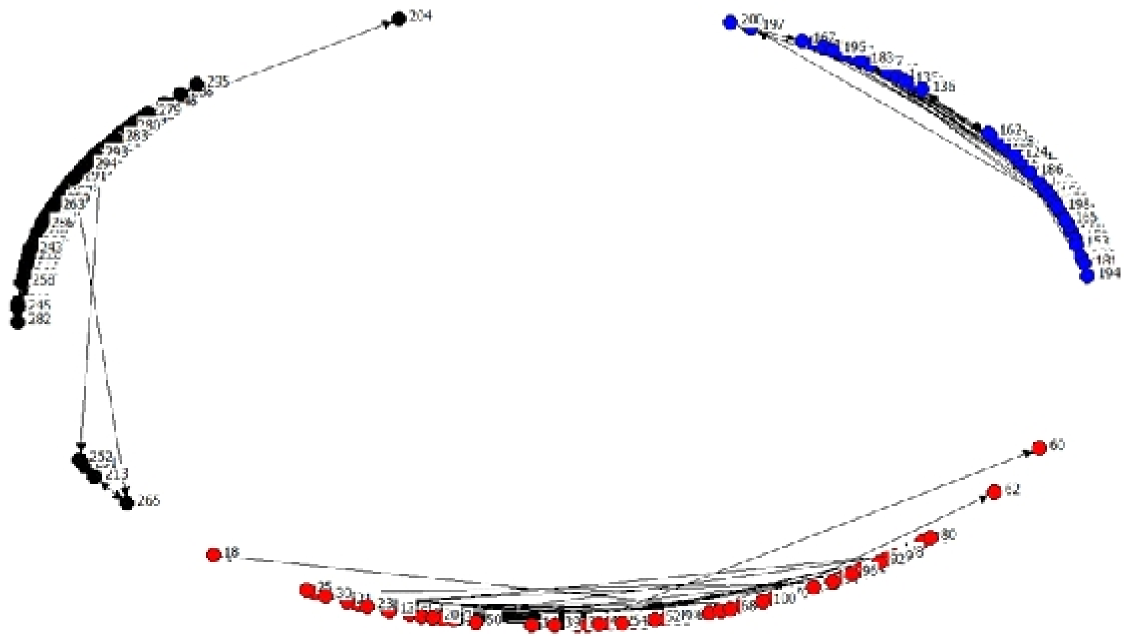

Figure 1: Three main components of the I.E. network resulting from a random sample assignment of endowments to 300 nodes partitioned into three communities $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$. Nodes in the same community $\mathbf{H}_w$ have endowment defined on the same field $\mathbf{X}_w$, $w = 1, 2, 3$. There are 200 known ideas defined on the aggregate knowledge space $\mathbf{\Omega} = \cup_w \mathbf{X}_w$. All nodes not shown in this figure are isolates. The simulation produces a perfect match between component and field: nodes belonging in the same commumity $\mathbf{H}_w$ belong to the same component, and vice-versa. Nodes in $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, are colored in red, black, and blue, respectively. $\mathbf{X}_2$, is adjacent to both $\mathbf{X}_1$ and $\mathbf{X}_3$, and there are 50 known ideas defined in the intersections $\mathbf{X}_2 \cap \mathbf{X}_1$ and $\mathbf{X}_2 \cap \mathbf{X}_3$. The sample realization of the network was produced by setting $\tilde{m}^I = 35$, $\tilde{n}^I = 4$.

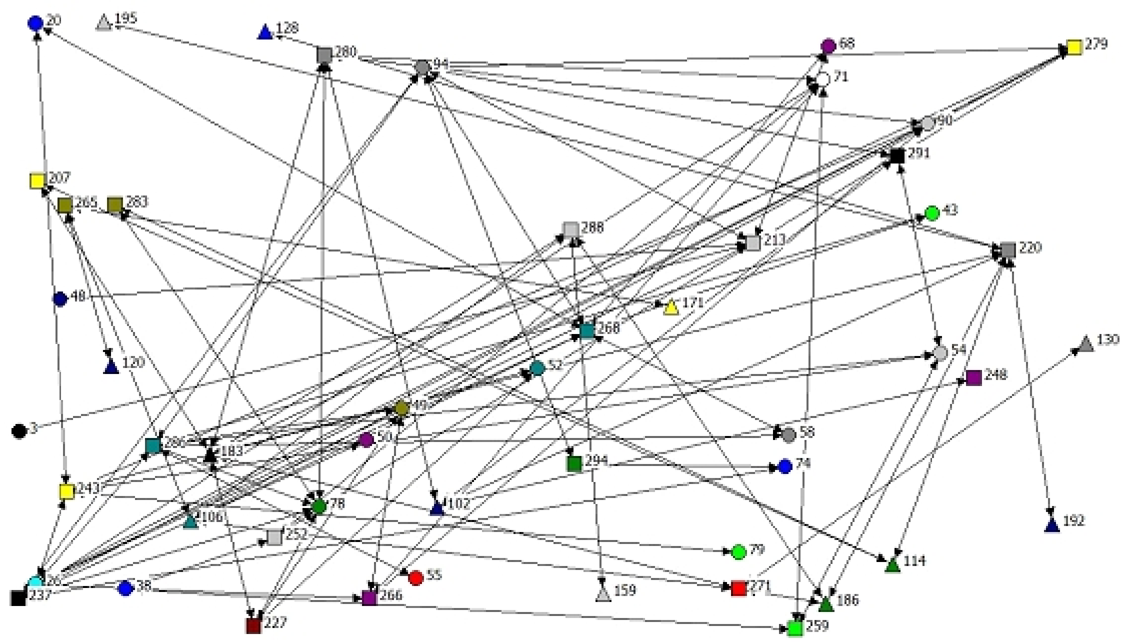

Figure 2: Giant and unique non-trivial component (52 nodes) of the R.E. network resulting from the sample assignment of endowments, presented in the caption text of Figure 1, and setting the overlap threshold $\tilde{m}^R = 20$, and the novelty threshold $\tilde{n}^R = 35$. There are not links connecting nodes with endowment in the same field, because novelty is below threshold, or connecting nodes with endowments in non-adjacent fields, because overlap is below threshold. In this picture, the field community of a node is identified by node shape (circle for $\mathbf{H}_1$, square for $\mathbf{H}_2$, triangle for $\mathbf{H}_3$).

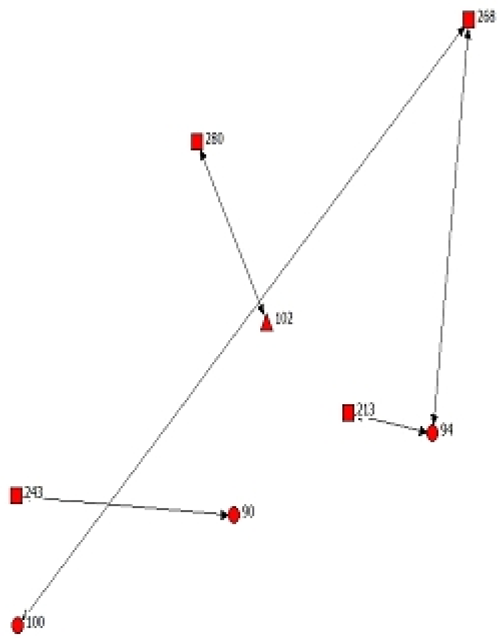

Figure 3: Non-isolated components of the R.E. network resulting from the same allocation of endowments and minimum novelty threshold $\tilde{n}^R$ supporting figure 2, and raising the minimum overlap threshold $\tilde{m}^R$ from 20 to 22. The field community of a node is identified by node shape (circle for $\mathbf{H}_1$, square for $\mathbf{H}_2$, triangle for $\mathbf{H}_3$). Notice how a modest rise of $\tilde{m}^R$ makes the R.E. network sparse. As in Figure 2, the qualitative structural property holds, that R.E. collaborations are agreed upon only by nodes belonging to adjacent fields.

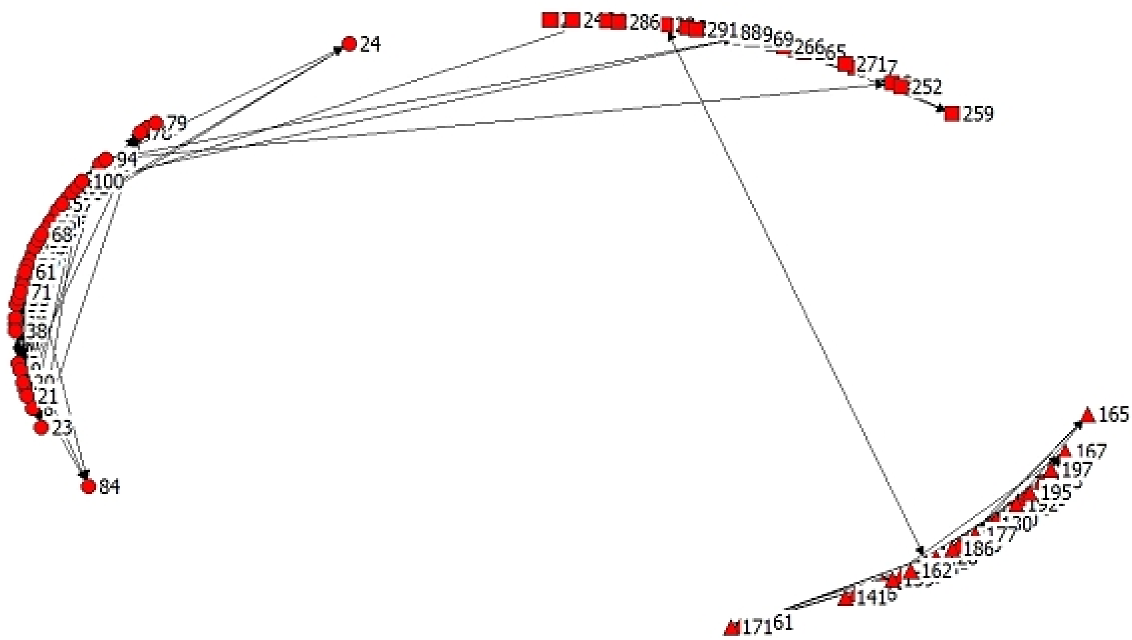

Figure 4: Giant and unique component of the mixed network resulting from the superposition of the I.E. network in Figure 1 and of the R.E. network in Figure 3, and subsequent delation of nodes with degree < 2. The the field community of a node is identified by node shape (circle for $\mathbf{H}_1$, square for $\mathbf{H}_2$, triangle for $\mathbf{H}_3$). The network is self-sustaining in the sense clarified in section 9.

9 Self sustaining networks

R&D activity makes the knowledge endowments endogenous. In this section we speculate on the architectures which may enable network survaival, under very weak assumptions concerning the rules that shape the change in endowments. To this end, we define a undirected R&D network $\mathbf{G} = \{\mathbf{H}, \mathbf{L}\}$ *self sustaining* if $\mathbf{L}$ is robust to the endogeneity of knowledge endowments.

Incremental innovation finds new ideas by exploring unknown configurations of an unchanging type space $\mathbf{\Gamma}$; radical innovation finds new ideas only through the discovery of new types, possibly embedded in knowledge space $\mathbf{\Omega}$ of higher dimension.

In what follows, we assume that any collaboration (i, j) causes a bi-directional finite flow of ideas between i and j . The flow does not directly extend (within the same time interval) to agents other than i and j, and there is a finite upper bound ω on the number of ideas which flow per unit of time. If (i, j) is a link in a R&D network, knowledge spillovers between i and j produce convergence of their knowledge bases, because the overlap λ_{ij} grows at the loss of novelty n_{ij} and n_{ji}. Without spillovers, innovation success by (i, j) causes weaker convergence, because discovery expands the overlap λ_{ij}, but leaves novelty n_{ij} and n_{ji} unchanged. Novelty is (ceteris paribus) enhanced by i's and j's simultaneous participation in collaborations (i, h) and (j, z). If $h \neq j$, and $z \neq i$[26], innovations by (i, h) and (j, z) expand i's and j's knowledge repertoires in non-overlapping directions.

In I.E. networks, the preservation of novelty n_{ij} and n_{ji} within a collaboration (i, j) finds its necessary eventual doom in the finite and fixed number of types in $\mathbf{\Gamma}$. In the short run, if both i and j have more than one link, they may be able to expand their knowledge repertoires in non-overlapping directions. If the expansion is fast enough to dominate the idea spillover between i and j, convergence between $\mathbf{A}_i$ and $\mathbf{A}_j$ will be postponed. But this countervailing force can not be everlasting. Search in a space of fixed and finite extension can not avoid the eventual decline of its success rate. Before success stops altogether, spillovers will eventually make novelty n_{ij} and n_{ji} between any two nodes i and j too low to justify continuation of collaboration $(i, j)^I$.

Proposition 9 *A undirected I.E. network is not self sustaining.*

Network survaival of a mixed (or R.E.) network is not always guaraneed either. To see this, consider the giant component $\mathbf{C} = \{\mathbf{H}_C, \mathbf{L}_C\}$ of the undirected mixed network $\mathbf{G} = \{\mathbf{H}, \mathbf{L}\}$, and suppose that there is some node i in $\mathbf{H}_C$ that is vertex to one and only one link in $\mathbf{L}_C$. This means that there is one and only one $j \in \mathbf{H}_C$ such that $(i, j) \in \mathbf{L}_C$. As a result, novelty n_{ji} is bound to fall through time, and j will eventually find collaboration with i unattractive.

Proposition 10 *A non-trivial component of a undirected R.E. or mixed network is self sustaining only if every link ij of the component is an element of*

[26] Notice that $z = h$ is not ruled out.

a cycle $\{za, ..., hi, ij, jk, ..., qz\}$. *An equivalent condition is that the component coincides with its 2-core decomposition*[27].

In any collaboration (i, j) embedded in a mixed (or R.E.) network meeting the necessary condition for self-sustainingness, there are forces acting counter to the novelty reducing effects caused by bidirectional knowledge flows between i and j. Such countervailing forces are sustained by innovations and spillovers which are produced by i's and j's simultaneous collaborations with other units.

The countervailing effect on $n_{ij} + n_{ji}$ is consistent with the possibility that i and j collaborate with the same unit z, and they do not have other research partners outside the set $\{i, j, z\} \subset \mathbf{H}$. To see this, consider the connected tryad formed by the nodes i, j, z and the links betwen them. At every time step, innovations produced by (i, j), by (i, z), and by (j, z) differ. As a result, for every couple of collaborators in the tryad, there is some new entry in the knowledge repertoire of one unit that is excluded (in the short-run) from the knowledge repertoire of the other. If knowledge spillovers are sufficiently slow relative to the rate of innovation, novelty may not fall over time, in spite of the fact that innovations and spillovers cause the growth over time of the overlaps λ_{ij}, λ_{iz}, and λ_{jz}. Moreover, the novelty-sustaining effects may be persistent, because the innovation rate is not forced to meet an eventual stop in a mixed network.

On the ground that every new type has an exponential effect on the number of ideas in the knowledge space, and that new types (basic ideas) would expand in a combinatorial way (Weitzman, 1998 [59]), it can be argued that any major slow down in the proliferation of ideas, and the consequent break to novelty, is only a remote possibility in a mixed network. We do not indulge in such speculations, here[28], and confine ourselves to the following observation.

Proposition 11 *On the assumption that the upper bound* ω *on knowledge spillovers is sufficiently low, relative to the innovation rate, the necessary condition for the self-sustainingness of a mixed network is also sufficient to prevent the fall of novelty below the minimum thresholds* $\tilde{n}^I$, $\tilde{n}^R$.

10 Concluding remarks

This section provides a brief summary of the main points raised in this paper and indicates some directions of further work.

[27] A *k-core* is a maximal subset of vertices such that each is connected to at least k other vertices in the subset. The word "maximal" means here that a group of vertices is only a k-core if it is not a subset of any larger group that is a k-core (Newman 2010 [39], p. 195). Notice that a 2-core decomposition of $\mathbf{G} = \{\mathbf{H}, \mathbf{L}\}$ may not be a connected subset of vertices, with the implication that a self sustaining network may host different components.

[28] Caminati (2006) [8] suggests that the arguments in favour of a non-declining innovation success rate have a long-run nature, and do not rule out the possibility of a temporary technological stasis.

It has been argued that knowledge diversity and similarity are complementary inputs to the R&D collaboration between any pair of heterogeneous organizations. The complementarity coefficients vary, according to the incremental or radical nature of the exploration activity under way. This provides a knowledge based explanation of R&D network formation, which is short-run, in that it is conditional on a given distribution of knowledge endowments, and makes abstraction from the indirect incentives to collaboration, that are related to network topology. The short-run nature of the model is the price to pay to the detailed analysis of the pairwise knowledge incentives to collaboration. Network formation broadly conforms to the empirically relevant properties of high clustering, asymmetry of link distribution, and short average distance, within the connected component of the network. In addition to these properties, that are now standard in the literature, the model predicts the structure of a mixed network, in which incremental and radical R&D collaborations co-exist. For appropriate values of the complementarity coefficients, a mixed I.R.E. network typically has a giant component, in which clusters of within-field I.E.collaborations are connected by the R.E. links of a number of high-knowledge firms, which are simultaneously engaged in both types of activity. The model further predicts that, on average, the relative cognitive distance $[1/2(n_{ij}/\lambda_{ij} + n_{ji}/\lambda_{ij})]$ of a collaboration pair (i, j) is correlated to the quality (incremental vs. radical) of its research activity. This provides a direct empirical test to corroborate or falsify the present theory. Indirect supporting evidence comes from empirical studies of R&D networks in the biotechnology, multi-media and other industries (Gilsing and Nooteboom 2006 [16], Gilsing and Duysters 2008 [15]). They reveal that within-cluster connections sent to firms working on the same well defined set of problems, coexist with between-cluster connections linking organizations which are mainly engaged in different areas of research. As is observed in such empirical studies, the diversity of knowledge across different clusters provides the variety which is *necessary* for recombination (Shilling and Phelps 2007 [51], Uzzi and Spiro 2005 [55]).

A direct benefit of this approach is that it lays the foundations of a dynamic extension in which network evolution is explicitly and rigorously tied to the processes of knowledge convergence and divergence. Only preliminary results of this dynamic extension are developed here. It is shown that network persistence crucially depends on the endogenous re-production of knowledge diversity, which may fall as a result of spillovers between collaborating organizations, but finds new possibility of expansion in the growing dimension of the knowledge space, produced by radical innovations. Here, and a fortiori in the planned fully developed dynamic extension, network topology and the intensity of spillovers matter[29].

[29] Other knowledge-based models of R&D network evolution, fully abstract from spillovers (Cowan and Jonard 2009 [12], König et al. 2011 [27]).

11 Appendix

11.1 Proof of proposition 1

The proof below follows the argument made in Cowan and Jonard (2009) [12]. We report expression (7) for convenience.

$$
\begin{aligned}
P(\lambda_{ij} &= m|K_i, K_j) = \\
&= \binom{\max(K_i, K_j)}{m} \binom{\delta - \max(K_i, K_j)}{\min(K_i, K_j) - m} \left[\binom{\delta}{\min(K_i, K_j)} \right]^{-1}
\end{aligned}
$$

For the sake of the argument, let $q \in \{i, j\}$ s.t. $K_q = \max(K_i, K_j)$. Since the ordering of ideas is arbitrary, we can conventionally order the ideas defined on $\mathbf{X}_w$, so that the endowment $\mathbf{K}_q^w$ is a list of length δ, such that its first K_q elements are equal to one, and the remaining $\delta - K_q$ elements are zero. The term $\binom{\max(K_i, K_j)}{m} \binom{\delta - \max(K_i, K_j)}{\min(K_i, K_j) - m}$ is the number of different strings of length δ, and containing $\min(K_i, K_j)$ components equal to 1, which have m ones in their first $\max(K_i, K_j)$ elements (and therefore contain $\min(K_i, K_j) - m$ ones in their last $\delta - \max(K_i, K_j)$ elements). The number of different strings of length δ, which have $\min(K_i, K_j)$ elements equal to one, is $\binom{\delta}{\min(K_i, K_j)}$. These strings are all equi-probable; hence the probability of each such a string is $\left[\binom{\delta}{\min(K_i, K_j)} \right]^{-1}$. This concludes the proof.

11.2 Proof of proposition 3

We report expression (9) for convenience.

$$
P_{w,w+1}(\bar{K}_h|K_h) = \binom{\gamma}{\bar{K}_h} \binom{\delta - \gamma}{K_h - \bar{K}_h} \left[\binom{\delta}{K_h} \right]^{-1} \qquad h = i, j
$$

We adopt two conventional re-orderings $(\mathbf{a}_1^w, \mathbf{a}_2^w, ..., \mathbf{a}_\delta^w)$, $(\mathbf{a}_1^{w+1}, \mathbf{a}_2^{w+1}, ..., \mathbf{a}_\delta^{w+1})$ of the known ideas defined on the fields $\mathbf{X}_w$ and $\mathbf{X}_{w+1}$(respectively), such that the ideas within $\mathbf{X}_w \cap \mathbf{X}_{w+1}$ occupy the first γ positions in these lists. With this notation in place, $\binom{\gamma}{\bar{K}_h} \binom{\delta - \gamma}{K_h - \bar{K}_h}$ is the number of different endowments (strings of length δ), with a total number K_h of ideas, which have $\bar{K}_h$ elements equal to 1 in their first γ positions, and $K_h - \bar{K}_h$ elements equal to 1 in their last $\delta - \gamma$ positions. The total number of different strings of length δ with K_h elements equal to 1 is $\binom{\delta}{K_h}$, and the random assignment of endowments implies that all such strings have the same probability. Conditional on endowment size K_h, this probability is therefore $\left[\binom{\delta}{K_h} \right]^{-1}$. This concludes the proof.

11.3 Proof of proposition 4

We adapt the logic of the argument supporting proposition 2, to the conditions considered in proposition 4. In addition to the incentive constraints $\lambda_{ij} \geq \tilde{m}^V$, $n_{ij} \geq \tilde{n}^V$, $n_{ji} \geq \tilde{n}^V$, overlap and novelty meet, by constuction, the feasibility constraints: $\lambda_{ij} \leq \gamma$, $\lambda_{ij} \geq \bar{K}_i + \bar{K}_j - \gamma$,and $n_{ij} \geq K_j - \bar{K}_j$, $n_{ji} \geq K_i - \bar{K}_i$. The last two inequalities yield $\lambda_{ij} \leq \min(\bar{K}_j, \bar{K}_i)$. Moreover, novelty from ideas within the intersection $\mathbf{X_w} \cap \mathbf{X}_{w+1}$ is needed to meet the incentive constraints only if $K_h - \tilde{n}^V < \bar{K}_h$, $h = i, j$. This means that, for $h = i, j$, the overlap is required to satisfy the constraints $\lambda_{ij} \leq \max[0, \min(\bar{K}_h, K_h - \tilde{n}^V)]$. We conclude that the lower bound $\lambda^V.\min$ and upper bound $\lambda^V.\max$, which delimit the the values of λ_{ij} that are consistent with the incentive and feasibility constraints above, are re-defined as follows. $\lambda^V.\min = \max(\tilde{m}^V, \bar{K}_i + \bar{K}_j - \gamma)$; $\lambda^V.\max = \max[0, \min(\bar{K}_i, \bar{K}_j, K_i - \tilde{n}^V, K_j - \tilde{n}^V)]$. The expression below gives the probability that $i \in \mathbf{H}_w$ and $j \in \mathbf{H}_{w+1}$ form a type-V collaboration, conditional on the size K_i, K_j of their endowments, and on the event the number of i's and j's ideas in the intersection $\mathbf{X}_w \cap \mathbf{X}_{w+1}$ are $\bar{K}_i$ and $\bar{K}_j$, respectively.

$$P_{w,w+1}((i,j)^V \in \mathbf{L}^V | \bar{K}_i, \bar{K}_j) = \sum\nolimits_{\lambda_{ij}=\lambda.\min}^{\lambda_{ij}=\lambda.\max} \binom{\max(\bar{K}_i, \bar{K}_j)}{\lambda_{ij}} \cdot \binom{\gamma - \max(\bar{K}_i, \bar{K}_j)}{\min(\bar{K}_i, \bar{K}_j) - \lambda_{ij}} \left[\binom{\gamma}{\min(\bar{K}_i, \bar{K}_j)} \right]^{-1} \quad (12)$$

Because only ideas defined on the competence fields $\mathbf{X}_w$ and $\mathbf{X}_{w+1}$ are randomly and independently assigned to i and j (respectively) with positive probability, it follows that, given the size K_i and K_j of their endowments, the condional probability of the event "i's and j's ideas in the intersection $\mathbf{X}_w \cap \mathbf{X}_{w+1}$ are $\bar{K}_i$ and $\bar{K}_j$, respectively, and $(i,j) \in \mathbf{L}^V$" is:

$$P_{w,w+1}((i,j)^V \in \mathbf{L}^V | \bar{K}_i, \bar{K}_j) \cdot P_{w,w+1}(\bar{K}_i | K_i) \cdot P_{w,w+1}(\bar{K}_j | K_j) \quad (13)$$

Expression (10) of proposition 4 in the text is obtained summing (13) over all the assignments $\bar{K}_i$, $\bar{K}_j$, such that $\tilde{m}^V \leq \bar{K}_h \leq K_h$, $h = i, j$, and makes use of the conventions $\binom{z}{m} = 0$ if $m > z$ and $\binom{z}{0} = 1$.

11.4 Conditional expected degree

Conditional on endowment size K_i, the expected number of type-V links, that $i \in \mathbf{H}_w$, forms with the nodes in the same community $\mathbf{H}_w$, which have endowment size K, $0 \leq K \leq \delta$, is

$$L_w^V(K_i, K) = P_w((i,j)^V \in \mathbf{L}^V | K_i, K_j = K) \cdot E(H_w(K)) \quad (14)$$

where $V = I, R$, and $E(H_w(K))$ is the expected number of nodes in community $\mathbf{H}_w$ which are assigned an endowment K by the random assignment rule.

The same argument, applied to crossfield links, yields, with obvious notation:

$$L^V_{w,w+1}(K_i, K) = P_{w,w+1}((i,j)^V \in \mathbf{L}^V | K_i, K_j = K) \cdot E(H_{w+1}(K)) \qquad (15)$$

Explicit solutions are obtained by substitution in (14) and (15), from (8) and (10), respectively. Conditional on endowment size K_i, the expected number of type-V links that $i \in \mathbf{H}_w$ sends to nodes in $\mathbf{H}_w$ and $\mathbf{H}_{w+1}$, respectively, are:

$$L^V_w(K_i) = \sum\nolimits_{K=0}^{K=\delta} L^V_w(K_i, K)$$

$$L^V_{w,w+1}(K_i) = \sum\nolimits_{K=0}^{K=\delta} L^V_{w,w+1}(K_i, K)$$

Our assumptions on overlap and novelty thresholds imply that organizations are only interested in collaboration with organizations in the same or in adjacent fields. This leads to the following formulation of the conditional expected degree:

$$L^V(K_i) = L^V_{w,w+1}(K_i) + L^V_w(K_i), \text{ if } w = 1$$

$$L^V(K_i) = L^V_{w-1,w}(K_i) + L^V_w(K_i), \text{ if } w = W$$

$$L^V(K_i) = 2L^V_{w,w+1}(K_i) + L^V_w(K_i), \text{ if } 1 < w < W$$

References

[1] Ahuja C. (2000): "Collaboration networks, structural holes, and innovation: A longitudinal study", *Administrative Science Quarterly*, 45, 425-455.

[2] Auerswald P., Kauffman S., Lobo J., and Shell K. (2000): "The production recipes approach to modeling technological innovation: An application to learning by doing", *Journal of Economic Dynamics and Control*, 24, 389-450.

[3] Barabasi A.L., and Albert R. (1999): "Emergence of scaling in random networks", *Science*, 286, 509-512.

[4] Barabasi A.L., Jeong H., Néda Z., Ravasz E., Schubert A., and Vicsek T. (2002): "Evolution of the social network of scientific collaboration", *Physica A*, 311, 590-614.

[5] Bogenrieder I. and Nooteboom B. (2004): "The emergence of learning communities: A theoretical analysis", in Tsoukas H. and Mylonopoulos N. (eds.), *Organizations as knowledge systems*, New York, Palgrave Macmillan, 46-66.

[6] Burt R.S. (1992): *Structural Holes. The Social Structure of Competition*, Cambridge, MA, Harvard University Press.

[7] Callon M. (2002): "From science as an economic activity to socioeconomics of scientific research. The dynamics of emergent and consolidated techno-economic networks", in Mirowski P.P. and Sent E.M. (eds.), *Science Bought and Sold: Essays in the Economics of Science*, Chicago, University of Chicago Press, 277-317.

[8] Caminati M. (2006): "Knowledge growth, complexity and the returns to R&D", *Journal of Evolutionary Economics*, 16, 207-229.

[9] Caminati M. (2009): "A knowledge based approach to collaboration in basic research", MPRA Paper n° 19521, online at http://mpra.ub.uni-muenchen.de/19521/.

[10] Carayol N. and Roux P. (2009): "Knowledge Flows and the geography of networks: A strategic model of small world formation", *Journal of Economic Behavior and Organization*, 71, 414-427.

[11] Cohendet P. (2005): "On knowing communities", paper prepared for the conference Advancing Knowledge and the Knowledge Economy, Washington,DC.

[12] Cowan R. and Jonard N. (2009): Knowledge portfolios and the organization of innovation networks, *Academy of Management Review*, 34, 320-342.

[13] Dosi G. (1982): "Technological paradigms and technological trajectories: A suggested interpretation of the determinants and directions of technological change", *Research Policy*, 11, 147-162.

[14] Frenken K., Marengo L. and Valente M. (1999): "Interdependencies, nearly-decomposability and adaptation", in Brenner T. (ed.), *Computational Techniques for Modelling Learning in Economics*, Dordrecht, Kluwer, 145-165.

[15] Gilsing V. and Duysters G.M. (2008): "Understanding novelty creation in exploration networks-Structural and relational embeddness jointly considered", Technovation, 28, 693-708.

[16] Gilsing V. and Nooteboom B. (2006): "Exploration and exploitation in innovation systems: The case of pharmaceutical biotechnology", *Research Policy*, 35, 1-23.

[17] Goyal S. (2007): *Connections. An Introduction to the Economics of Networks*, Princeton, Princeton University Press.

[18] Goyal S. and Joshi S. (2003): "Networks of collaboration in oligopoly", *Games and Economic Behavior*, 43, 57-85.

[19] Goyal S. and Joshi S. (2006): "Unequal connections", *International Journal of Game Theory*, 34, 319-349.

[20] Goyal S. and Moraga-González J.L. (2001): "R&D networks", *RAND Journal of Economics*, 32, 686-707.

[21] Goyal S., van der Leij M., and Moraga J. L. (2006): Economics: emerging small world, *Journal of Political Economy*, 114, 403-412.

[22] Jackson M. O. (2008): *Social and Economic Networks*, Princeton, Princeton University Press.

[23] Jackson M.O. and Rogers B.W. (2005): "The economics of small worlds", *Journal of the European Economic Association*, 3, 617-627.

[24] Jackson M.O. and Wolinsky A. (1996): A strategic model of social and economic networks", *Journal of Economic Theory*, 71, 44-74.

[25] Hansen M.T. (2002): "Knowledge networks: Explaining effective knowledge sharing in multiunit companies", *Organization Science*, 13, 232-250.

[26] Kauffman S. (1993): *The Origins of Order*, New York, Oxford University Press.

[27] König M.D., Battiston S., Napoletano M., and Schwitzer F. (2011): "Recombinant knowledge and the evolution of innovation networks", *Journal of Economic Behavior and Organization*, 79, 145-164.

[28] König M.D., Battiston S., Napoletano M., and Schweitzer F. (2012): "The efficiency and stability of R&D networks", *Games and Economic Behavior*,75, 694-713.

[29] Hagedoorn J. (2002): "Inter-firm R&D partnership: an overview of major trends and patterns since 1960", *Research Policy*, 31, 477-492.

[30] Hagedoorn J. and Roijakkers (2006): "Inter-firm R&D partnering in pharmaceutical biotechnology since 1975: Trends, patterns and networks", *Research Policy*, 35, 431-446.

[31] Hanaki N., Nakaijma R., and Ogura Y. (2010): "The dyanamics of R&D network in the IT industry", *Research Policy*, 39, 386-399.

[32] Holland J.H. (1992): *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, Artificial Intelligence*, 2nd ed., Cambridge MA, MIT Press.

[33] Lane D., Malerba F., Maxfield R., Orsenigo L. (1996): "Choice and action", *Journal of Evolutionary Economics*, 6, 43-76.

[34] March J. (1991): "Exploration and exploitation in organizational learning", *Organization Science*, 2, 71-87.

[35] Marengo L., Pasquali C., and Valente M. (2005): "Decomposability and Modularity of Economic Interactions", in Callebaut W. and Rasskin-Gutman D. (eds.), *Modularity. Understanding the Development and Evolution of Natural Complex Systems*, Cambridge MA., MIT Press, 383-408.

[36] Newman M.E.J. (2001): "The structure of scientific collaboration networks", *Proceedings of the National Academy of Sciences*, 98, 404-409.

[37] Newman M.E.J. (2004): "Coauthorship networks and patterns of scientific collaboration", *Proceedings of the National Academy of Sciences*, 101, 5200-5205.

[38] Newman M.E.J. (2006): "Modularity and Community Structure in Networks", *Proceedings of the National Academy of Sciences*, 103, 8577-8582.

[39] Newman M.E.J. (2010): *Networks. An Introduction*, New York, Oxford University Press.

[40] Newman M.E.J. and Girvan M. (2002): "Community structure in social and biological networks", *Proceedings of the National Academy of Sciences*, 99, 7821-7826.

[41] Newman M. E. J. and Girvan M. (2004): "Finding and Evaluating Community Structure in Networks", *Physical Review E*, 69, 026113.

[42] Nooteboom B. (1992): "Toward a dynamic theory of transactions", *Journal of Evolutionary Economics*, 2, 281-299.

[43] Nooteboom B. (1999): *Inter-firm Alliances: Analysis and Design*, London, Routledge.

[44] Nooteboom B. (2000): *Learning and Innovation in Organization and Economies*, Oxford, Oxford University Press.

[45] Nooteboom B. (2004): *Inter-firm Collaboration, Learning and Networks*, London, Routledge.

[46] Nooteboom B., Haverbeke W.V., Duysters G., Gilsing V., van den Oord A. (2007): "Optimal cognitive distance and absorptive capacity", *Research Policy*, 36, 1016-1034.

[47] Powell W.W., White D.R., Koput K.W., Owen_Smith J. (2005): "Network dyanamics and field evolution: the growth of interorganizational collaboration in the life-sciences, *American Journal of Sociology*, 110, 1132-1205.

[48] Reiter S. (1992): Knowledge, Discovery and Growth. Discussion Paper #1011, Northwestern University.

[49] Reiter S. (2001): "Knowledge, Discovery and Growth", in Olson G.M., Malone T.W., and Smith J.B. (eds.), *Coordination Theory and Collaboration Technology*, Mahwah, NJ, Lawrence Erlbaum Associates, 193-260.

[50] Saviotti P.P. (2009): "Knowledge networks:structure and dynamics", in Pyka A. and Scharnhorst A. (eds.), *Innovation Networks*, Berlin Heidelberg, Spinger-Verlag, 19-41.

[51] Shilling M. A. and Phelps C. C. (2007): "Interfirm collaboration networks: the impact of large-scale network structure on firm innovation", *Management Science*, 53, 11131126.

[52] Simon H. A. (1962): "The Architecture of Complexity", *Proceedings of the American Philosophical Society*, 106, 467-482; reprint in Litterer J. A. (ed.), *Organizations: Systems, Control and Adaptation*, 2, New York, J. Wiley, (1969), 98-114.

[53] Simon H. A. (1973): "The organization of complex ystems", in Patee H. (ed.), *Hierarchy Theory*, New York, Braziller, 1-27.

[54] Simon H. A. (2005): "The architecture of complexity in an evolving world: The role of near decomposability", in Callebaut W. and Rasskin-Gutman D. (eds.), *Modularity. Understanding the Development and Evolution of Natural Complex Systems*, Cambridge MA., MIT Press, ix-xiii.

[55] Uzzi B. and Spiro J. (2005): "Collaboration and creativity: The small world problem", *American Journal of Sociology*, 111, 447-504.

[56] Vega-Redondo F. (2007): *Complex Social Networks*, Cambridge UK, Cambridge University Press.

[57] Watson R. A. (2006): *Compositional Evolution. The Impact of Sex, Symbiosis, and Modularity on the Gradualist Framework of Evolution*, Cambridge MA, MIT Press.

[58] Watts D. (1999): *Small Worlds*, Princeton, Princeton University Press.

[59] Weitzman M. L. (1998): "Recombinant Growth", *Quarterly Journal of Economics*, 113, 331–360.

[60] Westbrock B. (2010): "Natural concentration in industrial research collaboration", *RAND Journal of Economics*, 41, 351-371.

[61] Wuyts S., Colombo M.G., Dutta S., Nooteboom B. (2006): "Empirical tests of optimal cognitive distance", *Journal of Economic Behavior and Organization*, 58, 277-302.