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A dynamic entry and price game with capacity indivisibility

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Abstract - Strategic market interaction is here modelled as a two-stage game in which potential entrants choose capacities and active firms compete in prices. Due to capital indivisibility, the capacity choice is made from a finite grid and there are substantial economies of scale. In the simplest version of the model assuming a single production technique, the equilibrium of the game is shown to depend on the market size - namely, on total demand at a price equal to the minimum average cost - relative to the firm minimum efficient scale: if the market is sufficiently large, then the competitive price (the minimum of average cost) emerges at a subgame-perfect equilibrium of the game; if the market is not that large, then the firms randomize in prices on the equilibrium path of the game. The role of the market size for the competitive outcome is even more important for the case of two production techniques.

JEL classification: D43, D44, L13

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1 Introduction

Research on Bertrand-Edgeworth competition with endogenous capacity has achieved important results on the relationship between price and Cournot competition. Under the efficient rationing rule, the subgame-perfect equilibrium of a duopolistic two-stage capacity and price game (henceforth, CPG) yields the Cournot outcome (Kreps and Scheinkman, 1983). This may not hold, though, under alternative rationing rules, where the equilibrium of the price subgame (henceforth, PS) may be in mixed strategies on the equilibrium path (Davidson and Deneckere, 1986). Madden (1998) has shown that a uniformly elastic demand curve is sufficient for the Cournot outcome under oligopoly, regardless of the rationing rule. According to Boccard and Wauthy (2000), while the Cournot result extends to oligopoly under KS's assumptions on cost and rationing rule, this need not be so if the firms can produce above "capacity" at a finite extra-cost.

Throughout this literature the cost of capacity has been viewed as a continuous convex function. Thus, at an equilibrium of the two-stage CPG identical potential entrants choose positive capacities and prices are above the competitive level.[1] Quite differently, in Yano (2008) average cost is U-shaped and potential entrants play a one-stage game: each firm chooses a "price/set of quantities" pair, the set including any quantity which it is indifferent to produce (on demand) at the chosen price. A competitive outcome obtains, i. e., prices equal to the minimum average cost.[2]

In the present paper, we assume that strategic interaction at the price-setting stage takes place among firms that have previously installed a positive capacity. However, we depart from the standard setup by assuming economies of scale due to capital indivisibility.[3] We model a two-stage CPG under efficient rationing and constant average variable cost below capacity. Then the equilibrium may yield the long-run competitive outcome.

The paper is organized as follows. Section 2 presents a model with a single production technique. At any equilibrium of the CPG, total capacity turns out to be equal to the quantity demanded at a price equal to the minimum average cost, while pricing on the equilibrium path depends on the market size relative to the firm minimum efficient scale: with a sufficiently

[1] See Section 4 below.

[2] At this price, each firm is willing to produce 0 as well as the average-cost minimizing output (q^*). Any firm producing q^* would sell nothing if raising its price: buyers would turn elsewhere and have their demand met by some firm that is otherwise producing 0.

[3] The role of indivisibility of productive factors (especially of capital equipment) for economies of scale has long been recognized (see Kaldor, 1934, and Koopmans, 1957).

large market, the competitive price (the minimum average cost) is charged, otherwise the PS has a mixed strategy equilibrium on the equilibrium path. Section 3 provides a simple generalization by showing how the competitive outcome can arise when two production techniques are available. Section 4 clarifies the key role of capacity indivisibility in our model.

2 A single production technique

In a homogeneous-product industry, let $D(p)$ and $P(Q)$ be the demand and the inverse demand function, respectively, p the market price, and Q the total quantity; $D'(p) < 0$ and $D''(p) \leq 0$ for $p \in (0, \overline{p})$, where $D(p) = 0$ at $p \geq \overline{p}$ and $D(p) > 0$ at $p < \overline{p}$.[4] At stage 1, set $\mathcal{Z} = \{1, ..., i, ..., z\}$ of potential entrants choose capacity: the capacity choice set is assumed to be $\mathcal{F}_+$ (the set of non-negative integers), due to indivisibility of capital. At stage 2, active firms (each i with capacity $\overline{q}_i > 0$) set prices. Assuming a constant cost per unit of capacity, active firm i's short-run cost is $c(q_i) = c\overline{q}_i$ for output $q_i \leq \overline{q}_i$ (we let 0 be the (constant) unit variable cost), and q_i cannot exceed $\overline{q}_i$. For each $i \in \mathcal{Z}$, long-run cost is thus $C(q_i) = c\overline{q}_i$, where $\overline{q}_i = [q_i, q_i + 1) \cap \mathcal{F}_+$ for $q_i \in \mathbb{R}_+$ ($\mathbb{R}_+$ being the set of non-negative reals): $C(q_i)$ is constant for $q_i \in (f, f+1]$ and jumps up by c as q_i marginally increases above f, hence $C(q_i)$ is not everywhere convex. Clearly, capacity indivisibility results in scale economies over any output interval $(f, f+1]$, where average cost decreases from $c(1 + 1/f)$ to c.

A deterministic capacity choice is assumed to be made by each $i \in \mathcal{Z}$ to maximize the expectation of profits $\pi_i = p_i q_i - c\overline{q}_i$. We denote by $\overline{\mathcal{Q}} = \{\overline{q}\}$ the set of feasible capacity vectors, where $\overline{q} = (\overline{q}_1, ..., \overline{q}_z)$ is a capacity vector resulting from stage-1 decisions. We also let $\mathcal{A} = \{i \mid \overline{q}_i > 0\}$ and $n = \#\mathcal{A}$ be the set and the number of active firms at $\overline{q}$, respectively, $\overline{q}_{-i}$ the capacity vector of i's rivals, $\overline{Q}$ total capacity, and g any firm with the largest capacity. At stage 2 every $i \in \mathcal{A}$ knows $\overline{q}$.

As a competitive benchmark we refer to the (long-run) competitive equilibrium (CE), i. e., the equilibrium of an industry where price-taking potential entrants make simultaneous capacity and quantity decisions. However, the CE does not usually exist here.[5] Total supply $S(p)$ is indefinitely large at $p > c$, and zero at $p < c$, while $S(c) \in \mathcal{F}_+$: at $p = c$, entrants choose any feasible capacity and supply it entirely (any lower output results in losses).

[4] For $Q > D(0)$ we let $P(Q) = 0$.

[5] For nonexistence under U-shaped average cost, see Mas-Colell et al. (1995), pp. 337-8.

Thus it can be $S(c) = D(c)$ only if $D(c) \in \mathcal{F}_+$.[6] To overcome non-existence we let $D(c) \in \mathcal{F}_+$: the competitive price and output are, respectively, $p^{ce} = c$ and $Q^{ce} = D(c)$.

At any $\overline{q}$, let $p^w(\overline{q})$ and $Q^w(\overline{q})$ be, respectively, the market-clearing price and total output with price-taking firms: $p^w(\overline{q}) = P(\overline{Q})$ and $Q^w(\overline{q}) = \overline{Q}$ if $\overline{Q} \leq D(0)$, while $p^w(\overline{q}) = 0$ and $Q^w(\overline{q}) = D(0)$ if $\overline{Q} \geq D(0)$. Further, $\pi_i^w(\overline{q}) = [p^w(\overline{q}) - c]\overline{q}_i$ denotes firm i's profit at $\overline{q}$ under market clearing and $\pi_i^w(\overline{q}_i, \overline{q}_{-i}) = [p^w(\overline{q}_i, \overline{q}_{-i}) - c]\overline{q}_i$ denotes firm i's profit under market clearing as a function of $\overline{q}_i$, given $\overline{q}_{-i}$. If $\overline{q}_i$ were continuous, then concavity of $\pi_i^w(\overline{q}_i, \overline{q}_{-i})$ would follow from $D''(p) \leq 0$.

A PS is played at any $\overline{q}$. Let $\mathbf{p} = (p_1, ..., p_n) = (p_i, p_{-i})$ be a pure strategy profile in the PS, p_{-i} being the strategy profile of i's rivals, and let $d_i(p_i, p_{-i}, \overline{q})$, $q_i(p_i, p_{-i}, \overline{q}), \pi_i(p_i, p_{-i}, \overline{q})$ and $\Pi_i(p_i, p_{-i}, \overline{q})$ be, respectively, firm i's demand, output, profit and revenue in the $\overline{q}$-PS at strategy profile $\mathbf{p}$: $\pi_i(p_i, p_{-i}, \overline{q}) = p_i q_i(p_i, p_{-i}, \overline{q}) - c\overline{q}_i = p_i \min\{d_i(p_i, p_{-i}, \overline{q}), \overline{q}_i\} - c\overline{q}_i$. With efficient rationing, $d_i(p_i, p_{-i}, \overline{q}) = \max\{0, D(p_i) - \sum_{j \neq i} \overline{q}_j\}$ when $p_i > p_j$ for any $j \neq i$. With $\sum_{j \neq i} \overline{q}_j < D(0)$, we let $\widetilde{q}_i = \widetilde{q}(\sum_{j \neq i} \overline{q}_j) = \arg\max_{q_i} P(q_i + \sum_{j \neq i} \overline{q}_j)q_i$, $\widetilde{\Pi}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j)\widetilde{q}_i$, and $\widetilde{p}_i = \widetilde{p}(\sum_{j \neq i} \overline{q}_j) = \arg\max_p p[D(p) - \sum_{j \neq i} \overline{q}_j]$. Clearly, $\widetilde{p}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j)$ and hence $\widetilde{\Pi}_i = \widetilde{p}_i \widetilde{q}_i$; also, $\max_i \widetilde{p}_i = \widetilde{p}_g$ since $\widetilde{p}'(\cdot) < 0$. So long as $\widetilde{q}_i \leq \overline{q}_i$, $\widetilde{q}_i$ is i's (short-run) Cournot best response to an output of $\sum_{j \neq i} \overline{q}_j$ by rivals: $\widetilde{q}'(\cdot) < 0$. Let $\pi_i(\overline{q})$ and $\Pi_i(\overline{q})$ be, respectively, i's expected profit and revenue at an equilibrium of the PS. The following fact is easily understood.

Lemma 1. *For any $i \in \mathcal{A}$, $\pi_i(\overline{q}) \geq \pi_i^w(\overline{q})$.*

Proof. This is obvious if $p^w(\overline{q}) = 0$. With $p^w(\overline{q}) > 0$, by charging $p^w(\overline{q})$ firm i fully utilizes capacity and hence earns $\pi_i^w(\overline{q})$, regardless of p_{-i}. ■

With $\overline{Q} \neq D(0)$, $\overline{q}_g/\overline{Q}$ must be sufficiently small in order for the market-clearing price to obtain at an equilibrium of the PS.

Lemma 2. *(i) If $p^w(\overline{q}) = 0$, $(p^w, ..., p^w)$ is an equilibrium of the $\overline{q}$-PS iff $\overline{q}_g/\overline{Q} \leq 1 - D(0)/\overline{Q}$. (ii) If $p^w(\overline{q}) > 0$, $(p^w, ..., p^w)$ is the equilibrium of the $\overline{q}$-PS $\overline{q}$ iff*

$$-\frac{p^w D'(p^w)}{\overline{q}_g} \geq 1. \tag{1}$$

Proof. (i) All prices equal to zero is an equilibrium iff $\sum_{j \neq g} \overline{q}_j \geq D(0)$, which leads to the stated condition.[7]

[6] Even then, some coordination is needed for the firms to exactly supply $D(c)$.

[7] Any strategy profile such that $\sum_{j \neq i : p_j = 0} \overline{q}_j \geq D(0)$ for any $i : p_i = 0$ is an equilibrium.

(ii) $(p^w, ..., p^w)$ is an equilibrium iff $[\partial(p(D(p)\text{-}\sum_{j\neq i}\overline{q}_j))/\partial p]_{p=p^{w(+)}} \leq 0$ for all $i \in \mathcal{A}$: this leads to $-p^w D'(p^w) \geq \overline{q}_i$, hence to (1). Uniqueness of equilibrium can be established straightforwardly. ■

Inequality (1) has a clear meaning: $p^w > 0$ is an equilibrium if and only if each firm's residual demand has elasticity not less than 1 when its price is raised above p^w. (1) can also be written $\overline{q}_g/\overline{Q} \leq \eta_{p=p^w}$, where $\eta_{p=p^w}$ is total demand elasticity at price p^w, or $\widetilde{p}_g \leq p^w$. A pure-strategy equilibrium (pse) does not exist when $p^w = 0$ and $\sum_{j\neq g}\overline{q}_j < D(0)$ or when $p^w > 0$ and $\widetilde{p}_g > p^w$. Then a mixed-strategy equilibrium (mse) exists: all the sufficient conditions of Theorem 5 of Dasgupta and Maskin (1986) for equilibrium existence are satisfied. It is a key property of mse that g's expected revenue equals the revenue of the Stackelberg follower when rivals supply their capacity.

Lemma 3. *At any $\overline{q}$ for which no pse exists, $\Pi_g(\overline{q}) = \widetilde{\Pi}_g = \widetilde{p}_g\widetilde{q}_g$.*

Proof. See Boccard and Wauthy (2000), De Francesco (2003) and, more recently, Hirata (2009). ■

Let $\pi_i^w(\widetilde{q}_i, \overline{q}_{-i}) = [P(\widetilde{q}_i + \sum_{j\neq i}\overline{q}_j) - c]\widetilde{q}_i$. By Lemma 3, $\pi_g(\overline{q}) = \pi_g^w(\widetilde{q}_g, \overline{q}_{-g}) - c(\overline{q}_g - \widetilde{q}_g)$ at a mse. We denote by $\overline{\mathcal{Q}}^* = \{\overline{q}^*\}$ the set of the least concentrated capacity configurations consistent with the CE-capacity: $\overline{q}^*$ is such that $n^* = \overline{Q}^* = D(c)$. We have this result.

Proposition 1 *(i) If $-cD'(c) \geq 1$, then any $\overline{q}^*$ is (part of) an equilibrium of the CPG where the competitive price c is charged on the equilibrium path; (ii) if $-cD'(c) < 1$, then any $\overline{q}^*$ is (part of) an equilibrium of the CPG where the firms randomize over prices on the equilibrium path. (iii) $\overline{Q} = D(c)$ at any equilibrium of the CPG.*

Proof. (i) At $\overline{q}^*$ inequality (1) reads $-cD'(c) \geq 1$: holding it, $(c, ..., c)$ is the equilibrium of the PS. An active firm (any $i \in \mathcal{A}^*$) has made a best capacity response to $\overline{q}^*_{-i}$. If $\widetilde{p}_i^* > P(D(c)+1)$,[8] a mse obtains if deviating to $\overline{q}_i' \geq 2$, resulting in $\pi_i(\overline{q}_i', \overline{q}^*_{-i}) = \widetilde{p}_i^*\widetilde{q}_i^* - c\overline{q}_i'$. This is negative because $\widetilde{p}_i^* \leq c$ and $1 \leq \widetilde{q}_i^* < 2 \leq \overline{q}_i'$. If $\widetilde{p}_i^* \leq P(D(c)+1)$, deviating to $\overline{q}_i' = 2$ leads to a pse, hence to a loss. A fortiori losses arise if deviating to $\overline{q}_i' > 2$. Finally, at $\overline{q}^*$ an inactive firm (any $u \notin \mathcal{A}^*$) has made a best response. Denote by $(\overline{q}_u', \overline{q}^*_{-u})$ the capacity vector when u deviates to $\overline{q}_u' > 0$. Obviously $\pi_u(\overline{q}_u', \overline{q}^*_{-u}) < 0$ if a pse obtains. At a mse, $\pi_u(\overline{q}_u', \overline{q}^*_{-u}) = \widetilde{p}_u\widetilde{q}_u - c\overline{q}_u'$; this is negative since $\widetilde{p}_u = P(\widetilde{q}_u + D(c)) < c$ and $\widetilde{q}_u = \widetilde{q}(D(c)) < \overline{q}_u'$.

[8] According to our notation, $\widetilde{p}_i^* = P(\widetilde{q}_i^* + \sum_{j\neq i}\overline{q}_j^*)$ and $\widetilde{q}_i^* = \arg\max_{q_i} P(q_i + \sum_{j\neq i}\overline{q}_j^*)q_i$.

(ii) A symmetric mse arises at $\overline{q}^*$ and $\pi_i(\overline{q}^*) = \widetilde{p}_i^*\widetilde{q}_i^* - c > 0$.[9] For $i \in \mathcal{A}^*$, deviating to $\overline{q}_i' > 1$ raises cost while, by Lemma 3, expected revenue does not change. For $u \notin \mathcal{A}^*$, deviating to $\overline{q}_u' > 0$ leads to a mse,[10] hence $\pi_u(\overline{q}_u', \overline{q}_{-u}^*) = \widetilde{p}_u\widetilde{q}_u - c\overline{q}_u' < 0$ since $\widetilde{p}_u = P(\widetilde{q}_u + D(c)) < c$ and $\widetilde{q}_u = \widetilde{q}(D(c)) < \overline{q}_u'$.

(iii) With $\overline{Q} < D(c)$, any $u \notin \mathcal{A}$ will profit by deviating to $\overline{q}_u' = 1$ and charging $P(\overline{Q} + 1)$.[11] With $\overline{Q} > D(c)$ and holding (1), any $i \in \mathcal{A}$ makes losses. If $\overline{Q} > D(c)$ and (1) does not hold, then g will profit by reducing capacity by one. This is immediate if $\overline{q}_i = 1$ for all $i \in A$: then $\pi_i(\overline{q}) = \widetilde{p}_i\widetilde{q}_i - c < 0$ since $\widetilde{p}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j) < c$ ($\sum_{j \neq i} \overline{q}_j > D(c)$) and $\widetilde{q}_i < 1$. With $\overline{q}_g > 1$, $\pi_g(\overline{q}) = \widetilde{p}_g\widetilde{q}_g - c\overline{q}_g$. Let $\widetilde{p}_g > c$ (otherwise we are already done), i. e., $\widetilde{q}_g + \sum_{j \neq g} \overline{q}_j < D(c)$. Note that $\overline{q}_g + \sum_{j \neq g} \overline{q}_j - D(c)$ is a positive *integer*, hence $\widetilde{q}_g < \overline{q}_g - 1$. Thus, firm g will profit by deviating to $\overline{q}_g' = \overline{q}_g - 1$: this lowers costs while expected revenue is not less than $\widetilde{p}_g\widetilde{q}_g$ at an equilibrium of the resulting PS.[12] ■

The condition in Proposition 1(i) can be written $D(c) \geq 1/\eta_{p=c}$, showing how the competitive outcome depends upon a sufficiently "large" market: the ratio between competitive industry output and the firm minimum efficient size (1, the minimum average-cost minimizing output) must be not less than the inverse of demand elasticity. To illustrate this point, take c as given and let an industry be identified by the value of parameter r in the family of demand functions $D^{(r)}(p) := rD(p)$, with $r \geq 1$ such that $rD(c) \in \mathcal{F}_+$. The industry is the larger as r is the higher ($D^{(r)}(c) = rD(c)$ increases with r), whereas, at any given p, demand elasticity is unaffected by r: hence $D^{(r)}(c) \geq 1/\eta_{p=c}$ for r sufficiently high.

Example. Let $c = 1.5$ and $D^{(r)}(p) = r(10.5 - p)/3$. At any equilibrium: if $r = 1$, then $n = \overline{Q} = D(c) = 3$ and, on the equilibrium path, $\Pi_i = \widetilde{\Pi}_i^* = 1.6875$, $\pi_i = .1875$, and $\phi(p) = \sqrt[2]{\frac{3(1.6875-p)}{p(1.5-p)}}$ for $p \in [1.6875, 2.25]$; if $r = 3$, then $n = \overline{Q} = D(c) = 9$ and price c obtains on the equilibrium path.

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[9] It can easily be checked that the equilibrium strategy is $\phi(p) = \sqrt[D(c)-1]{\frac{p - \widetilde{\Pi}_i^*}{p[D(c) - D(p)]}}$ for $p \in [\underline{p}^*, \overline{p}^*]$, where $\overline{p}^* = \widetilde{p}_i^*$ and $\underline{p}^* = \widetilde{\Pi}_i^*$.

[10] This is immediate if $\overline{q}_u' + D(c) \geq D(0)$ since then $p^w(\overline{q}_u', \overline{q}_{-u}^*) = 0$ while $\sum_{j \neq u} \overline{q}_j^* = D(c) < D(0)$. If $\overline{q}_u' + D(c) < D(0)$, then $d[p(D(p) - D(c))]/dp > 0$ at $p = p^w(\overline{q}_u', \overline{q}_{-u}^*)$: this follows since $d[p(D(p) - (D(c) - 1))]/dp > 0$ at $p = c$ and $D'' \leq 0$.

[11] With $\overline{Q} = \overline{Q}^* - 1$, this would result in zero profit if the resulting PS has a pse. Any such $\overline{q}$ is disposed of if, at zero profit, entering is preferred to not entering.

[12] Expected revenue is unchanged if a mse still obtains (i.e., $\widetilde{p}_g > P(\overline{Q} - 1)$) and g remains (one of the) largest firm(s).

We have seen that, at any equilibrium, $\overline{Q} = D(c)$ and that capacity vector $\overline{q}^*$ is always part of an equilibrium. Then it might be asked whether any $\overline{q}$ such that $\overline{q_g} > 1$ and $\overline{Q} = D(c)$ can be always ruled out as an equilibrium. The answer is definitely yes under linear demand.

Proposition 2 *If $D''(p) = 0$, then $\overline{q} \in \overline{\mathcal{Q}}^*$ at any equilibrium of the CPG.*

Proof. In the Appendix. ■

Unlike with linear demand, with $D''(p) < 0$ there might be equilibria with $\overline{q_g} > 1$. For example, let $p = 16.01 - Q^2$ and $c = 0.01$. Then $D(c) = 4$ and $-cD'(c) < 1$, hence any $\overline{q}^*$ ($\overline{q}_i^* = 1$ for all $i \in \mathcal{A}^*$ and $\overline{Q}^* = 4$) is an equilibrium where active firms randomize on the equilibrium path. However, one can check that any $\overline{q}$ such that $\overline{Q} = 4, n = 3, \overline{q}_g = 2$ is an equilibrium too, again with active firms randomizing on the equilibrium path.

3 Two production techniques

To see how the competitive outcome can arise under a plurality of techniques, suppose now that, at the time of entry-capacity decisions, firms can choose between two production techniques, α and β, entailing cost per capacity unit of c_α and c_β and capacity choice sets $\overline{\alpha}\mathcal{F}_+$ and $\overline{\beta}\mathcal{F}_+$, respectively. Let $\overline{\alpha} > \overline{\beta}$, $c_\alpha < c_\beta$ and $c_\beta\overline{\beta} < c_\alpha\overline{\alpha}$: while α is the average-cost minimizing technique, β is cheaper at a sufficiently low output since it involves a lower minimum capacity. Similarly as before, we let $D(c_\alpha) \in \overline{\alpha}\mathcal{F}_+$: hence a CE exists, where technique α is adopted, $p^{ce} = c_\alpha$, and $\overline{Q}^{ce} = D(c_\alpha)$. We define $\overline{\mathcal{Q}}^{(\alpha)} = \{\overline{q}^{(\alpha)} : n^{(\alpha)} = D(c_\alpha)/\overline{\alpha}, \overline{Q}^{(\alpha)} = D(c_\alpha)\}$, i.e., $\overline{q}^{(\alpha)}$ is any least concentrated industry configuration consistent with the competitive capacity. We also let $D(c_\beta) \in \overline{\beta}\mathcal{F}_+$ and define $\overline{\mathcal{Q}}^{(\beta)} = \{\overline{q}^{(\beta)} : n^{(\beta)} = D(c_\beta)/\overline{\beta}, \overline{Q}^{(\beta)} = D(c_\beta)\}$. For any $i \in \mathcal{A}^{(\alpha)}$ [13] we let $\widetilde{q}_i^{(\alpha)} = \widetilde{q}(\sum_{j\neq i}\overline{q}_j^{(\alpha)})$ and $\widetilde{p}_i^{(\alpha)} = P(\widetilde{q}_i^{(\alpha)} + \sum_{j\neq i}\overline{q}_j^{(\alpha)})$ (of course, $\sum_{j\neq i}\overline{q}_j^{(\alpha)} = D(c_\alpha) - \overline{\alpha}$) and similarly for any $i \in \mathcal{A}^{(\beta)}$ we let $\widetilde{q}_i^{(\beta)} = \widetilde{q}(\sum_{j\neq i}\overline{q}_j^{(\beta)})$ and $\widetilde{p}_i^{(\beta)} = P(\widetilde{q}_i^{(\beta)} + \sum_{j\neq i}\overline{q}_j^{(\beta)})$ (where $\sum_{j\neq i}\overline{q}_j^{(\beta)} = D(c_\beta) - \overline{\beta}$). We do not intend here to fully characterize the several patterns of equilibria which can arise according to circumstances. The results in the following Proposition call attention to two facts. First, there is an additional condition in order for the competitive outcome to arise in the CPG. Second, at an equilibrium of the CPG total capacity can be lower than the competitive capacity and the less efficient technique β can be adopted.

[13] $\mathcal{A}^{(\alpha)}$ ($\mathcal{A}^{(\beta)}$) is the set of active firms at some capacity configuration $\overline{q}^{(\alpha)}$ (resp., $\overline{q}^{(\beta)}$).

Proposition 3 *(i) Any* $\overline{q}^{(\alpha)}$ *is an equilibrium of the CPG where* c_α *is charged on the equilibrium path if and only if* $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$ *and* $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) \leq c_\beta$.[14] *(ii) Inequalities* $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$ *and* $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) \geq c_\beta$ *are sufficient for* $\overline{q}^{(\beta)}$ *to be an equilibrium of the CPG where* c_β *is charged on the equilibrium path. (iii) Any* $\overline{q}^{(\beta)}$ *is an equilibrium of the CPG where a mse for the PS is played on the equilibrium path if and only if* $-c_\beta D'(c_\beta) < \overline{\beta}$.

Proof. [15] (i) *[Sufficiency]* With $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$, a pse obtains at $\overline{q}^{(\alpha)}$. For any $i \in \mathcal{A}^{(\alpha)}$ it is not worth to reduce capacity, i. e., to deviate to technique β and install, say, capacity $\overline{\beta}$: at the new pse [16] it will sell $\overline{\beta}$ at price $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) \leq c_\beta$, hence losses (or no gains). *[Necessity]* With $-c_\alpha D'(c_\alpha) < \overline{\alpha}$ a mse obtains at $\overline{q}^{(\alpha)}$ and a fortiori at $\overline{q}$ such that $\overline{Q} = D(c_\alpha); \overline{q}_g > \overline{\alpha}$. With $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$ and $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) > c_\beta$, at $\overline{q}^{(\alpha)}$ it pays any $i \in \mathcal{A}^{(\alpha)}$ to deviate to technique β and capacity $\overline{\beta}$.

(ii) Since $D'' \leq 0$, it is also $-c_\beta D'(c_\beta) \geq \overline{\beta}$, hence at $\overline{q}^{(\beta)}$ prices are set equal to c_β. It does not pay any $i \in \mathcal{A}^{(\beta)}$ to deviate to technique α and, say, capacity $\overline{\alpha}$: the new market-clearing price $P(D(c_\beta) + \overline{\alpha} - \overline{\beta})$ is not higher than c_α, hence losses or at most zero profit at a pse. If a mse obtains,[17] expected profit is $\widetilde{p}_i^{(\beta)} \widetilde{q}_i^{(\beta)} - c_\alpha \overline{\alpha}$, less than 0 since $\widetilde{q}_i^{(\beta)} < \overline{\alpha}$ and $\widetilde{p}_i^{(\beta)} \leq c_\alpha$: the latter follows since $\widetilde{p}_i^{(\alpha)} \leq c_\alpha$ (by assumption, a pse obtains at $\overline{q}^{(\alpha)}$), $\widetilde{p}'(\cdot) < 0$, and $\sum_{j \neq i} \overline{q}_j^{(\beta)} = D(c_\beta) - \overline{\beta} \geq \sum_{j \neq i} \overline{q}_j^{(\alpha)} = D(c_\alpha) - \overline{\alpha})$.

(iii) A mse obtains at $\overline{q}^{(\beta)}$. It does not pay any $i \in \mathcal{A}^{(\beta)}$ to deviate to technique α and, say, capacity $\overline{\alpha}$: this raises capacity cost ($c_\alpha \overline{\alpha} > c_\beta \overline{\beta}$) while expected revenue is still $\widetilde{p}_i^{(\beta)} \widetilde{q}_i^{(\beta)}$ at the mse of the new PS.[18] ■

We can see the relevance of the size of market for the competitive outcome in terms of our family of demand functions, $D^{(r)}(p) = rD(p)$. Similarly

[14] The latter condition may well be more restrictive than the former. Let $D^{(r)}(p) = r(a - p)$. Then the former condition amounts to $r \geq \overline{\alpha}/c_\alpha$ and the latter condition to $r \geq (\overline{\alpha} - \overline{\beta})/(c_\beta - c_\alpha)$, hence the latter is more restrictive than the former if and only if $c_\alpha > \overline{\alpha} c_\beta / (2\overline{\alpha} - \overline{\beta})$.

[15] In the proof we will ignore deviations that can easily be ruled out. For example, as for part (i): deviating to capacity $\overline{q}_i' > \overline{\alpha}$ while sticking to technique α is ruled out as in the proof of Proposition 1(i); deviating to capacity $\overline{q}_i' > \overline{\alpha}$ *and* to technique β can easily be ruled out; and one can also easily rule out entry (with either technique) by any $u \notin \mathcal{A}^{(\alpha)}$.

[16] From $-c_\alpha D'(c_\alpha) \geq \overline{\alpha}$ and $D'' \leq 0$ it follows that (1) also holds at the new PS: $-pD'(p) \geq \overline{\alpha}$ at $p = P(D(c_\alpha) - \overline{\alpha} + \overline{\beta})$.

[17] In the given circumstances, while $\overline{q}^{(\alpha)}$ has a pse, the PS resulting from the deviation of $i \in \mathcal{A}^{(\beta)}$ under consideration *may* have a mse, so long as $P(D(c_\alpha) - \overline{\alpha} + \overline{\beta}) > c_\beta$.

[18] Any $u \notin \mathcal{A}^{(\beta)}$ faces an expected loss if entering. The proof goes as for Proposition 1(ii) if technique β is chosen by u. If choosing technique α, expected revenue is actually the same, by Lemma 3: thus, a fortiori an expected loss, since $c_\alpha \overline{\alpha} > c_\beta \overline{\beta}$.

as before, it must be $rD(c_\alpha)/\overline{\alpha} \geq 1/\eta_{p=c_\alpha}$ in order for a pse to obtain at $\overline{q}^{(\alpha)}$. Furthermore, $| \, rD'(p) \, |$ must be sufficiently high on a right neighbourhood of the competitive price c_α so that at $\overline{q}^{(\alpha)}$ it does not pay an active firm to shrink capacity, what it can do by deviating to technique β: the resulting increase in the market-clearing price (in fact, of the uniform price at the equilibrium of the new PS) must not exceed the increase in unit cost. Note that $P(D^{(r)}(c_\alpha) - \overline{\alpha} + \overline{\beta}) \leq c_\alpha + (1/ \, | \, rD'(c_\alpha) \, |)[\overline{\alpha} - \overline{\beta}]$ (strict equality holding iff $D'' = 0$), hence $P(D^{(r)}(c_\alpha) - \overline{\alpha} + \overline{\beta}) \to c_\alpha$ as $r \to \infty$. Consequently, $P(D^{(r)}(c_\alpha) - \overline{\alpha} + \overline{\beta}) < c_\beta$ for r sufficiently large.

Examples. 1: $D(p) = 32 - 2p$, $(c_\alpha = 1, \overline{\alpha} = 1)$, $(c_\beta = 1.2, \overline{\beta} = .8)$. The conditions of Proposition 3(i) hold: at $\overline{q}^{(\alpha)}$ the firms charge the competitive price c_α and $\overline{q}^{(\alpha)}$ is an equilibrium of the CPG.

2: $D(p) = 16 - p, (c_\alpha = 2, \overline{\alpha} = 2)$, $(c_\beta = 2.2, \overline{\beta} = .6)$. The sufficient conditions of Proposition 3(ii) hold: at $\overline{q}^{(\beta)}$ prices are set at the market-clearing level 2.2 and $\overline{q}^{(\beta)}$ is an equilibrium of the CPG. (If any $i \in \mathcal{A}^{(\beta)}$ raised capacity while deviating to α, this would result in losses at a mse of the new PS.)

3: $D(p) = 20.5 - p, (c_\alpha = .25, \overline{\alpha} = 2.25)$, $(c_\beta = .5, \overline{\beta} = 1)$. A mse obtains at $\overline{q}^{(\beta)}$ and $\overline{q}^{(\beta)}$ is an equilibrium of the CPG: it does not pay any $i \in \mathcal{A}^{(\beta)}$ to raise capacity and adopt α: this leads to a lower expected profit at a mse of the new PS.

4 Concluding remarks

The role of capacity indivisibility for the competitive outcome is easily understood. Under constant returns at full capacity utilization, long-run cost is $C(q_i) = c_\alpha q_i$[19] at any $q_i \in \mathbb{R}_+$ with perfect divisibility. Now, consider any capacity vector consistent with the competitive capacity $D(c_\alpha)$ and such that prices equal the competitive level c_α at an equilibrium of the PS. Then it pays an active firm to reduce capacity: this raises the market-clearing price above unit cost c_α, which leads to positive profit at an equilibrium of the PS. Things are quite similar if $C(q_i)$ is strictly convex. Then, at the CE each potential entrant is active with capacity q_i^{ce}, the solution of equation $P(zq_i) = C'(q_i)$. Suppose that, at the competitive capacity vector, the competitive price $P(zq_i^{ce})$ is charged at the equilibrium of the PS.[20] That capacity vector is not an equilibrium of the CPG, though: firm i will raise profits by mar-

[19] $C(q_i) = cq_i$, if, as in Section 2, a single technique is available.
[20] Otherwise the competitive outcome is immediately dismissed.

ginally reducing capacity while charging the market-clearing price (the rate of change of its profit will be $P'(zq_i^*)q_i^* + P(zq_i^*) - C'(q_i^*) = P'(zq_i^*)q_i^* < 0$).

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Proof of Proposition 2. Let $P(Q) = a - bQ$ for $Q \leq a/b$ $(a, b > 0)$. Then $D(c) = (a-c)/b$, $\widetilde{q}_i = \frac{a - b\sum_{j \neq i} \overline{q}_j}{2b}$ and $\widetilde{p}_i = \frac{a - b\sum_{j \neq i} \overline{q}_j}{2}$. Further, $\partial^2 \pi_i^w(\overline{q}_i, \overline{q}_{-i})/\partial \overline{q}_i^2 = -2b$ when $\overline{q}_i + \sum_{j \neq i} \overline{q}_j < D(0)$. Given Proposition 1, we just need to rule out any $\overline{q}$ such that $\overline{Q} = (a-c)/b$ and $\overline{q}_g > 1$. In fact, firm g can profitably deviate to $\overline{q}_g - 1$. This is immediate when the $\overline{q}$-PS has a pse: then $\pi_i(\overline{q}) = 0$ for any $i \in \mathcal{A}$, hence g will profit by deviating to $\overline{q}_g - 1$ and charging $P(D(c) - 1) = c + b$. If the $\overline{q}$-PS has a mse, then $\widetilde{p}_g > c$ and $\pi_g(\overline{q}) = \widetilde{\Pi}_g - c\overline{q}_g$. If $\widetilde{p}_g \geq c + b$, then deviating to $\overline{q}_g - 1$ would raise g's expected profit to at least $\widetilde{\Pi}_g - c(\overline{q}_g - 1)$: since rivals can produce $\sum_{j \neq g} \overline{q}_j$ at most, firm g will sell at least $\widetilde{q}_g = D(\widetilde{p}_g) - \sum_{j \neq g} \overline{q}_j \leq \overline{q}_g - 1$ when charging $\widetilde{p}_g$. If $\widetilde{p}_g < c + b$, then $\overline{q}_g - 1 < \widetilde{q}_g < \overline{q}_g$. Let $\overline{q}_i^\dagger = argmax_{\overline{q}_i \in \mathbb{R}_+} \pi_i^w(\overline{q}_i, \overline{q}_{-i})$. With $\sum_{j \neq i} \overline{q}_j \leq (a-c)/b$, then $\overline{q}_i^\dagger = \left(\frac{a-c}{b} - \sum_{j \neq i} \overline{q}_j\right)/2$ and one can write $\pi_i^w(\overline{q}_i, \overline{q}_{-i}) = \pi_i^w(\overline{q}_i^\dagger, \overline{q}_{-i}) - b\left(\overline{q}_i - \overline{q}_i^\dagger\right)^2$. The capacity reduction can be broken down in two virtual reductions, from $\overline{q}_g$ to $\widetilde{q}_g$ and then from $\widetilde{q}_g$ to $\overline{q}_g - 1$. It suffices to prove that g's profit will rise if, at each step, g is charging the market-clearing price. Assuming so, g's profit will rise to $\pi_g^w(\widetilde{q}_g, \overline{q}_{-g})$ in the first step. After the second step, g's profit will be $\pi_g^w(\overline{q}_g - 1, \overline{q}_{-g})$, higher than $\pi_g^w(\widetilde{q}_g, \overline{q}_{-g})$ because $\overline{q}_g^\dagger \leq \overline{q}_g - 1 < \overline{q}_g$ at any $\overline{q} : \overline{Q} = (a-c)/b; \overline{q}_g \geq 2$. ■