Journal of Economic Behavior and Organization 212 (2023) 1056–1085

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

The green-MKS system: A baseline environmental macro-dynamic model[☆]

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ARTICLE INFO

Article history:
Received 15 December 2022
Revised 25 May 2023
Accepted 23 June 2023

JEL classification:
C63
O11
O44
Q57

Keywords:
Sustainable development
Heterogeneous agents
Carbon tax
Hopf bifurcation
Growth-cycle dynamics

ABSTRACT

This paper extends the Marx-Keynes-Schumpeter model in Flaschel (2015) to study the social dimension of climate change. Agents are divided between those supporting and those opposing taxing Green House Gas (GHG) emissions. The composition of the population varies according to a continuous-time version of the discrete-choice approach. Conditional to the level of interaction between players, society chooses the respective carbon tax rate. Higher taxes reduce capital accumulation but support the development of energy-saving production techniques. Output growth and employment rates will be higher or lower depending on which effect prevails. Economic activity generates GHG emissions and determines the employment rate, which, in turn, endogenously feedback on environmental sentiments. Lower emissions reinforce sustainable attitudes, while falling employment increases households' concerns with more "urgent" needs, decreasing support for taxation. Hence, the model is compatible with a positive relationship between environmental attitudes and energy efficiency but not a clear association with output. A sufficiently strong response of attitudes to emissions combined with a "sentiments-autonomous" carbon tax may lead to the disappearance of the equilibrium in which most agents oppose regulation, controlling for multi-stability. Our 3-dimensional system admits endogenous persistent and bounded fluctuations, where the interaction between green attitudes and the macro-economy appears as a novel source of growth-cycle dynamics.

1. Introduction

In three different moments during his *Distinguished Lecture* at the Eastern Economic Association, Samuelson (1983) referred to Karl Marx, John M. Keynes, and Joseph Schumpeter as "heroes", "giants", and "prodigious workers" of the economic profession. Perhaps more than anybody else, they contributed to our understanding of the inherent instability of capitalist economies and the essential relationship between cycles and growth (see Vercelli, 1984). Marx died in 1883, the year Keynes and Schumpeter were born. One hundred years or so later, Goodwin (1986) presented his last fundamental contribution: the

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[☆] We would like to express our gratitude and pay our respects to Peter Flaschel, a dear friend whose contributions have profoundly influenced the model developed in this paper. An earlier version of it has been presented at the 11th MDEF Workshop, Urbino, Italy; at the 46th AMASES Annual Meeting, Palermo, Italy; at the 26th Forum for Macroeconomics and Macroeconomic Policies, Berlin, Germany; and at 49th EEA Annual Conference, New York, United States. We are grateful to the participants, particularly Christian Proaño, Beatrice Venturini, Tatyana Perevalova, Alejandro Gonzales, Roberto Dieci, Gerd Schuster, Mark Setterfield, and two anonymous reviewers for their helpful comments and suggestions. Usual caveats apply.

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https://doi.org/10.1016/j.jebo.2023.06.023

MKS System. Despite obvious differences and apparent incompatibilities between the three, Goodwin viewed their theories as complementary.

On the other hand, Peter Flaschel dedicated his entire academic career to developing a macro-dynamic framework for studying modern economies from a disequilibrium perspective. A fruitful collaboration with scholars from different parts of the world resulted in a series of papers and books (e.g. Chiarella and Flaschel, 2000; Chiarella et al., 2005; Asada et al., 2011; Flaschel et al., 2018) which some refer to as the “Bielefeld School”. He shared Goodwin’s (1967) understanding that growth and cycle are indissolubly fused and that the MKS system is a natural extension of the original growth-cycle model. From Marx, we have the correspondence between profits, income distribution, and accumulation. From Keynes, the principle of effective demand. Finally, Schumpeter presents innovation as the driving force of capitalism. Elements for a synthesis between these three authors can be found in Flaschel (2009) [1993]. The problem at hand is how to conceptualise the evolutionary aspect of capitalism. A baseline macro model was published a few years later, where the Marxian reserve army mechanism provides global stability while Keynesian demand and Schumpeterian process innovations work as destabilising forces (Flaschel, 2015; for recent developments, see Chiarella et al., 2021, pp. 271-410).

However, this literature remains silent on one of the main challenges of our generation: climate change. A fundamental contradiction exists between the economy’s current structure and the need to tackle global warming. Sustainable development seems to be a critical piece currently missing in the MKS framework. Our purpose in this paper is to extend the macrodynamic model in Flaschel (2015) to consider the feedback effects between the economy and the environment. In particular, we formalise into the model a mechanism that explains how people with different environmental attitudes influence each other and contribute to the design of environmental policies. Data from the World Risk Poll (2019) indicates that only 41% of the global population perceives climate change as a very serious threat and requires immediate action. The remaining 59% is a mix of those who see climate change only as a “somewhat serious threat” and those who are either unaware of it or do not see it as a problem at all. The difference between these shares provides an index of green attitudes similar to the one used to capture financial or business cycle sentiments.

Focusing on the European Union (EU) case, the World Risk Poll (2019) allows us to identify a positive relationship between energy efficiency and environmental attitudes. However, such correspondence disappears when we confront the latter with Gross Domestic Product (GDP). The proposed Green-MKS system innovatively explains those findings highlighting how policies are intrinsically subject to population heterogeneity. We differentiate between two types of agents along the same lines as previously described. The composition of the population varies according to a continuous-time version of the discrete-choice approach (Brock and Hommes, 1997; for a review of the related literature, see Franke and Westerhoff, 2017). Previous applications of the discrete-choice model in continuous-time to a range of different economic problems include theoretical (e.g. Dagsvik, 2002; Hommes and Ochea, 2012) and empirical considerations (Arcidiacono et al., 2016). The more traditional discrete-time version has been used to study climate transition, in particular, the choice between technologies (Hommes and Zeppini, 2014; Zeppini, 2015), general attitudes towards climate policies (Cafferata et al., 2021) and country participation in international environmental agreements (Galanis et al., 2022). Other scholars have used the analogous continuous-time transitional probabilities framework in an open economy environmental model (see Dávila-Fernández and Sordi, 2020). With respect to these previous studies, we further innovate by explicitly endogenising the choice of the tax rate on Green House Gas (GHG) emissions and by bringing income distribution into the discussion.

Conditional to the level of interaction between players, society chooses the carbon tax rate. Taxation is divided into two components. One is “autonomous” in the sense it does not depend on environmental sentiments. The other reflects policymakers’ response to the general public’s attitudes. Such a choice will influence the adopted production technology. Higher taxes reduce incentives to accumulate capital but can be used to support the development of energy-saving production techniques. Output growth and employment rates will be higher or lower depending on which effect prevail. A certain level of economic activity generates an amount of emissions and determines the employment rate, which in turn endogenously feedbacks the attitudes toward climate policies. Lower emissions reinforce sustainable attitudes. On the other hand, falling employment increases households’ concerns with more “urgent” or basic needs such as food and housing, decreasing the support for taxing pollution. As a result, we obtain a positive relationship between climate change attitudes and efficiency but not a clear association with GDP.

Our model is compatible with two stable equilibrium points. One with the majority of the population supporting environmental action and another in which most agents oppose strong regulation. Still, a sufficiently high response of sentiments to emissions combined with a partially autonomous tax rate on pollution may lead to the disappearance of the equilibrium in which most agents oppose taxation, thus controlling for the problem of multi-stability.[1] This result implies a cleaner and more energy-efficient economy but not necessarily higher growth. The latter will continue to depend on the structural conditions of each country and how dependent they are on capital accumulation. By applying the existence part of the Hopf bifurcation theorem, we show that our 3-dimensional system admits endogenous persistent and bounded fluctuations, where the interaction between attitudes towards climate policies and the macro-economy is a novel source of low-frequency growth-cycle dynamics.

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[1] Multistability consists in the coexistence of different attractors for a set of parameters (for a comprehensive review of theory and applications, see Pisarchik and Feudel, 2014). Given that the final state of a multistable system depends on initial conditions, developing control strategies to induce a definite switch to the desired equilibrium solution is a significant policy problem. The control should be robust against noise to avoid transitions between states.

To the best of our knowledge, our Green-MKS system is the first to use the discrete-choice approach to show it is possible to control for multi-stability and obtain a unique green equilibrium point. Such a result has important policy implications that open avenues for future research. The remainder of the paper is organised as follows. Section 2 relies on World Risk Poll (2019) data to build a green sentiments index for the EU and provide an initial assessment of its relationship with energy efficiency and output. Section 3 develops our 3-dimension dynamic model. In Section 4, we study the existence of equilibria and present the respective local stability analysis. A numerical experiment is reported in Section 5 to show the Hopf bifurcation is supercritical and assess the emerging cycle's robustness. Some final considerations follow.

2. Some empirical insights

Despite the near-global consensus among the scientific community, public perceptions of the urgency to fight climate change differ between nations and have fluctuated over time. This adds an interesting and important social dimension to the problem of tackling global warming. Formal assessments studying the economic implications of the coexistence of heterogeneous beliefs in ecological thinking go back to the seminal study by Janssen and de Vries (1998). They developed a multi-agent framework to investigate how people with different beliefs on economic growth and global warming adapt to environments corresponding to or failing to correspond to their views. Extensions to include more agent types and data updates can be found in Geisendorf (2016, 2018). With a somehow similar flavour, scholars such as Konc et al. (2022) have explored the dynamics of public support for climate policies using a model that combines general equilibrium and agent-based elements. Regulators adopt a certain climate policy conditional to public attitudes on the matter, which in turn a mediated by social influence (for a reference to confirmation bias, see Cafferata et al., 2021).

A detailed review of the agent-based literature goes beyond the scope of this paper (on the matter, see, e.g., Ciarli and Savona, 2019). Our purpose in this Section is to show how surveys on public attitudes toward climate change can be used to build an index similar to the one used to capture financial or business cycle sentiments. A large population of agents is usually divided into two main groups as, for simplicity, they face a binary decision. In principle, their options can be almost anything: strategies, rules of thumb to form expectations, and diffused beliefs. Individual agents switch from one to the other based on probabilities. Franke and Westerhoff (2017) distinguish between two main mechanisms: the discrete-choice approach (Brook and Hommes, 1997) and the transition probabilities approach (Lux, 1995). The present paper will adopt a continuous-time version of the former as part of the modelling strategy.

As mentioned in the Introduction, data from the WorldRiskPoll (2019) reveals that only 41% of the global population perceives climate change as a very serious threat.[2] The other 59% comprises people unaware of it, who do not see it as a threat or believe global warming is only somewhat serious. Of course, these figures hide significant heterogeneity between regions. Europe is not an exception. For instance, while more than 70% are highly concerned about the environmental challenge in Southern Europe, the number falls to less than half in Eastern Europe. If we compute the difference between those two groups for each country in the EU, we obtain an index between ± 1 of environmental sentiments (Φ). When $\Phi > 0$, most of the population firmly believes in the urgency of fighting climate change. On the other hand, $\Phi < 0$ corresponds to the opposite case. Finally, notice that $\Phi = 0$ captures a society somehow polarised, in the sense that it is equally divided between the two groups.

Fig. 1 reports our calculated Φ for each EU country, except for the Czech Republic, for which data is unavailable. A darker blue indicates regions with solid environmental attitudes. Spain, Portugal and Greece appear at the top of the list. Conversely, we colour red those countries where more indifferent sentiments prevail. A certain division of the continent between North and South is worth noting. Mediterranean countries in the South present more substantial favourable positions to green action, support that decreases as we move towards Nordic countries. For instance, Finland presents a $\Phi = -0.44$, which means that three of every four Finns believed climate change does not pose a "very serious" threat to people in their country. The prevalence of negative or less positive attitudes is also noticeable among Baltic states, particularly Lithuania.

In this paper, we are interested in two main links between environmental sentiments, the macroeconomy and the environment. On the one hand, it is reasonable to suppose that societies in which the population strongly supports tackling climate change will increase the number of resources dedicated to reducing emissions. This fundamentally requires the development of production techniques that are more energy efficient. Moreover, as cleaner technologies are adopted, reducing emissions will likely reinforce green attitudes mediated by social influence. As reported in Fig. 2 (a), data from the World Bank Indicators show a positive correspondence between green attitudes and energy efficiency. The dotted line and the correspondent 95% confidence interval are weighted by per capita GDP. We also report the level of emissions per capita. Despite its small size, Luxemburg is the highest polluter in per capita terms. On the other hand, Fig. 2 (b) depicts no clear-cut correspondence between sentiments and GDP.

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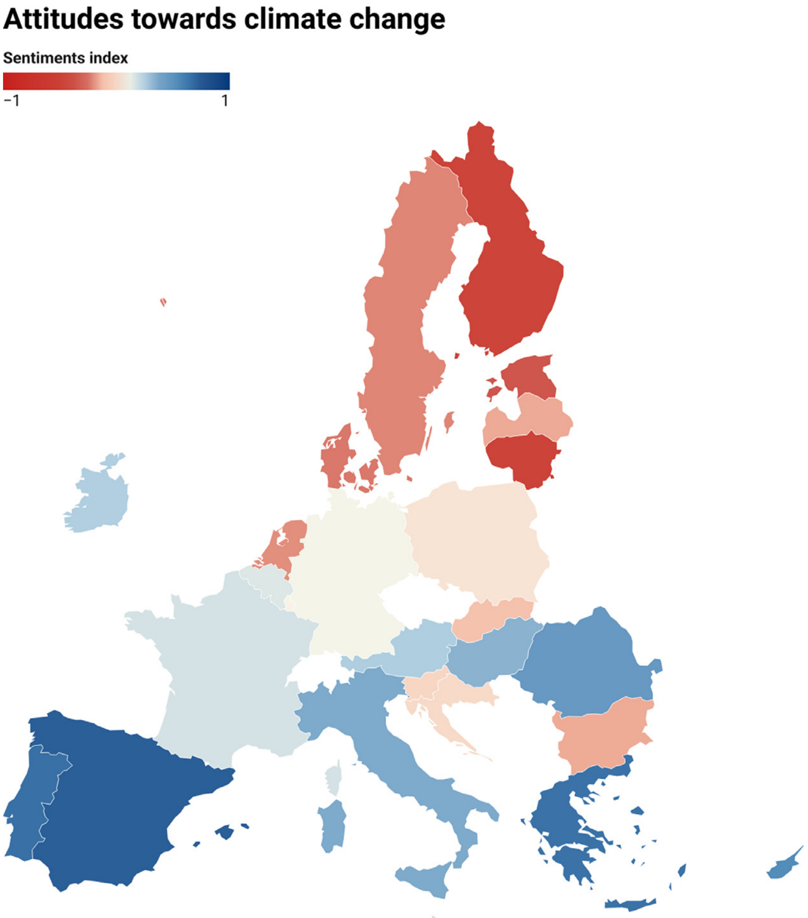
[2] The Lloyd's Register Foundation and Gallup published in 2019 the World Risk Poll as the first global picture of how the world's citizens see risk and safety and the differences between the perception of risk and actual experience. The research includes 150 000 people in 142 countries. They were interviewed face-to-face in the majority of cases. Regarding climate change, the question was: *Do you think that climate change is a very serious threat, a somewhat serious threat, or not a threat at all to the people in this country in the next 20 years? If you don't know, please just say so*. A second wave of the poll with data collected during 2021 was recently published. However, to avoid the influence of Covid-19-related issues, our analysis relies only on the first wave.

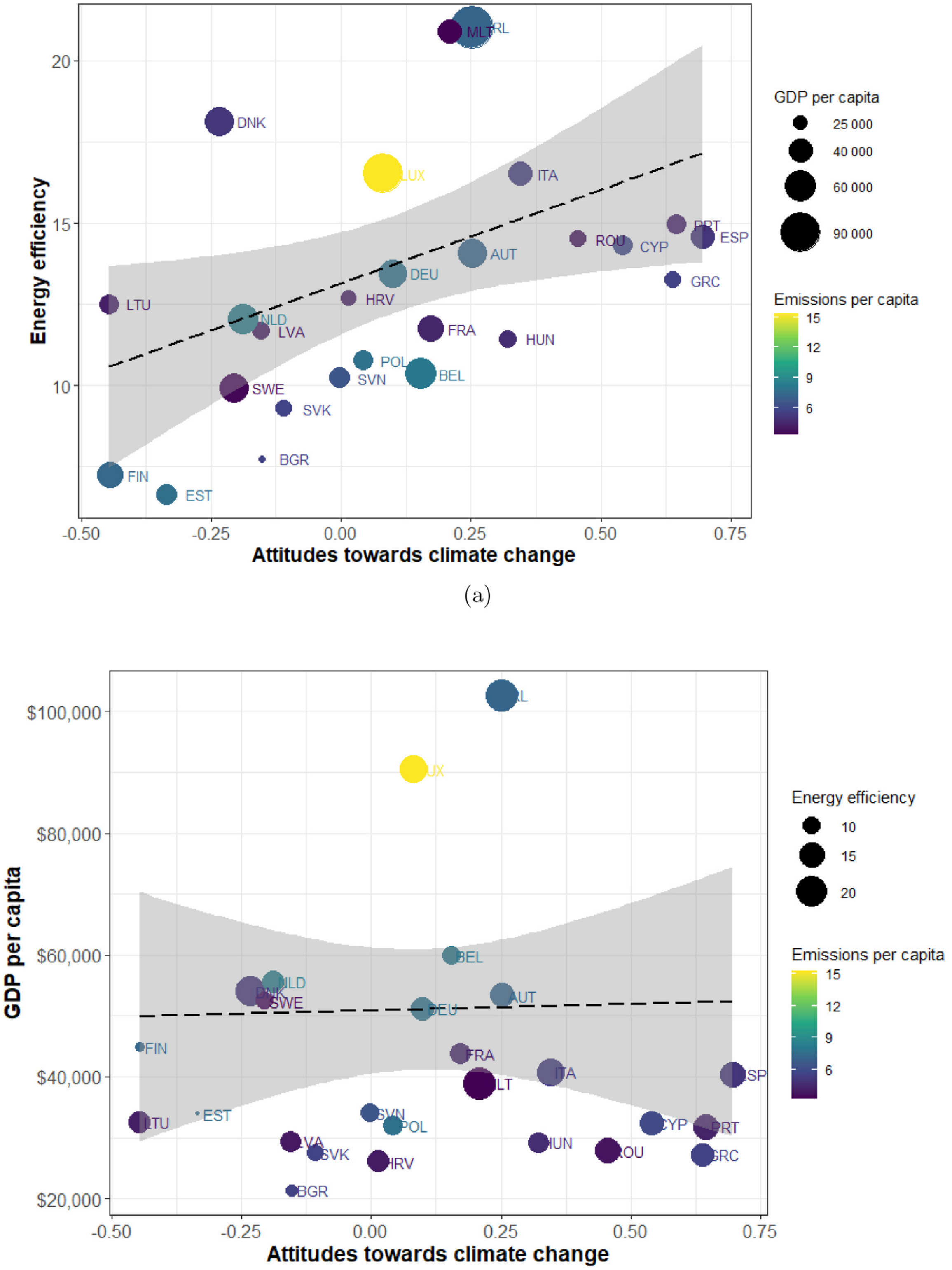


Source: World Risk Poll • Created with Datawrapper

Fig. 1. Attitudes towards climate change in the European Union. Blue colours represent the prevalence of positive attitudes, $\Phi > 0$, while in red, we have countries where more neutral or indifferent sentiments prevail, $\Phi < 0$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

A possible explanation could lie in two opposite effects of environmental regulation and, more specifically, taxing emissions. If policymakers respond to green awareness by increasing taxes on GHG emissions, they might favour the development of energy-saving production techniques but penalise capital accumulation in the very short term. The first result embodies two main mechanisms. First, and similar to the so-called induced technical change argument, firms have the incentive to develop energy-saving technologies to maintain their profit margins in response to the energy cost increase. Second, the public sector can use resources raised through taxing pollution directly to subsidise the adoption and development of innovations that increase energy efficiency. Still, during the transition to green technologies, economies must rely on "old" machinery. Even when this is not the case, adding a new capital unit comes along with energy requirements and leads to a certain amount of pollution in the form of emissions. Indirectly, taxing GHG emissions means taxing production, thus harming capital accumulation and output. Moreover, a reduction in GDP translates into firms needing fewer workers to produce, reducing the employment rate. At this point, households become increasingly concerned with more "urgent" needs such as food, housing, and paying utilities. This could lead to a reduction in the support for taxing pollution. A positive net effect of green attitudes on GDP would require that the energy-efficiency channel prevails over the capital accumulation one.

We believe these initial empirical insights require a more careful and formal assessment. For this purpose, the MKS system offers a handy platform that comes with the plus of bringing income distribution considerations into the discussion. In the next Section, we develop an extension of the model in Flaschel (2015) to study the feedback effects between the economy and the environment. In particular, we formalise into the model a mechanism that explains how people with different environmental attitudes influence each other and contribute to the choice of the rate of the emissions tax. The Green-MKS system adds itself to the attempts to provide a baseline environmental macrodynamic model with bounded rational decision-makers instead of the standard optimisation framework.



(a)

(b)

Fig. 2. The relationship between climate change attitudes, energy efficiency, and per capita GDP. Environmental sentiments are an index between [-1,1]. Energy efficiency consists of GDP per unit of energy use at constant 2017 PPP USD per kg of oil equivalent. GDP per capita is also measured in 2017 PPP USD. Emissions are reported as CO2 t per capita.

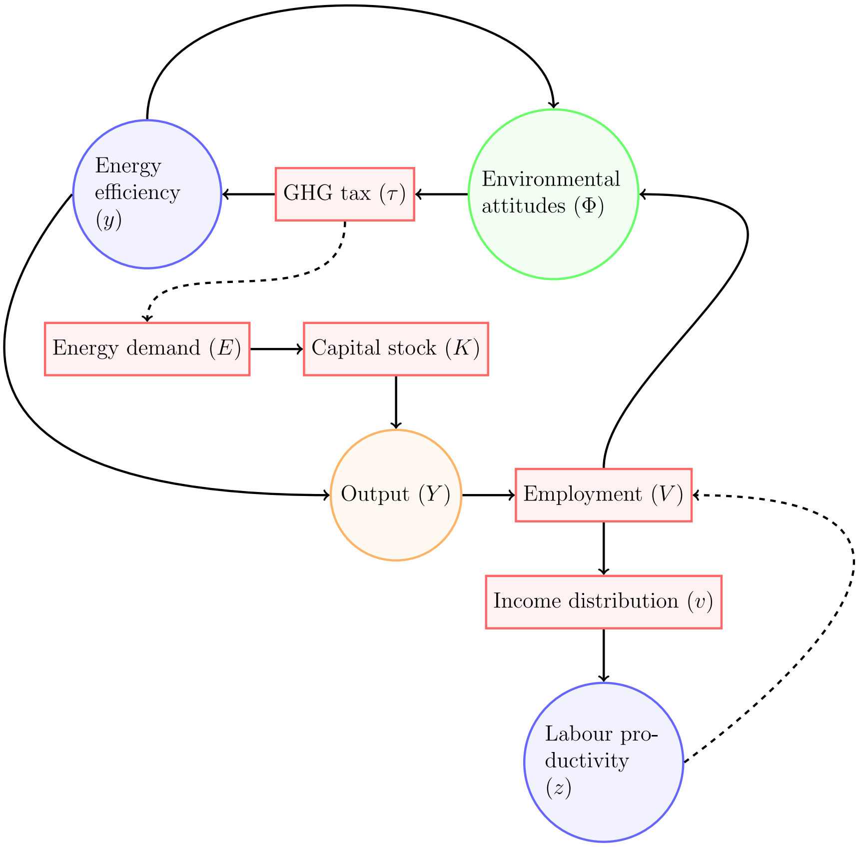


Fig. 3. A summarising diagram of the Green-MKS system. A continuous line between two nodes indicates a positive effect among the variables, while a dashed line indicates a negative one.

3. The model

During his academic career, Flaschel contributed to the development of two major families of models. The first is the Keynes-Metzler-Goodwin (KMG) framework, which results in a high-dimensional dynamic system used to study different macroeconomic problems, ranging from fiscal-monetary policy, real-financial market interactions, inventory cycles, and including considerations for open economies (see, for example, Chiarella and Flaschel, 2000; Chiarella et al., 2005; Asada et al., 2011; Flaschel et al., 2018). The KMG setup resulted from lifetime collaborations with other scholars directly or indirectly related to the Bielefeld School. The second family was mainly a personal project, summarised in his master-piece *The Macrodynamics of Capitalism: Elements for a Synthesis of Marx, Keynes and Schumpeter* (Flaschel, 2009 [1993]). Among other things, this book develops Goodwin's late fundamental contribution, namely, the MKS system. A workhorse model along these lines was then published in the special issue of the Cambridge Journal of Economics (CJE) entitled *Perspectives on the Contributions of Richard Goodwin* (Flaschel, 2015).

We build on this last article to study the social dimension of climate change. In principle, the KMG model could also provide a suitable structure for the problem. Still, given that innovation and technical change are fundamental elements for climate change transition, the Schumpeterian part of the MKS cannot be left aside. Furthermore, this model was originally conceived as a growth-cycle system, contrary to the KMG, which has a more business-cycle flavour. Given that sustainable development is deeply related to long-run economic performance, we believe the MKS is the more suitable choice. Finally, it allows us to advance a project Flaschel pursued on his own and more directly relates to one of the giants in the field of economic dynamics, namely, R. Goodwin.

The model is divided into three main blocks of equations to present it as a smooth extension of its original framework, trying to follow the original notation as closely as possible. The first block contains the basic structure and refers to some of Marx and Keynes's contributions to the economic profession. The second deals with a simplified reading of Schumpeterian innovation processes. Finally, we can introduce the coexistence of different environmental attitudes and how they interact between them as well as with the rest of the economy. Fig. 3 presents a summarising diagram of the Green-MKS system's main transmission mechanisms. The channels from output to employment and from there to income distribution are fundamentally the same as in Flaschel's CJE article. They correspond to the Marx-Keynes part of the story, while labour productivity was endogenised along Schumpeterian lines. We bring three main innovations into this framework:

1. Output is produced as a combination of energy and labour, the former being related to capital accumulation.
2. Firms can invest either in labour- or energy-saving production techniques conditional to the share of workers allocated to each task.
3. Environmental attitudes are introduced as a novel link connecting energy efficiency and the labour market to climate change.

3.1. A Marx-Keynes basic structure

Suppose output is determined by a Leontief production technology that combines energy (E) and labour (N) inputs:

$$Y = \min\{yE, zVN^s\} \tag{1}$$

where Y stands for output, y is the output-energy ratio and corresponds to a measure of energy efficiency, $z = Y/N^d$ is labour productivity, N^d is labour demand, $V = N^d/N^s$ stands for the employment rate, and N^s is labour supply. Notice that y can be rewritten as $y = (Y/P)(P/E)$, where P are GHG emissions. Adopting renewables reduces P/E while increasing Y/P and will impact output positively as long as there is an increase in energy efficiency.

Electricity is required to create and maintain capital. Following Daalgard and Strulik (2011), we adopt a measure of capital in energy units that allows us to avoid the well-known capital aggregation problem (see Cohen and Harcourt, 2003). In that case, energy conservation can be approximated by:

$$E = \mu K \tag{2}$$

where $\mu > 0$ captures the requirement to operate the generic capital good and for simplicity is assumed to be equal to one. The expression above captures the electricity demand at any given instant in time, summarising the instantaneous electricity requirements.

Substituting Eq. (2) into (1) and using the Leontief efficiency condition, we have that

$$\begin{aligned} Y &= yK \\ V &= \frac{Y}{zN^s} \end{aligned} \tag{3}$$

Output is determined by the amount of energy demand in the economy that depends on machinery and equipment used in the production process weighted by its productivity or efficiency, y. On the other hand, given that firms need to hire workers if they want to produce more, the employment rate is a positive function of the level of production; it is also negatively related to labour productivity and the labour supply because if the production technology allows firms to produce more with fewer workers, *ceteris paribus*, the higher z, the lower employment will be.

The relationship between demographic change and global warming has received some attention in the environmental macroeconomic literature but lies beyond the scope of this paper (see de la Croix and Gosseries (2012); O'Sullivan (2020)). To maintain the exercise as simple as possible, let us assume labour supply is equal to the population, which is constant. Taking log-derivatives with respect to time on both sides of (3), we obtain:

$$\begin{aligned} \frac{\dot{Y}}{Y} &= \frac{\dot{K}}{K} + \frac{\dot{y}}{y} \\ \frac{\dot{V}}{V} &= \frac{\dot{Y}}{Y} - \frac{\dot{z}}{z} \end{aligned} \tag{4}$$

where a dot over a variable indicates its time derivative. Combining the two expressions in (4), it follows that:

$$\frac{\dot{V}}{V} = \frac{\dot{K}}{K} + \frac{\dot{y}}{y} - \frac{\dot{z}}{z} \tag{5}$$

Thus, variations in the rate of employment depend on capital accumulation as well as on the rate of growth of both energy efficiency and labour productivity.

Marx and Keynes highlighted the role of profitability in capital accumulation. For instance, the former considered that "The rise of wages, therefore, is confined within limits that not only leave intact the foundations of the capitalist system but also secure its reproduction on a progressive scale" (Marx, 1909 [1867], p. 680). On the other hand, the marginal efficiency of capital is one of Keynes's central variables and turning points in his chapter on the trade cycle, which he considers "is best regarded, I think, as being occasioned by a cyclical change in the marginal efficiency of capital, though complicated and often aggravated by associated changes in the other significant short-period variables of the economic system" (Keynes, 1936, p. 313). Hence, as in Goodwin (1967), we assume the rate of growth of the capital stock is a positive function of the share of profits in income or negatively related to the wage-share (υ), such that:

$$\frac{\dot{K}}{K} = F(\upsilon, \tau) > 0, \ F_\upsilon < 0, \ F_\tau < 0 \tag{6}$$

where $F_\nu = \partial F/\partial \nu$ and $F_\tau = \partial F/\partial \tau$. We will maintain this notation all over the paper to indicate partial derivatives of a function. Producing requires using and expanding the capital stock of the economy. GHG emissions are a subproduct of this process. Therefore, taxing such emissions in the atmosphere implies, for a given technology, an implicit tax on production (τ). The higher such tax rate, the lower incentives to introduce new machinery into the economy because they need energy to operate, which is associated with a level of taxable pollution, i.e. $F_\tau < 0$.

Furthermore, Eq. (6) captures Keynesian demand restrictions on the evolution of economic activity. An increasing wage share means real wages are growing faster than labour productivity, signalling an increase in the bargaining power of labour. Wage shares and employment rates, in general, move together because increases in workers' negotiation power frequently follow a reduction in the unemployment rate. In this context, the economy might be subject to labour shortages that constrain investment. Flaschel (2015) explicitly included the employment rate in the accumulation function. For parsimony reasons, we avoid this route as we can consider ν captures both effects. Still, in line with the discussion above, we adopt a real wages (w) Phillips curve:

$$\frac{\dot{w}}{w} = G(V),\ \ G(0) < 0,\ \ G_V > 0 \tag{7}$$

such that for a very low V, real wages will fall. Empirical evidence supporting Eqs. (6) and (7) can be found in Grasselli and Maheshwari (2018) and in the profit-led *vs* wage-led literature (e.g. Kiefer and Rada, 2015 and Blecker et al., 2022; for a recent survey on the topic, also covering wavelet coherence and vector autoregressions, see Barrales-Ruiz et al., 2022). In a sample of 10 OECD countries, the response of $\dot{K}/K$ to ν was estimated between -0.2 and -0.3. On the other hand, the elasticity of income distribution with respect to the employment rate ranged between 0.11 in Canada and 0.98 in Italy, with an average of 0.42. These two expressions are at the core of the growth-cycle model.

3.2. Schumpeterian innovations

Schumpeter (1939, p. 84) defined "innovation as the setting up of a new production function". The two main variables defining the production technique in the model context are y and z, representing energy and labour productivity. An efficiency increase in the use of either input depends on resources directed to that purpose. Assume labour demand can perform two different tasks, production (N_Y^d) and R&D (N_R^d). Thus:

$$N^d = N_Y^d + N_R^d \tag{8}$$

where the subscript Y indicates workers allocated exclusively to the first task while R refers to those also involved in research activities. While the canonical education-race model (Tinbergen, 1974) differentiates between high and low-skill workers, a growing literature has adopted the task-based approach to study technology and its role in the labour market (for a review, see Acemoglu and Autor, 2011). Skills are applied to tasks to produce output, and workers of a given skill level can potentially perform various tasks. To avoid departing too much from the original MKS framework, we assume labour is relatively homogeneous regarding its skills. All workers perform production tasks, but N_R^d are also engaged in innovation. Though some might dedicate themselves solely to basic research, in general, even very specialised Science, Technology, Engineering and Mathematics (STEM) labour needs to maintain contact with direct production to avoid losing sight of firms' needs.

The share of wages in income is equal to the sum of the respective wage bills as a proportion of GDP. For simplicity, suppose that wages in the two tasks are equal, $w = w_Y = w_R$. This can be seen as following from our assumption that labour is homogeneous but able to perform both activities. Then, it is easy to see that:

$$\nu = \frac{w_Y N_Y^d + w_R N_R^d}{Y} = \frac{wN^d}{Y} = \frac{w}{z} \tag{9}$$

so increases in real wages above labour productivity result in a higher labour share in income. Therefore, taking log-derivates of Eq. (9) with respect to time:

$$\frac{\dot{\nu}}{\nu} = \frac{\dot{w}}{w} - \frac{\dot{z}}{z} \tag{10}$$

Notice that a stable income distribution requires wages to follow labour productivity. None of the two variables can permanently depart from the other without severe consequences for the organisation of the economic system.

Define the ratio between "researchers" and exclusively "production" workers (u) as an indicator of human resources dedicated to changing the shape of the production function:

$$\frac{N_R^d}{N_Y^d} = u \tag{11}$$

where $u \in (0, \infty)$. In one extreme, $u = 0$, nobody is involved in innovation, while in the other, $u \to \infty$, everyone dedicates at least part of her/his time to engage with process innovations. In this last case, pure assembly workers disappear; every worker is somehow performing tasks related to increasing productivity rates.

Labourers in research activities can choose between two different goals. They can either work on the development of labour-saving production techniques (u_N) or on increasing the output-energy ratio (u_E), that is:

$$u = u_N + u_E \tag{12}$$

such that the rate of growth of labour and energy productivity depends on the share of researchers in each activity:

$$\begin{aligned} \frac{\dot{z}}{z} &= H(u_N),\ H(0) = 0,\ H_{u_N} > 0 \\ \frac{\dot{y}}{y} &= J(u_E),\ J(0) = 0,\ J_{u_E} > 0 \end{aligned} \tag{13}$$

We do not impose any additional properties to functions $H(\cdot)$ and $J(\cdot)$ beyond assuming that they both increase in each argument. An S-shaped stands as a suitable functional form that accounts for the possibility of saturation in R&D efforts, as in semi-endogenous growth models (e.g. Jones, 1995). It also echoes the literature on technological revolutions and techno-economic paradigms (see Perez, 2010). Still, as it will become clear in what follows, we avoid this route to allow the dynamic properties of the model to depend on a natural non-linearity in environmental attitudes.

We assume technical change is fundamentally induced. This concept goes back at least to Hicks (1932), who suggested that a change in relative prices of factors of production can spur innovation directed at reducing the use of the factor that has become relatively more expensive. Using a neoclassical framework, Acemoglu (2010) showed that labour scarcity encourages technological advances if technology is strongly labour-saving. A cost component motivating innovation efforts has been included in part of the explanation of the phenomenon of secular stagnation from an evolutionary-Schumpeterian perspective (see Borsato, 2022). The latter provided empirical evidence that a 1% increase in the wage rate leads to 0.7-0.8% higher R&D funds. An increase in the wage share suggests increasing labour costs because real wages are growing faster than productivity. Firms respond by increasing the proportion of workers that are involved in research activities aiming for improvements in labour productivity:

$$u_N = u_N(v),\ 0 < u_{N_v} < 1,\ u_N(0) = 0 \tag{14}$$

Of course, zero labour costs imply no particular incentive to change technology in the direction of using less labour per unit of output, in which case $u_N = 0$.

Analogously, the tax rate on emissions increases the cost of accumulating new machinery and equipment. By reducing emissions, developing energy-saving production techniques is an alternative for firms to maintain profit margins. Furthermore, the public sector can directly apply resources raised through taxing emissions to pay researchers in increasing y. A classification framework for carbon tax revenue use can be found in Steenkamp (2021), who estimated global carbon tax revenues reached 24 trillion USD in 2019. In both cases, τ allows for raising the share of workers devoted to energy research:

$$u_E = u_E(\tau),\ 0 < u_{E_\tau} < 1,\ u_E(0) = 0 \tag{15}$$

Hence, taxing emissions has two opposing effects. It immediately reduces capital accumulation, negatively impacting output growth and employment. However, as long as those resources are redirected to R&D in energy, we might end up with higher growth and a more robust labour market.

3.3. Climate change attitudes

We are finally ready to present the last block of equations consisting of environmental attitudes. This last step is essential for three main reasons. First, it acknowledges the existence of heterogeneity in ecological thinking. As shown in the previous Section, an important part of the EU community does not believe climate change requires immediate action, which is likely to impact the implementation of environmental policies. Second, policymakers directly or indirectly reflect and respond to public views. For example, the decision of the tax rate on emissions is fundamentally endogenous to how society understands climate change as a collective. Finally, people create, live and share the economic system where they find themselves. This fact means macroeconomic conditions affect overall green attitudes.

Under the assumption that the population is constant and equals the labour supply, let us divide it between those who strongly believe in the urgency to fight climate change (N^+) and the "rest" (N^-), such that:

$$N^s = N^+ + N^- \tag{16}$$

Define the difference between the two groups (n):

$$n = N^+ - N^- \tag{17}$$

Taking the ratio between Eqs. (17) and (16) we can build the index (Φ) introduced in Section 2:

$$\Phi = \frac{n}{N^s}$$

where $\Phi \in (-1, 1)$ is equivalent to the difference between the shares of the two groups in society. Just as before, $\Phi > 0$ indicates the prevalence of sustainable attitudes in the economy. On the other hand, $\Phi < 0$ suggests most agents do not consider climate change an urgent matter. A society equally divided between the two groups will deliver $\Phi = 0$.

The fractions of each strategy, or in this case each attitude, evolve according to a smooth approximation of the Best Reply dynamics, the Logit dynamics, as in Brock and Hommes, 1997 and Hommes and Ochea (2012):

$$\begin{aligned}\frac{\dot{N}^+}{N^s} &= \frac{\exp\left(\beta U^+\right)}{\exp\left(\beta U^+\right)+\exp\left(\beta U^-\right)} - \frac{N^+}{N^s}\\ \frac{\dot{N}^-}{N^s} &= \frac{\exp\left(\beta U^-\right)}{\exp\left(\beta U^+\right)+\exp\left(\beta U^-\right)} - \frac{N^-}{N^s}\end{aligned} \tag{18}$$

where $\beta \in (0, +\infty)$ is the intensity of choice. Functions U^+ and U^- are usually interpreted as the success or fitness of the two attitudes. Although this is not done in typical applications, they could also take account of direct social interactions (for a review, see Franke and Westerhoff, 2017). We adopt this last position and suggest that β could somehow be seen as capturing the level of agent interaction. Specifically, when β is zero, agents act independently of each other. In that case, the switching rate is exogenously given, and the probability of belonging to each group is equal to 0.5. Hence, the population is fundamentally split and polarised between the two groups. On the other hand, for $\beta \to +\infty$, agents behave similarly and are more likely to follow the same belief. Therefore, almost all agents would either be strong climate supporters or environmentally indifferent. In both cases, there is more social cohesion, and people respond to whatever economic or ecological variables affect U^+ and U^-. Thus, we suggest that the change in the composition of the population at the macro level reflects stronger interactions between agents and their surroundings at a more micro dimension. Moreover, we impose it does not change over time, though such an assumption does not necessarily need to be the case (e.g. Chiarella and Di Guilmi, 2015).[3]

We suppose all agents look at the same set of macroeconomic variables. If an indicator increases the probability of adopting a positive attitude, it also reduces the probability of the opposite sentiment:

$$U = U^+ = -U^- \tag{19}$$

Thus, a crucial step consists in defining the determinants of U. The success or fitness of environmental attitudes depends on a vector of explanatory variables not limited to the economic realm.

Scholars such as Dávila-Fernández and Sordi (2020) have explored social influence's importance in generating polarisation or consensus in ecological thinking by making U an explicit function of Φ. On the other hand, Cafferata et al., 2021 expanded that framework to include labour market conditions, highlighting the role of confirmation bias in a set-up close to Hommes et al. (2005). In Konc et al. (2022), these elements are combined in a two-step process. An initial component captures opinions of climate policy without social influence as a function of personal well-being, distributional effects and effectiveness. The relative importance of such factors depends on political ideology. In a second step, social interactions influence policy opinions given a network structure (as in Konc and Savin, 2019). For the purposes of the present paper, we allow for two main factors to impact climate change attitudes. First, labour market conditions. An increasing employment rate allows agents to turn their attention from immediate needs – such as food, housing, rent, and education – to the climate change problem, whose costs are not daily visible (Hurst et al., 2013). The argument shares elements with the idea that environmental protection is a luxury good, very attractive but immediately set aside during recessions or periods of stagnation (e.g. Abou-Chadi and Kayser, 2017). Second, agents can perceive the negative consequences of emissions to some extent. The latter is negatively related to energy efficiency. Therefore, the efficient implementation of environmental policies can potentially affect overall sentiments.

We choose to work with variations in employment instead of its level for one main reason: a high (or low) occupation rate seems less relevant to us than employment prospects. For instance, over the business cycle, agents become pessimists while the labour market is still "hot". After a recession, expectations recover as employment conditions improve, even though the employment rate *per se* is still very low in comparison to the peak of the cycle. The unemployment rate might still be low, but as long as it is increasing, attitudes respond to the changing conditions of the labour market. As suggested by one of the reviewers, another way of thinking of this mechanism is that individuals have preferences such that they care about this variable relative to a reference point which is its value in the previous period. Using V in levels seems unsatisfactory because the perception that things are improving (or not) depends more on the direction of the movement rather than on the present state of affairs. Hence, the probability of having environmentally friendly sentiments starts improving with the variation of the employment rate rather than with its level.[4]

In mathematical terms, we have:

$$U = \rho\frac{\dot{V}}{V} + \frac{\dot{y}}{y} \tag{20}$$

---

[3] Notice that the level of interaction implicitly captures different network structures where agents are assumed to be not fully connected. As indicated by Grilli et al. (2020), dynamic network models using the switching behaviour show that by exogenously changing the intensity of choice parameter, credit linkages self-organize themselves into very different network architectures, ranging from random to scale-free topologies (see also Recchioni et al., 2015; Berardi and Tedeschi, 2017). Here, we do not explicitly model any form of network structure and only point out the possible connections between these two strands of the literature.

[4] A reader might wonder whether sentiments respond to income distribution rather than employment conditions. While we acknowledge the possible existence of this channel, we refrain from going in that direction for two reasons. First, in the context of the model, wage share and employment rate are already strongly correlated and move together. Second, variations in the employment rate capture the state of the labour market over the growth cycle more closely than the wage share.

where $\rho > 0$ captures how much agents value more employment perspectives rather than ecological conservation as captured by y.

Taking the difference between the two expressions in (18) and using Eqs. (19) and (20), we obtain:

$$\dot{\Phi} = \tanh\left(\beta\left(\rho\frac{\dot{V}}{V} + \frac{\dot{y}}{y}\right)\right) - \Phi \tag{21}$$

which describes the dynamics of attitudes as a function of the labour market and the energy efficiency growth rate. A positive employment growth rate leads to an increase/reduction in the probability of having positive/negative attitudes and, thus, a positive variation in Φ. Furthermore, falling emissions result from adopting cleaner, more energy-efficient technologies. Hence, a positive growth rate in y also brings an overall increase in environmentally friendly attitudes.

As a last modelling step, implementing an environmental policy occurs at a national level and depends on the share of agents that support environmental regulation.[5] Lab experiments suggest public acceptability of carbon taxation depends on its revenue use, where all revenues spent on support of climate projects is the most accepted option (Maestre-Andres et al., 2021). Scholars such as Drews et al. (2022) have found that the more one expects the tax to be accepted by others, the more one accepts it personally. Furthermore, their findings suggest opponents of a carbon tax tend to overestimate the prevalence of their opinion strongly. We assume the stringency of the emissions tax is an increasing function of Φ. A society with strong support for fighting climate change will adopt a higher tax rate on GHG emissions:

$$\tau = \tau(\Phi), \ \ \tau_{\Phi} > 0 \tag{22}$$

The share of $\tau(\cdot)$ captures the sensitivity of the tax rate to sentiments. One could argue that, to some extent, it also reflects the strength of democracy in a given society. In the limit, a dysfunctional democracy will correspond to a case in which policymakers do not care about public opinion and make decisions utterly independent from citizens.

4. Existence of equilibria and local stability analysis

Substituting Eqs. (14), (15), and (22) into (13), we obtain the rate of growth of labour productivity and energy efficiency. Substituting Eq. (22) into (6) gives us capital accumulation as a function of sentiments and the labour share. Thus, inserting these resulting expressions into Eq. (5), we have the dynamics of the employment rate as a function of income distribution and the GHG emissions tax. The latter depends on the composition of the population in terms of their environmental attitudes. On the other hand, by substituting Eq. (7) and the labour productivity growth rate into Eq. (10), it is possible to see how adjustments in income distribution respond to labour market conditions and induced technical change. Finally, substituting Eq. (22) into (15), and the resulting expression into Eq. (21), variations in climate attitudes depend on changes in the employment rate and energy efficiency, which in turn responds to the tax rate on emissions chosen by society. Our 3-dimension dynamic system is given by:

$$\begin{aligned}
\frac{\dot{V}}{V} &= F(v, \tau(\Phi)) + J(u_E(\tau(\Phi))) - H(u_N(v)) \\
\frac{\dot{v}}{v} &= G(V) - H(u_N(v)) \\
\Phi &= \tanh\left(\beta\left(\rho\frac{\dot{\mathrm{V}}}{\mathrm{V}} + \mathrm{J}(\mathrm{u}_{\mathrm{E}}(\tau(\Phi)))\right)\right) - \Phi \\
&= \theta(\mathrm{v}, \Phi)
\end{aligned} \tag{23}$$

In steady-state, $\dot{V} = \dot{v} = \dot{\Phi} = 0$. This results in the following equilibrium conditions:

$$\begin{aligned}
0 &= [F(v, \tau(\Phi)) + J(u_E(\tau(\Phi))) - H(u_N(v))]V \\
0 &= \left[G(\overline{V}) - H(u_N(v))\right]v \\
0 &= \tanh(\beta J(u_E(\tau(\Phi)))) - \Phi
\end{aligned}$$

Ruling out the trivial case in which $V = v = \Phi = 0$, the first expression in (23) shows that, in equilibrium, the rate of growth of the economy will be equal to the natural growth rate. The latter corresponds to the labour productivity growth rate because the population was assumed to be constant and equal to the labour force. Output expansions result either from increasing capital stock or improving energy efficiency, reducing emissions. From the second expression, we have that stable income distribution requires real wages to grow at the same pace of productivity. Finally, attitudes will stop changing when the probability of strongly supporting climate action equalises the probability of belonging to the opposite group. Given these conditions, we can state and prove the following Proposition regarding the existence of equilibria.

---

[5] A price on emissions can be achieved through an emission tax or permit trading. This paper focuses exclusively on the former, leaving the choice between these two measures to future research. For a recent formalisation with boundedly rational agents that compares both policies, see Foramitti et al. (2021). They show that the main difference between the two policies is that permit prices fall after successful abatement. This can lead to higher production levels under permit trading; however, it might drive emission-efficient firms out of the market.

Proposition 1. *The dynamic system (23) admits a set of equilibrium solutions defined by all triplets* $(\bar{V}, \bar{v}, \bar{\Phi})$ *that satisfy:*

$$\begin{aligned} &F\left(\overline{v}, \tau\left(\overline{\Phi}\right)\right) + J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right) = H(u_N(\overline{v})) \\ &\overline{V} = G^{-1}(H(u_N(\overline{v}))) \\ &\overline{\Phi} = \tanh\left(\beta J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right)\right) \end{aligned} \tag{24}$$

Proof. See Appendix A.1. □

Looking at (24), it is interesting to note that, in equilibrium, aggregate attitudes are determined independently of the other two endogenous variables. This result follows from our assumption that each strategy success, U^+ and U^-, responds to employment variations, $\dot{V}/V$, instead of its level. Such a hypothesis is plausible because boundedly rational households do not know a priori the equilibrium state of the economy. Moreover, they react to the prospects of being (un)employed. A person's concerns about losing her/his job begin when employment starts to fall, despite the fact that its level might still be high. Once environmental attitudes are decided, the GHG emissions tax rate and the energy efficiency growth rate are determined. Hence, income distribution is the adjusting variable between the actual and natural output growth rates. Knowing the tax rate and innovation prospects in energetic conditions, firms decide how much capital to accumulate and the share of workers engaged in the search for labour-saving production techniques.

A strong response of labour productivity to distribution results in a lower equilibrium wage share because slight variations in v are enough for firms to adopt a technology with a lower labour coefficient. Analogously, minor changes in v can halt accumulation if the investment is very sensitive to profitability. Therefore, the partial derivatives F_v and $H_{u_N} u_{N_v}$ are negatively related to the equilibrium share of workers in income. Finally, once firms decide how much to produce, they hire a certain amount of workers, thus determining the steady-state employment rate. As employment moves toward or away from equilibrium, the probability of becoming or not a strong climate action supporter changes. We are ready to turn to the investigation of the local stability properties of the equilibrium points defined by equations.

Proposition 2. *Each of the equilibrium points* $(\bar{V}, \bar{v}, \bar{\Phi})$ *is locally asymptotically stable in the region of the parameter space defined as:*

$$\begin{aligned} H_{u_N} u_{N_v} \overline{v} \;&>\; \theta_\Phi \\ (F_v - H_{u_N} u_{N_v})\theta_\Phi \;&>\; (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi \theta_v \end{aligned} \tag{25}$$

and

$$H_{u_N} u_{N_v}\left[(\theta_\Phi - H_{u_N} u_{N_v}\bar{v})\theta_\Phi - (F_v - H_{u_N} u_{N_v})\bar{V} G_V \bar{v}\right] + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \bar{V} \theta_v > 0$$

where all partial derivatives are calculated at $(\bar{V}, \bar{v}, \bar{\Phi})$.

Provided that (25) holds, a Hopf bifurcation can occur for values of the parameters such that:

$$H_{u_N} u_{N_v}\left[(\theta_\Phi - H_{u_N} u_{N_v}\bar{v})\theta_\Phi - (F_v - H_{u_N} u_{N_v})\bar{V} G_V \bar{v}\right] + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V V \theta_v = 0$$

Proof. See Appendix A.2. □

A special case of interest happens when $\partial\dot{\Phi}/\partial\Phi = \theta_\Phi < 0$ and $F_\tau + J_{u_E} u_{E_\tau} \lesseqgtr 0$. We are imposing that variations in attitudes are self-correcting and that the positive effect of taxation on output through energy efficiency, $J_{u_E} u_{E_\tau} > 0$, may or may not overcome its negative impact on capital accumulation, $F_\tau < 0$. Such a hypothesis opens the door to a positive or negative bridge between green attitudes and production. This is because taxes were assumed to be a function of public opinion, while output growth depends on capital accumulation and energy efficiency improvements. If firms are overwhelmed by the tax burden, or the effectiveness of efforts to reduce emissions is too low, $F_\tau + J_{u_E} u_{E_\tau} < 0$. Though it may be true in certain economies, we showed in the previous Section that such an assumption might not be valid for all EU members. Data suggests there is no clear-cut correspondence between output and attitudes. When the innovation effect overcomes the investment one, we actually have $F_\tau + J_{u_E} u_{E_\tau} > 0$. For clarity purposes, it is helpful to present a simplification of our previous Proposition that contemplates both cases.

Proposition 3. *A sufficient condition for the local stability of each equilibrium point* $(\bar{V}, \bar{v}, \bar{\Phi})$ *consists in constraining the region of the parameter space to:*

$$\theta_\Phi < 0$$

$$F_\tau + J_{u_E} u_{E_\tau} < 0 \tag{27}$$

and

$$(F_v - H_{u_N} u_{N_v})\theta_\Phi > (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi \theta_v$$

where all partial derivatives are calculated at $(\bar{V}, \bar{v}, \bar{\Phi})$.

When condition (27) does not hold, the solution is still locally stable provided that:

$$H_{u_N}u_{N_\nu}\left[(\theta_\Phi - H_{u_N}u_{N_\nu}\bar{\nu})\theta_\Phi - (F_\nu - H_{u_N}u_{N_\nu})\bar{V}G_V\bar{\nu}\right] + (F_\tau + J_{u_E}u_{E_\tau})\tau_\Phi G_V\bar{V}\theta_\nu > 0$$

However, if a change in one of the parameters determines the violation of this last inequality, the characteristic equation may have a pair of purely imaginary complex conjugate eigenvalues and no other eigenvalues with zero real part. Hence, a Hopf bifurcation might occur, and the system admits a family of periodic solutions.

Proof. See Appendix A.3. □

Suppose attitudes are self-correcting. When the investment response to taxes is stronger than the energy-innovation effect, the system will be locally stable conditional to variations in income distribution not having enough influence on agents' environmental decisions, i.e. $|\theta_\nu|$ sufficiently small. An initial increase in the wage share reduces profitability and, thus, firms lower capital accumulation. As a result, the employment rate falls, bringing attitudes towards climate policies down. Politicians respond by reducing the GHG emissions tax rate. However, such a measure will slow down improvements in energy efficiency, damaging output growth. Thus, firms will hire even fewer workers, leading Φ to spiral downward. This effect is mediated by how much importance individuals give to what is happening in the labour market, mediated by income distribution. If they care a lot, the equilibrium will be unstable.

On the other hand, we might have that the sensitivity of capital accumulation to taxes is weaker than the induced improvements in energy innovation. One could argue that this is not the most likely scenario, given that R&D takes time to mature and is under significant uncertainty. For example, if we understand renewables as a radical innovation that comes with a new techno-economic paradigm, an initial phase of exploratory improvements must be completed before a clear trajectory is defined (a comprehensive assessment of changes in technological paradigms can be found in Perez (2010)). Still, if such a trajectory is already explicit and firms know the direction R&D efforts should take, condition (27) is more likely to be violated. This last scenario is compatible with persistent endogenous fluctuations between environmental public opinion and the macroeconomy. The cycles' rationale and robustness will be assessed in the next Section.

To provide a more concrete view of the properties of the model, we adopt a set of functional forms for the behavioural equations of the model. This step allows us to restate the existence and local stability conditions of equilibria in terms of parameters that later on can be calibrated following empirical studies in the field and well-known time series:

$$\begin{aligned}
\frac{\dot{K}}{K} &= F(\nu,\tau) = (1-\tau)(\alpha_0 - \alpha_1\nu)\\
\frac{\dot{w}}{w} &= G(V) = -\gamma_0 + \gamma_1 V\\
\frac{\dot{z}}{z} &= H(u_N) = \sigma_1 u_N\\
\frac{\dot{y}}{y} &= J(u_E) = \mu_1 u_E\\
u_N &= u_N(\nu) = \sigma_2\nu\\
u_E &= u_E(\tau) = \mu_2\tau\\
\tau &= \tau(\Phi) = \tau_0 + \tau_1\Phi
\end{aligned} \tag{28}$$

where $\alpha_0 > 0$ is autonomous capital accumulation; $\alpha_1 > 0$ is the sensitivity of accumulation to income distribution; $\tau_0 > 0$ corresponds to autonomous taxes on emissions; $\tau_1 > 0$ stands for the sensitivity of the GHG tax rate to environmental attitudes; $\gamma_0 > 0$ captures the autonomous component of the real-wage Phillips curve; $\gamma_1 > 0$ represents the sensitivity of real wages to employment; $\sigma_1 > 0$ stands for the response of labour productivity to the share of workers performing production related R&D tasks; $0 < \sigma_2 < 1$ corresponds to this last group of workers as a proportion of labour costs; $\mu_1 > 0$ captures the sensitivity of energy efficiency to the share of workers performing energy-related R&D tasks; finally, $0 < \mu_2 < 1$ corresponds to workers performing energy-related R&D tasks as a proportion of the emissions tax.

Define ω_1 as the interaction between the intensity of choice, energy innovation, and attitudes-induced taxes:

$$\omega_1 = \beta\mu_1\mu_2\tau_1$$

Its economic intuition is the following. Recall that for a given composition of the population, GHG taxes will react with a sensitivity of τ_1 conditional to the strength of connections between agents (β). For each tax rate, firms respond by increasing the share of workers performing energy-saving innovation tasks – with a marginal effect of μ_2 – which will finally impact the energy efficiency rate of change by a factor of μ_1. On the other hand, ω_0 stands for the interplay between the component of the GHG emissions tax rate that does not depend on environmental sentiments, innovation in the energy segment, and the intensity of choice:

$$\omega_0 = \beta\mu_1\mu_2\tau_0$$

We thus can be more explicit about the conditions under which the model admits unique or multiple equilibria.

Proposition 4. *Whenever ω_1 is below a critical value, i.e. $\omega_1 < \tilde{\omega}_1$, or ω_0 is sufficiently high, $\omega_0 > \tilde{\omega}_0$, the dynamic system (23) admits a unique equilibrium solution that satisfies:*

$$\begin{aligned}\overline{V} &= \frac{\gamma_0 + \sigma_1\sigma_2\overline{v}}{\gamma_1}\\ \overline{v} &= \frac{\alpha_0 + (\mu_1\mu_2 - \alpha_0)\left(\tau_0 + \tau_1\overline{\Phi}\right)}{\sigma_1\sigma_2 + \alpha_1\left(1 - \tau_0 - \tau_1\overline{\Phi}\right)}\\ \overline{\Phi} &= \tanh\left(\omega_0 + \omega_1\overline{\Phi}\right) > 0\end{aligned}$$

On the other hand, when ω_1 is above a critical value, i.e. $\omega_1 > \tilde{\omega}_1$, and ω_0 is neglectable, $\omega_0 < \tilde{\omega}_0$, a Pitchfork bifurcation occurs, and two additional equilibria emerge, both satisfying:

$$\begin{aligned}\overline{V}_i &= \frac{\gamma_0 + \sigma_1\sigma_2\overline{v}_i}{\gamma_1}\\ \overline{v}_i &= \frac{\alpha_0 + (\mu_1\mu_2 - \alpha_0)\left(\tau_0 + \tau_1\overline{\Phi}_i\right)}{\sigma_1\sigma_2 + \alpha_1\left(1 - \tau_0 - \tau_1\overline{\Phi}_i\right)}\\ \overline{\Phi}_i &= \tanh\left(\omega_0 + \omega_1\overline{\Phi}_i\right)\end{aligned}$$

where $i = 1, 2$, $\bar{\Phi}_1 > 0$, and $\bar{\Phi}_2 < 0$.

Proof. See Appendix A.4. □

Let us spend some time on the particular case in which collective attitudes induce all taxes on emissions. So far, we have assumed that at least part of τ is not subject to changes in environmental sentiments, $\tau_0 > 0$. However, in the extreme situation where policymakers adopt a GHG tax rate entirely based on what the average citizen believes, we can be more explicit about the critical value of β related to the emergence of multiple equilibria. One could interpret it as a naive democratic regime in which politics perfectly reproduce the majoritarian views of the population.

Proposition 5. *Suppose all GHG tax emissions depend on environmental attitudes:*

$$\tau_0 = 0$$

When the parameter capturing the interaction between agents (β) is weak, below the critical value:

$$\beta \leq \frac{1}{\mu_1\mu_2\tau_1}$$

the dynamic system (23) admits a unique equilibrium solution defined and given by:

$$\begin{aligned}\overline{V} &= \frac{\gamma_0 + \sigma_1\sigma_2\overline{v}}{\gamma_1}\\ \overline{v} &= \frac{\alpha_0}{\sigma_1\sigma_2 + \alpha_1}\\ \overline{\Phi} &= 0\end{aligned}$$

On the other hand, when β is above the critical value:

$$\beta > \frac{1}{\mu_1\mu_2\tau_1}$$

a Pitchfork bifurcation occurs, and two additional equilibria emerge, both satisfying:

$$\begin{aligned}\overline{V}_i &= \frac{\gamma_0 + \sigma_1\sigma_2\overline{v}_i}{\gamma_1}\\ \overline{v}_i &= \frac{\alpha_0 + (\mu_1\mu_2 - \alpha_0)\left(\tau_0 + \tau_1\overline{\Phi}_i\right)}{\sigma_1\sigma_2 + \alpha_1\left(1 - \tau_0 - \tau_1\overline{\Phi}_i\right)}\\ \overline{\Phi}_i &= \tanh\left(\beta\mu_1\mu_2\tau_1\overline{\Phi}_i\right)\end{aligned}$$

where $i = 1, 2$, $\Phi_1 > 0$, and $\Phi_2 < 0$.

Proof. See Appendix A.5. □

The result above is central to our narrative. Previous studies have indicated that the group effect can generate the coexistence between states in which climate supporters and deniers/indifferent prevail (see Dávila-Fernández and Sordi, 2020; Cafferata et al., 2021). We innovate by explicitly referencing the GHG emissions tax rate and showing that $\tau_0 > 0$ might be a robust control to multi-stability. Comparing Propositions 4 and 5, the introduction of a tax component that does not depend on general environmental attitudes guarantees that $\omega_0 > 0$. If the interaction between τ_0, energy-innovation, and the intensity of choice is strong, i.e. $\omega_0 > \bar{\omega}_0$, it is possible to obtain a unique equilibrium solution in which positive environmental

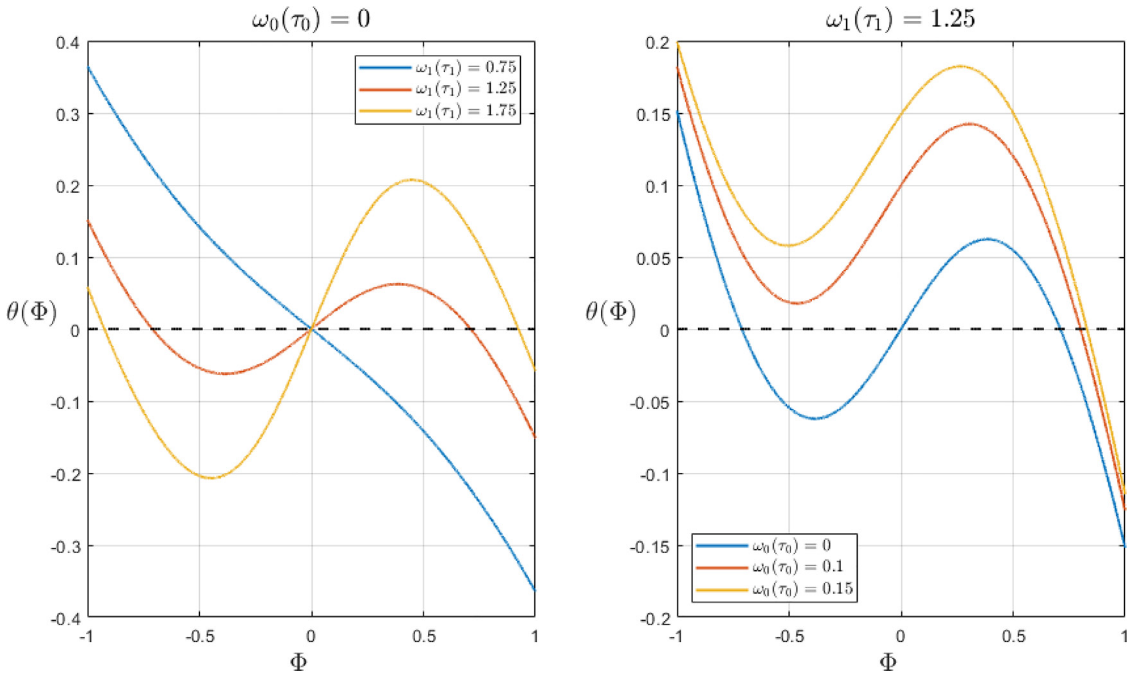


Fig. 4. Emergence and control of multi-stability. The intersections between the blue, orange and yellow colours with the black dotted line correspond to the equilibrium points. The left panel shows that raising ω_1 leads to two additional equilibria. The right panel indicates increasing ω_0 can restore a unique equilibrium solution. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

attitudes prevail. Such a result will hold independently of ω_1. Fig. 4, on the left, shows the emergence of the two additional equilibrium points as we increase ω_1 while fixing $\tau_0 = 0$. The intersection with the dotted black line corresponds to each equilibrium solution. We provide analogous comparative statics on the right panel, this time keeping $\omega_1 = 1.25$ constant and increasing ω_0 up to 0.15. In this case, the orange and yellow colours only intercept the black line at one point instead of two. Furthermore, notice that $\theta_\Phi < 0$ whenever the system admits a unique equilibrium. However, when we have three equilibrium points, $\theta_\Phi > 0$ in the central one, whereas $\theta_\Phi < 0$ in the others.

Before presenting our numerical exercise, we turn to the study of the local stability properties of the equilibria.

Proposition 6. *Whenever ω_1 is below a critical value, i.e. $\omega_1 < \tilde{\omega}_1$, or ω_0 is sufficiently high, $\omega_0 > \tilde{\omega}_0$, the unique international equilibrium solution of the dynamic system (23) is locally stable as long as:*

$$\mu_1\mu_2 < \alpha_0 - \alpha_1\bar{v} \tag{29}$$

and

$$[(1-\tau)\alpha_1 + \sigma_1\sigma_2]\bar{V}\gamma_1\theta_\Phi < (\alpha_0 - \alpha_1\bar{v} - \mu_1\mu_2)\tau_1\gamma_1\theta_v$$

However, when condition (29) does not hold, the solution is still locally stable provided that:

$$\sigma_1\sigma_2\Big\{(\theta_\Phi - \sigma_1\sigma_2\bar{v})\theta_\Phi + [(1-\tau)\alpha_1 + \sigma_1\sigma_2]\bar{V}\gamma_1\bar{v}\Big\} > (\alpha_0 - \alpha_1\bar{v} - \mu_1\mu_2)\tau_1\gamma_1\theta_v \tag{30}$$

Analogously, in the case ω_1 is above a critical value, i.e. $\omega_1 > \tilde{\omega}_1$, and ω_0 is neglectable, $\omega_0 < \tilde{\omega}_0$, the two additional equilibria are locally stable iff:

$$\mu_1\mu_2 < \alpha_0 - \alpha_1\bar{v}_i \tag{31}$$

and

$$[(1-\tau)\alpha_1 + \sigma_1\sigma_2]\bar{V}_i\gamma_1\theta_\Phi < (\alpha_0 - \alpha_1\bar{v}_i - \mu_1\mu_2)\tau_1\gamma_1\theta_v$$

A violation of condition (31) means the respective solution points will be locally stable provided that:

$$\sigma_1\sigma_2\Big\{(\theta_\Phi - \sigma_1\sigma_2\bar{v}_i)\theta_\Phi + [(1-\tau)\alpha_1 + \sigma_1\sigma_2]\bar{V}_i\gamma_1\bar{v}_i\Big\} > (\alpha_0 - \alpha_1\bar{v}_i - \mu_1\mu_2)\tau_1\gamma_1\theta_v \tag{32}$$

where $i = 1, 2$.

Moreover, suppose a change in one of the parameters determines the violation of (30) or (32), respectively. In that case, the characteristic equation may have a pair of purely imaginary complex conjugate eigenvalues and no other eigenvalues with zero real parts. A Hopf bifurcation might occur, and the system admits a family of periodic solutions.

Table 1
Parameter values.

Parameter	Definition	Value
α_0	Autonomous capital accumulation	0.025
α_1	Sensitivity of accumulation to income distribution	0.02
τ_0	Autonomous taxes on emissions	0.09
τ_1	Attitudes-induced tax on emissions	0.5
γ_0	Autonomous component real-wage Phillips curve	0.125
γ_1	Sensitivity of the real-wage Phillips curve to employment	0.2
σ_1	Labour productivity response to the share of workers performing output related R&D tasks	0.045
σ_2	Workers performing output related R&D tasks as a proportion of labour costs	0.58
μ_1	Energy efficiency response to the share of workers performing energy-related R&D tasks	0.66
μ_2	Workers performing energy related R&D tasks as a proportion of the emissions tax	0.06
β	Intensity of choice	2.5
ρ	How much agents value employment rather than ecological conservation	1

Proof. See Appendix A.6. □

Propositions 3 and 6 are equivalent. The main difference is that the latter follows from a specific set of functional forms and allows us to be more specific about the number of equilibrium points. Still, the message is fundamentally the same. When the sensitivity of investment to taxing emissions is stronger than efficiency gains in using energy resources, the system will be locally stable as long as the response of attitudes to employment is moderate. This posits a problem for policymakers because implies attitudes and output are negatively related, which is not desirable if the goal is to grow more sustainably. On the other hand, a stronger response of energy efficiency to τ reflects better-designed legislation on emissions and the capacity to translate those resources into productivity gains. Nonetheless, in this case, we might have to deal with endogenous fluctuations in public opinion that depend on and influence the macroeconomy over the medium and long run.

5. Numerical experiments

The existence part of the Hopf bifurcation theorem leaves us in the dark regarding the nature of the emerging cycles. They could be stable or unstable, characterising a super- or sub-critical bifurcation.[6] Hence, this Section presents different numerical experiments to assess the nature and robustness of the endogenous periodic fluctuations. This step is important because it allows us to separate two different effects. The first consists of waves in environmental attitudes, legislation, and employment that do not necessarily imply a permanent change in the economy's direction because the structural conditions are maintained. The second refers to the possibility of achieving a unique "green" equilibrium in which the consensus among the scientific community on the need to fight climate change is also reflected in the general society.

We choose parameter values such as to obtain economically meaningful results. Our selection, nonetheless, has only an illustrative purpose taking the EU as a general benchmark. Similar qualitative dynamics can be obtained for a wider range of values. Our reference values are reported in Table 1, where we assume the interaction between agents is weak, but they are not very sensitive to emissions. Furthermore, we demonstrated that persistent dynamics only emerge when $F_\tau + J_{u_E} u_{E_\tau} > 0$. Therefore, without loss of generality, our numerical experiments will be limited to that case. The three endogenous variables are the employment rate, wage share, and environmental attitudes. Under this baseline scenario, the system admits a unique internal equilibrium, such that:

$$\bar{V} \approx 0.7$$

$$\bar{\upsilon} \approx 0.6$$

and

$$\bar{\Phi} \approx 0$$

which are broadly in line with the case of the European Union. The employment rate as a percentage of the population aged 20 to 64 has fluctuated around 0.7 over the past ten years (Eurostat, 2022). On the other hand, the United Nations Economic Commission for Europe (UNECE, 2022) reports that the labour share of GDP, comprising wages and social protection transfers, has been close to 0.6, despite a clear negative trend. We set attitudes slightly above zero as an initial referential point that captures the heterogeneity in attitudes in the EU, as reported in Section 2.

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[6] A super-critical Hopf bifurcation occurs when the Maximum Lyapunov Exponent (MLE) is negative. A positive MLE is related to an unstable cycle, and we say the bifurcation is sub-critical. A degenerate case occurs when MLE = 0. However, given that it is not simple to provide an economic interpretation of the conditions required to calculate the MLE, we prefer to rely on numerical simulations. A rigorous reference to the topic can be found in (Kuznetsov, 2004, pp. 157–187).

Going back to (28) and substituting the values in Table 1, we obtain a general picture of the equilibrium magnitudes for the main variables in the model:

$$\tau = 0.09468, \qquad u_E = 0.00568, \qquad u_N = 0.34608$$

$$\frac{\dot{y}}{y} = 0.00375, \qquad \frac{\dot{z}}{z} = 0.01557, \qquad \frac{\dot{w}}{w} = 0.01557, \qquad \frac{\dot{E}}{E} = 0.01182$$

When society is almost equally divided between the two possible attitudes, the tax rate on GHG emissions mainly relies upon its autonomous component, which was assumed to be below 10 per cent by a small margin. According to the International Renewable Energy Agency IRENA (2021), 1.3 million people work in direct and indirect renewable energy jobs in the EU. Considering that the labour force in the EU is slightly above 215 million (World Development Indicators, WDI, 2022), we took the ratio between the two as a proxy of u_E. Analogously, Eurostat (2022) estimates that 74 million persons are employed in science and technology. Dividing again by the labour force, we have an approximation for u_N. Given that $\bar{\Phi}$ is very close to zero, the values of μ_1 and μ_2 are such that the rate of growth of energy efficiency is minor. Data from the WDI suggests the average rate of growth of labour productivity in the EU over the last 10 to 20 years has been 0.015. Hence, we chose σ_1 and σ_2 to deliver a compatible $\dot{z}/z$. Finally, our calibration of the capital accumulation and real-wage Phillips curve broadly follows the estimates by Grasselli and Maheshwari (2018). Still, to simplify the algebraic steps in the model, we assumed a constant energy-capital ratio. Therefore, we adjusted the respective coefficients, α_1 and α_2, to obtain $\dot{E}/E$ in line with data from the International Energy Agency (IEA, 2022).

In our baseline scenario, the unique internal equilibrium point is stable. Before investigating whether the Pitchfork and Hopf bifurcations we identified in the previous Section are supercritical, we perform some sensitivity analysis on the trajectories converging to the initial stable solution.[7] Two main parameters are of interest here. First, there is the response of attitudes to labour market conditions, ρ. We argued that people become more likely to pay attention to the environment when the employment rate increases because they expect their basic needs to be satisfied. By contrast, falling employment rates increase households' concerns about more urgent needs and raise the probability of downplaying the urgency of fighting climate change. Fig. 5 shows that doubling the value of ρ from 1 to 2 significantly impacts sentiments but has only minor effects on employment and income distribution. Moreover, the peak and valley in Φ become higher in absolute value when $\rho = 2$, and there is no change in V or f. Decisions to change taxes take time; reactions from capital accumulation, innovation and productivity are also not immediate; on top of that, the main effects are smoothed in the chain of events connecting Φ to the other two endogenous variables, thus explaining our findings.

A second parameter with policy interest is the energy efficiency response to the share of workers performing energy-related R&D tasks, μ_1. Implicitly, it captures the capacity of the national innovation system to achieve green growth. Taxing pollution is an indirect tax on production and is expected to harm accumulation, damaging output. However, suppose resources raised from τ are successfully applied to improve y, reducing GHG emissions. In that case, it might be possible to establish a positive correspondence between attitudes, taxing emissions, and growth. Fig. 6 plots the sensitivity of the trajectories generated by the dynamic system to this parameter. As in the previous diagram, the blue and red lines are relatively close for V and v, moving together. Employment leads the wage share because as unemployment rates fall, the bargaining power of workers increases, allowing workers to have real wage increases above improvements in labour productivity. The inverse reasoning also applies: Falling employment eventually results in a smaller labour share. On the other hand, attitudes are more volatile, alternating between periods with $\Phi \lessgtr 0$ before converging to $\bar{\Phi}$.

Taken together, Figs 5 and 6 also provide interesting insights into the sequence of changes in the respective variables. Improving employment rates lead to more support for environmental action. Policymakers respond by increasing GHG taxation, reducing investment but incentivising innovation in the energy segment. We are assuming the latter effect prevails. Therefore, there is an increase in output. Firms need to hire more workers, reinforcing the initial expansion of Φ. Moreover, a consequence of improvements in energy efficiency is a reduction in emissions, which further reinforces green attitudes:

$$V \uparrow \Longrightarrow \Phi \uparrow \Longrightarrow \tau \uparrow \Longrightarrow \begin{matrix} K \downarrow \\ y \uparrow \end{matrix} \overset{y_{\text{effect}} > K_{\text{effect}}}{\Longrightarrow} Y \uparrow \Longrightarrow V \uparrow$$

$$\Phi \uparrow \Longrightarrow \tau \uparrow \Longrightarrow y \uparrow \Longrightarrow \Phi \uparrow$$

These two instability channels are balanced by income distribution. We have previously discussed how a warming labour market improves the labour share. From the definition of v, the latter is the ratio between real wages and labour productivity. A rising v reduces the profitability of investment and implies an increase in production costs. Firms respond in two different ways. On the one hand, they reduce capital accumulation. On the other hand, they increase their search for labour-saving production techniques. This is done by augmenting the share of workers that also perform R&D. Both mechanisms reduce employment either because the investment was halted or because firms can produce more with fewer workers. They

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[7] Non-linear interactions between a Hopf bifurcation and a Pitchfork-type stationary bifurcation can produce secondary bifurcations of periodic solutions and tertiary bifurcations of periodic or aperiodic solutions lying on an invariant torus. In the present paper, we do not study these possibilities. A complete classification of the resulting bifurcation diagrams can be found in Langford and Iooss (1980, pp. 103–134).

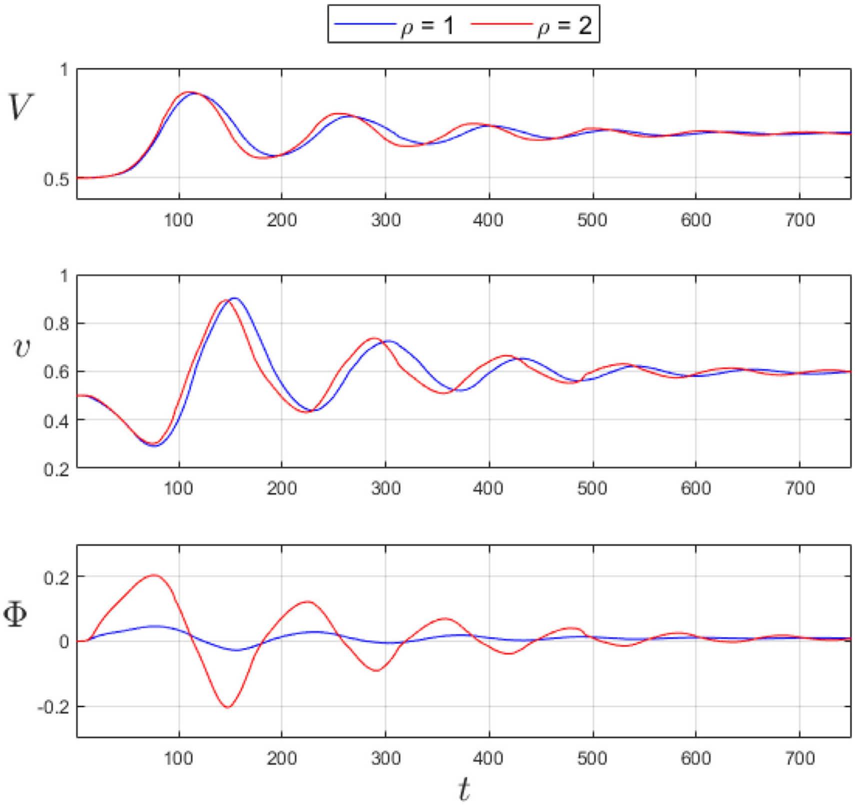

Fig. 5. Sensitivity of the dynamic system to changes in how much agents value employment when determining environmental attitudes ρ.

create a negative correspondence between wage share and attitudes, though the latter positively correlates to employment only when V is growing sufficiently fast:

$$v \uparrow \Longrightarrow \begin{matrix} K \downarrow \Rightarrow Y \downarrow \\ z \uparrow \end{matrix} \Longrightarrow V \downarrow \Longrightarrow \begin{matrix} \Phi \downarrow \\ v \downarrow \end{matrix}$$

Such interactions can also be appreciated in Fig. 3, which presents a summarising diagram of the Green-MKS system.

As we increase the intensity of choice, β, it is possible to show the emergence of a closed orbit around the equilibrium solution of the system. Fig. 7, on the left, indicates that trajectories depict cyclical convergence in our standard scenario, $\beta = 2.5$. However, for $\beta = 25$, we have a stable cycle characterising the Hopf bifurcation in the first part of Propositions 3 and 6 as supercritical. Orbits are anti-clockwise oriented in the phase space. The rationale for such persistent and periodic fluctuations follows the interaction between stabilising and destabilising forces discussed in previous paragraphs. The orbit itself results from the balance between the two of them. Fig. 8 assesses their robustness as we increase ρ and μ_1. When the equilibrium is stable, we showed that employment and income distribution dynamics are not very sensitive to those parameters. The picture is slightly different now. The amplitude of the cycle is significantly higher the higher are ρ and μ_1.

We are now ready to proceed with an initial assessment of the case in which the system admits multiple equilibria. As demonstrated in Propositions 4 and 5, conditional to ω_0 and ω_1, a Pitchfork bifurcation occurs, and we have two additional equilibrium points. Both ω_0 and ω_1 depend on the autonomous and attitudes-induced component of the emissions tax rate. To focus on economic intuition, we adopt the simplification of Proposition 5 and assume for a moment $\tau_0 = 0$. The intensity of choice was increased to $\beta = 53$, and we suppose that agents value a clean and improving sustainable environment – as captured by energy efficiency – much more than variations in employment, $\rho = 0.024$. Fig. 9 represents this situation. It is possible to appreciate the coexistence of two basins of attraction. In red, we have a trajectory converging to the equilibrium in which most agents support decisive climate action. Given our assumption that energy innovation responds stronger than an investment to pollution taxation, employment and the wage share will be higher than the blue alternative.[8]

Given that the two emerging equilibria are stable and the initial solution becomes unstable, we conclude our Pitchfork bifurcation is supercritical. From an economic point of view, this attribute is appealing but posits a problem. It is interesting

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[8] Such a hypothesis does not necessarily holds. When investment is more sensitive than innovation to taxation, the equilibrium in which “deniers” prevail has greater growth, employment, and labour share. However, we demonstrated in Section 2 that is not compatible with endogenous persistent dynamics. We thus do not further cover it in our numerical experiments.

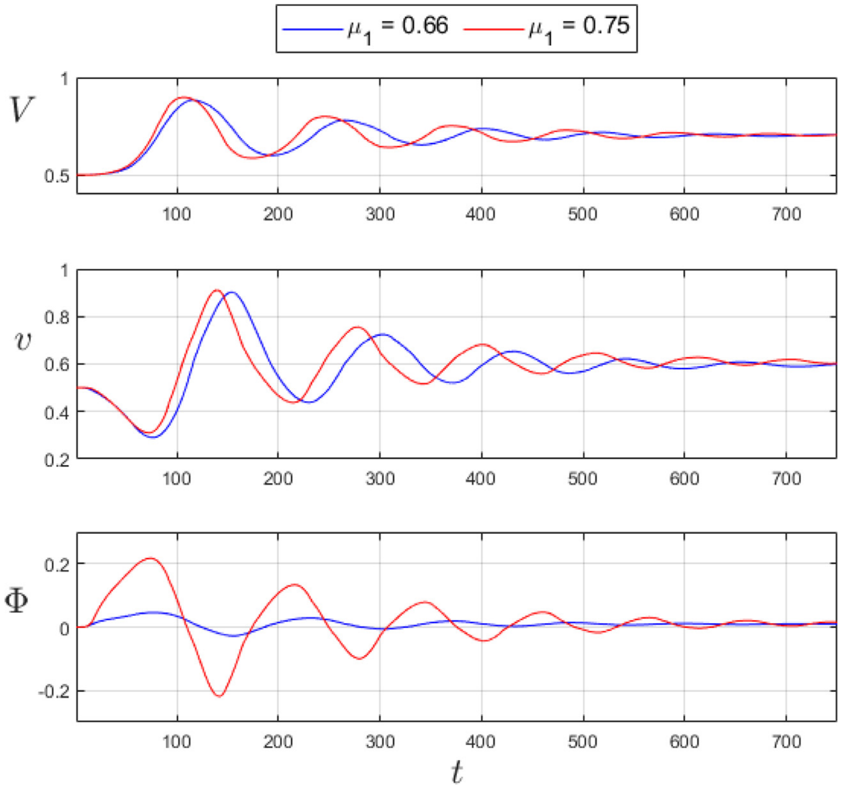

Fig. 6. Sensitivity of the trajectories generated by the dynamic system to changes in the coefficient capturing the response of energy-efficiency growth to GHG taxes.

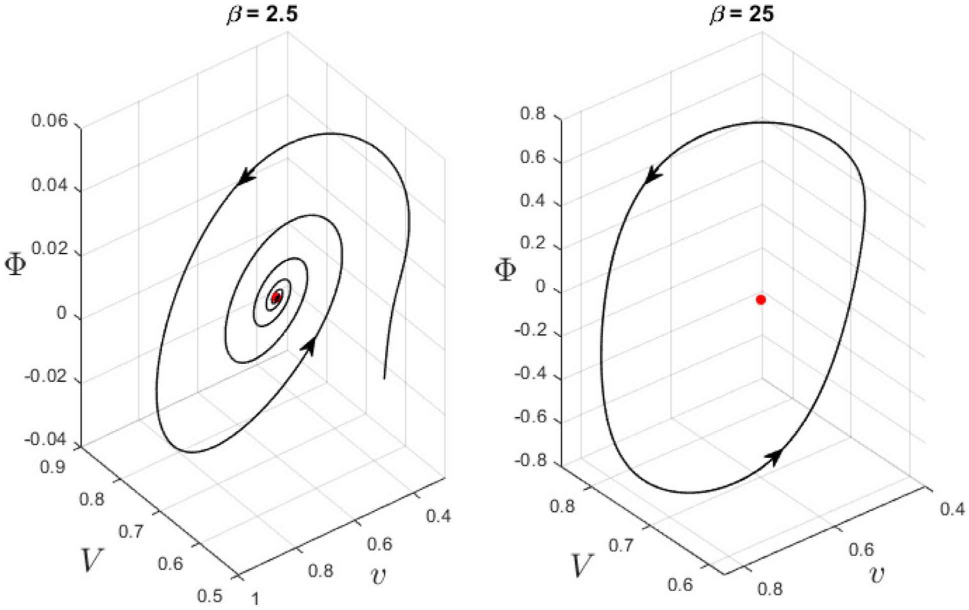

Fig. 7. Emergence of the Hopf bifurcation when the system admits a unique internal equilibrium point. The red dot corresponds to the equilibrium point. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

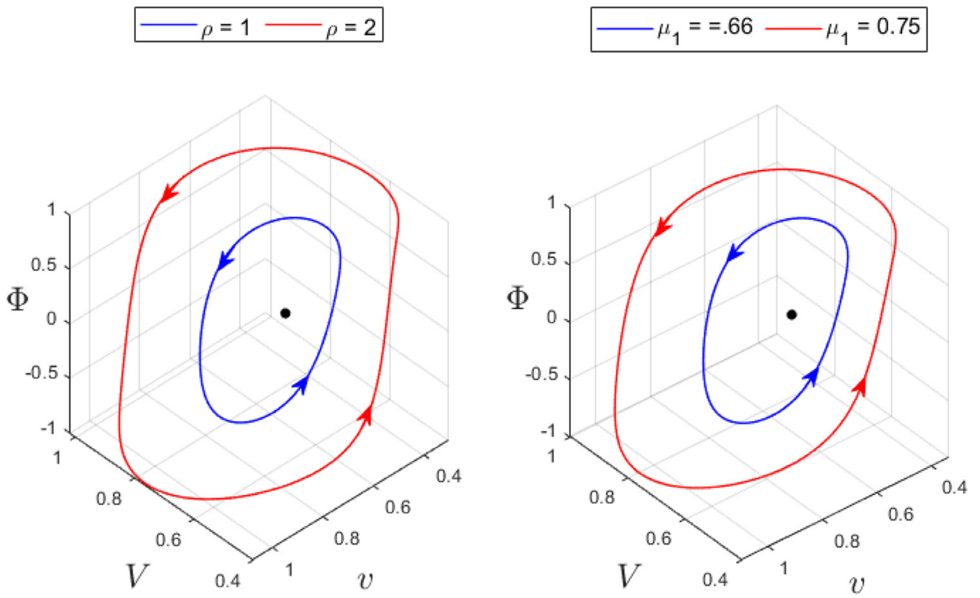

Fig. 8. Robustness of closed orbits to changes in attitudes' response to employment and to variations in the response of energy efficiency to taxing emissions. The black dot stands for the equilibrium point.

because it supports heterogeneity in ecological thinking, as captured in the World Risk Poll. However, if an economy finds itself in the "good" equilibrium, there is always the risk that an exogenous shock might permanently move the economy to the "bad" equilibrium. Moreover, one could wonder if it is not possible to find a more permanent solution for a country or region where not fighting climate change is the stable approach. Finally, we identify an anti-clockwise orbit surrounding the basins of attraction of the two stable points. The curve stresses the danger of falling into a loop in which society alternates phases of support and opposition or apathy to the climate issue.

The final state of a multi-stable system depends on initial conditions. An important policy problem is how to develop control strategies to induce a definite switch to the desired point. Such a control should be robust against noise to avoid transitions between states. Fig. 4 already provided some hints in that direction. We showed that *ceteris paribus* a higher ω_0 makes the equilibria with $\bar{\Phi} \leq 0$ disappear. Keeping everything else constant means, we cannot further raise β, μ_1, or μ_2 because doing so also brings ω_1 up. Fig. 10, on the left, reveals how increases in the intensity of choice lead to multiple equilibria. Taming β is not desirable, however, because it leads to a society equally divided between the two groups, which is not enough to design and consolidate the required environmental policies. We are left with the autonomous component of emissions taxation, τ_0. Fig. 10, on the left, indicates that a $\tau_0 = 0.045$ is already enough to obtain a stable unique equilibrium point in which strong environmental attitudes prevail.

Arguably, τ_0 responds to all other factors that matter to the choice of τ beyond domestic collective opinion. Here, we would like to highlight two main forces. First, we have the capacity of scientists to communicate with policymakers. The scientific community has almost a consensus that climate change is urgent and demands immediate action. If they can influence the decisions at a policy level, we may end up in a unique equilibrium in which green attitudes prevail in the overall population. Second, the international community also plays a relevant role in determining τ through τ_0. For example, the Paris Agreement is a legally binding international treaty on climate change. It was adopted by 196 Parties at the United Nations Climate Change Conference in Paris on December 12, 2015, and entered into force on 4 November 2016. Its goal is to limit global warming to below $1.5 - 2^{\circ}$ Celsius compared to pre-industrial levels. Not all countries signing the agreement have most of their populations supporting radical environmental action. Still, it acts as an exogenous force, especially in smaller nations, in the direction of increasing τ_0.

6. Final considerations

Peter Flaschel dedicated his entire academic career to the field of macrodynamics. His articles and books provide a solid framework for studying capitalist economies from a disequilibrium perspective (e.g. Chiarella and Flaschel, 2000; Chiarella et al., 2005; Flaschel, 2009 [1993]; Asada et al., 2011). Among his main contributions, the KMG and the MKS systems are notable baseline frameworks that can be applied to various economic problems. This paper developed a Green-MKS model incorporating what we consider a social dimension of climate change. Using data from the WorldRiskPoll (2019), we wanted to explain the positive correlation between environmental attitudes and energy efficiency without a clear correspondence between the former and GDP found in the EU. Our model shares the main characteristics of Flaschel (2015). From

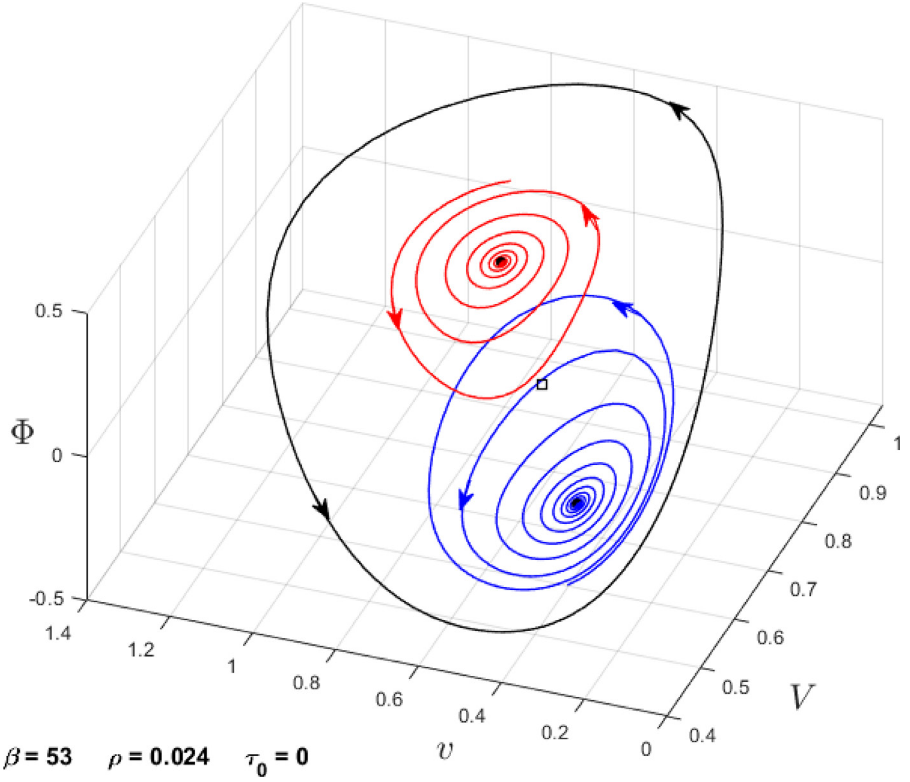

Fig. 9. Coexistence of two stable attractors with a periodic orbit. The black dots correspond to the stable equilibria, while a square represents the unstable point.

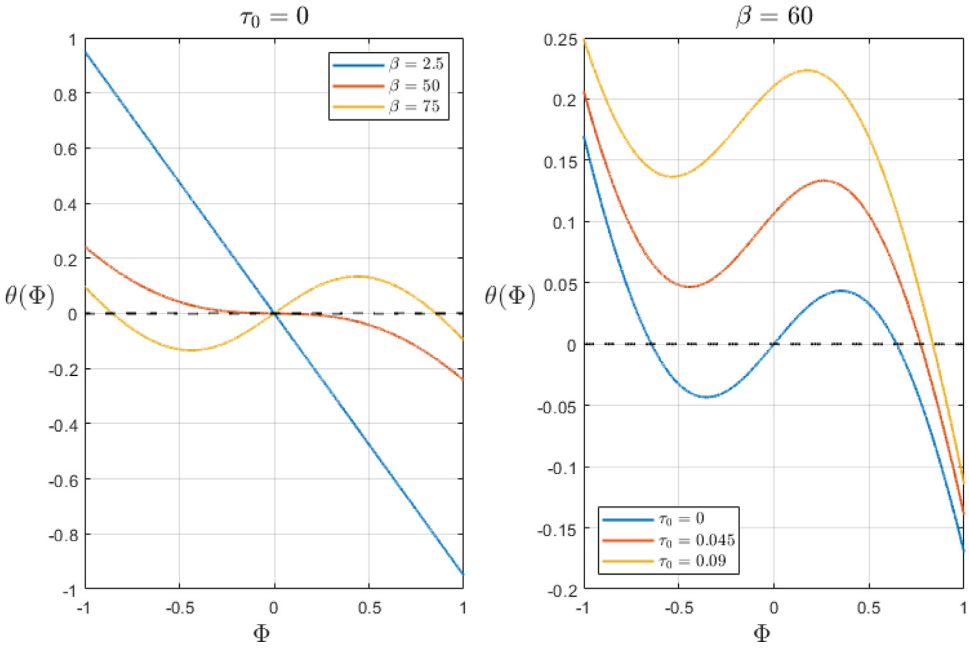

Fig. 10. Autonomous GHG tax rate (τ_0) as a control for multi-stability. The intersections between the blue, orange and yellow curves with the black dotted line correspond to the equilibrium points. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Marx, the correspondence between profits, income distribution, and accumulation. From Keynes, the principle of effective demand. Finally, from Schumpeter, innovation as a major force of long-run growth.

Furthermore, the population was divided between those supporting and opposing taxing emissions. Its composition varied according to a continuous-time version of the discrete choice approach. As the interaction between agents increases, a Pitchfork bifurcation occurs, resulting in two stable equilibrium points, one with most of the population supporting emission taxes and the other with most agents opposing such a policy. A sufficiently strong response of sentiments to emissions combined with partially autonomous pollution regulation may lead to the disappearance of the equilibrium in which most agents oppose GHG taxation, controlling for multi-stability. By applying the existence part of the Hopf bifurcation theorem, we show that our 3-dimensional system admits endogenous persistent and bounded fluctuations. Therefore, we demonstrated that the interaction between attitudes towards climate policies and the macro-economy might emerge as a novel source of low-frequency growth-cycle dynamics.

Conditional to the level of interaction between players, society chooses a given tax rate on pollution. The net effect of attitudes on growth is dubious because, through taxing emissions, they might enhance the development of energy-saving production techniques but penalise capital accumulation. Furthermore, a certain level of economic activity generates emissions and determines the employment rate. Both feedback on ecological sentiments. Lower emissions reinforce sustainable attitudes; however, a weak labour market increases the probability of not taking climate change seriously. This last effect follows from the fact that households become more concerned with basic needs such as housing, food, and utilities, leaving perceived long-run problems as a secondary issue. Hence, the model is compatible with a positive relationship between environmental attitudes and energy efficiency but not a clear association with GDP.

Declaration of Competing Interest

None.

Data availability

Data will be made available on request.

Appendix A. Mathematical appendix

A1. Proof of Proposition 1

To prove Proposition 1, recall the equilibrium conditions are given by:

$$0 = \left[F\left(\overline{v}, \tau\left(\overline{\Phi}\right)\right) + J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right) - H(u_N(\overline{v}))\right]\overline{V}$$

$$0 = \left[G\left(\overline{V}\right) - H(u_N(\overline{v}))\right]\overline{v}$$

$$0 = \tanh\left(\beta J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right)\right) - \overline{\Phi}$$

Disregarding the trivial case in which $\overline{V} = \overline{v} = \overline{\Phi} = 0$, we have that:

$$H(u_N(\overline{v})) = F\left(\overline{v}, \tau\left(\overline{\Phi}\right)\right) + J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right) \tag{A.1}$$

$$H(u_N(\overline{v})) = G\left(\overline{V}\right) \tag{A.2}$$

$$\overline{\Phi} = \tanh\left(\beta J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right)\right) \tag{A.3}$$

From Eq. (A.3), we can recursively prove the existence of a nontrivial solution. Start by noting that $\bar{\Phi}$ is determined independently of $\bar{V}$and $\bar{v}$. Furthermore, given that $\bar{\Phi} \in [-1, 1]$, then:

$$\tanh\left(\beta J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right)\right) = A\left(\overline{\Phi}\right) \in [-1, 1]$$

Hence, $A : [-1, 1] \to [-1, 1]$, we have that $\bar{\Phi}$ exists from Brouwer's fixed point theorem. Substituting this value into Eq. (A.1), we can solve for $\bar{v}$. Finally, substituting it into (A.2) and computing the respective inverse function, it follows that:

$$\overline{V} = G^{-1}(H(u_N(\overline{v})))$$

defining the equilibrium value of the employment rate.

A2. Proof of Propositions 2

To perform local stability analysis, we consider the following Jacobian matrix evaluated at $(\bar{V}, \bar{v}, \bar{\Phi})$:

$$\mathbf{Jac} = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix}$$

where

$$\begin{aligned} j_{11} &= 0 \\ j_{12} &= (F_v - H_{u_N} u_{N_v})\overline{V} < 0 \\ j_{13} &= (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi \overline{V} \gtreqless 0 \end{aligned}$$

$$\begin{aligned} j_{21} &= G_V \overline{v} > 0 \\ j_{22} &= -H_{u_N} u_{N_v} \overline{v} < 0 \\ j_{23} &= 0 \end{aligned}$$

and

$$\begin{aligned} j_{31} &= 0 \\ j_{32} &= \theta_v = \left[1 - \tanh^2\left(\beta J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right)\right)\right]\beta \rho (F_v - H_{u_N} u_{N_v}) < 0 \\ j_{33} &= \theta_\Phi = \left[1 - \tanh^2\left(\beta J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right)\right)\right]\beta[\rho (F_\tau + J_{u_E} u_{E_\tau}) + J_{u_E} u_{E_\tau}]\tau_\Phi - 1 \gtreqless 0 \end{aligned}$$

Thus, the characteristic equation can be written as

$$\lambda^3 + b_1 \lambda^2 + b_2 \lambda + b_3 = 0$$

where the coefficients are given by

$$\begin{aligned} b_1 &= -\mathrm{tr}(\mathbf{Jac}) \\ &= H_{u_N} u_{N_v} \overline{v} - \theta_\Phi \end{aligned} \tag{A.4}$$

$$\begin{aligned} b_2 &= \begin{vmatrix} j_{22} & 0 \\ j_{32} & j_{33} \end{vmatrix} + \begin{vmatrix} 0 & j_{13} \\ 0 & j_{33} \end{vmatrix} + \begin{vmatrix} 0 & j_{12} \\ j_{21} & j_{22} \end{vmatrix} \\ &= \begin{vmatrix} -H_{u_N} u_{N_v} \overline{v} & 0 \\ \theta_v & \theta_\Phi \end{vmatrix} + \begin{vmatrix} 0 & (F_v - H_{u_N} u_{N_v})\overline{V} \\ G_V \overline{v} & -H_{u_N} u_{N_v} \overline{v} \end{vmatrix} \\ &= -H_{u_N} u_{N_v} \overline{v} \theta_\Phi - (F_v - H_{u_N} u_{N_v})\overline{V} G_V \end{aligned} \tag{A.5}$$

and

$$\begin{aligned} b_3 &= -\det(\mathbf{Jac}) \\ &= -\begin{vmatrix} 0 & (F_v - H_{u_N} u_{N_v})\overline{V} & (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi \overline{V} \\ G_V \overline{v} & -H_{u_N} u_{N_v} \overline{v} & 0 \\ 0 & \theta_v & \theta_\Phi \end{vmatrix} \\ &= (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v} \theta_\Phi - (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \overline{v} \overline{V} \theta_v \end{aligned} \tag{A.6}$$

such that:

$$\begin{aligned} b_1 b_2 - b_3 &= (H_{u_N} u_{N_v} \overline{v} - \theta_\Phi)\left[-H_{u_N} u_{N_v} \overline{v} \theta_\Phi - (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v}\right] \\ &\quad + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \overline{v}\, \overline{V} \theta_v - (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v} \theta_\Phi \\ &= H_{u_N} u_{N_v} \overline{v}\left[-H_{u_N} u_{N_v} \overline{v} \theta_\Phi - (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v}\right] \\ &\quad + \theta_\Phi H_{u_N} u_{N_v} \overline{v} \theta_\Phi + \theta_\Phi (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v} \\ &\quad + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \overline{v} \overline{V} \theta_v - (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v} \theta_\Phi \\ &= H_{u_N} u_{N_v} \overline{v}\left[-H_{u_N} u_{N_v} \overline{v} \theta_\Phi - (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v}\right] \\ &\quad + \theta_\Phi H_{u_N} u_{N_v} \overline{v} \theta_\Phi + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \overline{v} \overline{V} \theta_v \\ &= H_{u_N} u_{N_v} \overline{v}\left[(\theta_\Phi - H_{u_N} u_{N_v} \overline{v})\theta_\Phi - (F_v - H_{u_N} u_{N_v})\overline{V} G_V \overline{v}\right] \\ &\quad + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \overline{v} \overline{V} \theta_v \end{aligned} \tag{A.7}$$

The necessary and sufficient conditions for the local stability of a given equilibrium point is that all roots of the characteristic equation have negative real parts. From the Routh-Hurwitz conditions, this requires:

$$\begin{aligned} b_1 &> 0 \\ b_2 &> 0 \\ b_3 &> 0 \\ b_1 b_2 - b_3 &> 0 \end{aligned}$$

where either the first or the second condition can be eliminated. For simplicity, we choose to evaluate conditions b_1, b_3, and $b_1 b_2 - b_3 > 0$.

Therefore, the equilibria will be locally stable, provided that:

$$\begin{aligned} H_{u_N} u_{N_v} \bar{v} \; &> \; \theta_\Phi \\ (F_v - H_{u_N} u_{N_v})\theta_\Phi \; &> \; (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi \theta_v \end{aligned}$$

and

$$H_{u_N} u_{N_v}\left[(\theta_\Phi - H_{u_N} u_{N_v}\bar{v})\theta_\Phi - (F_v - H_{u_N} u_{N_v})\bar{V} G_V \bar{v}\right] + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \bar{V}\theta_v > 0$$

On the other hand, as long as b_1, b_2 and $b_3 > 0$, a Hopf bifurcation can occur for values of the parameters such that $b_1 b_2 - b_3 = 0$. That is, when:

$$H_{u_N} u_{N_v}\left[(\theta_\Phi - H_{u_N} u_{N_v}\bar{v})\theta_\Phi - (F_v - H_{u_N} u_{N_v})\bar{V} G_V \bar{v}\right] + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \bar{V}\theta_v = 0$$

A3. Proof of Propositions 3

In the case in which $\theta_\phi < 0$, Eqs. (A.4) and (A.5) indicate it is always true that:

$$\begin{aligned} b_1 &> 0 \\ b_2 &> 0 \end{aligned}$$

From Eq. (A.7), a sufficient condition for $b_1 b_2 - b_3 > 0$ to be positive is:

$$F_\tau + J_{u_E} u_{E_\tau} < 0$$

It follows, from (A.6), that $b_3 > 0$ provided that:

$$(F_v - H_{u_N} u_{N_v})\theta_\Phi > (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi \theta_v$$

Moreover, when

$$F_\tau + J_{u_E} u_{E_\tau} > 0$$

b_3 will be necessarily greater than zero. We are left with:

$$b_1 b_2 - b_3 \gtreqless 0$$

according as to whether

$$H_{u_N} u_{N_v}\left[(\theta_\Phi - H_{u_N} u_{N_v}\bar{v})\theta_\Phi - (F_v - H_{u_N} u_{N_v})\bar{V} G_V \bar{v}\right] + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \bar{V}\theta_v \gtreqless 0$$

We can then conclude that when the latter condition is satisfied with the greater sign, the non-trivial solutions are locally stable, whereas a Hopf bifurcation can occur for values of the parameters such that:

$$H_{u_N} u_{N_v}\left[(\theta_\Phi - H_{u_N} u_{N_v}\bar{v})\theta_\Phi - (F_v - H_{u_N} u_{N_v})\bar{V} G_V \bar{v}\right] + (F_\tau + J_{u_E} u_{E_\tau})\tau_\Phi G_V \bar{V}\theta_v = 0$$

A4. Proof of Propositions 4

Disregarding the trivial case in which $\bar{V} = \bar{v} = \bar{\Phi} = 0$, the equilibrium conditions of the dynamic system (23) imply:

$$\begin{aligned} H(u_N(\bar{v})) &= F\left(\bar{v}, \tau\left(\overline{\Phi}\right)\right) + J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right) \\ H(u_N(\bar{v})) &= G\left(\overline{V}\right) \\ \overline{\Phi} &= \tanh\left(\beta J\left(u_E\left(\tau\left(\overline{\Phi}\right)\right)\right)\right) \end{aligned}$$

Using the functional forms in (28), we can rewrite them as:

$$\sigma_1 \sigma_2 \bar{v} = (1 - \tau)(\alpha_0 - \alpha_1 \bar{v}) + \mu_1 \mu_2 \left(\tau_0 + \tau_1 \bar{\Phi}\right) \tag{A.8}$$

$$\sigma_1 \sigma_2 \bar{v} = -\gamma_0 + \gamma_1 \bar{V} \tag{A.9}$$

$$\bar{\Phi} = \tanh\left(\omega_0 + \omega_1 \bar{\Phi}\right) \tag{A.10}$$

Fig. 4 plots $\theta = \tanh(\omega_0 + \omega_1 \Phi) - \Phi$. Hence, the intersection of the colour lines with the black dotted one corresponds to the values of Φ for which Eq. (A.10) is satisfied, $\theta = 0$. When ω_1 is below a critical value, i.e. $\omega_1 < \tilde{\omega}_1$, or ω_0 is sufficiently high, $\omega_0 > \tilde{\omega}_0$, a unique value of $\bar{\Phi}$ satisfies (A.10).

The analytical proof of this result is based on the continuity of the functional forms used. Notice that Eq. (A.10) can be rewritten as:

$$\tanh^{-1}(\Phi) = \omega_0 + \omega_1 \Phi$$

The inverse of tanh is the arctan. Using the definition of the latter, we have:

$$\frac{1}{2}\ln\left(\frac{1+\Phi}{1-\Phi}\right) = \omega_0 + \omega_1 \Phi$$

It immediately follows that:

$$\frac{1}{2\omega_0}\ln\left(\frac{1+\Phi}{1-\Phi}\right) = \ln(\exp(1)) + \frac{\omega_1}{\omega_0}\Phi$$

Moving $\ln(\exp(1))$ to the left-hand side, and collecting terms, we obtain:

$$\frac{1}{2\omega_0}\left[\ln\left(\frac{1+\Phi}{1-\Phi}\right) - 2\omega_0 \ln(\exp(1))\right] = \frac{\omega_1}{\omega_0}\Phi$$

from which, one gets:

$$\ln\left[\frac{1+\Phi}{e^{2\omega_0}(1-\Phi)}\right] = 2\omega_1 \Phi \tag{A.11}$$

Evaluating the limit of Eq. (A.11) when $\omega_1 \to 0$, we have:

$$\lim_{\omega_1 \to 0} \ln\left[\frac{1+\Phi}{e^{2\omega_0}(1-\Phi)}\right] = \lim_{\omega_1 \to 0} 2\omega_1 \Phi = 0 \tag{A.12}$$

Therefore, we must have:

$$\frac{1+\Phi}{e^{2\omega_0}(1-\Phi)} = 1$$

from which it follows that:

$$\Phi = \frac{e^{2\omega_0} - 1}{1 + e^{2\omega_0}} > 0$$

Therefore, there exists a $\tilde{\omega}_1$ such that for $\omega_1 < \tilde{\omega}_1$ the equilibrium value $\bar{\Phi}$ is unique and positive.

To show $\bar{\Phi}$ is unique and positive also in the case ω_0 is sufficiently high, define $C = \dot{\Phi}$ as an auxiliary variable:

$$C = \tanh(\omega_0 + \omega_1 \Phi) - \Phi$$

for which:

$$\frac{\partial C}{\partial \Phi} = \omega_1\left[1 - \tanh^2(\omega_0 + \omega_1 \Phi)\right] - 1$$

such that the critical points of Φ are:

$$\frac{\arctan\left(\sqrt{\frac{\omega_1 - 1}{\omega_1}}\right) - \omega_0}{\omega_1} \quad \text{and} \quad \frac{-\arctan\left(\sqrt{\frac{\omega_1 - 1}{\omega_1}}\right) - \omega_0}{\omega_1}$$

If $\omega_1 < 1$, the function C is decreasing, the critical point does not exist, and a unique $\bar{\Phi}$ satisfies the equilibrium condition $C = 0$. When $\omega_1 \geq 1$, we rely on the second derivative to assess whether these are maximum, minimum or inflexion points:

$$\frac{\partial^2 C}{\partial \Phi^2} = -2\omega_1^2 \tanh(\omega_0 + \omega_1 \Phi)\underbrace{\left[1 - \tanh^2(\omega_0 + \omega_1 \Phi)\right]}_{>0} \tag{A.13}$$

Evaluating the sign of (A.13) at each of them, we obtain:

$$2\omega_1^2\sqrt{\frac{\omega_1 - 1}{\omega_1}}\underbrace{\left[1 - \tanh^2(\omega_0 + \omega_1 \Phi)\right]}_{>0} \quad \text{and} \quad -2\omega_1^2\sqrt{\frac{\omega_1 - 1}{\omega_1}}\underbrace{\left[1 - \tanh^2(\omega_0 + \omega_1 \Phi)\right]}_{>0}$$

Therefore, when $\omega_1 = 1$, the unique critical point is an inflexion, function C is decreasing, and a unique Φ satisfies the equilibrium condition $C = 0$. Still, in the case $\omega_1 > 1$, the first point is a local minimum and the second a local maximum. If at its local minimum $C > 0$, it follows that there is a unique $\Phi > 0$ such that $C = 0$. This will be the case only when ω_0 is greater than a threshold value:

$$\omega_0 > \omega_1\sqrt{\frac{\omega_1 - 1}{\omega_1}} + \arctan\left(-\sqrt{\frac{\omega_1 - 1}{\omega_1}}\right)$$

Thus, there exists a $\tilde{\omega}_0$ such that for $\omega_0 > \tilde{\omega}_0$, the equilibrium value Φ is unique. This concludes the proof of the uniqueness of $\bar{\Phi}$ when ω_1 is sufficiently small, OR $\tilde{\omega}_0$ is sufficiently high.

Substituting $\bar{\Phi}$ into Eq. (A.8), and solving for income distribution, we have that:

$$\begin{aligned}
\sigma_1\sigma_2\overline{v} &= \left[1 - \left(\tau_0 + \tau_1\overline{\Phi}\right)\right](\alpha_0 - \alpha_1\overline{v}) + \mu_1\mu_2\left(\tau_0 + \tau_1\overline{\Phi}\right) \\
\sigma_1\sigma_2\overline{v} + \alpha_1\overline{v}\left[1 - \left(\tau_0 + \tau_1\overline{\Phi}\right)\right] &= \alpha_0\left[1 - \left(\tau_0 + \tau_1\overline{\Phi}\right)\right] + \mu_1\mu_2\left(\tau_0 + \tau_1\overline{\Phi}\right) \\
\overline{v} &= \frac{\alpha_0 + (\mu_1\mu_2 - \alpha_0)\left(\tau_0 + \tau_1\overline{\Phi}\right)}{\sigma_1\sigma_2 + \alpha_1\left[1 - \left(\tau_0 + \tau_1\overline{\Phi}\right)\right]}
\end{aligned} \tag{A.14}$$

Finally, substituting Eq. (A.14) into (A.9), we obtain the equilibrium value of employment:

$$\bar{V} = \frac{\gamma_0 + \sigma_1\sigma_2\bar{v}}{\gamma_1}$$

thus defining a unique equilibrium solution given by $(\bar{V}, \bar{v}, \bar{\Phi})$.

Turning to the case where ω_1 is above a critical value, i.e. $\omega_1 > \tilde{\omega}_1$, and ω_0 is sufficiently small, $\omega_0 < \tilde{\omega}_0$, two additional values of $\bar{\Phi}_i$ satisfy (A.10), where $i = 1, 2$, $\bar{\Phi}_1 > 0$ and $\bar{\Phi}_2 < 0$. This is clear from Fig. 4 and can also be shown analytically as follows. Evaluating the limit of Eq. (A.11) when $\omega_0 \to 0$, we have:

$$\lim_{\omega_0 \to 0} \ln\left[\frac{1 + \Phi}{e^{2\omega_0}(1 - \Phi)}\right] = \lim_{\omega_0 \to 0} 2\omega_1\Phi = 2\omega_1\Phi$$

from which it follows that:

$$\ln\left(\frac{1 + \Phi}{1 - \Phi}\right) = 2\omega_1\Phi \tag{A.15}$$

It is clear that $\Phi = 0$ is a solution of the above expression. Furthermore, notice that $\ln\left(\frac{1+\Phi}{1-\Phi}\right)$ is concave for $\Phi < 0$ and convex for $\Phi > 0$. Hence, there are at most three values of Φ for which Eq. (A.15) is satisfied. Derivating both sides with respect to Φ, we obtain:

$$\frac{2}{1 - \Phi^2} = 2\omega_1$$

from which:

$$\Phi = \pm\sqrt{\frac{\omega_1 - 1}{\omega_1}}$$

such that when $\omega_1 \leq 1$, the only solution is $\Phi = 0$, while when $\omega_1 > 1$, two additional roots emerge, one positive and the other negative. Therefore, there exist critical values $\tilde{\omega}_1$ and $\tilde{\omega}_0$ for which $\omega_1 > \tilde{\omega}_1$ and $\omega_0 < \tilde{\omega}_0$ lead to the emergence of two additional equilibrium solutions of system (23). In one of them, $\bar{\Phi}_1 > 0$ and in the other $\bar{\Phi}_2 < 0$. The corresponding values of the wage share and employment rate of these extra points are given by:

$$\begin{aligned}
\overline{v}_i &= \frac{\alpha_0 + (\mu_1\mu_2 - \alpha_0)\left(\tau_0 + \tau_1\overline{\Phi}_i\right)}{\sigma_1\sigma_2 + \alpha_1\left[1 - \left(\tau_0 + \tau_1\overline{\Phi}_i\right)\right]} \\
\overline{V}_i &= \tfrac{\gamma_0 + \sigma_1\sigma_2\overline{v}_i}{\gamma_1}
\end{aligned}$$

where $i = 1, 2$.

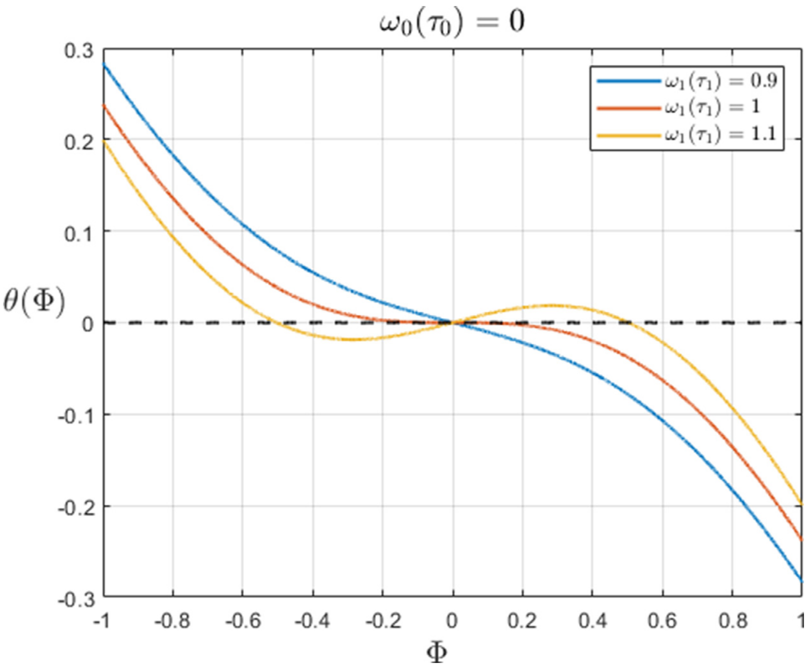

Fig. A1. Emergence of multiple equilibrium points when $\tau_0 = 0$.

A5. Proof of Propositions 5

The analytical proof of Proposition 5 is based on the continuity of the functional forms used. We will follow the steps of our previous demonstration, substituting $\tau_0 = 0$. It immediately follows that Eqs. (A.8)-(A.10) can be rewritten as:

$$\sigma_1\sigma_2\bar{v} = (1-\tau)(\alpha_0 - \alpha_1\bar{v}) + \mu_1\mu_2\tau_1\bar{\Phi} \tag{A.16}$$

$$\sigma_1\sigma_2\bar{v} = -\gamma_0 + \gamma_1\bar{V} \tag{A.17}$$

$$\bar{\Phi} = \tanh\left(\omega_1\bar{\Phi}\right) \tag{A.18}$$

Fig. A.11 plots $\theta = \tanh(\omega_0 + \omega_1\Phi) - \Phi$. The intersection of the colour lines with the black dotted one corresponds to the values of Φ for which Eq. (A.10) is satisfied, $\theta = 0$. Given that $\omega_0 = 0$, a unique value of $\bar{\Phi}$ satisfies (A.10) when $\omega_1 \leq 1$. This will happen as long as:

$$\beta \leq \frac{1}{\mu_1\mu_2\tau_1}$$

in which case $\bar{\Phi} = 0$. Substituting it into Eq. (A.16), and solving for income distribution, we have that:

$$\sigma_1\sigma_2\bar{v} = \alpha_0 - \alpha_1\bar{v}$$

$$\bar{v} = \frac{\alpha_0}{\sigma_1\sigma_2 + \alpha_1} \tag{A.19}$$

Finally, substituting Eq. (A.19) into (A.17), we obtain the equilibrium value of employment:

$$\bar{V} = \frac{\gamma_0 + \sigma_1\sigma_2\bar{v}}{\gamma_1}$$

Furthermore, when $\omega_1 > 1$ a Pitchfork bifurcation occurs, and two additional values of $\bar{\Phi}_i$ satisfy (A.10), where $i = 1, 2$, $\bar{\Phi}_1 > 0$ and $\bar{\Phi}_2 < 0$. Consequently, in equilibrium, the wage share and employment rate of these extra points are given by:

$$\bar{v}_i = \frac{\alpha_0 + (\mu_1\mu_2 - \alpha_0)\tau_1\overline{\Phi}_i}{\sigma_1\sigma_2 + \alpha_1\left(1 - \tau_1\overline{\Phi}_i\right)}$$

$$\overline{V}_i = \frac{\gamma_0 + \sigma_1\sigma_2\overline{v}_i}{\gamma_1}$$

A6. Proof of Propositions 6

Using the functional forms in (28), the various partial derivatives appearing in the Jacobian matrix become:

$$F_v = -\left(1 - \tau_0 - \tau_1 \overline{\Phi}\right)\alpha_1$$

$$F_\tau = -(\alpha_0 - \alpha_1 \overline{v})$$

$$G_V = \gamma_1$$

$$H_{u_N} = \sigma_1$$

$$J_{U_E} = \mu_1$$

$$u_{N_v} = \sigma_2$$

$$u_{E_\tau} = \mu_2$$

$$\tau_\Phi = \tau_1$$

$$\theta_\Phi = \left[1 - \tanh^2(\beta J(u_E(\tau(\Phi))))\right]\beta[\rho(F_\tau \tau_\Phi + J_{u_E} u_{E_\tau} \tau_\Phi) + J_{u_E} u_{E_\tau} \tau_\Phi] - 1$$

$$\theta_v = \left[1 - \tanh^2(\beta J(u_E(\tau(\Phi))))\right]\beta\rho(F_v - H_{u_N} u_{N_v})$$

Whenever ω_1 is below a critical value, i.e. $\omega_1 < \bar{\omega}_1$, or ω_0 is sufficiently high, $\omega_0 > \bar{\omega}_0$, Fig. 4 shows the system admits a unique equilibrium point such that in its neighbourhood $\theta_\Phi < 0$. Therefore, making use of Eqs. (A.4) to (A.7), we have:

$$b_1 = \sigma_1\sigma_2\overline{v} - \theta_\Phi > 0$$

$$b_2 = -\sigma_1\sigma_2\overline{v}\theta_\Phi + [(1-\tau)\alpha_1 + \sigma_1\sigma_2]\overline{V}\gamma_1\overline{v} > 0$$

whereas:

$$b_3 = (\alpha_0 - \alpha_1\overline{v} - \mu_1\mu_2)\tau_1\gamma_1\overline{v}\theta_v - [(1-\tau)\alpha_1 + \sigma_1\sigma_2]\overline{V}\gamma_1\overline{v}\theta_\Phi \gtreqless 0$$

such that

$$b_1 b_2 - b_3 = \sigma_1\sigma_2\overline{v}\left[(\theta_\Phi - \sigma_1\sigma_2\overline{v})\theta_\Phi + [(1-\tau)\alpha_1 + \sigma_1\sigma_2]\overline{V}\gamma_1\overline{v}\right] - (\alpha_0 - \alpha_1\overline{v} - \mu_1\mu_2)\tau_1\gamma_1\overline{v}\theta_v \gtreqless 0$$

The unique non-trivial solution is always locally stable as long as:

$$\mu_1\mu_2 < \alpha_0 - \alpha_1\bar{v} \tag{A.20}$$

and

$$[(1-\tau)\alpha_1 + \sigma_1\sigma_2]\bar{V}\gamma_1\theta_\Phi < (\alpha_0 - \alpha_1\bar{v} - \mu_1\mu_2)\tau_1\gamma_1\theta_v \tag{A.21}$$

This is because (A.20) guarantees $b_1 b_2 - b_3 > 0$ while (A.21) is necessary to have $b_3 > 0$.

However, when condition (A.20) does not hold, the solution is still locally stable provided that:

$$\sigma_1\sigma_2\left\{(\theta_\Phi - \sigma_1\sigma_2\bar{v})\theta_\Phi + [(1-\tau)\alpha_1 + \sigma_1\sigma_2]\overline{V}\gamma_1\overline{v}\right\} > (\alpha_0 - \alpha_1\bar{v} - \mu_1\mu_2)\tau_1\gamma_1\theta_v \tag{A.22}$$

If a change in one of the parameters determines the violation of this last condition, the characteristic equation may have a pair of purely imaginary complex conjugate eigenvalues and no other eigenvalues with zero real part. Hence, a Hopf bifurcation might occur, and the system admits a family of periodic solutions. The bifurcation condition can be written as

$$\sigma_1\sigma_2(\theta_\Phi - \sigma_1\sigma_2\bar{v})\theta_\Phi + \sigma_1\sigma_2\left[\left(1 - \tau_0 - \tau_1\bar{\Phi}\right)\alpha_1 + \sigma_1\sigma_2\right]\gamma_1\bar{V}\bar{v}(-\alpha_0 + \alpha_1\bar{v} + \mu_1\mu_2)\tau_1\gamma_1\bar{V}\theta_v = 0$$

Moreover, in the case ω_1 is above a critical value, i.e. $\omega_1 > \bar{\omega}_1$, and ω_0 is neglectable, $\omega_0 < \bar{\omega}_0$, the two additional equilibria are such that $\theta_\Phi < 0$ in their neighbourhood, as we can see in Fig. 4. Making use of Eqs. (A.4) to (A.7), we have again that iff:

$$\mu_1\mu_2 < \alpha_0 - \alpha_1\bar{v}_i \tag{A.23}$$

then $b_1 b_2 - b_3 > 0$. As long as:

$$[(1-\tau)\alpha_1 + \sigma_1\sigma_2]\bar{V}_i\gamma_1\theta_\Phi < (\alpha_0 - \alpha_1\bar{v}_i - \mu_1\mu_2)\tau_1\gamma_1\theta_v$$

then, $b_3 > 0$. A violation of condition (A.23) means the respective solution points will be locally stable provided that:

$$\sigma_1\sigma_2\left\{(\theta_\Phi - \sigma_1\sigma_2\bar{v}_i)\theta_\Phi + [(1-\tau)\alpha_1 + \sigma_1\sigma_2]\bar{V}_i\gamma_1\bar{v}_i\right\} > (\alpha_0 - \alpha_1\bar{v}_i - \mu_1\mu_2)\tau_1\gamma_1\theta_v \tag{A.24}$$

where $i = 1, 2$. A Hopf bifurcation might occur for the combination of parameters for which equality substitutes the inequality in Eq. (A.24).

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