Q-Bounded Maximum Directivity of Self-Resonant Antennas

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***Abstract*— The upper limit on the directivity of self-resonant antennas that fit within a minimum sphere is determined for a given quality factor. This formulation is obtained by analytically solving a rigorous convex problem and is expressed as a rapidly converging analytical series. The total quality factor, the inverse of the relative frequency bandwidth, is formulated by considering the quality factors of individual spherical waves. From the exact series, approximate closed-form formulas have been derived, which exhibit high accuracy in complementary ranges of the minimum circumscribed sphere's radius. These ranges encompass small antennas as well as intermediate to large antennas. Special emphasis is given to small antennas, where the solution is interpreted as a combination of dipolar and quadrupolar Huygens' source contributions with appropriate closed-form coefficients. The solution in this range provides continuity to the maximum directivity between 3 and 8 maintaining a constant Q.**

***Index Terms*— Antennas, bandwidths, fundamental limits, quality factor, spherical harmonics, super directivity.**

I. Introduction

GIVEN the recent requirements for highly directive electrically small radiators and scatterers in various communication and sensor applications, there has been a renewed interest in the concept of super directivity. Super directivity refers to the ability to achieve exceptionally high levels of directivity, beyond what is traditionally achievable with conventional antenna designs. This renewed interest is driven by the demand for improved performance in terms of range, resolution, and sensitivity in communication and sensing systems.

The definition of the maximum directivity for a given electrical size of the minimum sphere circumscribing the antenna requires a constraint to ensure a finite bound, as without it the directivity could be in principle infinite. This constraint can take the form of a lower limit on the efficiency or an upper limit on the Q-factor (i.e., minimum relative frequency bandwidth), among other possibilities. Alternatively, it is also possible to restrict the number of harmonics that can be excited over the minimum sphere based on the degrees of freedom (DoFs) of the field. In this article, our focus is on Q-bounded maximum directivity.

The problem of maximum bound of directivity and super directivity has been studied by many authoritative scientists [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]. In essence, the problem is convex, and therefore, our formulation can be aligned within the framework established in [1]. While the formulation in [1] is applicable to general shapes, the one presented here is focused on spherical source regions, with the advantage of resulting in a concise analytical formula. This formula is eventually approximated by a simple and appealing closed-form expression, which offers valuable physical insight, particularly for small antennas. The issue of losses arising from realistic conductivity is not treated here and will be addressed in [20].

Assuming sources fitting inside a minimum sphere of radius $r_{\min}$, the maximum directivity can be found as suggested by Harrington [3], [4], [5]. His method is based on the expansion of the radiated field in a finite number of spherical waves (SWs), and on the maximization of the directivity with respect to the coefficients of the expansion. This procedure leads to

$$D_{\max} = \sum_{n=1}^{N_{\max}} (2n+1) = (N_{\max})^2 + 2N_{\max} \tag{1}$$

where $N_{\max} \geq 1$ is the maximum polar index of the SWs that contribute to the far field for the given minimum sphere. Hence, the maximum directivity depends on the value set for $N_{\max}$. It was suggested by Harrington that the maximum polar index is the largest integer smaller than $kr_{\min}$, where k is the free-space wavenumber, i.e., $N_{\max} = \lfloor kr_{\min} \rfloor$ (where $\lfloor . \rfloor$ is the entire part of the argument). This assumption invokes the difficulty to excite SWs with polar index $n > kr_{\min}$ with a sufficiently high intensity over the minimum sphere to significantly contribute to the far field; namely, as underlined in [6], it relies on the finiteness of the number of DoFs of the field in the far zone. To put it differently, Harrington's procedure is grounded in the understanding that the SWs are below cutoff as long as the order of the spherical Hankel function is larger than its argument. This is actually the same concept invoked to establish the number of DoF of the field radiated by sources inside a minimum sphere. We note that

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according to (1), when $N_{\max} = 1$, the maximum directivity is equal to 3, and it is associated with Huygens' source.

Since the formula in (1) is discontinuous, Harrington suggested to give it continuity just posing $N_{\max} = kr_{\min}$, thus leading to

$$D_{\max} = (kr_{\min})^2 + 2kr_{\min}. \tag{2}$$

This extension, while in asymptotic agreement with the DoF concept of truncation, is empirical and manifests its inaccuracy, especially in the range $kr_{\min}$ between 1 and 2. In this range, the truncation of the series in (1) leads to only two potential outcomes for maximum directivity, namely, 3 (for $N_{\max} = 1$) and 8 (for $N_{\max} = 2$). The interpolation in (2) represents just one possible approach to transition from one to the other with continuity. Other possible formulas have been proposed for small antenna sizes. It was suggested in [6] that the maximum directivity for nonsuper reactive antennas can be heuristically defined as $D_{\max} = (kr_{\min})^2 + 2kr_{\min}$ for $kr_{\min} > 1.5$ and $(kr_{\min} + 3)$ for $kr_{\min} < 1.5$; this heuristic extension was compared with some available literature data for very small antennas.

It is well-known that there exist antennas, sometimes called "superdirective," with directivity larger than the limit in (2) even if with a small bandwidth. The possibility to exceed the limit in (2) derives from the fact that its derivation does not consider the possibility to excite, with sufficiently large intensity over the minimum sphere, SWs with polar index larger than $kr_{\min}$. As a matter of fact, increasing the number of super reactive harmonics over the minimum sphere leads to a diverging Q-factor, which is eventually useless for practical antenna applications, since the bandwidth goes to zero. Vice versa, allowing for a certain desired maximum Q may imply a bound of directivity larger than the one derived in (2). The main objective of this article is, therefore, to obtain the maximum directivity for a given bandwidth and minimum sphere, and a closed-form approximation for it.

Yaghjian [18] presented simplified formulas for sampling the minimum sphere. He distinguished nonresonant antennas from resonant antennas and provided a link between the equivalent radius associated with the storage of reactive energy and the maximum equivalent area of the antenna, therefore establishing a link with the maximum directivity. Fante [17] proposed a maximization of the product directivity–bandwidth (note that although the title in [17] mentions "gain," the treatment there is relevant to directivity). The coefficients of the SWs found by Fante [17] are different from the ones found by Harrington, and the series is not truncated like in (1). However, the D/Q bound obtained this way is always relevant to small Q (large bandwidth) and moderate D, which is of practical interest only for ultrawideband antennas. In [13], the minimum Q-factor for a given directivity of a small antenna of arbitrary shape is obtained by setting a convex problem and solving it by a semidefinite relaxation technique. In this article, instead, Q is fixed a priori, and the directivity is maximized.

This article is structured as follows. Section II derives the limit for Q-bounded maximum directivity. Section III focuses on small resonant antennas, where the maximum directivity is obtained by the combination of dipolar and quadrupolar resonant sources. The Chu limit of bandwidth is extended to the presence of quadrupoles and a closed-form formula for small antennas is provided, which allows to have a continuous description of maximum directivity from 3 to 8 for constant Q and for any antenna size. Section IV presents a closed-form approximation of the maximum directivity for intermediate to moderately large antennas. Section V shows the maximum directivity in terms of an equivalent radius that contains all the reactive energy. Finally, Section VI gathers the conclusion.

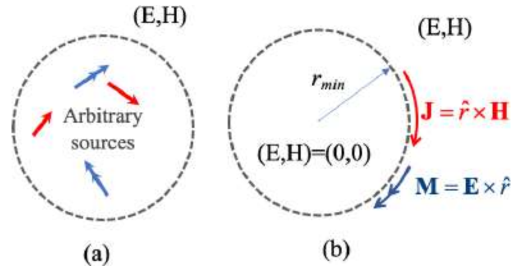

Fig. 1. Application of the equivalence principle to the minimum sphere surface: (a) original problem and (b) equivalent electric and magnetic surface currents radiating in free space with zero field inside (Love formulation).

II. Q-Bounded Maximum Directivity

Let us first refer to the problem in Fig. 1, where the equivalence theorem is applied to the minimum spherical surface including all the sources. The Love formulation of the equivalence principle relates the equivalent electric (**J**) and magnetic (**M**) currents to arbitrary magnetic (**H**) and electric (**E**) Maxwellian's fields, though $\mathbf{J} = \hat{r} \times \mathbf{H}$, $\mathbf{M} = \mathbf{E} \times \hat{r}$, where $\hat{r}$ is the normal to the surface. In Love's formulation, the field inside the minimum sphere is assumed to be zero, and thus, it is not equivalent in terms of stored energy to the initial problem. However, since the energy of the equivalent problem is zero inside the sphere, any other source generating the same external fields will lead to a higher Q. Therefore, the bounds we will find here are more optimistic than the one obtained by electric currents only over a sphere [2], in which the energy can be stored even inside the sphere.

In the rest of this article, the currents will be expanded in terms of SW harmonics, with coefficients C_i, adopting the normalization of Hansen's [21] book, for which the radiated power is given by $P_r = (1/2)\sum_i |C_i|^2$.

A. Total Q-Factor

The Q-factor can be defined in two different ways depending on whether one assumes to have a self-resonant antenna or an antenna that is made resonant by providing an external reactive energy from a lossless tuning circuit. In the first case, one has $W_e = W_m$, $Q = 2\omega W_e/P_r = 2\omega W_m/P_r$, where ω is the angular frequency, P_r is the radiated power, and W_e and W_m are the electric and magnetic stored energies, respectively. For nonresonant antennas, one has $Q = 2\omega W_e/P_r$ for capacitive antennas and $Q = 2\omega W_m/P_r$ for inductive antennas. This definition assumes that an external energy has been added to the system to get the resonance. In both cases, the Q-factor can be interpreted as the reciprocal of the fractional bandwidth $BW = 1/Q$ when it is larger than 10 [15].

The calculation of the stored energies $W_{e,m}$ of a general SW expansion is an old and debated topic [3], [16], [22],

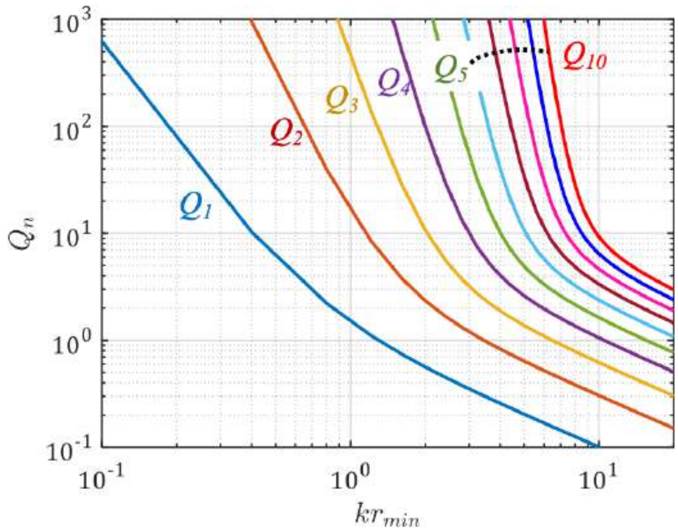

Fig. 2. Fante's Q_n coefficients. The used log–log scale emphasizes the different behavior of Q_n with corner point at $kr_{\min} \approx n$.

[23], [24], [25]. Essentially, the most used approaches are the ones provided by Chu [26], Collin and Rothschild [15], and Fante [22], and the latter generalized to the case of arbitrary field internal to the minimum sphere in [23]. It turns out [17] that the condition of maximum directivity implies that transverse electric (TE) and transverse magnetic (TM) spherical modes with the same indices have coefficients of equal amplitude; under this condition, the Q of the antenna reduces to

$$Q = \frac{\sum_n |C_n|^2 Q_n}{\sum_n |C_n|^2} \tag{3}$$

where Q_n are defined in Appendix A and C_n are the field expansion coefficients for the maximum directivity. The adoption of (3) inherently assumes self-balancing of reactive energy. Consequently, the limit we obtain in the subsequent analysis is relevant to self-resonant antennas, namely, antennas characterized by achieving resonance without requiring the provision of energy through an external circuit.

For the sake of convenience, the Fante's Q_n are plotted in Fig. 2 as a function of $kr_{\min}$ for some values of n. The asymptotic behavior for small and large values of $kr_{\min}$ is given by [23]

$$Q_n \sim \begin{cases} \dfrac{n}{2}\left(\dfrac{(2n)!}{n!2^n}\right)^2 \dfrac{1}{(kr_{\min})^{2n+1}}, & \text{for } kr_{\min} \to 0 \\ a_n (kr_{\min})^{-1}, & \text{for } kr_{\min} \to \infty \end{cases} \tag{4}$$

with $a_n = (a_{n-1} + n)$ and $a_0 = 1$ by definition. It is seen that Q_n exhibits a drastic change of slope for $kr_{\min}$ around n. It is important to note that Fante's Q_n for $n = 1$ gives exactly $Q_1 \equiv (1/2)(kr_{min})^{-3} + (kr_{min})^{-1}$, which will be shown later to be related to the Chu limit of bandwidth.

B. Analytical Form for Q-Bounded Maximum Directivity

The maximum directivity for a given quality factor Q is formulated in terms of the minimization of a convex function. The optimization problem consists on finding the SW coefficients that maximize U/P_r where $P_r = (1/2)\sum_i |C_i|^2$ and $U = (1/4\pi)|\sum_i C_i K_i|^2$ with

$$K_i = \sqrt{2n+1} \cdot \begin{cases} 0, & \text{if } |m| \neq 1 \\ -(-j)^n, & \text{TE}, \ m = \pm 1 \\ -m(-j)^n, & \text{TM}, \ m = \pm 1 \end{cases} \tag{5}$$

where the "polar" index n refers to the order of the Hankel function and the index m refers to the azimuthal angular wavenumber of the SW expansion. The index $i = 2[n(n+1) + m - 1] + s$ collects the three indexes n, m, s, where s assumes values 1 and 2 for TE and TM polarizations, respectively. We note that (5) is congruent with the normalization of Hansen's [21] book, for which the power radiated by an individual harmonic is $(1/2)|C_i|^2$. The maximization of U/P_r bounded by a certain value of Q is given by

$$\max_{C_i} \frac{|\sum_i C_i K_i|^2}{\sum_i |C_i|^2}; \quad \frac{\sum_n Q_n |C_n|^2}{\sum_n |C_n|^2} \le Q \tag{6}$$

where Q_n are Fante's quality factors (Appendix A), and Q is the maximum accepted Q-factor (minimum relative bandwidth). It is important to observe that the inequality in (6) is actually equivalent to an equality since it comes out that any increase of the amplitude of the higher order coefficients that provides larger directivity also implies an increase of Q. Therefore, finding the maximum directivity for a given maximum Q means in practice finding it for a constant Q.

Since the constraint depends only on the amplitude of the coefficients, the maximum in (6) is achieved by $\angle C_i = -\angle K_i$ and $C_i K_i = |C_i||K_i| = |C_n|\sqrt{2n+1}$. From now on, we can, therefore, use the polar index n only since the amplitude of K_i does not depend on the indexes m and on the polarization. Setting the radiated power so that $\sum_n |C_n|^2 = 1$, the problem in (6) is equivalent to

$$\max_{|C_n|} \left(\sum_n |C_n|\sqrt{2n+1}\right)^2 \text{ with } \sum_n |C_n|^2 = 1$$
$$\sum_n Q_n |C_n|^2 = Q. \tag{7}$$

We can now reformulate (7) by using its dual problem obtained by combining the constraints with a scalar parameter ξ; i.e.,

$$\min_{\xi} \max_{C_n} \left(\sum_n |C_n|\sqrt{2n+1}\right)^2$$
$$\text{s.t.} \sum_n [\xi(Q_n - Q) + 1]|C_n|^2 = 1. \tag{8}$$

The solution of this problem for $Q > Q_1$ is given in Appendix B by the Lagrange multiplier method. The result is

$$D_{\max}(Q, kr_{\min}) = \min_{\xi \in [0, \xi_{\max}]} \sum_{n=1}^{\infty} \frac{2n+1}{\xi(Q_n - Q) + 1} \tag{9}$$

where $\xi_{\max} = 1/(Q - Q_1)$. Equation (9) explicitly relates the maximum directivity to the bandwidth for any given antenna size. The value $\bar{\xi} = \bar{\xi}(Q, kr_{\min})$ that minimizes the series in (9) is represented in Fig. 3 as a function of the $kr_{\min}$ for

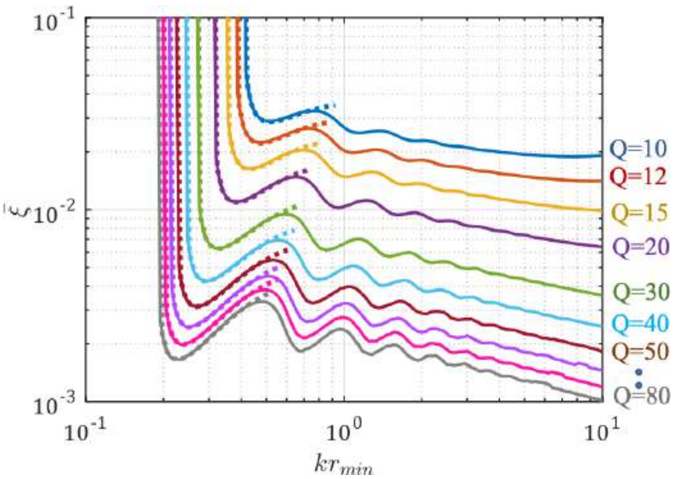

Fig. 3. Parameter $\bar{\xi}$ that minimizes the summation in (9) (continuous line) and its approximation in (10) (dashed line) truncated at $Q = Q_2$, for various values of Q ($Q = 10, 12, 15, 20, 30, 40, 50, 60, 70, 80$).

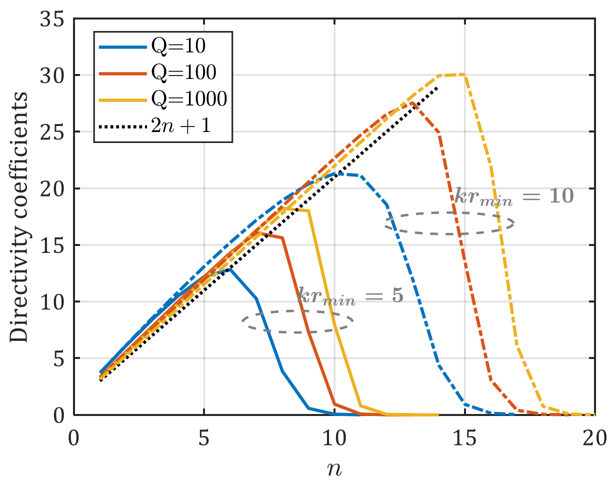

Fig. 4. Envelope of the maximum directivity summation terms in (9) for three values of Q, namely, $Q = 10$, $Q = 100$, and $Q = 1000$, for two values of $kr_{\min}$. The black dotted line represents the envelope of the corresponding Harrington coefficients for maximum directivity.

some fixed values of Q. An excellent approximation of $\bar{\xi}$, valid for $Q_1 \leq Q \leq Q_2$, is given by

$$\bar{\xi} = \frac{8}{3(Q_2 - Q)}\left(-1 + \sqrt{1 - \frac{60(Q_2 - Q)}{256(Q_1 - Q)}}\right). \qquad (10)$$

We note that $\bar{\xi}$ tends to diverge when $Q = Q_1$. It can be easily seen, however, that $\bar{\xi}(Q_1 - Q)$ in the denominator of (9) goes to zero, while all the other terms of the series are negligible, due to the high values of $\bar{\xi}(Q_n - Q)$ for $n \neq 1$. Therefore, $D_{\max}$ tends to 3 for any Q that tends to Q_1. This is expected since, in the quasi-static limit, a sphere can contain only one self-resonant source, which is Huygen's dipole.

The envelope of the terms inside the summation in (9) is plotted in Fig. 4. For low values of Q, the coefficients envelope reaches a maximum close to $n = kr_{\min}$, while for large Q, the maximum moves toward higher values, being defined by $Q = Q_n$. Let us define the quantity kr_n, as follows:

$$kr_n \text{ such as } Q = Q_n. \qquad (11)$$

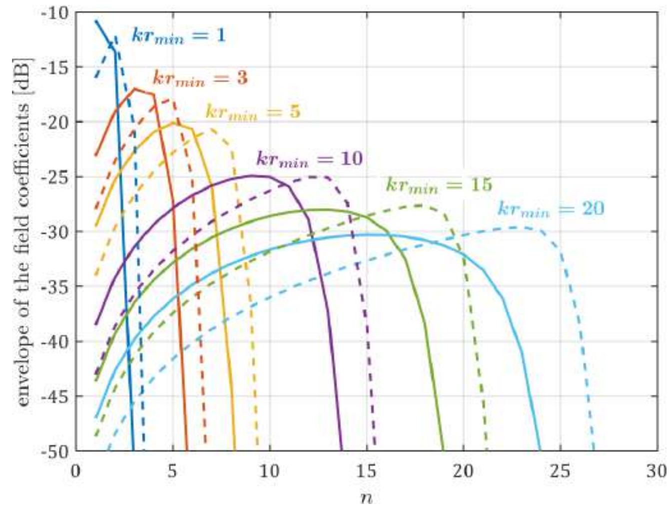

Fig. 5. Envelope of the amplitude of the coefficients in (13) for various values of $kr_{\min}$ and two values of Q, namely, $Q = 10$ (continuous line) and $Q = 100$ (dashed line). The arbitrary constant C_0 has been set to unity.

For $n > r_n$, the coefficients exhibit a fast decay, with a rate that depends on $kr_{\min}$ and Q. This behavior is due to the change of the decay rate of Q_n when crossing kr_n (see Fig. 2). An approximation of kr_n for values of Q larger than 100, $kr_{\min} < 2$ and $n < 12$, is

$$kr_n \approx \frac{n}{1.356Q^{1/(2n+1)}}. \qquad (12)$$

The above formula is useful for the approximation that will be presented in Section IV.

C. SW Coefficients for Maximum Directivity

The maximum directivity for constant Q is obtained with field coefficients

$$C_{i,\max}^{(Q)} = \frac{C_0\sqrt{2n+1}}{\bar{\xi}(Q_n - Q) + 1} \cdot \begin{cases} 0, & \text{if } |m| \neq 1 \\ -(j)^n, & \text{TE, } m = \pm 1 \\ -m(j)^n, & \text{TM, } m = \pm 1 \end{cases} \qquad (13)$$

where C_0 is an arbitrary constant. Note that for $\bar{\xi}(Q_n - Q) \ll 1$, the coefficients $C_{i,\max}^{(Q)}$ recover the coefficients obtained by Harrington for the case of the finite number of SWs without imposing the Q-bound, and just invoking the truncation of the series. It should be stressed that the set of SW coefficients in (13) implies that electric and magnetic energies are equal to each other for $r > r_{\min}$ for any polar index; namely, the maximization is relevant to self-resonant antennas. The envelope of the amplitude of the coefficients in (13) in dB scale is given in Fig. 5 for two values of Q ($Q = 10$ and $Q = 100$) and various values of $kr_{\min}$; the arbitrary constant C_0 is set to unity in the calculations.

D. D_{max} Versus kr_{min} for Constant Q

Fig. 6 shows the maximum directivity $D_{\max}$ in a range of $kr_{\min} \in (0.1; 10)$ for various values of Q. The directivity value is truncated at 3, which corresponds to the condition $Q = Q_1$. This situation is associated with the Chu limit, as will be discussed next. For the sake of comparison, the Harrington formula in (2) is also included, represented by the

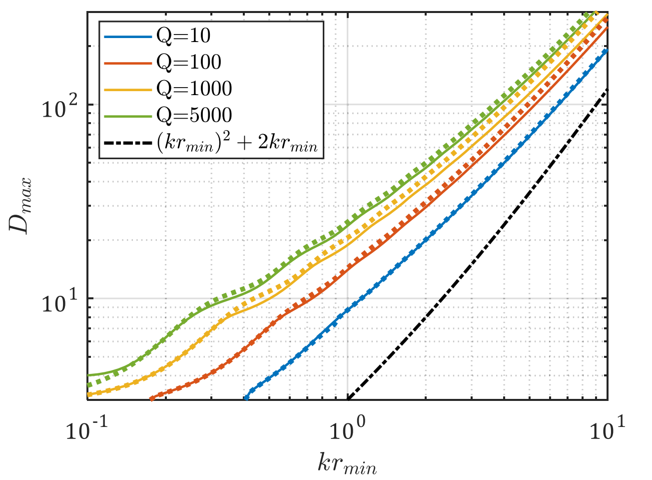

Fig. 6. Maximum directivity (log–log scale) for constant Q. The curves are truncated at the corresponding Chu limit radius, where $D_{\max}(Q, kr_{\min}) \approx 3$. Comparisons of the exact formula (9) (continuous line) and the combination between (23) and (28) (dotted lines) for $Q = 10$, $Q = 100, 1000, 5000$.

black dashed-dotted line. However, we should consider that the latter does not provide the same Q for any value of the minimum radius. The dotted lines correspond to a closed-form approximation provided in Sections III-D and IV in their respective fields of validity.

III. Small Antennas: Dipolar and Quadrupolar Resonant Sources

From the previous analysis, it can be seen that the nth spherical harmonics, either TE and TM with azimuthal index $m = \pm 1$, have the same Q_n and the same coefficients' amplitude for providing maximum directivity. This is true for both Harrington's coefficients [conjugate of (5)] and the Q-bounded ones derived in (13). For $n = 1$, the resulting radiated field can be interpreted as the one produced outside the minimum sphere by an elementary Huygens' dipole (HD) located at the origin [Fig. 7(a)]. The latter is constituted by a couple of horizontal electric and magnetic dipoles with momenta related by the free-space impedance ζ (i.e., $I\Delta\ell = M\Delta\ell/\zeta$). By duality, the energy density of the HD is balanced outside the minimum sphere aligning with the self-resonant formulation we have adopted. Huygens' source antennas have been successfully implemented in practical applications, ranging from electrically small packages [27], [28], [29] to larger ones [30], [31]. Their significance as a research area has been acknowledged, particularly for Internet-of-Things (IoT) applications [32]. Furthermore, Huygens' metasurfaces have already demonstrated their effectiveness in antenna and scattering problems [32], [33], [34], [35], [36].

The combination of the SW harmonics for $n = 2$ provides the field of a Huygens' quadrupole (HQ) combined with the one of a dual vertical quadrupole (DVQ). Graphical representations of the HQ and DVQ are given in Fig. 7(b) and (c), respectively. The HQ is composed by pairs of counterdirected HDs separated by a vanishingly small electrical distance. The HQs have been recently studied in the problem on needle radiation from an array [37]. The DVQ is a combination of close by counterdirected vertical magnetic and electric dipoles, where the displacement of the electric dipoles is along x

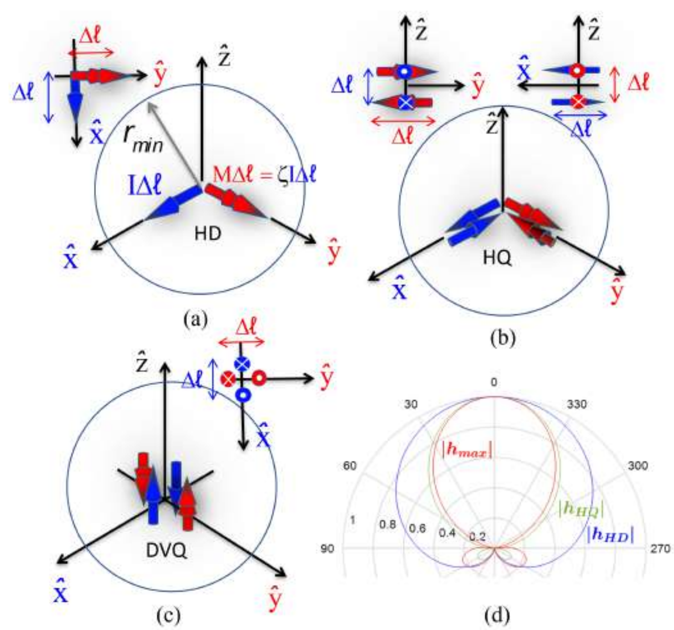

Fig. 7. Graphical representation of a: (a) HD; (b) HQ; and (c) DVQ. (d) Normalized radiation patterns of HD, HQ, and the combination of HD, HQ, and DVQ for maximum directivity.

and the one of the magnetic dipoles is along y. Like the HD, the HQ and DVQ provide balanced energy outside the minimum sphere, due to duality. The normalized far-field radiation pattern expressions of HD, HQ, and DVQ are given by

$$\mathbf{h}_{\mathrm{HD}} = \frac{1}{2}(\cos\theta + 1)\hat{\mathbf{p}} \tag{14}$$

$$\mathbf{h}_{\mathrm{HQ}} = \frac{1}{2}(\cos\theta + 1)\cos\theta\hat{\mathbf{p}} \tag{15}$$

$$\mathbf{h}_{\mathrm{DVQ}} = \sin^2\theta\hat{\mathbf{p}} \tag{16}$$

$$\hat{\mathbf{p}} = \left[\cos\phi\hat{\theta} - \sin\phi\hat{\phi}\right] \tag{17}$$

where $\hat{\mathbf{p}}$ is the unit polarization vector. The individual directivity of isolated $\mathbf{h}_{\mathrm{HD}}$ and $\mathbf{h}_{\mathrm{HQ}}$ are 3 and 7.5, respectively. We note that the condition of maximum directivity for $n = 2$ links the coefficients of HQ and DVQ each other combining them as follows:

$$\mathbf{h}'_{\mathrm{HQ}} = \mathbf{h}_{\mathrm{HQ}} - \frac{1}{2}\mathbf{h}_{\mathrm{DVQ}} = \frac{1}{2}(\cos\theta + \cos 2\theta)\hat{\mathbf{p}}. \tag{18}$$

The normalized far-field radiation patterns of $\mathbf{h}_{\mathrm{HD}}$, $\mathbf{h}_{\mathrm{HQ}}$, and $\mathbf{h}_{\mathrm{HQ}}$' are shown in Fig. 7(d). We note that the exact form of Q_1 and Q_2 are given by

$$Q_1 \equiv \frac{1}{2(kr_{\min})^3} + \frac{1}{(kr_{\min})} \tag{19}$$

$$Q_2 \equiv \frac{9}{(kr_{\min})^5} + \frac{9}{2(kr_{\min})^3} + \frac{3}{kr_{\min}}. \tag{20}$$

These two expressions can be interpreted as the quality factors of isolated HD and HQ, respectively.

A. Minimum Q for Isolated HD (Chu Limit)

Before proceeding further, we should note that the condition $Q \geq Q_1$ provides a lower bound for the Q-factor (upper bound for the maximum bandwidth) for small antennas; this can be identified as the well-known Chu limit. However, we note that Chu [26] defined the limit for omnidirectional antennas by considering only the TM modes, thus providing $Q \geq Q_{Chu}^{(TM)} = 1/(kr_{\min})^3 + 1/(kr_{\min})$. McLean [38] emphasizes that the limit changes when considering both TE and TM modes, with special emphasis on circular polarization. The latter corresponds to two vertical electric and magnetic dipoles with balanced energy. In this case, the dominant quasi-static term is weighted by a factor 2 at the denominator, which identifies $Q \geq Q_1$ as the Chu limit. Indeed, Chu [26] and McLean [38] did not consider the case of maximum directivity but the case of antenna isotropic in the azimuthal plane. In the case of maximum directivity and self-resonant antennas, the present formulation naturally recovers an HD instead of two vertical dipoles since the HD gives the maximum directivity. With the above clarification, we will refer anyway to the condition $Q \geq Q_1$ as the Chu limit.

B. Minimum Q for Isolated HQ

It is seen from (20) that the generalization of the Chu limit to isolated HQ leads to $Q \geq Q_2$. This is much more restrictive in terms of bandwidth than $Q \geq Q_1$; that is, it provides a relative bandwidth $0.11(kr_{\min})^5$ in the dominant low-frequency term; however, HQ gives a higher directivity with respect to HD (7.5 versus 3). However, the maximum possible directivity is obtained by a proper combination of dipolar and quadrupolar contributions, as shown in the following.

C. Combination of Dipolar and Quadrupolar Contributions for Maximum Directivity

Let us combine the dipolar and quadrupolar contributions $\mathbf{h}_{\text{HD}}$ and $\mathbf{h}'_{\text{HQ}}$ with arbitrary coefficients, namely, $\mathbf{h} = \mathbf{h}_{\text{HD}} + \gamma \mathbf{h}'_{\text{HQ}}$, where γ is a real number. The directivity can be calculated as $\text{D} = 2/\int_0^{\pi} |\mathbf{h}|^2/|\mathbf{h}|^2_{\max} \sin\theta d\theta$, leading to

$$\text{D} = 3\frac{(1+\gamma)^2}{1+\frac{3}{5}\gamma^2}. \tag{21}$$

This directivity exhibits a maximum at $D_{\max} = 8$ for $\gamma = 5/3$. This result is in agreement with the results obtained by Harrington's coefficients and leads to a total Q equal to

$$Q^{(2)} = \frac{3Q_1 + 5Q_2}{8} = \frac{45}{8(kr_{min})^5} + \frac{3}{(kr_{min})^3} + \frac{2}{(kr_{min})}. \tag{22}$$

The latter is the quality factor that can be obtained with dipolar and quadrupolar contributions combined for maximum directivity. This result implies a relative bandwidth of $0.17(kr_{\min})^5$ at low frequency, which is larger than that of the HQ alone. In Fig. 8, the points described by $Q = Q_1$, $D_{\max} = 3$, and $Q = Q^{(2)}$, $D_{\max} = 8$ are set in a diagram $D_{\max}$ versus Q for several values of kr_{min}. The dots are connected with straight dotted lines. For comparison, our solution in (9) is plotted in Fig. 8. The latter covers with continuity the range from max directivity 3–8 with minimum Q (continuous line).

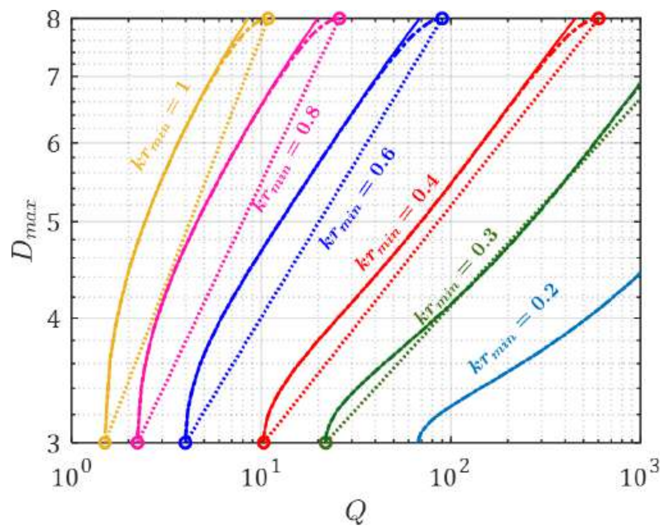

Fig. 8. Maximum directivity as a function of the Q. Dots (connected by dotted lines) are obtained by $Q = Q^{(2)}(D_{\max} = 8)$ and by $Q = Q_1(D_{\max} = 3)$. Continuous lines are obtained through (9). Dash-dotted lines are obtained by using (23), namely, setting Q constant for dipolar and quadrupolar contributions. The solution minimally deviates around $Q = Q^{(2)}$ with respect to (9) since the latter includes the contribution of order 3 (hexapoles).

D. Combination of Dipolar and Quadrupolar Contributions With Q Bound

Imposing a bound to Q and truncating the series in (9) at the first two terms lead to the combination $\mathbf{h} = \mathbf{h}_{\text{HD}} + \gamma \mathbf{h}'_{\text{HQ}}$, resulting in $Q = (Q_1 + (3/5)\gamma^2 Q_2)/(1 + (3/5)\gamma^2)$; deriving γ from the latter leads to $\gamma = \sqrt{5(Q - Q_1)/(3(Q_2 - Q))}$, which substituted in (21) yields

$$\text{D}_{max} = 3\frac{\left(\sqrt{Q_2 - Q} + \sqrt{\frac{5}{3}(Q - Q_1)}\right)^2}{Q_2 - Q_1}; \quad Q_1 \leq Q \leq Q^{(2)}. \tag{23}$$

Note that, even if the above equation exists for $Q_1 \leq Q \leq Q_2$, the result is representative of the maximum achievable directivity for Q bound only in the range $Q_1 \leq Q \leq Q^{(2)}$. In $Q^{(2)}$, the directivity assumes the maximum value equal to 8, touching the Harrington point and then decreases (see Fig. 8). Note that the same result can be obtained by truncating the series in (9), setting to zero the derivative, and solving the resulting second-order equation with the proper branch in ξ. Therefore, it results that the combination of dipolar and quadrupolar contributions that leads to the minimum Q and maximum directivity for any antenna size is

$$\mathbf{h} \approx \mathbf{h}_{HD} + \sqrt{\frac{5(Q - Q_1)}{3(Q_2 - Q)}}\mathbf{h}'_{HQ}, \quad Q_1 \leq Q \leq Q^{(2)}. \tag{24}$$

The accuracy of (23) when compared with the exact solution is good in the range of maximum directivity $3 \leq D_{\max} \leq 8$ (dashed-dotted line in Fig. 8, in comparison with the exact solution). We stress that this range of directivity corresponds to $Q_1 \leq Q \leq Q_2$, or in terms of antenna size

$$kr_1 \leq kr_{\min} \leq kr^{(2)} \approx 0.9kr_2 \tag{25}$$

where kr_1, kr_2, and $kr^{(2)}$ are the values for which $Q = Q_1$, $Q = Q_2$, and $Q = Q^{(2)}$, respectively. The values of

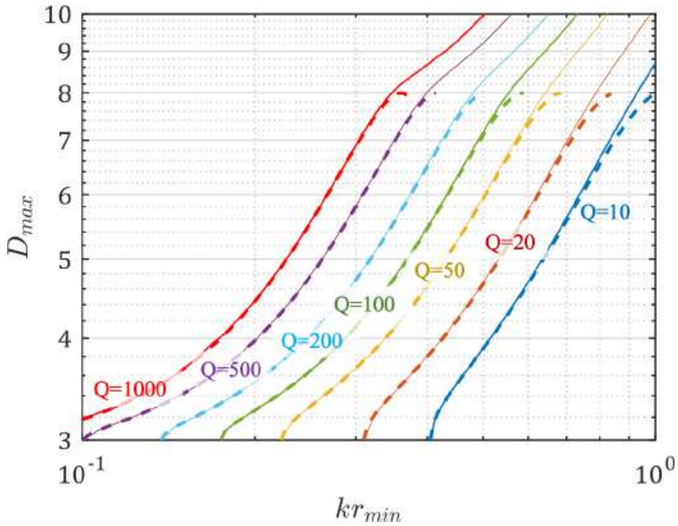

Fig. 9. Bound of frequency bandwidth constrained maximum directivity (log–log scale) for different values of Q, truncated at the corresponding Chu limit radius, where $D_{\max}(Q, kr_{\min}) = 3$. Comparison between the exact formula (9) (continuous line) and the approximate formula for small antennas in (23) (dashed lines). The dashed curves are plotted for $kr_1 \leq kr_{\min} \leq 0.8kr_2$.

$kr_1, kr^{(2)}$, and kr_2 a function of Q are obtained by inverting (19), (20), and (22) and can be approximated as

$$kr_1 \approx 0.4\left(\frac{1}{Q} + \frac{2}{Q^{1/3}}\right) \tag{26}$$

$$kr^{(2)} \approx 1.42\left(\frac{1}{Q} + \frac{1}{Q^{1/5}}\right) = 0.9kr_2. \tag{27}$$

Comparison of the exact form (9) (continuous line) with the two terms approximation in (23) (dashed lines) is shown in Fig. 9 for several values of Q. The percentage relative error of (23) with respect to the full expansion in the range $kr_1 \leq kr_{\min} \leq 0.8kr_2$ is less than 1% for $100 \leq Q \leq 1000$ and less than 4% for $10 \leq Q \leq 100$, with maximum error always obtained close to $0.8kr_2$.

E. Multipole Contributions Without and With Q Bounds

For completeness, Fig. 10 shows D_{max} as a function of Q for various values of $kr_{\min}$ associated with a finite number $N_{\max}$ of multipole orders. To this end, the expression $D_{\max} = (N_{\max})^2 + 2N_{\max}$ is evaluated as a function of $Q = \sum_{n=1}^{N_{\max}} Q_n(2n+1)/(N_{\max}^2 + 2N_{\max})$. The latter expression corresponds to the finite number of harmonics weighted by the Harrington coefficients [the latter given by the conjugate of (5)]. The Fante's Q_n have been used in the calculations. These plots can be obtained only by discrete points since $N_{\max}$ can assume only integer values. These points are connected by straight lines in Fig. 10. The continuous curve is obtained by the exact formula (9). It is seen that the discrete points are reasonably close to the continuous curve obtained by (9), and always below the continuous curve.

IV. Closed-Form Formulas for $0.6kr_2 < kr_{\min} < 10$

It is seen from Fig. 3 that the value of $\bar{\xi}$ is smoothly varying for $kr_{\min} \leq kr_2$. Just adopting the value $\bar{\xi}$ possesses in $0.6kr_2$ and maintaining it all-over the range $0.6kr_2 < kr_{\min} < 10$ provides a quite accurate solution for the directivity. This

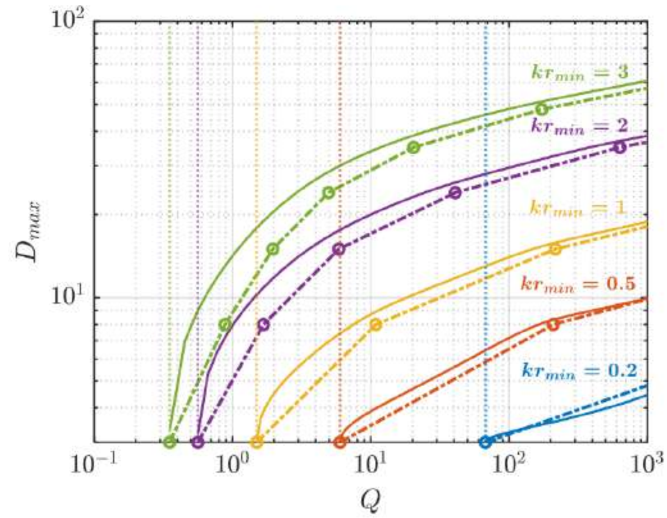

Fig. 10. Maximum directivity as a function of Q from the Harrington's coefficients (dots, connected by dashed lines) derived from (9) (dash dotted lines) for different values of $kr_{\min}$ from 0.2 to 3. The vertical dotted lines correspond to the Chu limit $Q = Q_1$.

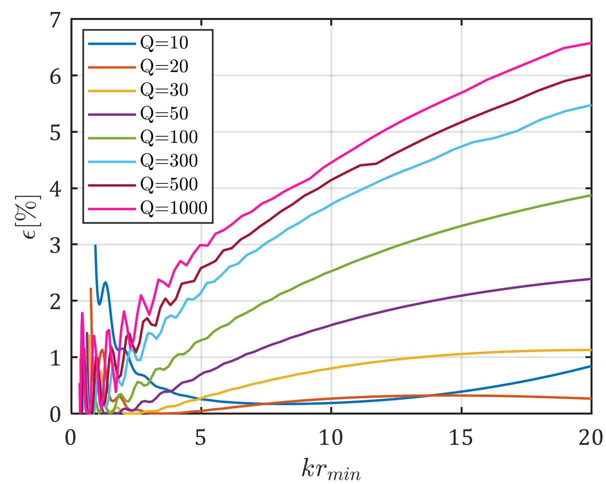

Fig. 11. Percentage error between the approximation in (28) and the exact formula (9) for $Q = 10, 100, 1000$ in the range of antenna size $0.8kr_2 \leq kr_{\min} \leq 20$.

value, derived directly from (10), leads to $\bar{\xi} \approx 0.16/Q$, thus providing the compact closed form

$$D_{\max}^{(approx)} \simeq \sum_{n=1}^{\infty} \frac{(2n+1)}{0.16\left(\frac{Q_n}{Q} - 1\right) + 1}. \tag{28}$$

We observe that the summation can be truncated at $[kr_{\min}] + 10$ without compromising the accuracy. Comparison between the exact form in (9) and the one in (28) is given in Fig. 6. It is seen that this formula is accurate for $0.8kr_2 \leq kr_{\min} \leq 20$ and $10 < Q < 5000$. In particular, the percentage error $\varepsilon\% = (D_{\max}^{(approx)} - D_{\max})/D_{\max}$ in the above range for various values of Q is presented in Fig. 11. It can be seen that the percentage error is less than 5% for $Q < 100$ and less than 7% for $Q < 1000$.

V. Equivalent Radius Containing the Reactive Energy

We can evaluate the ratio between the equivalent radius obtained by $D_{\max} = (kr_{eq})^2$ and the radius of the minimum

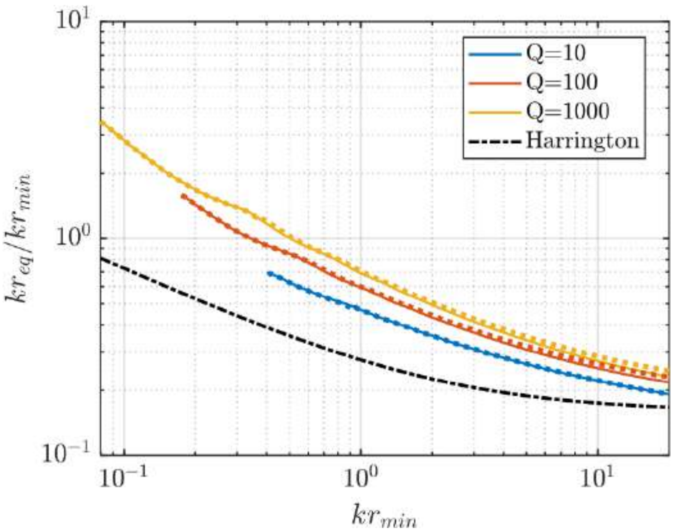

Fig. 12. Ratio between r_{eq} and $r_{\min}$ as a function of $kr_{\min}$ for different values of Q (the curves are truncated at a radius corresponding to the Chu limit). The equivalent radius is obtained from the exact formula (9) (continuous lines) and the approximated ones in (28) (colored dotted lines). The black dashed line is obtained with the Harrington formula in (2), i.e., $(kr_{\min})^2 + 2(kr_{\min})$.

sphere; r_{eq} may be interpreted as the equivalent radius at which the reactive field becomes negligible [18].

Fig. 12 shows for different values of Q the comparison of the results with the ones obtained from the directivity estimated by Harrington.

VI. Conclusion

We have presented an exact analytical expression [see (9)] for the maximum antenna directivity with a limit of bandwidth, as a function of the antenna electrical size. The expression has been obtained by solving a convex optimization problem formulated in terms of an SW expansion of the radiated field, and it is expressed in the form of a series that converges rapidly in a large range of the parameters' variation. The main achievements connected with this expression are summarized here.

1) The coefficients of the SW expansion providing the maximum directivity are given in analytical form, thus allowing for the derivation of the optimal radiation pattern for any radius of the minimum sphere.
2) The maximum directivity limit goes to 3 for the values of $kr_{\min}$ that respect the Chu limit for dipolar Huygens' sources. There, the exact formula predicts maximum directivity equal to 3 independently of the value of Q.
3) For small antennas, the results are interpreted in terms of a combination of the field radiated by dipolar and quadrupolar Huygens' sources outside the minimum sphere. This interpretation leads to the simple formula (23) which provides an accurate continuous description of directivity in the range $3 \le D_{\max} \le 8$ as a function of the minimum Q for any fixed antenna size.
4) As an intermediate step of the solution for small-size antennas, we have also found the relation $Q > Q_2$ that established the limit of bandwidth for HQs alone, which is an extension of the Chu limit to isolated resonant quadrupoles.
5) By using the exact formula, a simple analytical closed-form expression has been derived, which complements the expression in the quadrupole range for electrical size till $kr_{\min} = 20$ and $Q < 5000$.

We should stress that being the limit obtained with zero field inside the minimum sphere, this limit could be very difficult to approach for large antenna sizes. For small- to intermediate-size antennas, the simplicity of the final formulas together with their interpretation renders this work useful for antenna engineers. The extension of this work to account for losses will be carried out in a dedicated article.

Appendix A
Q-Factors for SWs

The analytical expression of Fante's Q_n is $Q_n = 1/2(Q'_n + Q''_n)$, where

$$Q'_n = x - |h_n(x)|^2\left[\frac{1}{2}x^3 + x(n+1)\right] - \frac{1}{2}x^3|h_{n+1}(x)|^2 + \frac{1}{2}x^2(2n+3)\left[j_n(x)j_{n+1}(x) + y_n(x)y_{n+1}(x)\right] \quad (29)$$

$$Q''_n = x - \frac{1}{2}x^3\left[|h_n(x)|^2 - j_{n-1}(x)j_{n+1}(x) - y_{n-1}(x)y_{n+1}(x)\right] \quad (30)$$

where $x = kr_{\min}$ and h_n, j_n, and y_n are the spherical Hankel of the second kind, Bessel, and Neumann functions of order n, respectively.

Appendix B
Solution of (8)

We can solve the problem in (8) by using the Lagrange multiplier method [39]. To this end, we define the Lagrangian function

$$\Lambda(A_n, \lambda) = \left(\sum_n A_n|K_n|\right)^2 - \lambda\left(\sum_n [\xi(Q_n - Q) + 1]A_n^2 - 1\right) \quad (31)$$

where $A_n = |C_n|$ and $|K_n| = \sqrt{2n+1}$. The problem is formulated so that the minimum with respect to ξ of the value λ which maximizes the Lagrangian represents the maximum directivity with a certain Q-bound. In order to find this value, (31) is differentiated with respect to λ and to A_m and the derivative is set to zero

$$\frac{\partial\Lambda(A_n, \lambda)}{\partial A_m} = 2|K_m|\sum_n A_n|K_n| - 2\lambda[\xi(Q_m - Q) + 1]A_m = 0 \quad (32)$$

$$\frac{\partial\Lambda(A_n, \lambda)}{\partial\lambda} = \sum_n [\xi(Q_n - Q) + 1]A_n^2 - 1 = 0 \quad (33)$$

while (33) ensures the respect of the bound in Q after minimization, (32) is satisfied if and only if

$$\sum_n A_n|K_n| = \lambda \quad \text{and } |K_m| = [\xi(Q_m - Q) + 1]A_m. \quad (34)$$

Substituting the second equality in the first equality of (34) and using $|K_n| = \sqrt{2n+1}$ leads to

$$\lambda = \sum_n \frac{2n+1}{[\xi(Q_n - Q) + 1]} \tag{35}$$

from which the maximum directivity is obtained by minimizing with respect to ξ. The range of variation of ξ is determined by imposing that the constraint terms are nonnegative, $\xi(Q_n - Q) + 1 \geq 0\ \forall n$; i.e.,

$$\begin{aligned} Q_n > Q, \quad & \xi \geq \max\left\{-\frac{1}{(Q_n - Q)}\right\} = 0 \\ Q_n < Q, \quad & \xi \leq \min\left\{\frac{1}{(Q - Q_n)}\right\} = \frac{1}{(Q - Q_1)} \end{aligned} \tag{36}$$

which leads to (9).

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