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The Economic Value of Dual-Token Blockchains

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Abstract: It is standard for blockchain platforms to issue native tokens, or crytpocurrencies, that users must own to operate within the platform. Some blockchains, however, decided to issue two tokens, establishing a dual system, with one token typically for governance and the other for implementing functions on the blockchain, such as executing transactions or smart contracts. Therefore, the two tokens are used for different activities. Typically, owning the governance tokens gives the right to receive the other token for free, as a reward for participating in the blockchain decision-making and voting processes. However, both tokens can also be traded on some exchange nodes, which means that platform functions could be implemented even without owning governance tokens. In this paper, we discuss some economic fundamentals of dual-token blockchain platforms—in particular, how to establish their economic value and the market *relative attractiveness* of the two tokens. We do so by introducing some simple numerical indicators, based on prices, and traded circulating monetary quantities. Such indicators, which are meant to reflect the platform's view on the tokens' market desirability, could be computed in real time and used to support the platform's policy making.

Keywords: dual-token blockchains; economic value

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1. Introduction

As a follow-up to the appearance of Bitcoin [1] in recent years, there has been a remarkable growth in the number of blockchain platforms, providing a wide range of services. Typically, these platforms are endowed with a native currency, or token, which is used to perform a variety of functions: implement transactions and smart contracts, obtain voting rights for governance decisions, and other activities. The question of how to issue, and manage, tokens originated the whole new area of tokenomics [2–6]. In such blockchains, market demand for a unique token can be considered as a *desirability* indicator of the platform. However, with a unique token, the market cannot distinguish, for example, between a request for tokens for implementing monetary transactions from a demand for voting rights.

Some platforms, such as [7,8], opted instead for a dual-token economy—that is, for introducing two different tokens performing different functions. One of the reasons for doing so has been to isolate the transaction fees from market oscillations separating the main native token, needed for governance participation, from the one used to pay for operating on the platform [9–11] This idea originated in Ethereum [12], which, however, does not allow the token needed to perform platform operations to be traded. Indeed, it is important to observe that, if both tokens are traded on the market, the stabilization of the transaction fees' values paid in a fiat currency may be hindered, since both of their prices may oscillate. In any case, unlike the one-token blockchains, if traded on the market, the dual-token model can provide more detailed information on the *attractiveness* and desirability of the platform's different activities. Indeed, some users could be more interested in governance, while others are only interested in implementing services on the blockchain.

In this paper, we investigate some economic fundamentals of dual-token blockchains.

In particular, we shall discuss a methodology on how to define the economic value of the two tokens, to quantify their *absolute* and *relative* attractiveness—desirability. The methodology is based on numerical indicators which, we believe, may provide useful representations of the degree of the economic success of such tokens and, more in general, of the entire platform. Our ambition is not to present an exhaustive list of possible indicators but, rather, to suggest a possible method for a blockchain to evaluate its tokens' attractiveness. The challenge with this approach is to find indicators that are both sufficiently simple while, at the same time, effective in expressing the platform's view on the tokens' desirability. Indeed, if so, they could be easy to compute, as well as useful for supporting the platform policy making. This is because we envisage the indicators as possible tools for the blockchain when deciding the token supply, the quality of services provided, the fees charged for operating on the platform, the governance rules, etc. More specifically, if, for example, based on the indicators, the platform would consider that the tokens' relative attractiveness is meaningfully "unbalanced" in favour of the governance token, then the blockchain may consider improving the quality of the technology behind smart contracts, increasing the involvement of the potential ecosystem and other activities, to improve the attractiveness of the on-chain operations.

Some *natural* variables to consider for constructing such indicators are the market prices and the exchanged quantities on the market. Yet, several other variables could also be informative and of interest, such as the block size, the average transaction size, the amount of transaction fees, the average time that tokens are held in the users' wallets, and others.

In this work, we shall focus on market prices together with circulating and traded quantities, as privileged variables to build indicators that might represent a platform's view on the attractiveness of the two tokens. As mentioned, such indicators should reflect what the blockchain thinks is the current degree of desirability of the tokens, so that they could possibly act consequently. The paper is structured as follows: In Section 2, we discuss the materials and methods. In Section 3, we present the results, developing our proposal for evaluating the *economic attractiveness* of the two tokens, and discussing some economic indicators, based on price and quantities taken separately, as well as in combination. Section 4 discusses the findings, while Section 5 concludes the paper.

2. Materials and Methods

The methodology adopted in this work can be considered theoretical, although with empirical and policy-making implications. Indeed, the aim of the paper is to develop a proposal for a blockchain to formulate an economic evaluation of the two tokens. The evaluation would be *absolute*, that is, how one token is considered by the market, as well as *relative*, when comparing the two tokens. Though based on a simple framework, to our knowledge, this paper is the first contribution attempting to do so. The importance of such an evaluation is practical as it could suggest, for the blockchain, if and how to intervene in the market, as well as when to implement new on-chain services, change the governance criteria, etc. As mentioned above to pursue the goal, in this paper, our suggestion is to construct numerical indicators based on market prices and traded quantities. Indeed, besides data being available, we also find them to be easy to understand and effective for informing the platform about the tokens' desirability. The paper exposition follows a gradual approach; we first consider price-only indicators, which we then extend to price and traded quantity, combined, indicators. Indeed, as we argue below, quantities may provide complementary information to prices, which however may sometimes be inconsistent. This is why a combined indicator might provide a more balanced, as well as comprehensive, evaluation of the two tokens' attractiveness. A *natural-price-only* indicator that we consider is the tokens' price ratio, where prices are computed with respect to a third currency, of the two tokens. In fact, this tells us how many units of one token trade in the market with one unit of the other token. Combined, price–quantity, indicators that we consider are extensions of it, weighted by the traded quantities. To focus on the tokens' relative desirability, the traded quantities that we use are not absolute, but rather relative to the number of currently circulating tokens. Indeed, we consider this as the proper way to introduce quantities for comparison purposes. Although we discuss few indicators, our goal is not to propose an exhaustive list of indexes, but rather to suggest how such indicators could be built and used to represent a platform's view on the tokens' desirability. In fact, based on economic data, different blockchains may have different views on the economic value of their tokens and, so, adopt alternative, *tailor-made* indicators to represent their perspectives. Most of the examples that we provide illustrate two main classes of indicators. Those representing blockchains which consider prices and quantities to be equally important for evaluating the tokens' desirability, such as the linearly weighted exchange rate indicator; and, alternatively, those which represent blockchains considering prices as the priority when establishing which token is more desirable, such as, for instance, the exponentially weighted exchange rate. The ultimate goal of this paper would be to embody this methodology in the blockchain operations, supporting the platform's policy making.

3. Results

In this section, we are going to formulate our proposal, beginning with price-only indicators, then introducing traded quantities as well, as informative elements for formulating a token economic value.

3.1. Two-Token Economies

Two-token economies (TTEs), such as NEO, exhibit some resemblances to standard economies and with other blockchain platforms, but also differences. We begin the section by discussing, in this part, some of the main economic features of TTEs. Inspired by the NEO blockchain, we shall indicate the two tokens, respectively, with N(eo) and $G(as)$, where N is the governance token and G the token needed to operate on the platform. The choice was taken to make the exposition connected to reality.

The Economic Meaning of N and G

A standard economic interpretation, or evaluation, of N and G hinges on their market price, where the price is typically computed in terms of fiat currencies, or in terms of the main cryptocurrencies. Indeed, their price is supposed to embody the degree of the *absolute desirability* of the two tokens by the market; that is, the desirability is expressed in terms of a currency *external* to, or outside, the platform.

Therefore, it seems natural to think that appropriate combinations of the two prices, such as their ratio, could represent, for a blockchain, an indication of the *relative desirability* of the two tokens, that is, how much the market is valuing one token as compared to the other. Both absolute and relative desirability may be useful information in blockchain policy making.

However, price combinations neither contain explicit information on the exchanged volumes of tokens that induced that price, that i the traded quantities, nor on the number of circulating quantities, which may also inform us about the tokens' attractiveness. In what follows, we introduce the above indicators and discuss their meaning.

3.2. The "Absolute Desirability" of N and G

Let $t = 0, 1, 2, \ldots$ be the time index expressed in days, months, etc., or, alternatively, $t \geq 0$, in case where time changes continuously. Furthermore, define $p_{N\$}(t)$ and $p_{G\$}(t)$ as the price of \$ in terms of, respectively, N and G, with the units of measurement given by, again respectively, $\frac{\$}{N}$ and $\frac{\$}{G}$—that is, how many \$ are exchanged against, respectively, one unit of N and one unit of G. In general if C, with $C = 1, .., M$, is a generic fiat currency/cryptocurrency traded in the market, then $p_{NC}(t)$ and $p_{GC}(t)$ indicate the prices of the two tokens with respect to such a currency. Clearly, in general, for a pair of different currencies $C' \neq C$, $p_{NC'}(t) \neq p_{NC}(t)$ and $p_{GC'}(t) \neq p_{GC}(t)$ with $p_{GC'}(t) = p_{GC}(t)p_{CC'}(t)$ and $p_{NC'}(t) = p_{NC}(t)p_{CC'}(t)$. Therefore, as indicators of the absolute desirability of the tokens, the prices $p_{GC}(t)$ and $p_{NC}(t)$ are *not invariant* with respect to the currency of reference.

Thus, we define the inverse prices as $p_{CN}(t) = \frac{1}{p_{NC}(t)}$ and $p_{CG}(t) = \frac{1}{p_{GC}(t)}$. If $p_{NC}(t) = 0$ then, according to the standard definition, we call N a *free good*, since N tokens can be obtained against 0 units of C. Alternatively, with any amount of C, it is possible to obtain ∞ units of N. Similar considerations hold for $p_{GC}(t) = 0$.

3.3. The "Relative Desirability" of N and G

A natural way to evaluate the desirability of N relative to G would be to consider the market price $p_{NG}(t)$, expressed in terms of $\frac{G}{N}$, which takes place in the market. However, the exchange nodes may not have a market where N and G are exchanged *directly*, but only through a third *indirect* market. In what follows, we shall focus the discussion on indicators in the absence of a *direct* market.

In this case, an indicator $\varphi_{NGC}(t)$, based on prices only can, in general, be defined as a function f of the prices:

$$\varphi_{NGC}(t) = f(p_{NC}(t), p_{GC}(t)) \tag{1}$$

where $\varphi_{NGC}(t)$ may be required to fulfil some properties. In particular the following two are quite natural, though not always satisfied:

(i) $(Equal\ Desirability\ of\ Tokens) \varphi_{NGC}(t) = 1$, if $p_{NC}(t) = p_{GC}(t)$ for any C
(ii) $(Currency\ Independence\ of\ the\ Indicator) \varphi_{NGC}(t) = \varphi_{NGC'}(t)$ for any $C \neq C'$

As we shall see, property (i) is inspired by the fact that, when $p_{NC}(t) = p_{GC}(t)$, then, except for transaction fees, in the market one unit of N trades with one unit of G. Property (ii) means that $\varphi_{GC}(t)$ is independent of C. That is, whatever the currency of reference, the index takes the same value.

Notice that (ii) could be reformulated as

(iia) $f(p_{NC}(t), p_{GC}(t)) = f(kp_{NC}(t), kp_{GC}(t))\ for\ any\ k > 0$

Since, as above, changing the reference currency amounts to multiplying the two prices by the same number.

Posing $k = \frac{1}{p_{GC}(t)}$, it follows that (ii)–(iia) imply:

$$f(p_{NC}(t), p_{GC}(t)) = f(\frac{p_{NC}(t)}{p_{GC}(t)}, 1) \tag{2}$$

That is, $f(p_{NC}(t), p_{GC}(t))$ should depend on the prices only through their ratio rather than on their absolute values.

It is easy to see that the number of indicators satisfying (i) and (ii) are virtually infinite. For instance,

$$f(p_{NC}(t), p_{GC}(t)) = (\frac{p_{NC}(t)}{p_{GC}(t)})^{(\frac{p_{NC}(t)}{p_{GC}(t)})} \text{ and } f(p_{NC}(t), p_{GC}(t)) = \left(1 + \frac{p_{NC}(t)}{p_{GC}(t)}\right)^2 \frac{1}{4\left(\frac{p_{NC}(t)}{p_{GC}(t)}\right)} \tag{3}$$

are just two examples. However, both of them have a non-obvious and, possibly, non-useful interpretation for evaluating the relative desirability of N and G, a point which we shall further develop below.

Since, according to (i)–(ii), the price ratio $\frac{p_{NC}(t)}{p_{GC}(t)}$ plays a major role in (1), in the next paragraph, we will discuss its main features.

3.4. The "Price Ratio"

Let us indicate the two relevant markets for trading the tokens as NC and GC, which are available in one, or more than one, exchange node. Hence, disregarding the transaction fees the price ratio, or exchange rate, is defined as

$$\varphi_{NGC}(t) = e_{NGC}(t) = \begin{cases} \frac{p_{NC}(t)}{p_{GC}(t)} & if\ p_{NC}(t) \neq 0, p_{GC}(t) \neq 0\ or\ both \\ 1 & if\ p_{N\$}(t)\ and\ p_{G\$}(t) = 0 \end{cases} \quad (4)$$

and expressed in terms of $\frac{G}{N}$, represents the number of G units that can be purchased with 1 unit of N in the market, by selling and buying C. Besides such a natural interpretation, notice that (4) satisfies both (i) and (ii), provided that the market is *well-functioning* and arbitrage-free. That is, the number of G tokens that can be purchased with 1 unit of N tokens are the same, if, rather than buying and selling C, one would buy and sell any other currency C', with $C \neq C'$. Indeed, since $p_{NC'}(t) = p_{NC}(t)p_{CC'}(t)$ and $p_{GC'}(t) = p_{GC}(t)p_{CC'}(t)$, it would immediately follow that $\frac{p_{NC}(t)}{p_{GC}(t)} = \frac{p_{NC'}(t)}{p_{GC'}(t)}$.

For this reason, henceforth, $e_{NGC}(t)$ will be written as $e_{NG}(t)$.

However, it is worth anticipating that, informative as it may be, below, we shall discuss that $e_{NG}(t)$ could be an incomplete, or partial, indicator, since it does not take explicitly into account the volumes of tokens exchanged in the market, which could be useful elements for the tokens' economic evaluation.

It is also important to point out again that $e_{NG}(t)$ does not derive from the quantities of N and G *directly* traded with each other in the market. Indeed, it could be $e_{NG}(t) > 0$, even if no unit of N is effectively exchanged against any unit of G, via any currency C. In this case, $e_{NG}(t)$ should be interpreted as a *hypothetical* price, if a user wanted to sell one token to buy the other token, and vice versa.

Hence the following basic, and intuitive, interpretations of (4) can also be made. Broadly speaking, the larger $e_{NG}(t)$ is, the *stronger* and more desirable N is as compared to G, while the opposite holds true the smaller $e_{NG}(t)$ is. Moreover, if $e_{NG}(t) < 1$, then one could claim that G is *more desirable* than N; if $e_{NG}(t) > 1$, N is *more desirable* than G, while in the limiting case of $e_{NG}(t) = 1$, they are *equally desirable*.

It is appropriate to point out that such an interpretation certainly gains value when the circulating number of both tokens are sufficiently large, and the markets (in principle) *thick*, that is exhibiting some meaningful volumes of trades. In that case, the market prices and traded quantities can be appropriate signals of the tokens' desirability. Instead, when the circulating quantity of a token is low—in the extreme case, just one unit—then care is required in interpreting the price ratio. Later, we shall come back to the issue when introducing quantities.

In the NEO blockchain, intuitively, one would expect $e_{NG}(t) > 1$, because of the intrinsic *asymmetric* relationship between the two tokens. Indeed, G is distributed to $N's$ holders for voting participation, without any *out-of-pocket* payment, while the contrary is not true. That is, G holders cannot obtain N unless they pay for them, while N holders can obtain G also without explicitly paying for them. It is true that voting participation requires attention and is time-consuming, and, for this reason, it bears an *opportunity cost*. However, this is not an *out-of-pocket*, or explicit, disbursement of money.

Finally, observe that posing $f(p_{NC}(t), p_{GC}(t)) = f(\frac{p_{NC}(t)}{p_{GC}(t)}, 1) = g(e_{NG}(t))$ implies that choosing $e_{NG}(t)$ as an indicator of the relative desirability of the tokens means choosing $g(e_{NG}(t))$ as the identity function $g(e_{NG}(t)) = e_{NG}(t)$.

A simple indicator, analogous to (4), still based only on prices, could also be the following:

$$\varphi_{NGC}(t) = d_{NGC}(t) = p_{NC}(t) - p_{GC}(t) \quad (5)$$

that is, the difference between the amount of currency that, respectively, a single unit of N and a single unit of G can buy. However, as compared to (4), indicator (5) does not satisfy (i)–(ii) and its interpretation requires some care, since $p_{NC}(t)$ is expressed in terms of $\frac{C}{N}$ and $p_{GC}(t)$ in terms of $\frac{C}{G}$. Hence, to make sense of $d_{NGC}(t)$, one may assume that $p_{NC}(t)$ is multiplied by one unit of N and $p_{GC}(t)$ by a unit of G, so that $d_{NGC}(t)$ is simply expressed in terms of C. In the case where the prices are the same, it is $d_{NGC}(t) = 0$, which corresponds to $e_{NG}(t) = 1$ in (4), while $d_{NG}(t) > 0$ corresponds to $e_{NG}(t) > 1$ and $d_{NG}(t) < 0$ to $e_{NG}(t) < 1$. Based on the above discussion, when considering prices only, we find $e_{NG}(t)$ to be the most intuitive index for evaluating the relative desirability of the two tokens. For this reason, in the rest of this paper, $e_{NG}(t)$ will represent a reference in the analysis.

Following the above considerations, in general, we expect N to be somehow more *attractive* than G; hence, $e_{NG}(t) > 1$. Yet, the level of $e_{NG}(t)$ can be affected by several factors, some of which we discuss later.

As an example of the above considerations, data from Coinmarketcap indicate that, on 18 August 2022, it was $p_{N\$}(t) = 10.45$ and $p_{G\$}(t) = 2.88$, while on 18 August 2023, it was $p_{N\$}(t) = 7.01$ and $p_{G\$}(t) = 2.22$. Therefore, the exchange rate on those two dates was, respectively, about $e_{NG}(t) = 3.62$ and $e_{NG}(t) = 3.15$. This implies that, over that period, the price ratio decreased by roughly 13%, suggesting that, although N remained stronger, its relative importance with respect to G decreased. Hence, these empirical observations are consistent with the intuition that $e_{NG}(t) > 1$, but that $e_{NG}(t)$ can oscillate. This means that the two tokens' market prices may not always move synchronically and, even when they do, not necessarily to the same extent.

Finally, an interesting and important question related to the above values could be anticipated at this point: which value, between $e_{NG}(t) = 3.62$ and $e_{NG}(t) = 3.15$, is preferable by NEO? More generally, is there an *optimal* value of the exchange rate that the platform would want to target? A discussion on this question will be deferred until later.

Indeed, in what follows, we shall consider how the indications provided by the prices can be complemented with quantities, for the blockchain to extract additional information from the data on the attractiveness of N and G.

3.5. The Absolute Supply–Demand Ratio of N and G

To gain further insights on the interpretation of $e_{NG}(t)$ and discuss how quantities could be informative on the desirability of the two tokens, consider the limiting case $e_{NG}(t) = 1$—that is, $p_{NC}(t) = p_{GC}(t)$—which means that, with one unit of C, it is possible to buy the same number of N and G units. Notice again that, because of the arbitrage activity, it will also have to be $p_{NC'}(t) = p_{GC'}(t)$ for any other currency $C' \neq C$.

Suppose, for example, $C = \$$, that $p_{N\$}(t) = 2 = p_{G\$}(t)$, and assume that both prices are equilibrium prices; that is, they equalize the supply and demand in the $N\$$ and $G\$$ markets. However, before proceeding, a note on terminology is in order, to point out that, for example, *at the equilibrium price*, the market supply $S_{N(\$)}(t)$ of N against \$, in the $N\$$ market, coincides with the market demand of N against \$, in the same market, that is with the supply $S_{\$(N)}(t)$ of \$ against N in that market. Namely, the exchange of those two quantities effectively takes place at the prevailing price. The same holds for the $G\$$ market.

Consider first the $N\$$ market, where $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)}$. As above, if $p_{N\$}(t) = 0$, it follows that $S_{\$(N)}(t) = 0$, while $S_{N(\$)}(t)$ could be any non-negative number.

Then, of course, $p_{N\$}(t) = 2$ can obtain if $S_{N(\$)}(t) = 10$ and $S_{\$(N)}(t) = 20$, so that $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{20}{10} = 2$, or, alternatively, it could be $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{400}{200} = 2$, etc. Namely, the value $p_{N\$}(t) = 2$ can be generated by, possibly, very different supply and demand volumes in the $N\$$ market, exhibiting the same proportion. Indeed, in general, any pair $S_{\$(N)}(t)$ and $S_{N(\$)}(t)$ satisfying the equation

$$S_{\$(N)}(t) = 2S_{N(\$)}(t)$$

would generate the same price $p_{N\$}(t) = 2$.

Likewise, the value $p_{G\$}(t) = 2$ may also, in principle, be generated by any suitable supply–demand pair, in the $G\$$ market. Suppose now, for instance, that $p_{N\$}(t) = \frac{400}{200} = \frac{4}{2} = p_{G\$}(t)$; can one really claim that, in general, N and G are *equally strong*, or *equally desirable*, in the market? Based on the demand–supply *quantities* providing the two prices, the answer may be *dubious*. This is because the prices are simple demand–supply *ratios* and, therefore, do not embody the information on the *absolute volume* of the transactions executed. But the traded volumes could be informative on the tokens' desirability.

Therefore, to take into account the volumes, in what follows, we introduce some simple quantity indicators which, however, as we shall discuss, are also not exempt from interpretational ambiguities.

To see why, consider, for example, *basic* indicators such as the quantity ratios:

$$Q_{\$}(t) = \frac{S_{\$(N)}(t)}{S_{\$(G)}(t)} = \frac{400}{4} = 100 = \frac{200}{2} = \frac{S_{N(\$)}(t)}{S_{G(\$)}(t)} = Q_{NG}(t) \tag{6}$$

that is the ratios of the supplied \$, N and G volumes, which could be used to argue about the *desirability* of N as compared to G. Namely, quite simply, the absolute volume of transacted currencies may also be informative on the two tokens' attractiveness. By considering the ratio $\frac{400}{4}$ in (6) we observe that, at the market equilibrium, the traded volume of \$ against N is a hundred times the traded volume of \$ against G, which may be interpreted as a much larger market *willingness to buy*, or desirability for purchasing, N.

However, at the same time, in (6) the ratio $\frac{200}{2}$ may also be interpreted as a higher *willingness to sell* N, instead of G, against \$, and so of a stronger preference by the platform users for keeping G, instead of N. But, of course, the willingness to sell may also be affected by the tokens held by a user, and, in general, by the number of tokens circulating in the systems—those issued by the platform. We shall defer the discussion of this point until later.

The above considerations suggest that the interpretation of quantity ratios may be approached from *two perspectives*: the point of view of the *buyers*, who induce the demand for the tokens in terms of \$, and that of the *sellers*, who provide the supply for N and G against \$. Indeed, in the above example, the buyers seem to be more interested in N, while the sellers appear more interested in G. Moreover, since $p_{N\$}(t) = p_{G\$}(t)$, one may also claim that the preferences, N for the buyers and G for the sellers, are of the same extent, or degree.

To further develop the above discussion, based on quantities, consider now the case of $p_{N\$}(t) \neq p_{G\$}(t)$. As an example, suppose again $p_{N\$}(t) = \frac{400}{200} = 2$ but $p_{G\$}(t) = \frac{10}{2} = 5$, so that, according to the price ratio $e_{NG}(t) = \frac{2}{5} < 1$, one would argue that G is stronger, or relatively more desirable, than N.

The interpretation based on the quantity ratios

$$q_{\$}(t) = \frac{400}{10} = 40 < 100 = \frac{200}{2} = q_{NG}(t)$$

would be analogous, but not identical, to the previous one. While $q_{\$}(t) = 40$ suggests that the volume of the exchanged \$ against N is 40 times larger than the one exchanged against G, the number of N supplied against \$ is 100 times the number of G supplied against \$. Therefore, one may observe that N is preferred by the *buyers* and G by the *sellers*—however, with the latter preference being stronger than the former. That is, the quantity ratios may complement (4) with interesting information on which side of the market may explain the level of $e_{NG}(t)$.

Finally, notice that the left-hand side of (6) is a pure number, since it is the ratio of $\frac{\$}{\$}$, while the right-hand side is expressed in terms of $\frac{N}{G}$.

3.6. Arbitrage: Direct vs. Indirect Markets for N and G

Before proceeding, it is worth reminding the reader that the price ratio (4), expressed in terms of $\frac{G}{N}$, *cannot* be interpreted as the quantity of N traded against G, since we assumed no *direct* exchange market for that. It only represents the ratio between the two quantities traded in the market against \$. Likewise, in the above example, the ratio $\frac{N}{G} = \frac{200}{2} = 100$ *cannot* be considered as the number of N tokens exchanged against G tokens.

However, if a direct (NG) exchange market exists then disregarding, for simplicity, the transaction fees, due to the arbitrage activity, the price $p_{NG}(t)$ could not differ from $e_{NG}(t)$, and, so, $p_{NG}(t) = e_{NG}(t)$.

Indeed, suppose $p_{NG}(t) = \frac{1}{100} = \frac{G}{N}$, while $e_{NG}(t) = 1$, with $p_{N\$}(t) = \frac{400}{200} = 2 = \frac{4}{2} = p_{G\$}(t)$. Then, a user owning $1G$ could sell it in the NG market to obtain 100 units of N tokens. Subsequently, by selling these $100N$ units against \$, she would obtain 200\$, which, in turn, when sold against G tokens, would generate $100G$.Therefore, by performing this sequence of trades, the user could obtain a very large number of G tokens with an initial single G token. But, of course, by replicating the same procedure more than once, the supply of G tokens in the NG direct market will increase, and also, possibly, the supply of N will decrease, and the price $p_{NG}(t)$ will tend to increase. Analogous considerations apply for the other two markets, until the equality $p_{NG}(t) = e_{NG}(t)$ tends to prevail.

In the case where a direct market is introduced, with the arbitrage activity inducing

$$p_{NG}(t) = e_{NG}(t)$$

then this *non-arbitrage* equation poses some conditions on the traded relevant quantities.

To clarify this point, in what follows, we discuss a simple example. Consider the three markets (1) $N\$$, (2) $G\$$, and (3) NG, and indicate with $\$_i$, G_i, and N_i, with $i = 1, 2, 3$, the quantities of the three currencies exchanged in the three markets, where $G_1 = N_2 = \$_3 = 0$. Finally, suppose $\$_T = \$_1 + \$_2;\ G_T = G_2 + G_3;\ N_T = N_1 + N_3$ are the total quantities of the three currencies exchanged in the three markets since, additionally, for the time being we assume the two tokens are not traded in other markets. Then, $p_{NG}(t) = e_{NG}(t)$ implies

$$p_{NG}(t) = \frac{(G_T - G_2)}{(N_T - N_1)} = \frac{G_3}{N_3} = \frac{\frac{\$_1}{N_1}}{\frac{\$_2}{G_2}} = \frac{\$_1}{N_1}\frac{G_2}{\$_2} = e_{NG}(t) \tag{7}$$

Equation (7) includes many variables so that none of them, alone, could be fully determined unless we fix all the others. Therefore, there could be several, in fact, unlimited, combinations of the relevant quantities which can satisfy (7). To gain some insights, below we take, as a given, $r = \frac{\$_1}{\$_2}$, N_T, and G_T to investigate the relationship between N_1 and G_2. Indeed, after appropriate rearrangement, (7) can be written as

$$N_1 = \frac{rG_2N_T}{(G_T - G_2(1-r))} \tag{8}$$

In absence of arbitrage possibilities, the above expression (8) provides some interesting indications on N_1. First, for any $r > 0$, it is increasing in G_2, and as $G_2 \to G_T$, then $N_1 \to N_T$, while as $G_2 \to 0$, then also $N_1 \to 0$. Additionally, it is increasing in r, converging to N_T as r goes to infinity, as well as increasing in N_T, but decreasing in G_T.

Notice that, for the given N_1, N_T, G_2, and G_T, in (8), the value of r is the same for any currency C. Indeed, since N_1, N_T, G_2, and G_T are uniquely determined quantities in the market, independently of the third currency, it follows that the ratio r must be the same for any C. For instance, if, rather than \$, we would consider €, then the ratio $r' = \frac{€_1}{€_2}$ will be such that $r' = \frac{p_{\$€}(t)\$_1}{p_{\$€}(t)\$_2} = r$, where $p_{\$€}(t)$, expressed in terms of $\frac{€}{\$}$, is the price of € in terms of \$.

As a simple numerical illustration, suppose $r = 1$, $N_T = 1000$, and $G_T = 100$; then (8) would lead to $N_1 = 10G_2$, regardless of the absolute size of $\$_1$ and $\$_2$, since what it counts in (7) is their ratio only. Hence, in this case,

$$p_{NG}(t) = \frac{(100 - G_2)}{(1000 - 10G_2)} = \frac{1}{10} \tag{9}$$

Expression (8) is, of course, an identity which endows $G_2 > 0$ with the freedom to take any value no larger than G_T, leaving indeterminate also the absolute levels of $p_{N\$}(t)$ and $p_{G\$}(t)$.

If $G_2 = 1$, then $G_3 = 99, N_1 = 10$, and $N_3 = 990$. Since $r = 1$, then $\$_1 = \$_2$, so that, if $\$_1 = \$_2 = 100$, it follows that $p_{N\$}(t) = 10$ and $p_{G\$}(t) = 100$, while if $\$_1 = \$_2 = 1000$, then $p_{N\$}(t) = 100$ and $p_{G\$}(t) = 1000$. Therefore, the amount of \$ determines the absolute level of the two prices, while the arbitrage activity determines their ratio, which, indeed, could now inform us about the number of G tokens exchanged against N tokens, in the direct market.

To conclude this part, we extend the above analysis to more than one currency. Suppose there are M currencies, with which the two tokens could be directly exchanged. Hence, markets $1, .., M$ are for N against the currencies, markets $(M+1), \ldots, 2M$ are for G against the same currencies, and the $(2M+1)th$ market is the NG market. In particular, C, N_C, and G_C stand, respectively, for the currency units, the N units, and the G units exchanged in market C.

Moreover, for any market $C = 1, , .., M$ where N is traded against the relevant currency, there is a corresponding market $(C + M)$ where G is traded against the same currency.

So, in total, there would be $2M + 1$ markets, and, assuming non-arbitrage, the following conditions must hold:

$$\frac{G_{2M+1}}{N_{2M+1}} = \frac{\frac{C}{N_C}}{\left(\frac{(C+M)}{G_{C+M}}\right)} \quad with\ C = 1, .., M \tag{10}$$

Since the left-hand side of (10) must be the same for all markets, it follows that

$$\frac{\frac{C}{N_C}}{\left(\frac{C+M}{G_{C+M}}\right)} = \frac{\frac{C'}{N_{C'}}}{\left(\frac{C'+M}{G_{C'+M}}\right)} \tag{11}$$

for any pair of markets $C \neq C'$. As well as for (8), Equation (11) suggests that the exchanged quantities are not free to take any value, since they must comply with the proportions *defined* by the non-arbitrage condition.

3.7. The Relative Supply–Demand Ratio of N and G

The tokens' market price, being defined as the ratio between the absolute levels of supply and demand, does not consider the number of circulating tokens. For example, suppose again that, in the $N\$$ market prices, $p_{N\$}(t) = \frac{400}{200} = 2 = \frac{4}{2} = p_{G\$}(t)$. Of course, the number of traded N tokens, that is, 200, being much larger than the number of traded G tokens, may induce one to think that N is more attractive than G for the buyers, and the opposite for the sellers.

However, as far as the two tokens' market attractiveness is concerned, such a direct comparison between the absolute quantities may be deceiving. Indeed, what may be more interesting/informative to consider is the proportion between the traded and circulating tokens, where, by circulating, we mean the total number of tokens issued by the platform.

Therefore if at time t, for instance, the number of N circulating tokens is $N_c(t) = 400,000$ and the number of circulating G tokens is $G_c(t) = 200$, then

$$s_{N(\$)}(t) = \frac{S_{N(\$)}(t)}{N_c(t)} = \frac{200}{400,000} = \frac{1}{2000} < \frac{2}{200} = \frac{1}{100} = \frac{S_{G(\$)}(t)}{G_c(t)} = s_{G(\$)}(t) \quad (12)$$

That is, the relative number of supplied G tokens $s_{G(\$)}(t)$ would be higher than the relative number of supplied N tokens $s_{N(\$)}(t)$, and the ratio of these two relative quantities is equal to

$$q_{NG\$}(t) = \frac{s_{G(\$)}(t)}{s_{N(\$)}(t)} = \frac{0.01}{0.002} = 5 \quad (13)$$

Notice that such a ratio would be a pure number, that is, independent of the measurement units; in addition, it is the ratio $q_{\$}(t) = \frac{400}{4} = 100$ between the traded dollars. However, relative trades concerning different currencies, in general, differ.

Hence, comparing now the two relative-quantity ratios, we observe that $q_{\$}(t) = 100 > 5 = q_{NG\$}(t)$, and, so, despite the price ratio being equal, it seems to suggest that, in fact, N is more desirable than G, since the former is *relatively* less frequently traded. The relative supplies will be used latter to build combined price–quality indicators for the tokens' attractiveness. However, prior to doing so, it is worth mentioning an additional notion of price, as well as discussing how a notion of the optimal $\varphi_{NGC}(t)$ may be introduced.

3.8. The "Virtual" Price of G and N

In the above discussion, we took, as a reference for the economic value of the two tokens, their prices against \$, when considering indirect exchanges with respect to a generic currency, or the price of N against G in a direct market. Then, the arbitrage activity led to

$$p_{NG}(t) = \frac{p_{N\$}(t)}{p_{G\$}(t)} = e_{NG}(t)$$

The relevant prices $p_{NG}(t),\ p_{N\$}(t),\ and\ p_{G\$}(t)$ are all computed in the three *bilateral* markets $NG, N\$,\ and\ G\$$ on the basis of the demand and supply—hence, the quantities exchanged in those markets. However, whether or not a direct GN market exists, it is always possible to compute a ratio between the total number of N, $S_N(t)$, exchanged against all currencies and the total number of G, $S_G(t)$, traded against all currencies. That is, the ratio $v_{NG}(t)$ is defined as

$$v_{NG}(t) = \frac{S_G(t)}{S_N(t)}$$

which we call a *virtual price,* since, typically, it is not explicitly computed, and, yet, it may also be a useful indicator for evaluating the relative desirability of the two tokens.

To see how informative it may be, as compared to the previous indicators, consider the following very simple example. Suppose there are only two currencies to trade the two tokens with: \$ and €. Moreover, assume $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{400}{200} = 2$ and $p_{G\$}(t) = \frac{S_{\$(G)}(t)}{S_{G(\$)}(t)} = \frac{4}{2} = 2$, so that $e_{NG}(t) = \frac{p_{N\$}(t)}{p_{G\$}(t)} = \frac{2}{2} = 1 = p_{NG}(t)$. Furthermore, suppose that $p_{N€}(t) = \frac{S_{€(N)}(t)}{S_{N(€)}(t)} = \frac{100}{20} = 5$ and $p_{G€}(t) = \frac{S_{€(G)}(t)}{S_{G(€)}(t)} = \frac{20}{4} = 5$, so that $e_{NG}(t) = \frac{p_{N€}(t)}{p_{G€}(t)} = \frac{5}{5} = 1 = p_{NG}(t)$. Therefore, according to (4), and considering the arbitrage activity, the two tokens are equally desirable by the market.

However, computing the *virtual* price, we obtain $v_{NG}(t) = \frac{S_G(t)}{S_N(t)} = \frac{2+4}{200+20} = \frac{6}{220} \sim 0.03$, suggesting that G is a stronger token than N, because the total number of traded Gs is much lower. Again, $v_{NG}(t)$ is not a *proper* price, since quantities are supplied and demanded in separate markets and not in a single global market. Hence, it can only be interpreted as a *hypothetical* price in the following way: if the total traded quantities were exchanged as a whole, rather than on bilateral markets, then $v_{NG}(t)$ would be the equilibrium price. Though not computed in practice, $v_{NG}(t)$ may be informative as a ratio of the total quantities traded on the market. The example shows a major difference between the indicators based on bilateral markets and the virtual price. Again, this may be because, in $v_{NG}(t)$, we considered the absolute instead of the relative (to the circulating quantities) exchanged volumes. With relative quantities, we may expect a reduction in the difference, as compared to bilateral markets; yet, there is no *a priori* reason to believe that such a difference would be eliminated.

3.9. The "Optimal" Level of $\varphi_{GC}(t)$

As anticipated, upon having defined $\varphi_{GC}(t)$, an additional, interesting question to ask is whether there is an optimal level of $\varphi_{NGC}(t)$ for the platform to target. This is what we discuss in this section. The starting point is to take a user with one unit of a fiat currency, say \$, considering the possibility of buying N tokens or G tokens, or even both. In the first case, she would receive $p_{\$N}(t)$ units of N tokens, plus $p_{\$N}(t)\rho$ units of G tokens, where ρ, expressed in terms of $\frac{G}{N}$, is the number of G tokens obtained by the user in a time period, by holding $p_{\$N}(t)$ units of N tokens and participating in governance and voting sessions. If $U(N, G)$ is the user's utility obtained by holding N and G tokens then, in this case, her utility would be $U(p_{\$N}(t),\ p_{\$N}(t)\rho)$. In the second case, the user's utility level would be $U(0,\ p_{\$G}(t))$. It follows that, if $U(p_{\$N}(t),\ p_{\$N}(t)\rho) > U(0,\ p_{\$G}(t))$, then the user would prefer to buy N.

In particular, assuming $\varphi_{NG}(t) = e_{NG}(t)$, as well as the utility function to be increasing in both arguments, if $p_{\$N}(t)\rho > p_{\$G}(t)$, then purchasing N would be a *dominant* action for the user, that is, if $\rho > e_{NG}(t)$.

Instead, if $U(p_{\$N}(t),\ p_{\$N}(t)\rho) < U(0,\ p_{\$G}(t))$, the user will purchase G, while if $U(p_{\$N}(t),\ p_{\$N}(t)\rho) = U(0,\ p_{\$G}(t))$, she would be indifferent to how to allocate the dollar between the two tokens. Therefore, broadly speaking, if the above individual represents an *average* user, or a representative agent, for the two markets $N\$$ and $G\$$ to be up and running, the condition $U(p_{\$N}(t),\ p_{\$N}(t)\rho) = U(0,\ p_{\$G}(t))$ would be likely to prevail. If this is acceptable then, for any given ρ, one hopes to be able to solve the above equality to obtain a relationship between $\varphi_{GC}(t)$ and ρ. Admittedly, this may be a demanding task; yet, conceptually, this should be the way to follow for a platform, and, in some specific circumstances, an explicit form could be found in a relatively simple way. For example, with a particularly simple form such as $U(p_{\$N}(t),\ p_{\$N}(t)\rho) = p_{\$N}(t) + p_{\$N}(t)\rho$ and $U(0,\ p_{\$G}(t)) = p_{\$G}(t)$, in analogy with the above discussion, it will have to be

$$e_{NG}(t) = 1 + \rho \tag{14}$$

It follows that, once ρ is fixed, if the goal of the platform is to have both markets functioning, then $e_{NG}(t)$ as in (14) would be *optimal*, and market policies should be pursued to target that value. It is clear that a crucial role is played by the users' preferences, and utility functions may easily be more complex than the one above. For example, if

$$U(p_{\$N}(t),\ p_{\$N}(t)\rho) = \sqrt{p_{\$N}(t)} + p_{\$N}(t)\rho$$

that is, when the user would be risk-averse with respect to $p_{\$N}(t)$, then it is easy to verify that the optimal level of $e_{NG}(t)$ depends not only on ρ but also on the prices, and it is a much more involved expression than (14).

3.10. The Relative Desirability of N and G as a Combination of Prices and Quantities

In the previous sections, we discussed some alternative criteria for evaluating the relative attractiveness of the two tokens, based on price and quantity market data, on a separate basis. We have also seen that the suggestions emerging from different criteria may sometimes be consistent, while, in other circumstances, they could differ. Because of this, in this section we propose some composed indicators, combining prices and quantities to embody the above considerations. We shall then compare such indicators to $e_{NG}(t)$.

However, prior to entering the discussion, it is useful to introduce the following quantities

$$\pi_{NC} = \begin{cases} \frac{s_{N(C)}(t)}{s_{N(C)}(t)+s_{G(C)}(t)} & if\ s_{N(C)}(t) \neq 0, s_{G(C)}(t) \neq 0\ or\ both \\ 1 & if\ s_{N(C)}(t), s_{G(C)}(t) = 0 \end{cases} \tag{15}$$

and

$$\pi_{GC} = \begin{cases} 1 - \pi_{NC} = \frac{s_{G(C)}(t)}{s_{N(C)}(t)+s_{G(C)}(t)} & if\ s_{N(C)}(t) \neq 0, s_{G(C)}(t) \neq 0\ or\ both \\ 1 & if\ s_{N(C)}(t), s_{G(C)}(t) = 0 \end{cases} \tag{16}$$

The above expressions, (15) and (16), inform us about the relative proportions of the traded tokens against currency C. For this reason, and also because they are pure numbers, free of units of measurement, they could be conveniently used as weights in indicators evaluating the relative attractiveness of the two tokens. For completeness, it is appropriate to point out that we should have written $\pi_{N\$}$ as $\pi_{N\$}(t)$ and $\pi_{G\$}$ as $\pi_{G\$}(t)$, since both of them are time-dependent. However, to save on notation, we omitted the time index, although we should keep in mind that (15) and (16), as well as the quantities, vary with time. Notice also that, in general, $\pi_{NC} \neq \pi_{NC'}$ and $\pi_{GC} \neq \pi_{GC'}$ with $C \neq C'$, which implies that, as weights, they are currency-specific. Hence, one simple way to obtain a weight which is currency-independent could be to take

$$\pi_N = \frac{\sum_{C=1}^{M} \pi_{NC}}{M};\ \pi_G = 1 - \pi_N = \frac{\sum_{C=1}^{M} \pi_{GC}}{M} \tag{17}$$

Considering the above weights, perhaps an extended class of indicators may be formulated as

$$\varphi_{NGC}(t) = f(p_{NC}(t), p_{GC}(t), \pi_N, \pi_G) \tag{18}$$

which would combine prices and quantities to inform us about the relative attractiveness of the two tokens.

One may also require (18) to satisfy the following properties:

(iii) $(Equal\ Desirability\ of\ Tokens) \varphi_{NGC}(t) = 1$, if $p_{NC}(t) = p_{GC}(t)$ for any C.

(iv) $(Currency\ Independence\ of\ the\ Indicator) \varphi_{NGC}(t) = \varphi_{NGC'}(t)$ for any $C \neq C'$

Since π_N, π_G are currency-independent, also, in this case, (iv) could be re-written as

(iva) $f(kp_{NC}(t), kp_{GC}(t), \pi_N, \pi_G) = f(p_{NC}(t), p_{GC}(t), \pi_N, \pi_G) = f(e_{NG}(t), 1, \pi_N, \pi_G)$ for all $k > 0$

An additional, desirable property may be the following:

(v) $(Quantity\ Independence) f(p_{NC}(t), p_{GC}(t), \pi_N = \pi = \pi_G) = f(p_{NC}(t), p_{GC}(t))$

While we already commented on (iii)–(iv), property (v) is new. It requires that quantities do not affect the value of the indicator only if the weights π_N, π_G, based on relative trades, are equal.

Before proceeding, it is appropriate to remind the reader that, with quantities, indicators may take a dual perspective: the one of the *sellers*, or suppliers, of tokens, and the one of the *buyers*, who demand tokens. The supplier's perspective represents the tokens' owner desirability; that is, how many tokens she is willing to keep or get rid of. The buyers' perspective, instead, represent the non-owners' tokens' desirability. For this reason, it seems intuitive for a proper discussion of the issue to consider both perspectives. In what follows, we shall start with the sellers, assuming $\pi_N, \pi_G > 0$.

3.10.1. Combined Price–Quantity Indicators: The Sellers' Perspective

It is easy to observe that, in principle, one could conceive an infinite number of indicators, combining prices and quantities, to evaluate the two tokens' relative attractiveness. However, since such indicators should represent the platform's view on the tokens' relative desirability, in principle, they could be built by the blockchain, by comparing the price–quantity profiles in the following way: For example, a blockchain may envisage the pair of market prices $p_{N\$}(t) = 2,\ p_{G\$}(t) = 1$, together with the pair of traded quantities $\pi_N = \frac{2}{3}, \pi_G = \frac{1}{3}$, so that $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$, as a situation for which the sellers consider the two tokens to be equally attractive. However, the same blockchain may not consider the situation $\pi_N = \frac{3}{4}, \pi_G = \frac{1}{4}$, $e_{NG}(t) = 3$, and $\frac{\pi_G}{\pi_N} = \frac{1}{3}$ as indicating equally attractive tokens. Other blockchains, in turn, may have different views. That is, it would be possible to evince the platforms' view on the tokens' relative desirability by asking the blockchain to compare alternative situations, as in the above example. Such views could be conveniently summarised by numerical indicators, to define the price–quantity situations for which tokens may, or may not, be considered as equally desirable by the platform.

Indicators might be useful tools to support blockchain policy making. For example, suppose an indicator chosen by a blockchain suggests that the sellers are having a much higher preference for the N tokens, as compared to the G tokens. This situation may be considered inappropriate by the platform, being a possible path to a power concentration in governance, as well as limited interest from the sellers for operating on the blockchain. In this case, the platform may intervene in the markets, for example, by increasing the supply of N, in so doing lowering its market price and, possibly, the indicator value, and mitigating the risk of power concentration. The platform may also react by increasing the range and quality of the services provided on the chain, in this way trying to reduce the sales of G.

As an example, two simple indicators combining prices and quantities that may, perhaps, capture some platform's view on the tokens' relative desirability are the following: (the *linearly weighted exchange rate, $LWE_{NGCs}(t)$—the sellers' perspective*)

$$LWE_{NGCs}(t) = \left(\frac{\pi_G}{\pi_N}\right)\left(\frac{p_{N\$}(t)}{p_{G\$}(t)}\right) = \left(\frac{\pi_G}{\pi_N}\right)e_{NG}(t) \tag{19}$$

and
(the *exponentially weighted exchange rate, $EW_{NGs}(t)$—the sellers' perspective*)

$$EWE_{NGCs}(t) = \begin{cases} e_{NG}(t)^{\left(\frac{\pi_G}{\pi_N}\right)} & if\ e_{NG}(t) \geq 1 \\ e_{NG}(t)^{\left(\frac{\pi_N}{\pi_G}\right)} & if\ e_{NG}(t) < 1 \end{cases} \tag{20}$$

The above indicators embody the same information, however, composed differently, to put a different emphasis on the role of the prices and relative traded quantities, which, in fact, reflect their different roles according to the platform's view. While market prices reflect more the token holders' relation to currencies *outside* the platform, the relative traded quantities is more concerned with the token holders' relation *within* the platform, namely, with respect to the circulating stock of domestic currencies. For these reasons, they inform the blockchain on the *external* and *internal* tokens' desirability. Notice, however, that the external and internal levels are not independent of each other, since they are linked by the quantity of tokens traded against a currency, which, indeed, appears in both expressions.

Prior to considering the essential features of (19) and (20), it is worth stressing a point which they share—namely, that both indicators increase with π_G and decrease with π_N. The intuition is simple: for the sellers, N can be relatively more attractive than G not only if $p_{N\$}(t) > p_{G\$}(t)$, but also if G is relatively more traded than N. Indeed, this means that the tokens' holders prefer to sell a larger share of circulating G tokens, rather than N tokens.

Below, we are going to discuss some of the main differences between (19) and (20).

As for (19), property (iii) is, in general, not satisfied, since $LWE_{NG\$s}(t)$ could differ from 1, even when $e_{NG}(t) = 1$, unless $\pi_N = \pi_G$. Hence, since, when both the prices and traded quantities are symmetrical it is $LWE_{NG\$s}(t) = 1$, also in this case it may be natural for the platform to take the unit value of the indicator as the level signalling the equal desirability of the two tokens. However, as compared to the case in which only the prices are considered, $e_{NG}(t) = 1$ is neither a necessary nor a sufficient condition for $LWE_{NG\$s}(t) = 1$. Indeed, any combination of quantity and price ratios satisfying

$$\left(\frac{\pi_G}{\pi_N}\right) = \frac{1}{e_{NG}(t)}$$

will convey the same attractiveness, for the platform, of the two tokens. Hence, for example, if $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$, then the fact that one unit of N can be exchanged with two Gs is counterbalanced by the fact that N is traded twice as much as G which means that, in terms of (relative) sales, token owners preferred to sell N rather than G. As a follow-up to the above considerations, we interpret $LWE_{NG\$s}(t) > 1$ as the sellers' finding N more attractive than G, and the opposite for $LWE_{NG\$s}(t) < 1$.

Prior to proceeding, it is also interesting to observe that, for example, values such as $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$—that is, with a high $e_{NG}(t)$ and a low $\frac{\pi_G}{\pi_N}$, and vice versa, providing $LWE_{NG\$s}(t) = 1$—can effectively take place, because they refer to different situations. Indeed, suppose, for simplicity, there is just one currency $C = \$$ to exchange the tokens with, that $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{180}{90} = 2$ and $p_{G\$}(t) = \frac{S_{\$(G)}(t)}{S_{G(\$)}(t)} = \frac{10}{10} = 1$, so that $e_{NG}(t) = 2$. Additionally, assume $N_c(t) = 450$ and $G_c(t) = 100$, so that $s_{N(\$)}(t) = \frac{90}{450} = \frac{1}{5}$ and $s_{G(\$)}(t) = \frac{10}{100} = \frac{1}{10}$, which implies $\frac{\pi_G}{\pi_N} = \frac{1}{2}$.

In general, for any given level of $LWE_{NG\$s}(t) = L > 0$, equations of the type

$$\left(\frac{\pi_G}{\pi_N}\right) = \frac{L}{e_{NG}(t)} \tag{21}$$

represent the so-called *iso-score* curves in the two-dimensional space $\left(\frac{\pi_G}{\pi_N}\right), e_{NG}(t)$, that is, the set of pairs providing the same level of score L.

To summarise, (19) reflects the view of a platform for which prices are as important as quantities to establish the tokens' attractiveness.

Instead, $EWE_{NGCs}(t)$ satisfies all the properties (iii)–(iv) and (v). For this reason, $EWE_{NGCs}(t) = 1$ if and only if $e_{NG}(t) = 1$, regardless of the ratio $\frac{\pi_G}{\pi_N} > 0$. In this case, the *iso-score* curves $EWE_{NGCs}(t) = E$ would be given by the expression

$$\left(\frac{\pi_G}{\pi_N}\right) = \begin{cases} \frac{logE}{loge_{NG}(t)} & if\ e_{NG}(t) > 1 \\ (0, \infty) & if\ e_{NG}(t) = 1 \\ \frac{loge_{NG}(t)}{logE} & if\ e_{NG}(t) < 1 \end{cases} \tag{22}$$

where (22), with $e_{NG}(t) < 1$, is, indeed, positive, since in this case it is also $E < 1$. Therefore, a main difference between (19) and (20) is that, unlike $EWE_{NGCs}(t)$, $LWE_{NGCs}(t)$ can take any value for any level of $e_{NG}(t)$, as long as $\left(\frac{\pi_G}{\pi_N}\right)$ compensates appropriately, while $EWE_{NGCs}(t) > 1$ if and only $e_{NG}(t) > 1$, $EWE_{NGCs}(t) = 1$ if and only $e_{NG}(t) = 1$, and $EWE_{NGCs}(t) < 1$ if and only $e_{NG}(t) < 1$. Therefore, if $EWE_{NGCs}(t) = 1$ is also taken as a threshold for equal attractiveness, with $EWE_{NGCs}(t) > 1$ indicating that N is more attractive than G, and the opposite for $EWE_{NGCs}(t) < 1$, then it follows that the tokens' attractiveness is only determined by the value of $e_{NG}(t)$, with the level of $\left(\frac{\pi_G}{\pi_N}\right)$ affecting only the degree of desirability, but not its direction.

For the above reasons, (20) would reflect the view of a platform which considers the exchange rate as the main source of information to establish which token is more attractive, while the degree of desirability is defined by the traded quantities.

Obviously, in general, neither $LWE_{NGCs}(t)$ nor $EWE_{NGCs}(t)$ can be interpreted in terms of the number of G exchanged against N, but rather as a function of it. As a matter of fact, the difference between the indicators and the exchange rate could be thought of as the additional contribution of the quantities to the exchange rate, in forming the indicator's value.

In particular considering, for example, $C = \$$, one can rewrite them as

$$LWE_{NG\$s}(t) = [LWE_{NG\$s}(t) - e_{NG}(t)] + e_{NG}(t)$$

and

$$EWE_{NG\$s}(t) = [EWE_{NG\$s}(t) - e_{NG}(t)] + e_{NG}(t)$$

where the above squared brackets contain the additional contribution of the traded quantities to the indicator, added to the prices' contribution as formalized by the exchange rate.

For example, suppose $e_{NG}(t) = 16$ and $\left(\frac{\pi_G}{\pi_N}\right) = \frac{1}{2}$. Then, $LWE_{NG\$s}(t) = 8$ and $EWE_{NG\$s}(t) = 4$. Therefore, as for $LWE_{NG\$s}(t)$, the quantities' contribution is -8, while, for $EWE_{NG\$s}(t) = -12$; that is, in both indicators, the higher (proportional) trades of N compensated for the price ratio, although to a different degree. Which one, between (19) and (20) and possibly other indicators, is chosen by the platform to quantify the tokens' relative desirability depends on the blockchain, its preferences and policy targets.

There could certainly be other ways to combine the prices and quantities for representing the platform views. For example,

$$f(p_{NC}(t), p_{GC}(t), \pi_N, \pi_G) = e_{NG}(t)^{(\pi_G - \pi_N)} \tag{23}$$

satisfies (iii)–(iv), and, for this reason, may look like a promising candidate as an indicator. However, for $\pi_G = \pi_N$, it would become equal to 1, regardless of the exchange rate value. That is, for a platform adopting (23), equal attractiveness could also depend on quantities only, irrespective of the prices. Therefore, (23) could represent the views of a blockchain putting additional emphasis on quantities, as compared to (20).

3.10.2. Combined Price–Quantity Indicators: The Buyers' Perspective

As well as for the suppliers, below, we define the indicators formalising the buyers' perspective on the relative attractiveness of the two tokens. For example, indicators (19) and (20) become, respectively,

(the *linearly weighted exchange rate,* $LWE_{NGCb}(t)$*—the buyers' perspective*)

$$LWE_{NGCb}(t) = \left(\frac{\pi_N}{\pi_G}\right) e_{NG}(t) \tag{24}$$

and

(the *exponentially weighted exchange rate,* $EWE_{NGb}(t)$*—the buyers' perspective*)

$$EWE_{NGCb}(t) = \begin{cases} e_{NG}(t)^{\left(\frac{\pi_N}{\pi_G}\right)} & if\ e_{NG}(t) \geq 1 \\ e_{NG}(t)^{\left(\frac{\pi_G}{\pi_N}\right)} & if\ e_{NG}(t) < 1 \end{cases} \tag{25}$$

That is, to capture the buyers' perspective, we simply switch the quantity weights from the sellers' perspectives.

It follows that, for both indicators, the sellers' perspective would prevail if $\left(\frac{\pi_G}{\pi_N}\right) > \left(\frac{\pi_N}{\pi_G}\right)$, and the opposite for the buyers' perspective. In the case where $\left(\frac{\pi_G}{\pi_N}\right) = \left(\frac{\pi_N}{\pi_G}\right)$, the prevailing perspective will be determined by the exchange rate only.

The above considerations imply some additional, interesting consequences. In particular, if $LWE_{NGCs}(t) = 1$—that is, the sellers find the two tokens equally attractive—then $LWE_{NGCb}(t) \neq 1$, unless $\left(\frac{\pi_G}{\pi_N}\right) = \left(\frac{\pi_N}{\pi_G}\right)$. Considering again $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$, we saw that $LWE_{NGCs}(t) = 1$ but $LWE_{NGb}(t) = 4$; that is, the buyers are more attracted by N.

Taking the same numerical example, we obtain $EWE_{NGCs}(t) = \sqrt{2}$ and $EWE_{NGCb}(t) = 4$, which, as we saw, implies that both the buyers and the sellers are more attracted by N, although to a different extent. This means that, depending upon the platform's view, there would be a variety of ways to formalize the tokens' relative attractiveness.

The previous observations suggest that both $LWE_{NGC}(t)$ and $EWE_{NGC}(t)$ lack symmetry, in the sense that if they indicate one of the two tokens to be more attractive for the sellers–buyers, it does not follow that the buyers–sellers would be more attracted by the other token. Indeed, the indicator $EWE_{NGC}(t)$ captures a platform's view for which sellers and buyers *must* have the same preferences regarding the two tokens, though to a different extent which, in general, is not the case with $LWE_{NGC}(t)$.

3.11. Combined Price–Quantity Currency-Dependent Indicators: The Sellers' Perspective

We conclude by considering an indicator, which may be more flexible than the previous ones, but whose value could differ across different currencies. Then, if of interest, a unique indicator could be obtained by aggregating the single indicators across currencies.

Suppose again, for the sake of exposition, that $C = \$$, $p_{N\$}(t), p_{G\$}(t) > 0$, and $\pi_{N\$}, \pi_{G\$} > 0$; then, the indicator

(the *exponentially weighted prices,* $EWP_{NG\$s}(t)$*—the sellers' perspective*)

$$EWP_{NG\$s}(t) = \frac{[p_{N\$}(t)]^{\pi_{G\$}}}{[p_{G\$}(t)]^{\pi_{N\$}}} \tag{26}$$

is inspired by (20), where, however, in the ratio, prices have different weights. Since, as well as for $LWE_{NGCs}(t) = 1$, also $EWP_{NG\$s}(t) = 1$ when both $p_{N\$}(t) = p_{G\$}(t)$ and $\pi_{N\$} = \pi_{G\$}$, it makes sense to take $EWP_{NG\$s}(t) = 1$ as the value for equal attractiveness, with $EWP_{NG\$s}(t) > 1$ indicating a preference for N, while $EWP_{NG\$s}(t) < 1$ indicates a preference for G. Though similar to (20), (26) does not embody the same prominent role played by $e_{NG}(t)$, in particular, preventing $e_{NG}(t) = 1$ from becoming the critical threshold for both the sellers' and the buyers' preferences.

Indeed, as well as (19), it could be $EWP_{NG\$s}(t) = 1$ also for $p_{N\$}(t) \neq p_{G\$}(t)$, as long as the value of $\pi_{N\$}$ appropriately compensates for the price difference.

For example, suppose $p_{N\$}(t) = 10$ and $p_{G\$}(t) = 2$; then, if $\pi_{N\$} = \frac{lnp_{N\$}}{lnp_{G\$}+lnp_{N\$}} \sim 0.77$, it is $EWP_{NG\$s}(t) = 1$—that is, if tokens N are relatively more frequently traded than G. Therefore, with the above values, $e_{NG}(t) = 5$ would suggest that N is more desirable than G, while $EWP_{NG\$s}(t) = 1$ suggests that they are equally desirable, from the sellers' perspective.

Therefore, $EWP_{NG\$s}(t) \geq 1$ if

$$p_{N\$}(t) \geq [p_{G\$}(t)]^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)} \tag{27}$$

where, in (27), the expression $[p_{G\$}(t)]^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}$ is linear in $p_{G\$}(t)$ for $\pi_{G\$} = \frac{1}{2}$, convex if $\pi_{G\$} > \frac{1}{2}$, and concave if $\pi_{G\$} < \frac{1}{2}$.

Likewise, we can define

(the *exponentially weighted prices,* $EWP_{NG\$b}(t)$*—the buyers' perspective*)

$$EWP_{NG\$b}(t) = \frac{[p_{N\$}(t)]^{\pi_{N\$}}}{[p_{G\$}(t)]^{\pi_{G\$}}} \tag{28}$$

and so $EWP_{NG\$b}(t) \geq 1$ if

$$p_{N\$}(t) \geq [p_{G\$}(t)]^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)} \tag{29}$$

Therefore, the following holds:

Proposition 1. *Both buyers and sellers find N at least as attractive as G if* $p_{N\$}(t) \geq max [p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}]$, *and find N no more attractive than G if* $p_{N\$}(t) \leq min [p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}]$. *If* $p_{G\$}(t) \leq 1$ *and* $min[p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}] \leq p_{N\$}(t) \leq max[p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}]$, *then sellers(buyers) would prefer N and buyers(sellers) G, while the opposite is true if* $p_{G\$}(t) > 1$.

Therefore, although $EWP_{NGC}(t)$ resembles $EWE_{NGC}(t)$, it is more flexible, since it allows all possible combinations of preferences towards N and G, across buyers and sellers, depending on the value of $\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)$.

Finally, it is worth noting that $EWP_{NG\$s}(t)$, in (26), has been defined considering $p_{N\$}(t)$ and $p_{G\$}(t)$, namely referring to the indirect markets $N\$$ and $G\$$, rather than to the direct market NG, that is, to the price $p_{NG}(t)$. However, in principle, it would make perfect sense to consider $p_{NG}(t)$ as a reference for the combined price–quantity indicator for the values of N and G. Below, we briefly discuss how $EWP_{NG\$s}(t)$ relates to $p_{NG}(t)$, under the $p_{NG}(t) = e_{NG}(t)$ non-arbitrage condition. Indeed,

$$\begin{aligned} EWP_{NG\$s}(t) &= \frac{[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}+\pi_{N\$}}}{[p_{G\$}(t)]^{\pi_{N\$}}} = \frac{[p_{N\$}(t)]^{\pi_{N\$}}}{[p_{G\$}(t)]^{\pi_{N\$}}}[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}} \\ &= [p_{NG}(t)]^{\pi_{N\$}}[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}} \end{aligned} \tag{30}$$

Namely, $EWP_{NG\$s}(t)$ is positively related to $p_{NG}(t)$, according to the function $[p_{NG}(t)]^{\pi_{N\$}}$, scaled by the quantity $[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}}$.

It follows that it is also $EWP_{NG\$s}(t) = [e_{NG}(t)]^{\pi_{N\$}}[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}}$, which means that it does not depend on prices only through $e_{NG}(t)$, except for when $\pi_{G\$} = \frac{1}{2} = \pi_{N\$}$, in which case

$$EWP_{NG\$s}(t) = \sqrt{p_{NG}(t)} = \sqrt{e_{NG}(t)}$$

Likewise, from the buyers' perspective, we now obtain

$$\begin{aligned} EWP_{NG\$b}(t) &= \frac{[p_{N\$}(t)]^{\pi_{N\$}-\pi_{G\$}+\pi_{G\$}}}{[p_{G\$}(t)]^{\pi_{G\$}}} = \frac{[p_{N\$}(t)]^{\pi_{G\$}}}{[p_{G\$}(t)]^{\pi_{G\$}}}[p_{N\$}(t)]^{\pi_{N\$}-\pi_{G\$}} \\ &= [p_{NG}(t)]^{\pi_{G\$}}[p_{N\$}(t)]^{\pi_{N\$}-\pi_{G\$}} \end{aligned}$$

and so, also $EWP_{NG\$b}(t)$, does not depend on prices only through $[p_{NG}(t)]^{\pi_{G\$}}$, unless $\pi_{G\$} = \frac{1}{2} = \pi_{N\$}$.

4. Discussion

In this paper, to our knowledge for the first time, we addressed a discussion on the economic fundamentals of a TTE. In particular, we have introduced a methodology based on a number of economic indicators that might be considered to define the absolute and relative economic values for the tokens, as well as for the whole platform. Our main goal has not been to propose a complete list of indicators, but rather to suggest an approach on how to evaluate the two tokens. To construct such indicators, we used the market prices, the traded and circulating quantities of the tokens. These are only a subset of the possible metrics that one could consider and, for this reason, our proposed indicators should, by no means, be considered the only ones. Indeed, we mentioned that the number of transactions and their average monetary size, the block size, and others could also be informative variables to consider.

The analysis on the traded quantities suggested the introduction of two different perspectives, the *buyers*' and the *sellers*', for the combined price–quantity indicators. We envisage the proposed indicators, computed in real time, as composing a *dashboard* for the blockchain policy-makers, that can enjoy the continuous observation of the absolute and relative economic values of the tokens, as well as of the platform. Decision-making systems, based on artificial intelligence methods, can then be used to associate the indicators' values to the platform policy making, such as changing the token supply, calibrating the number of tokens obtained for free by holding the other token, etc. Because of its focus, this paper does not discuss a number of interesting elements of TTEs—more specifically, the dynamics of users' monetary holdings, the stability of transactions fees paid in fiat currency, and the possibility of speculative trades due to one token being obtained for free by owning the other token. Yet, despite these limitations, we believe that this contribution can convey interesting insights for the platform's decision-makers, in support of their policies.

5. Conclusions

This work considers dual-token blockchains, posing the question of their economic evaluation, in absolute as well as relative terms. We believe this to be an important aspect of two-token platforms, as the blockchain may decide to intervene in the market, or change its services, depending upon the tokens' evaluation. Different platforms may have different criteria for formulating such values and, in this article, we suggest a possible avenue by proposing numerical indicators based on market prices and traded quantities. We consider this paper to be a first step towards a more comprehensive investigation of the issue, that may be based also on other metrics such as the number of transactions in a block, the number of bytes occupied in a block, the waiting time between two validated blocks, the transaction fees, and others.

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