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A model of network formation for the overnight interbank market: When is core-periphery an illusion?

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ABSTRACT

We develop a theoretical model of network formation in the overnight interbank market, where banks manage liquidity under reserve uncertainty by strategically forming bilateral lending relationships. The model incorporates counterparty risk and the central bank's corridor system, yielding endogenously determined equilibrium networks. A key result, relevant for systemic stability policy, is that the equilibrium network is bipartite: active banks act either as lenders or borrowers, and no strategic (interbank) intermediation arises. We also show that, via temporal aggregation of equilibrium networks, apparent intermediation and a core-periphery structure emerge. We validate these predictions using e-MID market data, showing that the model reconciles the frequency-dependent network features documented in the empirical literature for this market.

1. Introduction

The overnight interbank market is a key component of the financial system, where banks lend and borrow unsecured funds to manage their reserves. By reallocating liquidity among banks, this market helps smooth out short-term liquidity fluctuations, ensuring that banks can manage their liquidity efficiently and meet regulatory requirements. Beyond liquidity management, the overnight interbank market is crucial for monetary policy transmission and financial stability. Central banks target overnight interbank interest rates, thereby guiding short-term interest rates in the broader banking system and economy.

We contribute to the understanding of the overnight interbank market by developing a tractable model of network formation for a single trading period and analyzing its implications.[1] The key implication of our model is the absence of strategic intermediation. Whether intermediation occurs in a network of financial institutions has significant implications for systemic stability. Financial market regulators, in particular, may be concerned about "too-connected-to-fail" or "too-central-to-fail" banks and institutions, i.e.,

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[1] The network literature is extensive (see comprehensive treatments Jackson, 2008; Bramoullé et al., 2016). Our model contributes to the stream of *strategic network formation* models, in which link formation is endogenously determined by agents, see e.g., Jackson and Wolinsky (1996) and Bala and Goyal (2000), and is related to the literature on trading in networks (Kranton and Minehart 2001, Corominas-Bosch 2004, Blume et al. 2009, Manea 2011, Nava 2015).

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those whose large intermediation role in the network of exposures makes their failure likely to disrupt market functioning and exacerbate systemic risk (see, e.g., Haldane and May, 2011 and Battiston et al., 2012).[2] While intermediation can pose risks in other markets, such as longer-term lending, our result suggests that this is not the case for the overnight interbank market.

The absence of intermediation in our model aligns with some empirical studies of the overnight market (e.g., Iori et al., 2008) but may appear to contrast with others. For example, Fricke and Lux (2014) document intermediation in the overnight interbank market and identify a *core-periphery* structure, in which 'core' banks transact heavily among themselves while 'periphery' banks primarily trade with core banks.[3] In our setting, the absence of intermediation rules out a core-periphery network. We reconcile these differing empirical findings with our theory by demonstrating that the intermediation observed in the data may emerge in our model through *temporal aggregation*.

We extend the foundational framework of stochastic liquidity management by Poole (1968) to a bilateral exchange setting that incorporates counterparty risk. Our approach contrasts with the recent study by Whitesell (2006), which, also building on Poole's framework, models the equilibrium between the demand and supply for liquidity among homogeneous banks within a single competitive market.[4] *Heterogeneous* counterparty risk in our setting gives rise to variation in loan quality, impeding centralized market clearing. Based on individual banks' incentives to transact in the interbank market, we characterize the equilibrium network. Our network approach is well-suited for bilateral transactions and, *a priori*, does not exclude intermediation. We adopt the equilibrium concept of *pairwise stability*, introduced by Jackson and Wolinsky (1996), and extend it by allowing pairs of banks to determine both loan amounts and interest rates. We then find that these equilibrium networks exhibit a *bipartite* structure, with banks divided into three sets of lenders, borrowers, and inactive banks. This implies that intermediation does not occur in equilibrium. This result is primarily driven by the inclusion of counterparty risk in our model, which sets our work apart from much of the existing literature on the overnight interbank market. Specifically, we account for the risk that lenders may not fully recover their loans and interest. This risk makes transacting through an intermediary more costly without yielding additional benefits; therefore, in the absence of other frictions, intermediation is avoided.

Recent influential papers on the interbank market by Afonso and Lagos (2015) and Bech and Monnet (2016) employ a search and matching framework, well suited for decentralized markets such as the US Fed funds market. Our model, in contrast, captures settings where counterparties are easily observable and search frictions are limited, thereby allowing us to focus on how counterparty risk shapes equilibrium network structures. A leading example of such a transparent market is the Italian Electronic Market for Interbank Deposits (e-MID).[5]

We compare the model's implications with e-MID data. Our analysis indicates that, consistent with the model, strategic intermediation is rare on a daily basis. We also find that intermediation increases when data are aggregated over monthly, quarterly, or yearly frequencies. We argue that this pattern is consistent with a statistical phenomenon arising from the temporal aggregation of our model's equilibria, where banks with more operations trade more frequently in any given period. Thus, we contribute to the literature by demonstrating that temporal aggregation provides a sufficient condition for the appearance of intermediation in lower-frequency data.

Our model yields several policy insights on how central bank instruments shape overnight interbank rates and trading volumes.[6] In particular, we show that narrowing the policy corridor reduces volatility and lowers trading volume, that higher reserve requirements push up overnight rates, and that increases in policy rates can depress overall trading activity. Finally, the model identifies conditions under which high default risk generates a market freeze, and shows how widening the corridor or reducing losses given default can reactivate trading.

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[2] The literature on the consequences of interdependencies for contagion risk in financial markets dates back to Rochet and Tirole (1996) and Allen and Gale (2000). Several network models analyzing systemic risk and shock propagation in financial systems emerged following the global financial crisis (GFC), see, e.g., Gai et al. (2011), Elliott et al. (2014), Acemoglu et al. (2015), and Glasserman and Young (2015). In contrast to these studies, which assume an exogenous network structure of banks' exposures, our model endogenously forms the network and focuses exclusively on the overnight market. Other recent literature on endogenous network formation in financial institutions includes Babus (2016), Babus and Hu (2017), in 't Veld et al. (2020), and Erol and Vohra (2022). While these models target conditions under which intermediation emerges in equilibrium, we base our assumptions on a specific market and do not assume any inherent benefits or costs of intermediation *a priori*.

[3] Borgatti and Everett (2000) formally define the core-periphery structure, originally described informally in various studies on social and organizational networks. Craig and Von Peter (2014) apply this concept to interbank networks and provide empirical support for the tiered structure using the German interbank exposure network. Several other studies (e.g., van Lelyveld and in 't Veld, 2014, Langfield et al., 2014 and Fricke and Lux, 2014) have found core-periphery structures in the banking networks of other countries. Recent theoretical models of core-periphery structures include Hojman and Szeidl (2008), Galeotti and Goyal (2010), and Erol (2019), and, for financial networks specifically, Wang (2016), Farboodi (2023) and Farboodi et al. (2023).

[4] Whitesell's study focuses on the comparison of a corridor system with period-average reserve requirements. Other recent studies in Poole's framework include Berentsen and Monnet (2008), which investigates monetary policy in a corridor system, and Afonso et al. (2019), which develops a unified framework to capture the characteristics of the US Fed funds market before and after the GFC. However, none of these papers employ a network approach or incorporate counterparty risk, both of which are crucial for capturing the features of the overnight interbank market.

[5] E-MID is a major platform for bilateral transactions among banks in the Euro area. The market operates as an electronic order book, where any participating bank can select an existing quote or place a new order. Typically, the identities of the banks are shown in the order book or, if not, they are revealed to a counterparty before a transaction is executed.

[6] This relates to an extensive literature on banking regulations; see, e.g., Afonso et al. (2011), Heider et al. (2015), Erol and Ordoñez (2017), and Blasques et al. (2018).

The rest of the paper is organized as follows. In Section 2, we introduce the model and present our equilibrium concept of pairwise stability. Section 3 characterizes the equilibria, deriving the key result of a bipartite equilibrium network. Section 4 shows how temporal aggregation of equilibrium networks generates apparent intermediation and compares the model with e-MID data. Section 5 discusses the central bank's objectives and monetary policy tools through the lens of our framework. Section 6 concludes. Proofs of all results are collected in Appendix A. Appendix B discusses a special case in which interest rates are set competitively. Appendix C demonstrates the versatility of our framework by presenting two model extensions in which intermediation arises in equilibrium. Appendix D provides comparative statics analyses for policy implications.

2. Model

Consider a finite set $\mathcal{N}$ of banks and the central bank (CB). The CB establishes the reserve holding requirement at the end of each day for every bank $i \in \mathcal{N}$, denoted by T_i. If bank i's reserve holdings at the end of the day are lower than T_i, it must borrow the difference at a *penalty rate* r^p. If, instead, reserves are in excess of the required level, the bank i deposits any excess reserves with the CB at *deposit rate* r^d. The interest rates r^p and r^d with $r^p > r^d$ efficiently establish the upper and lower bounds for the interbank interest rates and are called the ceiling and floor in the so-called corridor system.[7]

We model bank operations as follows. Before the overnight market opens, each bank predicts its net cash position at the end of the day. The expected reserves, without overnight interbank borrowing or lending, are denoted as c_i^0. Given these *initial* expected reserves, banks access the decentralized interbank market to lend or borrow overnight funds, optimizing their expected profit by adjusting their reserves. Let c_i denote this adjusted position, which we refer to as *interim expected reserves*, i.e., the expected reserves after interbank lending or borrowing. However, reserves remain uncertain, and the *actual* reserves are given by $c_i + \varepsilon_i$, where ε_i is the *reserve prediction error*, a realization of a random variable that follows a continuous distribution with mean 0 and a strictly monotonic cumulative distribution function (CDF) F_i.

Formally, the model consists of three periods. In period 0, banks learn their initial expected reserves, c_i^0. In period 1, the only period during which decisions are made, they participate in the interbank market and determine their interim expected reserves, c_i. In period 2, uncertainty regarding actual reserves is resolved as ε_i is realized for each bank. This error accounts for all transactions and other discrepancies that were unknown in period 0 when initial expected reserves were determined. At this stage, a bank either deposits excess funds with the CB or borrows from it to meet the minimum reserve requirements. Additionally, banks pay interest on overnight transactions and may default, as explained later. Thus, all payoffs are realized.[8]

Our main focus is the interbank market for overnight funds. In this market, any pair of banks may enter into a bilateral agreement, where, say, bank i agrees to lend bank j a quantity $\ell_{ij} > 0$ at an interest rate r_{ij}.[9] We introduce a *lending matrix* $\mathbf{L} = \{\ell_{ij}\}_{i,j\in\mathcal{N}}$ with $\ell_{ij} \in \mathbb{R}_+$ and $\ell_{ii} = 0$ for all i. Each positive element of $\mathbf{L}$ corresponds to a loan ℓ_{ij} with an associated interest rate r_{ij}. Together, the lending matrix and the corresponding interest rate matrix induce a two-layered network $g = (\mathbf{L}, \mathbf{r})$.

Given network g, each bank i's interim expected reserves are

$$c_i = c_i^0 - \sum\nolimits_{k\in\mathcal{N}} \ell_{ik} + \sum\nolimits_{m\in\mathcal{N}} \ell_{mi}\,, \tag{1}$$

where the first sum is over all borrowers of i and the second sum is over all lenders to i. Once the interbank market closes, uncertainty regarding actual reserve holdings, captured by the reserve prediction error, is resolved. Thus, the actual reserves of bank i are $c_i + \varepsilon_i$.

The expected payoff from bank i's overnight position in the CB is

$$\pi_i^{\mathrm{CB}} = \int_{-\infty}^{T_i - c_i} \big((c_i + \varepsilon_i) - T_i\big) r^p \mathrm{d}F_i(\varepsilon_i) + \int_{T_i - c_i}^{\infty} \big((c_i + \varepsilon_i) - T_i\big) r^d \mathrm{d}F_i(\varepsilon_i)\,. \tag{2}$$

The first term represents the expected loss from failing to meet the reserve requirement, while the second term represents the expected gain from depositing excess reserves with the CB.

Banks participate in the interbank market to adjust their reserves and improve their expected profits π^{CB} given by (2). In this market, borrowing banks bear interest expenses, while lending banks earn interest but face counterparty default risk. The overnight interbank market is unsecured, meaning that with some (small) probability, a borrower may default, failing to repay its lender(s). We

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[7] Reserve requirements is one of the monetary policy instruments. Its exact implementation differs by country. Australia, Canada, Hong Kong, Sweden, and the UK do not have explicit reserve requirements, but all banks still need to balance their holdings (that is, every night their accounts with the CB must remain non-negative, so that $T_i = 0$). In addition, many central banks, such as the Federal Reserve (the Fed) and the European Central Bank (ECB), require banks to hold an *average amount of reserves* over the maintenance period. In such cases, our model is most applicable on the settlement day, that is, the last day of the maintenance period. The target interest rate is the primary monetary policy instrument. The corridor system is now used by most central banks, with the target rate set in the middle of the corridor. The width of the corridor, the difference between the penalty and deposit rates, is typically kept stable but can also be adjusted by the CB. For example, before the 2007-2008 GFC, the ECB maintained the width of the corridor at 200 basis points. Due to zero lower bound, the ECB reduced the width to 50 basis points by 2014. When negative deposit interest rates were introduced, the width increased back to 75 basis points. Our model allows for negative deposit interest rate.

[8] In practice, period 0 corresponds to the late afternoon, about two hours before the end of the operating day. At this point, each bank has a list of pending transactions, which are used to determine expected reserve holdings. Period 1 takes place from this moment until the end of operations. Period 2 occurs after operations are closed and interest payments are settled, aligning with the start of the next operating day.

[9] We do not *a priori* rule out that i and j have multiple agreements, e.g., where i lends to j and simultaneously borrows from j. However, we later prove that this cannot occur in equilibrium.

focus exclusively on the overnight market and do not model the causes of defaults, assuming that they originate outside the interbank market.[10] We make the following assumption:

Assumption 1. Each bank i has an exogenous default probability $q_i \in [0, 1)$, which is common knowledge among all banks in the market.

Since defaults originate outside the interbank market, a bank's probability of default is not affected by overnight market operations. The assumption that default probabilities are common knowledge is motivated by the fact that banks in this market know their counterparties' identities.[11]

The next assumption addresses the consequences of default in the interbank market.

Assumption 2. If a bank defaults, its lenders will *not* recover a fraction $\gamma \in [0, 1]$ of their loans, including both principal and interest. Additionally, its borrowers must repay their full loans to a default administrator.

With this assumption, the expected profit from the transactions on the interbank market of bank i, which has not defaulted, is given by

$$\begin{aligned}\pi_i^{\mathrm{IM}} &= \underbrace{\sum_{k\in\mathcal{N}} r_{ik}\ell_{ik}\big((1-q_k)+(1-\gamma)q_k\big)}_{\text{expected interest fromrepaying borrowers}} \quad - \quad \underbrace{\sum_{k\in\mathcal{N}} \ell_{ik}\gamma q_k}_{\text{expected loss of principal due to borrowers at default}} \quad - \quad \underbrace{\sum_{m\in\mathcal{N}} \ell_{mi} r_{mi}}_{\text{interest due tolenders}} \\ &= \sum_{k\in\mathcal{N}} r_{ik}\ell_{ik}(1-\gamma q_k) - \sum_{k\in\mathcal{N}} \ell_{ik}\gamma q_k - \sum_{m\in\mathcal{N}} \ell_{mi} r_{mi}\,. \end{aligned} \tag{3}$$

Assumption 2 stresses that the lenders face a risk on the interbank unsecured market. The parameter γ is known as the *loss given default*, while $\mathrm{EL}_k \equiv \gamma q_k$ is the *expected loss rate.*[12]

Remark 1. Under a more general interpretation of the model, the revenue from lending by i to k is the interest rate collected, $r_{ik}\ell_{ik}$, while the cost, fully borne by the lender, is proportional to the total loan including the interest, $\mu_k \ell_{ik}(1 + r_{ik})$. In this interpretation, the expression in (3) still holds, but the expected loss rate γq_k is replaced by the marginal cost μ_k. This marginal cost may arise not only from the expected loss but also from other factors, e.g., due diligence, clearinghouse commission, and other associated costs.

The total expected profit that bank i derives from the overnight market is

$$\pi_i = \pi_i^{\mathrm{CB}} + \pi_i^{\mathrm{IM}}\,, \tag{4}$$

where the two terms are given by (2) and (3). Bank i receives this profit only when it is not in default (i.e., with probability $1 - q_i$). Since defaults are exogenous to the overnight market operations, we assume that the objective of bank i is to maximize π_i in (4).[13]

Maximizing (4) involves the following trade-offs. When bank i borrows funds in the interbank market, it increases its probability of meeting the reserve requirements (thereby increasing π_i^{CB}), but at the cost of paying interest on the loans, which decreases π_i^{IM}. The trade-off is more complex when bank i lends funds, as it reduces its probability of meeting the reserve requirements (thereby decreasing π^{CB}), but may either increase or decrease the net cash flow from the borrowers (the first two terms in (3)), depending on the interest rate and the probability of counterparty default.

To summarize, banks participate in the interbank market and form bilateral agreements based on several exogenous variables: their initial expected reserves c_i^0, probabilities of default q_i, loss given default γ, the distribution of reserve shocks characterized by the CDF F_i, reserve requirements T_i, the deposit rate r^d, and the penalty rate r^p. These agreements collectively form the endogenous network g. For this network g, we can calculate each bank's interim expected reserves $c_i(g)$ using (1), and the expected payoff $\pi_i(g)$ by applying (2)–(4).

2.1. Pairwise stable trading equilibrium networks

To study the equilibrium networks of the interbank market, we focus on the standard notion of "pairwise stable" networks, as introduced by Jackson and Wolinsky (1996). However, in our setting, every link has *three* payoff-relevant variables: the direction of the loan, the loan amount, and the interest rate. Therefore, we adapt the concept of pairwise stability and propose the following definition.

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[10] A bank may default in the overnight market due to issues elsewhere in its balance sheet, such as a run on deposits, financial asset devaluation, or nonperforming loans unrelated to overnight transactions.

[11] In practice, banks actively monitor each other (Allen, 2002). Some monitoring costs are incurred regardless of whether a bank participates in the market on a given day, so we do not explicitly model these costs. We assume that banks generally agree on their peers' default probabilities, q_i. However, periods of heightened uncertainty may lead to heterogeneous beliefs due to asymmetric information and varying monitoring efforts. We discuss this case in Appendix C.2. Finally, note that we do not assume that the CB knows the banks' default probabilities.

[12] Under the Foundation Internal Ratings-Based (IRB) approach introduced in the Basel II Accord, the loss given default γ is set at 0.45 for senior unsecured exposures, including overnight interbank loans.

[13] This assumption implies risk neutrality of banks. Risk neutrality is a common assumption in the economic literature; see, for example, Assumption A4 in Greenwald and Stiglitz (1993), or Acemoglu et al. (2020). Risk neutrality implies that correlations of defaults of different borrowers do not enter the optimization problem of a lender. Therefore, in our setting, lenders do not have motives to diversify over several borrowers.

Definition 1. A network g associated with the lending matrix $\mathbf{L}$ is a **pairwise stable trading equilibrium (PSTE)** if the following conditions are met:

(1) no banks i and j with $\ell_{ij} = 0$ can find an agreement $(\tilde{\ell}_{ij}, \tilde{r}_{ij})$ with $\tilde{\ell}_{ij} > 0$ that would make one bank strictly better off without making the other bank strictly worse off;
(2) no single bank i can remove an existing link in which it is involved (that is, for any $k \in \mathcal{N}$ with $\ell_{ik} > 0$ and for any $m \in \mathcal{N}$ with $\ell_{mi} > 0$) and be strictly better off;
(3) no pair of banks i and j with a link $\ell_{ij} > 0$ can change their agreement to $(\tilde{\ell}_{ij}, \tilde{r}_{ij})$ in a way that would make bank i strictly better off without making bank j strictly worse off.

According to this definition, as is standard for pairwise stability, if any bank benefits from either creating a new link (without making a counter-party worse off) or unilaterally removing an existing link, the network is not a PSTE. An additional condition *(3)* specifies that link creation follows a bargaining process between pairs of banks, ensuring that, given the pattern of trade on all other links, the gains from trade are fully realized.

Pairwise stability is a relatively general (and therefore weak) concept of network stability, as it does not preclude profitable deviations that require coordinated changes involving more than one link or more than two players. This contrasts with alternative concepts such as strong (Dutta and Mutuswami, 1997) or bilateral (Goyal and Vega-Redondo, 2007) stability. Nevertheless, as we will see, this general notion is sufficient to provide a rather precise characterization of trading in our setting. Furthermore, given the bilateral nature of interbank agreements in this market, which operates through a transparent order book, this concept appears to be the most suitable.

3. Characterization of the PSTE networks

The goal of this section is to characterize the PSTE. To achieve this, we first introduce an important quantity, the *expected marginal rate* of banks with the CB, in Section 3.1. Then, in Section 3.2, we examine the incentives of a pair of banks to establish a link, given that the rest of the network is fixed. Finally, Section 3.3 identifies the network types compatible with the PSTE, establishes necessary and sufficient conditions for its existence, and discusses which variables are uniquely determined and which are not.

3.1. Expected marginal rate

Let us define the *expected marginal rate* of bank i's transaction with the CB as

$$\rho_i \equiv \frac{\partial \pi_i^{\mathrm{CB}}}{\partial c_i} = r^p\, F_i(T_i - c_i) + r^d \big(1 - F_i(T_i - c_i)\big) = r^d + (r^p - r^d)\, F_i(T_i - c_i)\,. \tag{5}$$

The only endogenous variable affecting ρ_i is the level of expected interim reserves, c_i, which itself depends on the entire interbank network. In the absence of interbank trade, when the network is empty, $c_i = c_i^0$, which represents the initial expected reserves. We denote the corresponding initial expected marginal rate as ρ_i^0. When bank i lends to other banks, its reserves fall and the expected profit from interactions with CB drops marginally by ρ_i. Conversely, if bank i borrows funds from the interbank market, both its reserves and the expected profit from the CB increase. The following lemma summarizes the elementary properties of the expected marginal rate.

Lemma 1. *Consider an arbitrary (not necessarily equilibrium) network g and bank i. Then the expected marginal rate of bank i, $\rho_i(g) = \rho_i$, has the following properties:*

1. *ρ_i is a strictly decreasing function of the expected interim reserves c_i.*
2. *$\rho_i \to r^p$ when $c_i \to -\infty$ and $\rho_i \to r^d$ when $c_i \to +\infty$, so that $r^d < \rho_i < r^p$.*

Fig. 1 (left panel) illustrates the expected marginal rate as a function of the expected interim reserves, given a specific distribution of the reserve prediction error and specific reserve requirements. Banks i and j differ in their initial reserve predictions, with $c_i^0 > c_j^0$, leading to different initial expected marginal rates, namely, $\rho_i^0 < \rho_j^0$.

3.2. Bilateral trading incentives

We analyze the incentives for two individual banks to establish a lending link. Given a network g and a pair of banks i and j, let $g + ij$ denote the network obtained by adding a link where bank i lends an amount ℓ to bank j at an interest rate r. To assess the impact of this new link, we define the changes in the expected profits for the lender and borrower as: $\Delta_i^{g+ij}(\ell, r) = \pi_i(g + ij) - \pi_i(g)$, and $\Delta_j^{g+ij}(\ell, r) = \pi_j(g + ij) - \pi_j(g)$, respectively. The *feasibility sets* of i as a lender to j and of j as a borrower from i, are defined as

$$F^{\mathrm{L}}_{i\to j}(g) = \left\{(\ell, r) \,:\, \ell > 0, \Delta_i^{g+ij}(\ell, r) \geq 0\right\}, \quad F^{\mathrm{B}}_{j\leftarrow i}(g) = \left\{(\ell, r) \,:\, \ell > 0, \Delta_j^{g+ij}(\ell, r) \geq 0\right\}.$$

They contain all loans (ℓ, r) that make lender i and borrower j, respectively, better off by forming a lending relationship. Finally, let $\rho_i(0)$ and $\rho_j(0)$ denote the expected marginal rates of banks i and j when $\ell = 0$. The feasibility sets can be characterized as follows.

Lemma 2. *Consider a network g, where bank i considers lending reserves to bank j. Then:*

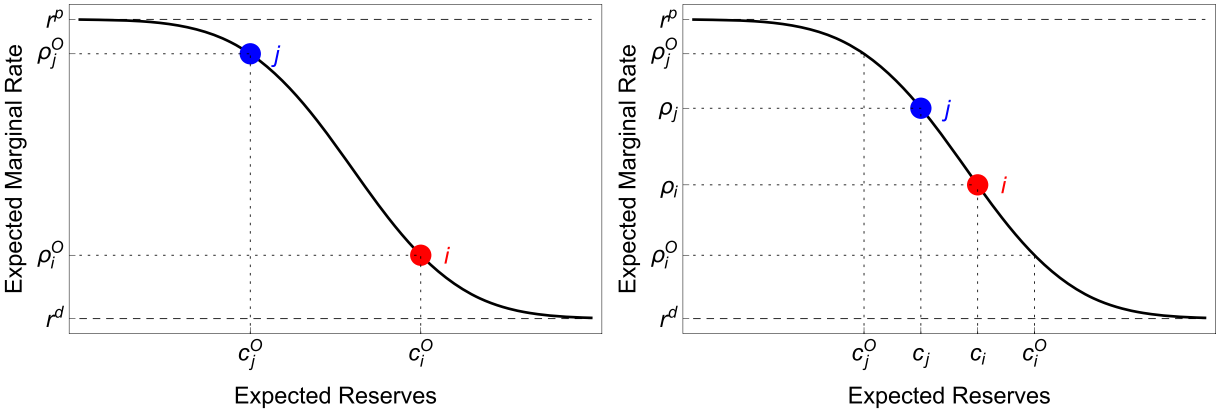

Fig. 1. The expected marginal rates of two banks before (*left*) and after (*right*) interbank trade. In equilibrium, bank i lends reserves to bank j.

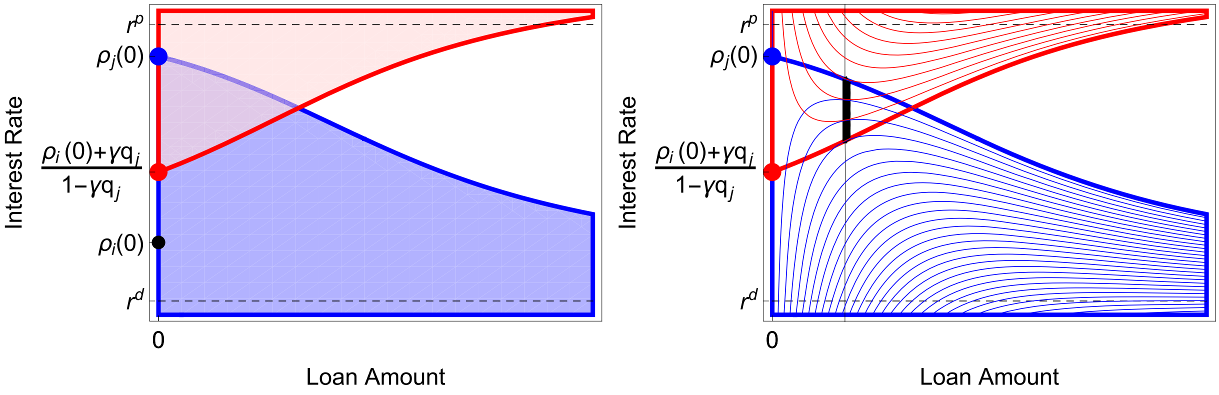

Fig. 2. *Left:* Feasibility sets of a lender (red) and a borrower (blue). *Right:* Indifference curves of the lender and the borrower with the contract curve (vertical solid black line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

1. *the feasibility set of the lender is the epigraph of the function* $h^L : \mathbb{R}^+ \to \mathbb{R}$,

   $$F^L_{i\to j}(g) = \left\{(\ell, r) : \ell > 0, r \geq h^L(\ell)\right\}.$$

   The function h^L *is continuous, strictly increasing, and such that*

   $$h^L(0) = \frac{\rho_i(0) + \gamma q_j}{1 - \gamma q_j}. \tag{6}$$

2. *the feasibility set of the borrower is the hypograph of the function* $h^B : \mathbb{R}^+ \to \mathbb{R}$,

   $$F^B_{j\leftarrow i}(g) = \left\{(\ell, r) : \ell > 0, r \leq h^B(\ell)\right\}.$$

   The function h^B *is continuous, strictly decreasing, and such that*

   $$h^B(0) = \rho_j(0). \tag{7}$$

Proof. See Appendix A.2. ☐

The results of Lemma 2 are illustrated in the left panel of Fig. 2. The feasibility sets of a lender i and a borrower j are shown in the (ℓ, r) coordinate space, with the rest of the network fixed. On the vertical axis, the higher (blue) dot represents the highest interest rate that the borrower can offer, as given by $h^{\mathrm{B}}(0)$ in (7). The middle (red) dot represents the lowest interest rate at which the lender is willing to provide a loan, as given by $h^{\mathrm{L}}(0)$ in (6). The lender's lowest acceptable rate increases with the borrower's default risk q_j. In the absence of default risk, this rate would be $\rho_i(0)$, as indicated by the lower (black) dot. In the example, the feasibility sets intersect, revealing a region of (ℓ, r) combinations where both banks would benefit from forming a link.

It follows from Lemma 2 that the feasibility sets of a borrower and a lender intersect if and only if $h^{\mathrm{B}}(0) > h^{\mathrm{L}}(0)$. Rewriting this inequality, we obtain the following result:

Proposition 1. *Consider two banks* i *and* j *in a network, and denote their expected marginal rates as* $\rho_i(0)$ *and* $\rho_j(0)$*, respectively. A new loan* (ℓ, r) *from* i *to* j *with* $\ell > 0$ *that makes each bank better off exists if and only if* $\rho_i(0) < \rho_j(0) - \gamma q_j(1 + \rho_j(0))$.

Applying this result to the equilibrium network, requirement *(1)* of Definition 1 implies:

Corollary 1. *If, in the PSTE, bank i does not lend to bank j, then their expected marginal rates satisfy:* $\rho_i \geq \rho_j - \gamma q_j(1+\rho_j)$.

To analyze requirement *(2)* of Definition 1, assume that a PSTE network g' contains a link (ℓ_{ij}, r_{ij}). Let $g = g' - ij$ denote the network obtained from g' by deleting this link. The changes in the expected profits for banks i and j when the link is removed are given by $\pi_i(g) - \pi_i(g') = -\Delta_i^{g+ij}(\ell_{ij}, r_{ij})$ and $\pi_j(g) - \pi_j(g') = -\Delta_j^{g+ij}(\ell_{ij}, r_{ij})$. Thus, bank i will not benefit from deleting the link if $\Delta_i^{g+ij}(\ell_{ij}, r_{ij}) \geq 0$, and similarly, bank j will not benefit from deleting the link if $\Delta_j^{g+ij}(\ell_{ij}, r_{ij}) \geq 0$. In the PSTE network, neither bank is willing to remove the link, implying that both inequalities must hold. We conclude that if the PSTE network contains a link from i to j, this link must lie within the intersection of the feasibility sets of lender i and borrower j in network $g = g' - ij$.

Returning to the left panel of Fig. 2, with the feasibility sets of banks i and j drawn given the rest of the network, Corollary 1 implies that a network without a link from i to j cannot be a PSTE. Moreover, if the network with a link (ℓ_{ij}, r_{ij}) is a PSTE, this link must lie within the intersection of the feasibility sets of i and j.

If i lends to j in the PSTE, the question is where exactly within the intersection of the sets $F^{\mathrm{L}}_{i\to j}$ and $F^{\mathrm{B}}_{j\leftarrow i}$ the link (ℓ_{ij}, r_{ij}) lies. Requirement *(3)* of Definition 1 imposes that, in the PSTE, the banks must be at a Pareto-optimal point. That is, the link must belong to the *contract curve*, where the slopes of the banks' indifference curves coincide. Calculations (see Appendix A.1) show that the expected marginal rates of i and j on the contract curve satisfy the condition $\rho_i = \rho_j - \gamma q_j(1+\rho_j)$. The next result establishes both the existence of loans on the contract curve and uniqueness of the loan amount.

Proposition 2. *Consider a network with banks i and j having a non-empty intersection of the feasibility sets $F^L_{i\to j}$ and $F^B_{j\leftarrow i}$. Then, there exists a loan on the contract curve, with the loan amount ℓ_{ij} defined uniquely.*

Proof. See Appendix A.3. □

The right panel of Fig. 2 shows the lender's and borrower's iso-profit curves, which belong to their respective feasibility sets. As the intersection of the feasibility sets is not empty, the contract curve exists. It is shown by the thick line, which is vertical, indicating the uniqueness of the loan amount but an interval of possible interest rates.

Since the condition for the contract curve must hold for the equilibrium marginal rates at *every* link existing in the PSTE, Proposition 2 implies the following.

Corollary 2. *If in the PSTE bank i lends to bank j, the expected marginal rates of banks satisfy the condition:*

$$\rho_i = \rho_j - \gamma q_j(1+\rho_j)\,. \tag{8}$$

In summary, our analysis of banks' incentives establishes Corollary 2, which characterize the expected marginal rates in any PSTE. Specifically, if bank i lends to bank j in the PSTE, the rates are related through Eq. (8). Conversely, if $\rho_i > \rho_j - \gamma q_j(1+\rho_j)$, then there is no link from i to j in the PSTE.

The condition in (8) indicates that the expected marginal rate of the lender is equal to the *risk-adjusted* expected marginal rate of the borrower. In the absence of expected losses (i.e., if there is no default risk, $q_j = 0$, or if all losses are recovered, $\gamma = 0$), banks i and j would have incentives to establish a link from i to j whenever $\rho_i < \rho_j$. This link would equalize their expected marginal rates. However, the presence of counterparty risk reduces opportunities for interbank trade, as the intersection of the banks' feasibility sets shrinks, and also prevents the equality of expected marginal rates in equilibrium.[14]

In practice, the overnight interbank market allows banks with excess reserves to lend to those with insufficient reserves. In our model, when two banks have initial expected reserves such that $c_i^0 > c_j^0$, they may or may not redistribute reserves from i to j in the equilibrium. The outcome depends on exogenous parameters, including their specific reserve requirements, distribution of their prediction errors, the risk of default and the loss given default. For identical distributions F_i and F_j and equal reserve requirements $T_i = T_j$, the bank with higher expected reserves has a smaller expected marginal rate, meaning that $\rho_i^0 < \rho_j^0$, as shown in the left panel of Fig. 1. Provided that γq_j is sufficiently low, bank i will lend to bank j in the equilibrium, as illustrated in the right panel of Fig. 1.

3.3. Equilibrium network configurations

We now characterize possible network structures in the pairwise stable trading equilibria (PSTE). Recall that the expected loss rate from lending to bank k is denoted as $\mathrm{EL}_k = \gamma q_k$. The next lemma allows us to derive the equilibrium network properties.

Lemma 3. *Assume that $\gamma > 0$. The following three properties hold in any PSTE:*

1 ***No two-cycles:*** *if $\ell_{ij} > 0$, and if either $q_i > 0$ or $q_j > 0$, then $\ell_{ji} = 0$.*
2 ***No intermediaries:*** *there cannot be three banks i, j, k, such that $q_j > 0$, $\ell_{ij} > 0$ and $\ell_{jk} > 0$.*
3 ***Equal ρ for any two lenders:*** *there cannot be four banks i, j, k, h, such that $\ell_{ij} > 0$, $\ell_{kh} > 0$, and $\rho_i \neq \rho_k$.*

Proof. See Section A.4. □

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[14] In the left panel of Fig. 2, the lower bound of the lender's feasibility set shifts upward with higher γq_j. When $\gamma q_j = 0$ (no expected losses), this bound starts at $\rho_i(0)$. Also note that if $\rho_i(0) \geq \rho_j(0)$, then also $\rho_i(0) \geq \rho_j(0) - \gamma q_j(1+\rho_j(0))$, and in this case, the link will not be created *irrespective* of the counterparty risk level.

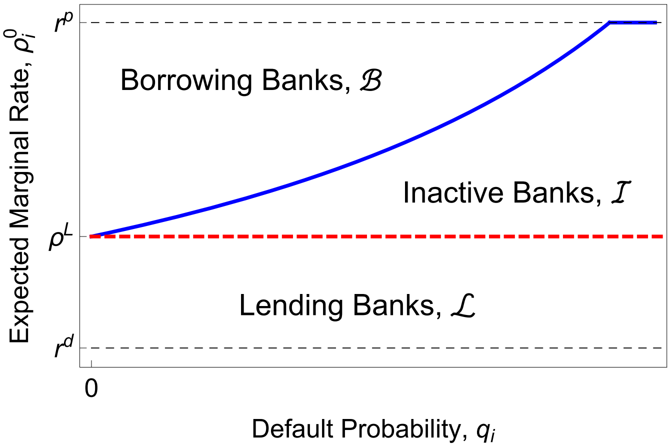

Fig. 3. Partitioning of all banks based on their exogenous characteristics: the probability of default, q_i, and their initial expected marginal rate, ρ_i^0, given the endogenous value of ρ^{L}. Two plotted functions represent the equilibrium expected marginal rates for borrowers (blue, solid) and lenders (red, dashed). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The second property of the lemma is crucial for the network configuration of the PSTE, as it rules out intermediation. To sketch the proof, recall that along any existing link of the equilibrium network, the positive expected loss faced by a lender creates a wedge between the expected marginal rates of the lender and borrower, see Eq. (8). If a chain with intermediation existed in the PSTE, these wedges would accumulate along such a chain, implying that the first and last banks in the chain would have a direct link. However, with this direct link, at least one bank would find it profitable to drop a link from the original chain, leading to a contradiction.

Lemma 3 implies the following result.

Proposition 3. *Assume that $\gamma > 0$ and every bank has a positive default probability. Then, in any PSTE, all banks can be partitioned into three sets: (1) the set of borrowers, $\mathcal{B}$ (banks that borrow reserves from one or several lenders and do not lend); (2) the set of lenders, $\mathcal{L}$ (banks that lend reserves to one or several borrowers and do not borrow); and (3) the set of inactive banks, $\mathcal{I}$ (banks that neither borrow nor lend). The equilibrium expected marginal rate is the same for all lenders.*

Proof. The partitioning part of the statement is a direct consequence of the fact that the PSTE cannot have cycles of any length that follows from Lemma 3, parts 1 and 2. Part 3 implies that all lenders in such PSTE must have the same expected marginal rate. □

Consider any PSTE with interbank trade (if it exists). It follows that all banks, except inactive ones, form a *bipartite* network, with links from the set of lenders to the set of borrowers. Bipartite networks arise in equilibrium due to counterparty risk, which makes intermediation costly. In a sense, the configuration of equilibrium networks reflects banks' incentives by ruling out intermediation that would accumulate default risk costs without providing any additional gains.

Propostion 3 also states that, in equilibrium, all lending banks have the same expected marginal rate, which we denote by ρ^{L}. Given ρ^{L}, the equilibrium expected marginal rate for *every* bank can be found. Specifically, for any bank i, define

$$\rho_i^{\mathrm{B}} = \frac{\rho^{\mathrm{L}} + \gamma q_i}{1 - \gamma q_i}\,. \tag{9}$$

For any borrowing bank j, the quantity ρ_j^{B} is its expected marginal rate, as follows from Eq. (8). All inactive banks, by definition, retain their initial expected marginal rates.

By inverting the definition of the expected marginal rate in Eq. (5), we obtain

$$c_i = T_i - F_i^{-1}\left(\frac{\rho_i - r^d}{r^p - r^d}\right). \tag{10}$$

This formula determines the equilibrium reserves of any bank based on their equilibrium expected marginal rates.

The relatively simple structure of the equilibrium networks we have established allows for a detailed characterization of equilibria. First, we show that, given ρ^{L}, the exact roles of banks in a PSTE are uniquely determined by their initial expected marginal interest rates, ρ_i^0, and default probabilities, q_i.

Corollary 3. *Consider the PSTE with interbank trade and let ρ^L be the expected marginal rate of lenders in this equilibrium. Define ρ_i^B for each bank according to (9). Then, the sets of lenders, inactive banks, and borrowers are*

$$\mathcal{L} = \left\{i \in \mathcal{N} \,:\, \rho_i^0 < \rho^L\right\}, \quad \mathcal{I} = \left\{i \in \mathcal{N} \,:\, \rho^L \le \rho_i^0 \le \rho_i^B\right\}, \quad \mathcal{B} = \left\{i \in \mathcal{N} \,:\, \rho_i^0 > \rho_i^B\right\}.$$

Proof. See Appendix A.5. □

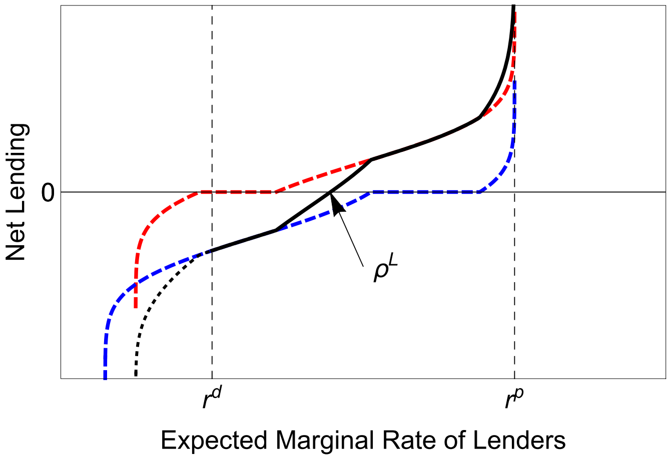

Fig. 4. Functions ℓ_i as defined in Eq. (11) for two banks (dashed curves). Their sum (solid black curve), defined on (r^d, r^p), intersects the horizontal axis at ρ^{L}.

Fig. 3 illustrates this result. The dashed (red) horizontal line shows the value of ρ^{L}, while the thick (blue) curve plots the function ρ_i^{B} as defined in Eq. (9) (bounded by r^p). These two curves partition the space of banks, characterized by pairs (q_i, ρ_i^0), into three sets: $\mathcal{L}$, $\mathcal{I}$, and $\mathcal{B}$. The curves also represent the equilibrium expected marginal rates of lenders and borrowers, respectively. Interbank trade increases the expected marginal rates of all lenders, as they all end up on the horizontal line, and decreases the expected marginal rates of all borrowers, as they all end up on the increasing curve.

Our characterization of the PSTE, including the partitioning illustrated in Fig. 3, depends on the value of ρ^{L}, which is endogenous. We now show how this value can be determined. For any bank i, we define the following real-valued function ℓ_i:

$$\ell_i(\rho) = \begin{cases} F_i^{-1}\left(\dfrac{\dfrac{\rho + \gamma q_i}{1 - \gamma q_i} - r^d}{r^p - r^d}\right) + c_i^0 - T_i & \text{if } \rho \in \left(r^d - \gamma q_i(1 + r^d), \rho_i^0 - \gamma q_i(1 + \rho_i^0)\right) \\ 0 & \text{if } \rho \in \left[\rho_i^0 - \gamma q_i(1 + \rho_i^0), \rho_i^0\right] \\ F_i^{-1}\left(\dfrac{\rho - r^d}{r^p - r^d}\right) + c_i^0 - T_i & \text{if } \rho \in \left(\rho_i^0, r^p\right). \end{cases} \tag{11}$$

The function defined in this piecewise manner allows us to describe the bank's net lending position in the equilibrium, as the next result shows.

Lemma 4. *Expression (11) defines a continuous and monotonically increasing function on the domain $D_i = \left(r^d - \gamma q_i(1 + r^d), r^p\right)$. At the PSTE's ρ^L, its value, $\ell_i(\rho^L)$, represents the equilibrium lending amount (if positive) or borrowing amount (if negative) for bank i.*

Proof. See Appendix A.6. □

Lemma 4 suggests an algorithm for computing the equilibrium expected marginal rate, ρ^L, and total lending (or borrowing) amounts for each bank i in the PSTE. Fig. 4 illustrates the function ℓ_i for two banks as two dashed curves. The solid curve represents their *sum*, defined on the interval (r^d, r^p). Intuitively, given the bipartite network structure, summing the functions ℓ_i across all banks yields the total net lending as a function of the expected marginal rate of lenders. The equilibrium value ρ^{L} is where the total net lending is zero, which corresponds to the point where the sum-function intersects the horizontal axis. With this insight, we can establish our main result, providing a necessary and sufficient condition for the existence of interbank trade in equilibrium.

Theorem 1. *Assume that $\gamma > 0$ and every bank has a positive default probability.*

(i) *If the initial marginal rates are such that $\rho_i^0 \geq \rho_j^0 - \gamma q_j(1 + \rho_j^0)$ for any pair of banks i and j, then there will be no interbank trade in the PSTE. Thus, $\mathcal{N} = \mathcal{I}$.*

(ii) *If there exist two banks i and j with $\rho_i^0 < \rho_j^0 - \gamma q_j(1 + \rho_j^0)$, then there are (possibly multiple) PSTE with interbank trade. The partitioning $\mathcal{N} = \mathcal{L} \cup \mathcal{I} \cup \mathcal{B}$, and banks' expected marginal rates and interim expected reserves are the same in all PSTE.*

Proof. See Appendix A.7. □

According to Theorem 1, the banks' roles and final allocations in equilibrium are uniquely determined. However, when interbank trade exists, there are generally multiple ways in which reserves can be redistributed from lenders to borrowers. As a result, if at least two borrowers and two lenders trade in equilibrium, there are infinitely many PSTE consistent with the final allocations. That is, there are multiple equilibrium networks, though all of them are bipartite. Generically, all active banks form a single connected component.[15]

The equilibrium interest rates along the links are also not uniquely determined. As shown in the right panel of Fig. 2, there is a range of rates consistent with the PSTE for any existing link. Thus, additional assumptions concerning the relative bargaining power of any pair of banks with a link are needed to pin down their interest rate.

One possibility, specified in Appendix B in the form of a *competitive PSTE*, is to assume that if bank i lends to bank j in the PSTE, the interest rate, r_{ij}, is set as if both banks behave competitively. In this equilibrium, $r_{ij} = \rho^{\mathrm{B}}_j$. Intuitively, for each borrower, competition with other equally risky borrowers pushes the rate to the point where the borrower is indifferent between transacting in the interbank market and an outside option, i.e., transacting with the CB. With these borrower-specific rates, lenders are exactly compensated for the counterparty default risk. Referring to Fig. 3, in the competitive PSTE, all interest rates lie on the upper (blue) curve. If the banks are not fully competitive, then for a given loan from i to j, we may observe a higher (if the lender has higher bargaining power) or lower (if the borrower has higher bargaining power) interest rate than ρ^{B}_j. The exact range of the interest rates consistent with PSTE for a given borrower-lender pair depends on their utilities in the absence of such a link, and thus on the realized rates on all other links.[16]

We conclude this section with a comparative statics result showing how exogenous variables influence the equilibrium marginal rate ρ^{L}.

Corollary 4. *Assume that $\gamma > 0$ and that every bank has a positive default probability. Consider the marginal rate of lenders as a function $\rho^{\mathrm{L}}(\gamma, r^d, r^p, q_1, \ldots, q_n, T_1, \ldots, T_n)$, defined for any set of parameter values such that a PSTE with interbank trade exists. This function is continuous in its arguments, strictly decreasing in γ, and strictly increasing in r^d and r^p. Moreover, it is weakly decreasing in q_i and weakly increasing in T_i for every i.*

Proof. See Appendix A.8. ☐

The proof of this result builds on the same geometric intuition as the implicit function theorem but does not assume the differentiability of the F_i functions. This result will be useful in Section 5, where we discuss the CB's policy.

4. Apparent intermediation and core-periphery

The empirical literature on interbank trading networks frequently reports core-periphery structures (Borgatti and Everett, 2000), in which a small number of banks ('core') are highly interconnected, while the remaining banks ('periphery') have relatively few links, primarily to the core banks. Intermediation is therefore channeled through the core. This raises a natural question: can our model, which predicts no intermediation, be reconciled with such empirical findings? Importantly, the empirical literature often relies on regulatory data available at *quarterly* or *annual* frequencies, see Craig and Von Peter (2014) and Craig and Ma (2022) for Germany, Langfield et al. (2014) for the UK, and van Lelyveld and in 't Veld (2014) for the Netherlands. These studies infer networks from banks' bilateral exposures, *aggregating* loans across maturities.

Our model focuses on a narrower setting: we examine an overnight interbank market within a single period. The model is tailored to markets with bilateral trading on a centralized, transparent platform, such as e-MID, which facilitates visibility of transactions.[17] For e-MID, Fricke and Lux (2014) document a core-periphery structure using quarterly data. By contrast, studies using higher-frequency e-MID data show that, on a daily basis, intermediaries, and hence core-periphery structures, are essentially absent (see, for example, Iori et al., 2008; Barucca and Lillo, 2016), consistent with our theoretical prediction. This suggests that core-periphery patterns can emerge as an artifact of temporal aggregation, rather than reflecting the structure of the underlying daily network.

Finger et al. (2013) seek to reconcile these findings by identifying the aggregation frequency that best captures key features of the e-MID trading network. Examining how network statistics change with aggregation, they hypothesize that there exists a "latent" network from which daily links are drawn. Because daily networks contain few links, they argue that at least monthly aggregation is necessary to reveal the underlying latent network. This, in turn, shifts attention to the core-periphery structures observed at lower frequencies rather than to the very limited intermediation in daily networks.

In this section, we show that no assumption of a latent network of preferential relationships is required to generate intermediation and core-periphery structures. Instead, these features arise from bank heterogeneity and temporal aggregation of our PSTEs, and thus become only *apparent* at lower frequencies.[18]

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[15] In any PSTE, total net lending is zero, allowing for a continuum of possible loan distributions across active banks (as soon as there are at least two borrowers and two lenders). For a disconnected network to emerge in equilibrium, it must be feasible in the first place. This requires that the active banks include a proper subset where the total net lending is also zero. If initial expected reserves are drawn from a continuous distribution, the probability of this occurring is zero. Even if this event were to occur, distributing loans in such a way that the subset remains separate would require satisfying a finite number of restrictions over a continuum of possible ways to redistribute reserves.

[16] It follows that in the competitive PSTE, the variability of the interest rates in the market is exclusively due to the variability in the banks' default probabilities. But in a general PSTE, there is additional variability caused by bargaining processes on the established links. The range on each link is obtained from the feasibility sets, as shown in the right panel of Fig. 2.

[17] See footnote 5 and Iori et al. (2008) for a detailed institutional description of the e-MID market.

[18] In discussing Theorem 1, we noted that our framework admits a multiplicity of equilibrium networks (each bilateral) consistent with uniquely determined equilibrium reserves. Thus, our model does not rule out the existence of a latent structure. If such a structure exists, the network

Before presenting the main results on temporal aggregation in Section 4.2 and comparing them with data in Section 4.3, we introduce additional assumptions that allow us to derive closed-form solutions and gain insights into the PSTE.

4.1. *Stochastic liquidity and large market limit*

In Section 3, we fully characterized the PSTE and provided an algorithmic procedure to determine all equilibrium quantities (using Lemma 4 and Eqs. (9) and (10)). This analysis assumes that the banks' initial expected reserves, c_i^0, are given. To prepare for a multi-period extension of the framework, we model bank i's initial expected *shortfall*, $T_i - c_i^0$, as a realization of a continuous random variable with CDF Φ_i. Importantly, this extension does not change the preceding analysis: realizations of these random variables determine the initial expected reserves, so all previous results continue to apply.

When the shortfalls are random, the initial expected marginal rate becomes a random variable as well:

$$\rho_i^0 = r^d + (r^p - r^d)F_i(T_i - c_i^0) = r^d + (r^p - r^d)\psi_i\,,$$

where ψ_i is a random variable with a CDF given by $\Phi_i \circ F_i^{-1}$, the composition of Φ_i and the inverse of F_i. The random variable ψ_i takes values in the interval $[0,1]$, and, consequently, ρ_i^0 lies within $[r^d, r^p]$.

By Theorem 1, for a given realization of ψ_i for the set of n banks $\mathcal{N}_n$, there exists a PSTE with interbank trade if and only if

$$\min_{i\in\mathcal{N}_n}\left\{r^d + (r^p - r^d)\psi_i\right\} < \max_{j\in\mathcal{N}_n}\left\{(1-\gamma q_j)\big(r^d + (r^p - r^d)\psi_j\big) - \gamma q_j\right\}. \tag{12}$$

Intuitively, this inequality may become less restrictive when the minimum and maximum are taken over a larger set of banks.

To gain further insights, we consider the case where the number of banks grows without bound, which we refer to as the *large market limit* (LML). For analytical tractability, we further restrict the analysis to the case where all banks have the same default probability, denoted by q. When $q = 0$, inequality (12) is trivially satisfied for any set of banks. However, when $q > 0$, the possibility of no interbank trade, i.e., a *market freeze*, arises.

Proposition 4. *Assume that all banks have the same default probability, q, and ψ_i's are realizations of independent random variables having a full support on* $[0,1]$. *Let*

$$q^c = \frac{1}{\gamma}\,\frac{r^p - r^d}{1 + r^p}\,. \tag{13}$$

If $q < q^c$, then in the LML the PSTE with interbank trade exists almost surely. If $q > q^c$, then for any n, in any PSTE there will be no interbank market.

Proof. See Appendix A.9. □

When the q_i's differ across banks, a sufficient (but not necessary) condition for trade in the LML is that their maximum is less than q^c. The threshold value q^c in (13) depends on the width of the corridor, the penalty rate and the loss given default.

The next result derives the equilibrium expected marginal rate of lenders explicitly in the LML, provided that a market freeze does not occur. To obtain an analytical expression for this rate, we assume that the distributions F_i and Φ_i are identical across all banks and symmetric around a mean of zero.[19]

Proposition 5. *Assume that $\gamma > 0$, and that all banks have the same default probability, $q > 0$; the same distribution of reserve prediction errors, F; and the same distribution of the initial expected shortfall, Φ. Furthermore, assume that both F and Φ are symmetric distributions with a mean of zero.*

Let ρ_n^L denote the equilibrium expected marginal rate of lenders in the PSTE (when trade exists) and assume that the trade exists in the LML. Then, ρ_n^L has a subsequence that converges, as $n \to \infty$, to

$$\rho^L = \frac{(r^p + r^d)(1-\gamma q) - \gamma q}{2 - \gamma q}\,. \tag{14}$$

Proof. See Appendix A.10. □

This result relates the equilibrium lenders' marginal rate to the key variables of the model. To illustrate this result, we fix the interest rates at their competitive values, i.e., we consider the competitive PSTE defined in Appendix B. Recall from Section 3.3 (see also Eq. (B.10)) that in this case, the interest rates of all loans are given by $(\rho^{\mathrm{L}} + \gamma q)/(1-\gamma q)$. The two solid lines in Fig. 5 illustrate how the lenders' marginal rate (lower red line) and the competitive interest rate (upper blue line) vary with q in the LML. When $q = 0$, both rates lie at the midpoint of the corridor. As q increases, the loan rate rises, reflecting the need to compensate lenders for the higher default risk of borrowers. This, in turn, causes more banks to become inactive, until the market ultimately freezes at $q = q^c$. As lenders provide less volume, they are left with larger reserves in equilibrium, which implies a lower marginal rate, ρ^{L}.[20]

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realized in practice may be sampled from this structure. However, we do not consider this to be the primary driver, as it would require an additional assumption that some links are more preferred for banks than others. That said, during periods of heightened uncertainty about counterparty risk, preferential relationships may play a more significant role (Temizsoy et al., 2015).

[19] By definition, a distribution with CDF G is *symmetric* if there exists a constant v such that $G(v-x) + G(v+x) = 1$ for any x. If G is symmetric, the constant v from this definition is the mean of the distribution. For the PDF g, this definition implies that $g(v-x) = g(v+x)$.

[20] While this discussion focuses on the competitive PSTE, recall that the rates derived through bilateral bargaining in any PSTE lie in an interval around points on the upper blue curve in Fig. 5.

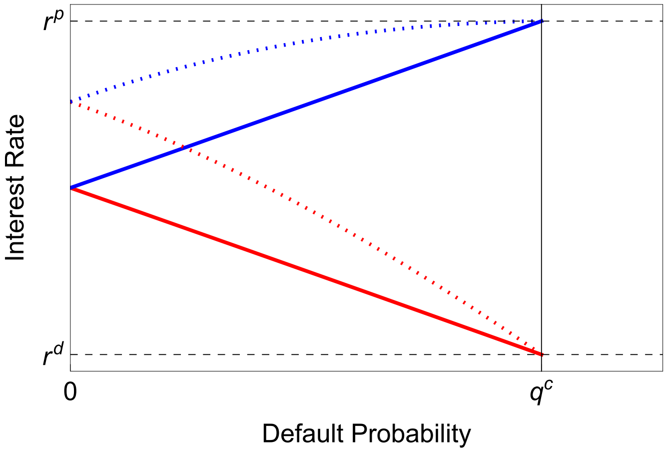

Fig. 5. The lenders' marginal rate in the PSTE (red) and the rate on loans in the competitive PSTE (blue) as functions of the banks' default probability in the LML. The solid lines correspond to the case of the zero mean of the shortfall distribution Φ. The dotted curves correspond to the case of the positive mean of Φ (i.e., liquidity shortage). The vertical black line shows the threshold value, q^c, above which the market freezes. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Propostion 5 can be generalized in different directions (see Appendix A.10, after the proof). For example, when the symmetric shortfall distribution Φ has a non-zero mean μ, the equilibrium ρ^{L} in the LML increases with μ. The dotted curves in Fig. 5 illustrate an example with $\mu > 0$, that is, when there is an overall liquidity shortage in the market. In this case, both the lenders' marginal rates and the rate on loans in the competitive PSTE increase relative to the case of $\mu = 0$.

Inverting the relation derived in Proposition 5, the default probability of the banks can be inferred from the interest rate charged on the interbank loans. Specifically,

$$q = \frac{1}{\gamma}\frac{r^p + r^d - 2\rho^{\mathrm{L}}}{1 + r^p + r^d - \rho^{\mathrm{L}}} = \frac{1}{\gamma}\frac{2r - r^d - r^p}{1 + r}, \tag{15}$$

where the last equality holds in the competitive PSTE with the interest rate on loans r.

4.2. Heterogeneous banks and temporal aggregation

Suppose that banks interact in the overnight market over multiple time periods, say, days. Each bank i begins each day with an expected shortfall $T_i - c_i^0$, drawn from the distribution Φ_i. The draws are independent across banks and over time. Given the realization of the expected shortfalls, a PSTE of the baseline model is realized.[21]

Let us assume that banks' distributions Φ_i, all centered at zero, differ in their scale.[22] This setup reflects a market with balanced overall liquidity, where variation in banks' expected shortfall distributions arise from systematic differences in the *scale of operations*. Indeed, since a bank's initial expected reserves, c_i^0, build up from the daily payment flows of its clients, we may expect a positive relationship between the scale of operations and the scale of the expected shortfall distribution.[23] To be specific, we assume that there are several types of banks, each characterized by its own distribution of expected shortfall. We can now apply the LML, as introduced in Section 4.1, and obtain the following.

Proposition 6. *Assume the conditions of Proposition 5, except that the distributions of expected shortfalls are heterogeneous. Suppose there are K bank types, indexed by k, with a fixed fraction f_k of banks of type k, and with type-k initial shortfalls following the symmetric, zero-mean distribution Φ_k. Then, under the LML, the result of Proposition 5 holds, with the same expression for ρ^L as in (14).*

Let k and k' be two types of banks such that Φ_k is more peaked than $\Phi_{k'}$. Then, in the PSTE under the LML, the proportion of banks that trade among the type-k banks is smaller than the proportion of banks that trade among the type-k' banks.

Proof. See Appendix A.11. ☐

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[21] Since the draws of the banks are independent over time and the default probabilities are exogenous, there is no loss of generality in solving for the PSTE on a daily basis.

[22] In the next proposition, we will use a technical concept of *relative peakedness*, as introduced in Birnbaum (1948). Let X and Y be random variables with mean 0. We say that X is *more peaked* than Y if, for all $z > 0$, $P(|X| \leq z) > P(|Y| \leq z)$. While relative peakedness is closely related to a mean-preserving spread, it additionally requires that the probability mass is redistributed in a monotonic manner. In many cases (e.g., for normal distributions), the ordering by peakedness is determined solely by the relative scale (as expressed by the standard deviation): a smaller scale indicates a more peaked distribution.

[23] To illustrate, consider the *sum* of a bank's daily payment flows, $\sum_{i=1}^{N} \zeta_i$. Assuming that transactions are IID with standard deviation σ_ζ, the standard deviation of the sum is $\sqrt{N}\sigma_\zeta$, increasing with the number of transactions N.

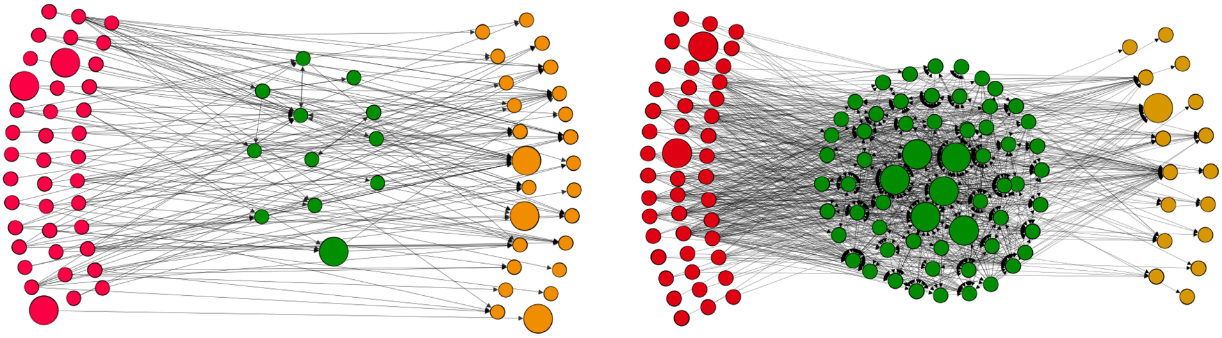

Fig. 6. Network of lending and borrowing on the e-MID market: Transactions over one day (left panel) and one month (right panel). Arrows indicate the direction of the loan. Red nodes represent lenders only, yellow nodes represent borrowers only, and green nodes represent intermediaries. Node sizes reflect the relative assets of the banks. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Recall that in the PSTE of our model (corresponding to a day), a bank can act as a lender, a borrower, or remain inactive. These roles can shift day by day, as any new realization of initial shortfalls will change the PSTE. Proposition 6 focuses on comparing the likelihood of trading for different types of banks on a given day, depending on the relative peakedness of their shortfall distributions. It shows that banks with a higher peakedness (and thus a smaller scale, see footnote 22) in their shortfall distribution are less likely to trade (either as borrowers or as lenders) than banks with a lower peakedness. Using an interpretation of the heterogeneity based on the scale of operations, one can refer to the k and k' types from the proposition as ‘smaller-scale’ and ‘larger-scale’ banks, respectively.

It follows that, when aggregated over many days, larger-scale banks are expected to be active more frequently than smaller-scale banks. Intuitively, this is because they are more likely to experience larger absolute values of initial expected shortfalls compared to smaller-scale banks. Banks with initial expected shortfalls close to zero will be inactive in a market with balanced liquidity (due to counterparty risk). In contrast, banks with high expected shortfalls or surpluses will be active in the PSTE, either transacting with a counterparty capable of matching their needs (likely a larger-scale bank) or establishing multiple links, including those with smaller-scale counterparties.

Therefore, aggregating interbank transactions over multiple days reveals a pattern in which larger-scale banks frequently trade with one another and with multiple smaller-scale banks, although the specific smaller-scale banks vary from day to day because more of them remain inactive. Meanwhile, smaller-scale banks trade more often with larger-scale banks. The resulting aggregate structure will resemble a core-periphery network, where larger-scale banks form the core and smaller-scale banks are on the periphery. Larger-scale banks are more likely to appear to function as intermediaries, but this is only due to aggregation and the fact that the same larger-scale banks show up repeatedly on both sides of the market over time.

4.3. *Empirical support from E-MID market*

We now turn to an empirical analysis of the e-MID market to confront our theoretical predictions on network structure with data. To illustrate the market’s network structure, Fig. 6 displays the interbank trading network for a single day (left panel) and for transactions aggregated over one month (right panel). Nodes represent banks and are color-coded as lenders only (red), borrowers only (yellow), and intermediaries (green). We observe that while some banks act as intermediaries on a daily basis, they represent only a small fraction of the network. In contrast, the number of intermediaries increases substantially when we consider the monthly aggregation.

The banks in the e-MID dataset are anonymized, with each bank assigned a unique ID. However, the data still allows for distinguishing between domestic Italian and foreign banks. Domestic banks are further classified into five categories based on their total assets: major, large, medium, small, and minor.[24] The sizes of the nodes in Fig. 6 reflect this categorization, with the larger nodes representing the major and large categories, and the smaller nodes representing the remaining categories.

Table 1 reports key statistics for the activity and intermediation of banks at a *daily frequency*, broken down by this classification and averaged over the entire sample period. Our dataset covers the period from January 1999 to December 2008.[25]

When bringing the data to support our temporal aggregation result of Section 4.2, it is natural to assume that banks with larger asset holdings tend to operate on a larger scale. However, Proposition 6 relies on the variability in the scales of shortfall distributions across banks. We therefore estimate the scale parameters. Column two reports the standard deviation of banks’ net positions at the

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[24] Specifically, major banks have total assets exceeding EUR 60bn; large banks have assets between EUR 26bn and EUR 60bn; medium banks have assets between EUR 9bn and EUR 26bn; small banks have assets between EUR 1.3bn and EUR 9bn; and minor banks have assets below EUR 1.3bn. See Kapar et al. (2020) for more details about the dataset. As the foreign banks are not categorized by their assets, we report them in the tables, but do not discuss them in details.

[25] In the beginning of 2009, an alternative, additional interbank market was created (see footnote 37). We do not observe data for the MIC, and since banks may operate in both markets, we limit our analysis to the period prior to 2009.

Table 1
Activity and intermediation on the e-MID by bank type at a daily frequency.

Bank	Net Pos	Active		Intermediary			
type	std dev	ave #	%	ave #	%	% vol	% profitable
Minor	162.29	31.23	76.35	1.95	6.24	4.83	43.70
Small	233.28	53.72	84.06	5.47	10.18	5.38	53.68
Medium	331.49	15.67	85.16	4.34	27.72	13.56	63.51
Large	496.24	7.73	89.88	2.75	35.58	21.07	64.95
Major	890.69	5.39	89.83	2.62	48.58	17.42	75.22
Foreign	856.49	18.53	40.45	1.31	7.06	5.21	50.55
Overall	438.61	132.28	72.06	18.44	13.94	9.63	59.46

end of the trading day, i.e., the difference between the total amount borrowed and the total amount lent during the day. We use a bank's net position as a proxy for its initial expected shortfall.[26] We observe that the standard deviation increases with the size of the bank. This supports the link between the size of the bank and the variability of its initial expected shortfall.

Columns three and four report the average number of 'active' banks (i.e., banks that have at least one transaction in a day) and their proportion across all banks (both active and inactive) of a given type.[27] The Italian banking sector, like many others, is characterized by a larger number of small banks relative to large ones, a pattern that is reflected in the distribution of the average number of active banks by size. However, the proportion of active banks within each size category increases with size. This tendency is consistent with the result of Proposition 6. Note also that the proportion of active foreign banks is relatively small, possibly because they have access to trading opportunities in other overnight markets.

The last four columns illustrate the extent of intermediation in the market. They report: (i) the average number of intermediary banks per day, (ii) their proportion among all active banks, (iii) the proportion of total trade volume attributable to intermediation, and (iv) the proportion of profitable intermediary transactions. To compute the final measure, we identified all intermediaries and calculated the difference between their total lending revenue and borrowing costs.[28] We take this value as their revenue from intermediation and report the proportion of intermediation instances that resulted in positive revenue.

The overall proportion of intermediaries on a daily basis is relatively small, just below 14 %. While this percentage is not directly explained by our model, the share of volume attributed to intermediation is even smaller, just below 10 %. Importantly, the probability of an intermediary making a profit on any given day is less than 60 %, based on our revenue measure. Actual profitability, accounting for default risks, would likely be even lower. This suggests that the observed low level of intermediation on a daily basis is *non-strategic*. For example, it may result from adjustments in expected shortfalls as more information about payment flows becomes available throughout the day, which is not accounted for by our simple model.

Regarding variability in observed intermediation across types, small banks show the highest average numbers. However, this is largely due to their prevalence in the market. When considering the proportion of intermediaries relative to the number of active banks of a specific type, the proportion increases with bank size. A similar trend is seen in intermediation volume. This, again is consistent with possible within-day adjustments to expected shortfalls: larger banks tend to have higher variability in these shortfalls and therefore get more such non-strategic intermediation. Finally, profitability from intermediation increases with bank size. This might be due to larger banks having better bargaining power over rates and our measure underestimating the real cost to a greater extent for larger banks.

To sum up, intermediation is relatively rare on any given day, and when it does occur, it is likely non-strategic. We next contrast these daily intermediation patterns with those observed at lower-frequency data.

Table 2 reports the average number of active banks and the proportion of intermediaries among them at daily, monthly, quarterly, and yearly time intervals. A bank is considered active if it has at least one transaction within the specified time interval. Naturally, the average number of active banks increases with the interval, but the largest increases occur among smaller-sized banks (consistent with Proposition 6).

As we move from daily to monthly aggregation, the overall proportion of intermediaries sharply increases from around 14 % to 64 %. Moreover, an increasing number of larger banks become intermediaries, as our theory predicts (see the discussion following Proposition 6). When we consider even lower frequencies, such as quarterly and yearly, intermediation increases further. These

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[26] More precisely, in light of our model, net positions represent a *lower bound* for initial expected shortfalls. In equilibrium, shortfalls are close to, but not exactly, zero due to market rationing caused by counterparty default risk. As for the calculations, we pool all banks within each type across all days in the sample on which they traded. We exclude banks that did not trade on a given day, as the proportion of such banks varies across types, and including them could introduce bias, especially for smaller types.

[27] Banks may enter and exit the market over time, but we do not directly observe this. Therefore, we calculate the proportion of active banks for each year using a proxy for the total number of banks, defined as the number of banks that were active on at least one day during the year. We then compute a weighted average of these yearly proportions over the 10-year sample period, using the relative number of active banks in each year as weights.

[28] In the context of our model, this provides an upper bound on profitability from intermediation, as it does not account for potential costs related to expected counterparty default, see Eq. (3). Note that under non-strategic intermediation, we would expect this number to be 50 %.

Table 2
Average number of active banks (**Act** column) and percentage of intermediary banks (**% Int** column) by bank type across various frequencies.

Bank	Day		Month		Quarter		Year	
type	Act	% Int	Act	% Int	Act	% Int	Act	% Int
Minor	31.23	6.24	36.22	44.82	37.33	60.88	40.90	79.46
Small	53.72	10.18	61.17	64.31	61.88	80.04	63.90	91.39
Medium	15.67	27.72	17.73	85.90	17.93	94.42	18.40	97.28
Large	7.73	35.58	8.23	93.12	8.28	97.28	8.60	100.00
Major	5.39	48.58	5.70	98.98	5.75	100.00	6.00	100.00
Foreign	18.53	7.06	36.73	58.48	39.83	74.01	45.80	84.72
Overall	132.28	13.94	165.78	63.69	170.98	77.47	183.60	88.34

tendencies remain stable across different years in our sample. In this way, we reconcile the findings obtained in the literature for different frequencies.

5. Central bank and policy discussion

The CB uses a range of policy instruments that influence interest rates and trading volume in the overnight interbank market. In turn, banks pass through the interest rates from this market to the rest of the economy. In deciding policy parameters, the CB may pursue several objectives (see Bindseil, 2014, particularly Section 6.2, and references therein). First, since the CB implements monetary policy by targeting a specific interest rate on this overnight interbank market, it prefers interbank rates to remain close to this target and to exhibit low volatility around it. Second, the CB aims for an active overnight interbank market, as this market can provide valuable information regarding both individual banks' creditworthiness and broader market conditions. Specifically, the CB may be concerned about banks with substantial liquidity needs that are unable to borrow from the overnight interbank market.[29] Additionally, the average interbank rate provides information about market conditions, indicating liquidity shortages (when approaching the penalty rate) or excess liquidity (when approaching the deposit rate). Moreover, the CB aims to avoid excessive expansion of its balance sheet resulting from direct lending to or borrowing from banks. It is impossible to achieve all these policy objectives simultaneously: for example, setting both r^d and r^p equal to the target rate (i.e., setting a zero-width corridor) would eliminate volatility but entirely suppress market activity. Therefore, the optimal policy must strike a balance that reflects the CB's preferences.[30]

The model's equilibrium, as described in Section 3.3 depends on both the variables directly controlled by the CB and the factors describing general economic conditions and individual banks' characteristics. The former include the deposit and penalty rates, r^d and r^p, which determine the *corridor width*, $r^p - r^d$, as well as the banks' minimum reserve requirements, T. The latter include banks' default probabilities, q, loss given default, γ, the distribution of reserve prediction errors, F, and banks' initial expected reserves, c^0, which in principle can be influenced by the CB through its liquidity policies. The comparative statics result in Corollary 4 shows the dependence of the lender's expected marginal rate, ρ^{L}, on these variables. We now discuss how each of these variables impact interbank market rates and activity (volume and number of active banks), *ceteris paribus*.

An increase in either r^d or r^p raises both the initial and equilibrium values of the expected marginal rates, and consequently, the interbank interest rates.[31] Tightening the corridor (by reducing r^p or increasing r^d) compresses the range of PSTE rates available to any pair of trading banks. This results in lower volatility. However, it also leads to a higher number of inactive banks and lower trading volume, as the feasibility sets for trading between any pair of banks diminish (see the left panel of Fig. 2).[32] This illustrates a clear trade-off between achieving low volatility and sustaining high trading volume. Finally, when both r^d and r^p increase synchronously, thus maintaining a constant corridor width, the volume of trade decreases.[33] Intuitively, this is because lenders require higher compensation for expected losses, which rise with higher interest rates.

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[29] In the context of our model such a bank would be inactive, which implies (see Fig. 3) that it has a relatively high default probability, q_i, for the initial expected marginal rate, ρ_i^0.

[30] An example of how the CB's preferences can be set explicitly is provided in Bindseil and Jablecki (2011), who simulate various scenarios within a stylized two-bank framework. They model the CB's preferences as a weighted function of the objectives that we discussed above.

[31] Eq. (9) implies an increasing relationship between ρ^{L} and ρ_i^{B}, which represents the competitive PSTE interest rate for all loans involving a borrowing bank i. In any PSTE, the rates will lie within intervals around these competitive rates (see the discussion after Proposition 4 and the right panel of Fig. 2 for illustration).

[32] The total effect of changes in r^p and r^d on trading volume arises both from their direct impact on individual lending and borrowing functions and from the indirect effect through changes in the equilibrium ρ^{L}. By applying the implicit function theorem, one can show that, locally, the total volume of trade increases with r^d and decreases with r^p. See Appendix D for these derivations and additional comparative statics results.

[33] This effect can be illustrated using the two-bank case shown in the left panel of Fig. 2. A synchronous increase in both r^d and r^p keeps the relative positions of ρ_i^0 (black dot) and ρ_j^0 (blue dot) within the corridor unchanged. However, with a higher ρ_i^0, the relative position of $(\rho_i^0 + \gamma q_j)/(1 - \gamma q_j)$ (red dot) increases. Consequently, the overlapping area between the opportunity sets shrinks and may eventually disappear.

Minimum reserve requirements, together with initial expected reserves, determine the initial expected *shortfall* of bank i, defined as $T_i - c_i^0$. When a bank's shortfall increases, due to stricter reserve requirements and/or lower expected liquidity, the equilibrium marginal rate tends to rise (Corollary 4), resulting in higher average overnight market rates. However, a uniform change in reserve requirements and/or liquidity across all banks generally has an ambiguous effect on total trade volume, since larger shortfalls increase borrowers' demand but simultaneously reduce lenders' supply, and *vice versa* for lower shortfalls. When the expected shortfalls of all banks are either extremely high or extremely low, no trade occurs.[34]

Increasing bank i's uncertainty about its expected reserves, as captured by a higher variance in its reserve prediction error distribution, F_i, shifts its initial expected marginal rate toward the midpoint of the corridor (the curve in Fig. 1 flattens). When such a change occurs across all banks, interbank rates become less volatile, but interbank trade also tends to decline.

An increase in a borrower's default probability, q_i, leads to higher interest rates for that borrower (but not for others) and reduces both its individual trading volume and the total volume. If q_i rises beyond a certain threshold, this bank becomes inactive. When economic conditions deteriorate, default probabilities may rise simultaneously across all banks.[35] This results in higher equilibrium overnight market rates and a reduced number of transactions. For sufficiently high default probabilities, a "market freeze" occurs (see Theorem 1, case *i*). Although such a freeze is an equilibrium outcome in the interbank market, it is clearly undesirable from the CB's perspective, given its policy objectives.

While all previous discussion applied to the general version of the model (as solved in Section 3), in Proposition 4 of Section 4.1 we provided a closed-form solution for the threshold on default probabilities that leads to a market freeze. From a policy perspective, restarting a frozen interbank market can be achieved by widening the interest-rate corridor or by shifting both rates downward while keeping the corridor width unchanged. In contrast, lowering both rates while narrowing the corridor may fail to revive the market.[36] The market can also be supported by measures that reduce the loss given default, γ.[37]

Finally, and most importantly, a key implication of our model is the absence of intermediation in the overnight interbank market. As shown in Section 4, the core-periphery structure observed in low-frequency data may be an illusion created by temporal aggregation. When strategic intermediation does arise in markets, it can generate long and intertwined chains, giving rise to complex networks. In particular, if the network displays a core-periphery structure, the institutions occupying the core can be viewed as systemically important financial institutions (SIFIs). This perspective supports the use of network-based prudential regulations intended to limit the propagation of shocks across such highly interconnected systems. Our model, however, shows that the overnight market operates as a bi-partite market, a substantially different and far less complex architecture. Applying network-based prudential regulations in this setting, under a misidentification of a core-periphery structure, would impose net costs on banks' liquidity management without addressing any identifiable inefficiency.[38]

This argument does not imply that network-based prudential tools are unwarranted in general; such instruments may be appropriate where complexity is intrinsic (e.g., OTC derivatives networks), but they are poorly targeted to the overnight interbank market considered here. We also note that, under certain frictions, strategic intermediation can arise even within the model we study. In Appendix C, we present two extensions showing how frictions, such as a restricted network of potential loans and uncertainty about default risk, can generate intermediation in our baseline framework, even at a daily horizon.

6. Conclusions

In this paper, we develop a model of endogenous network formation in the overnight interbank market. Extending the concept of pairwise stability, we showed that the resulting lending-borrowing network is bipartite, which is a direct consequence of unsecured default risk in this market. The bipartite structure that arises, regardless of how small the default risk may be, rules out intermediation.

A further insight of our analysis concerns how the structure of the overnight market network depends on the horizon of data aggregation. At high frequency (e.g., daily), which our model captures, all transactions occur directly between ultimate borrowers and lenders. However, when trades are aggregated over time, some banks may appear as intermediaries. Specifically, we show that heterogeneity in the variances of expected initial shortfalls is sufficient to generate intermediation and a core-periphery network structure, with banks facing greater variance more likely to appear as intermediaries in the aggregated network.

We also provide empirical support for our theoretical predictions using data from the e-MID market. Although we find little evidence of strategic intermediation at the daily level, intermediation becomes more prominent at lower frequencies, i.e., monthly, quarterly, and annually. These findings speak directly to the debate on the role of network structure in systemic risk. Although interbank exposure networks are generally useful for identifying systemically important banks, our analysis suggests that this may not apply to the overnight interbank market. Even when a core in the network emerges at lower frequencies, it may simply reflect bank

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[34] In such cases, the initial ρ_i^0 for all banks converges to either r^p or r^d, respectively. In either scenario, this is the case (*i*) of Theorem 1, implying a complete absence of trade. The effect can be illustrated in the left panel of Fig. 1 as an extreme shift of both banks' positions along the curve, either far to the left or far to the right. A real-life example occurred in response to the initial shock of the GFC, when central banks flooded the market with liquidity, which substantially reduced interbank trading volume.

[35] Copeland (2019) estimates sharp increases in the default probabilities of all banks in the Federal Funds Market during the GFC.

[36] This is similar to the situation the ECB faced in 2013, when the penalty rate stalled at the zero lower bound.

[37] For example, in the wake of the GFC, policymakers proposed insurance schemes to protect banks against counterparty defaults and stabilize short-term funding markets. One such scheme, the Collateralized Interbank Market (MIC), was introduced in early 2009 by the Bank of Italy in collaboration with e-MID and the Italian Banking Association, with the explicit aim of encouraging the resumption of interbank trading.

[38] We thank an anonymous referee for their helpful insights used in this paragraph.

size and the statistical properties of equilibrium aggregation, rather than deeper, more fundamental sources of systemic importance. Mis-modeling a bipartite market as core-periphery can generate efficiency losses at the regulatory margin.

Our model offers a tractable framework for analyzing how central bank policy instruments influence outcomes in the overnight interbank market. By endogenizing the formation of lending relationships, it captures how policy parameters and individual banks' conditions, including default rates, affect both interbank rates and the level of trading activity. We discuss the trade-off central banks may face between reducing interest rate volatility and increasing market activity. Ultimately, the central bank's preferences will guide the setting of these parameters, and our framework can be used for such calibration.

Our framework can be extended in several directions. For instance, one could explore stricter network equilibrium configurations, such as a scenario where banks reconsider multiple links simultaneously. This refinement is similar to the concept of bilateral stability, studied in a different context by Goyal and Vega-Redondo (2007). Another extension involves banks not acting myopically but instead anticipating how their decisions to establish or sever links will affect the network structure. This could be analyzed using the framework of farsighted stability, as proposed by Herings et al. (2009). In addition, studies on financial regulation, policy interventions, and monetary policy can leverage our network framework.

CRediT authorship contribution statement

Mikhail Anufriev: Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization; **Andrea Deghi:** Writing – review & editing, Writing – original draft, Software, Methodology, Formal analysis, Data curation; **Valentyn Panchenko:** Writing – review & editing, Writing – original draft, Software, Methodology, Formal analysis, Data curation, Conceptualization; **Paolo Pin:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization.

Data availability

The authors do not have permission to share data.

Declaration of competing interest

All authors declare no relevant or material financial interests that relate to the research described in this paper.

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Appendix A. Proofs

A.1. Derivatives of borrowers and lenders

For the following reference, we compute the derivatives of π_i defined in (4). Recall that F_i is the CDF of the reserve prediction error, ε_i. Let f_i denote its PDF.

Borrower. When bank i borrows from bank m, the i's marginal profit is

$$\frac{\partial \pi_i}{\partial \ell_{mi}} = \frac{\partial \pi_i^{\mathrm{CB}}}{\partial \ell_{mi}} + \frac{\partial \pi_i^{\mathrm{IM}}}{\partial \ell_{mi}} = r^p F_i(T_i - c_i) + r^d(1 - F_i(T_i - c_i)) - r_{mi} = \rho_i - r_{mi}\,. \tag{A.7}$$

The second derivative of profit with respect to the loan amount is

$$\frac{\partial^2 \pi_i}{\partial \ell_{mi}^2} = \frac{\partial \rho_i}{\partial \ell_{mi}} = -(r^p - r^d) f_i(T_i - c_i)\frac{\partial c_i}{\partial \ell_{mi}} = -(r^p - r^d) f_i(T_i - c_i)\,.$$

The derivative of profit with respect to the interest rate is

$$\frac{\partial \pi_i}{\partial r_{mi}} = -\ell_{mi}\,.$$

Lender. When bank i lends to bank j, the i's marginal profit is

$$\frac{\partial \pi_i}{\partial \ell_{ij}} = \frac{\partial \pi_i^{\mathrm{CB}}}{\partial \ell_{ij}} + \frac{\partial \pi_i^{\mathrm{IM}}}{\partial \ell_{ij}} = -\rho_i + r_{ij}(1-\gamma q_j) - \gamma q_j = -\rho_i + r_{ij} - \gamma q_j(1+r_{ij})\,. \tag{A.8}$$

The second derivative of profit with respect to the loan amount is

$$\frac{\partial^2 \pi_i}{\partial \ell_{ij}^2} = -\frac{\partial \rho_i}{\partial \ell_{ij}} = (r^p - r^d) f_i(T_i - c_i)\frac{\partial c_i}{\partial \ell_{ij}} = -(r^p - r^d) f_i(T_i - c_i)\,.$$

The derivative of profit with respect to the interest rate is

$$\frac{\partial \pi_i}{\partial r_{ij}} = \ell_{ij}(1-\gamma q_j)\,.$$

Contract Curve. When bank i lends bank j a loan ℓ_{ij} at rate r_{ij}, the slopes of banks' indifference curves are $-\frac{\partial \pi_i}{\partial \ell_{ij}}\Big/\frac{\partial \pi_i}{\partial r_{ij}}$ and $-\frac{\partial \pi_j}{\partial \ell_{ij}}\Big/\frac{\partial \pi_j}{\partial r_{ij}}$. Setting them equal to each other gives

$$-\frac{-\rho_i + r_{ij} - \gamma q_j(1+r_{ij})}{\ell_{ij}(1-\gamma q_j)} = -\frac{\rho_j - r_{ij}}{-\ell_{ij}}\,,$$

which simplifies to (8).

A.2. Proof of Lemma 2 (page 5)

Proof. The proofs of the two statements are similar, so we present only the first (lender's) part. Consider how the expected marginal rate of lender i depends on the potential link (ℓ, r) with borrower j. According to Eq. (5), this rate does not depend on r, so we denote its dependence on the link as $\rho_i(\ell)$. Using the derivatives computed in Appendix A.1, we find $\partial\Delta_i^{g+ij}/\partial\ell = -\rho_i(\ell) + r - \gamma q_j(1+r)$ and $\partial\Delta_i^{g+ij}/\partial r = \ell(1-\gamma q_j)$.

Let r^* be defined as the right-hand side of (6). We first show that no point (ℓ, r) with $r \le r^*$ belongs to the lender's feasibility set $F^{\mathrm{L}}_{i\to j}(g)$. Indeed, Lemma 1 implies that $\rho_i'(\ell) > 0$. Then, for $\ell > 0$, and since $1-\gamma q_j > 0$ (as $q_j < 1$), we obtain

$$\frac{\partial\Delta_i^{g+ij}}{\partial\ell} = -\rho_i(\ell) + r - \gamma q_j(1+r) < -\rho_i(0) + r^* - \gamma q_j(1+r^*).$$

The right-hand side equals 0 by definition of r^*. Thus, for any $r \le r^*$, Δ_i^{g+ij} strictly decreases with ℓ, and since $\Delta_i^{g+ij}(0, r) = 0$, the statement follows.

Fix an arbitrary $\tilde{\ell} > 0$. We have shown that $\Delta_i^{g+ij}(\tilde{\ell}, r^*) < 0$. Since Δ_i^{g+ij} is strictly increasing in r (as $\partial\Delta_i^{g+ij}/\partial r = \ell(1-\gamma q_j) > 0$) and diverges to $+\infty$ as $r \to \infty$ (according to (3) and because (2) does not depend on r), there exists a unique value $\tilde{r} > r^*$ for which $\Delta_i^{g+ij}(\tilde{\ell}, \tilde{r}) = 0$. The point $(\tilde{\ell}, \tilde{r})$ belongs to the boundary of the feasibility set, while all points $(\tilde{\ell}, r)$ with $r > \tilde{r}$ lie inside the feasibility set. Therefore, the feasibility set is the strict epigraph of a function defined by mapping $\tilde{\ell} > 0$ to $\tilde{r}$. Let h^{L} be this function. We know that $h^{\mathrm{L}}(\ell) > r^*$ for any $\ell > 0$. Moreover, by the implicit function theorem, this function is continuous for $\ell > 0$: it is the iso-profit function of i, and since $\partial\pi_i^{g+ij}/\partial r = \ell(1-\gamma q_j) > 0$, it follows that the function is continuous.

To show that h^{L} strictly increases with ℓ, assume the opposite. Then, there exist positive $\ell_1 < \ell_2$ such that $h^{\mathrm{L}}(\ell_1) \ge h^{\mathrm{L}}(\ell_2) > r^*$, where the last inequality follows from the first step of the proof. Then, for $\hat{r} \in [h^{\mathrm{L}}(\ell_2), h^{\mathrm{L}}(\ell_1)]$, there exist at least two positive values of ℓ where $\Delta_i^{g+ij}(\ell, \hat{r}) = 0$. In addition, $\Delta_i^{g+ij}(0, \hat{r}) = 0$. Such a situation is impossible, as the function $\Delta_i^{g+ij}(\ell, \hat{r})$ is concave in ℓ, since $\partial^2\Delta_i^{g+ij}/\partial\ell^2 = -\rho_i'(\ell) < 0$.

Finally, we extend h^{L} by continuity to define its value at $\ell = 0$. From the first step of the proof, $h^{\mathrm{L}}(0) \ge r^*$. Moreover, by the continuity of $\rho(\ell)$, for any $r > r^*$, there exists a sufficiently small ℓ such that $\partial\Delta_i^{g+ij}/\partial\ell > -\rho_i(0) + r^* - \gamma q_j(1+r^*) = 0$, that is there exists a small positive ℓ within the feasibility set. It follows that $h^{\mathrm{L}}(0) = r^*$. This completes the proof of the first part of the lemma. □

A.3. Proof of Theorem 2 (page 7)

Proof. From definition (5), the expected marginal rates depend on the loan amounts but not on the interest rates. Thus, the condition for the loan being on the contract curve between i and j can be expressed as an equation in terms of the loan amount ℓ_{ij}:

$$\rho_i(\ell_{ij}) = \rho_j(\ell_{ij}) - \gamma q_j(1 + \rho_j(\ell_{ij}))\,. \tag{A.9}$$

Since the intersection of the feasibility sets is non-empty, it follows from Proposition 1 that $\rho_i(0) < \rho_j(0) - \gamma q_j(1+\rho_j(0))$. As ℓ_{ij} increases, Lemma 1 implies that the left-hand side of Eq. (A.9) strictly increases, approaching r^p as $\ell_{ij} \to \infty$. In contrast, the right-hand side strictly decreases (since $1-\gamma q_j > 0$), approaching $(1-\gamma q_j)r^d - \gamma q_j$ as $\ell_{ij} \to \infty$. Since $r^p > r^d > (1-\gamma q_j)r^d - \gamma q_j$, it follows that the Eq. (A.9) has a unique positive solution for ℓ_{ij}.

We now show that there exists an interest rate r_{ij} such that (ℓ_{ij}, r_{ij}) lies in the intersection of the feasibility sets. Set $r_{ij} = \rho_j(\ell_{ij})$. Since ℓ_{ij} satisfies (A.9), we can also write $r_{ij} = (\gamma q_j + \rho_i(\ell_{ij}))/(1-\gamma q_j)$. It follows that at the point (ℓ_{ij}, r_{ij}), the indifference curves of both the lender and the borrower are horizontal, verifying that this point lies on the contract curve. From Lemma 2, the function $h^{\mathrm{L}}(\ell)$, whose slope at ℓ_{ij} is positive, corresponds to the condition $\Delta_i^{g+ij}(\ell, r) = 0$. But the slope of the lender's indifference curve for a given ℓ decreases with r. Therefore, $r_{ij} > h^{\mathrm{L}}(\ell_{ij})$, and so $(\ell_{ij}, r_{ij}) \in F^{\mathrm{L}}_{i\to j}$. A similar argument shows that $(\ell_{ij}, r_{ij}) \in F^{\mathrm{B}}_{j\leftarrow i}$, completing the proof. ☐

A.4. Proof of Lemma 3 (page 7)

Proof. To prove the first property, assume the contrary, that is, banks i and j lend funds to each other. Applying Corollary 2 to both links, from i to j and from j to i, we obtain:

$$\rho_i = (1-\gamma q_j)\rho_j - \gamma q_j = (1-\gamma q_j)(1-\gamma q_i)\rho_i - (1-\gamma q_j)\gamma q_i - \gamma q_j < (1-\gamma q_j)(1-\gamma q_i)\rho_i\,,$$

where in the last step we used the fact that at least one bank has non-zero probability of default. The final inequality implies that $\rho_i < \rho_i$, which is a contradiction.

To prove the second property, assume the contrary, that is, there are three banks with $\ell_{ij} > 0$ and $\ell_{jk} > 0$. Then

$$\rho_i = (1-\gamma q_j)\rho_j - \gamma q_j < \rho_j = (1-\gamma q_k)\rho_k - \gamma q_k\,,$$

where both equalities follow from Corollary 2 and the inequality holds because $q_j > 0$. Thus, $\gamma q_k + \rho_i < (1-\gamma q_k)\rho_k$ and from Corollary 1, there must be a link from i to k in the PSTE, i.e., $\ell_{ik} > 0$. Applying Corollary 2 again, we get $\rho_i = (1-\gamma q_k)\rho_k - \gamma q_k$, which contradicts the earlier inequality.

To prove the third property, assume the contrary, i.e., for these four banks, say, $\rho_i > \rho_k$. From Corollary 2 applied to the link ij, we have $(1-\gamma q_j)\rho_j - \gamma q_j = \rho_i$. Therefore, $\rho_k < (1-\gamma q_j)\rho_j - \gamma q_j$, and Corollary 1 implies that then it must be a link from k to j in the PSTE, i.e., $\ell_{kj} > 0$. However, applying Corollary 2 along this link, we find $\rho_k = (1-\gamma q_j)\rho_j - \gamma q_j$, which contradicts the earlier inequality. ☐

A.5. Proof of Corollary 3 (page 8)

Proof. Consider a PSTE with ρ^{L} and partition all banks on sets $\mathcal{L}$, $\mathcal{I}$ and $\mathcal{B}$ of lenders, inactive banks, and borrowers as in Proposition 3. As established in the main text, if bank i is a lender in the PSTE, then $\rho_i = \rho^{\mathrm{L}}$. Monotonicity of the expected marginal rate implies that for this bank $\rho_i^0 < \rho^{\mathrm{L}}$. Similarly, if bank j is a borrower in the PSTE, $\rho_j = \rho_j^{\mathrm{B}}$ as defined in (9). Again, by monotonicity of the expected marginal rate, $\rho_j^0 > \rho_j^{\mathrm{B}}$.

Finally, if bank k is inactive in this PSTE (where i is a lender and j is a borrower), then, on the one hand, $\rho_k = \rho_k^0$, and, on the other hand, links ik and kj do not exist. Corollary 1 then implies that $\rho^{\mathrm{L}} \geq (1-\gamma q_k)\rho_k^0 - \gamma q_k$ and that $\rho_k^0 \geq (1-\gamma q_j)\rho_j - \gamma q_j$. The former inequality can be rewritten as $\rho_k^0 \leq \rho_k^{\mathrm{B}}$, using (9). The latter inequality is simply $\rho_k^0 \geq \rho^{\mathrm{L}}$ due to (8). ☐

A.6. Proof of Lemma 4 (page 9)

Proof. Because of the strict monotonicity of F_i, the formulas in both the low-ρ (first) case and in the high-ρ (third) case in (11) define strictly increasing functions of ρ. In the first case, given the interval for ρ, we have

$$r^d \leq \frac{\rho + \gamma q_i}{1-\gamma q_i} \leq \rho_i^0.$$

By the definition of ρ_i^0, it follows that in this case the function ℓ_i maps ρ into $(-\infty, 0)$. Similarly, in the third case, the function ℓ_i maps ρ into $(0, +\infty)$. We have verified that $\ell_i(\rho)$ is well defined and continuous. Furthermore, it is clearly monotonically increasing.

To establish that the function, evaluated at ρ^{L}, gives the equilibrium net lending position of bank i, we apply Corollary 3. If the bank is a lender, then $\rho_i^0 < \rho^{\mathrm{L}}$, so we are in the third case of the function's definition. There, the equilibrium lending amount is $c_i^0 - c_i$ with c_i given by (10). This coincides with the value of $\ell_i(\rho^{\mathrm{L}})$ in the third case.

If the bank is inactive, then $\rho^{\mathrm{L}} \leq \rho_i^0 \leq \rho_i^{\mathrm{B}}$. Using (9), we verify that this corresponds to the middle-ρ (second) case in (11), where the function indeed yields a net lending of 0.

Finally, if the bank is a borrower, then $\rho_i^0 > \rho_i^{\mathrm{B}}$. By (9), this corresponds to the first case. The net lending (i.e., the negative of the borrowed amount) is

$$c_i^0 - c_i = c_i^0 - T_i + F_i^{-1}\left(\frac{\rho_i^{\mathrm{B}} - r^d}{r^p - r^d}\right),$$

which coincides with the value of $\ell_i(\rho^{\mathrm{L}})$ in the upper case, in light of (9). ☐

A.7. Proof of Theorem 1 (page 9)

Proof. In the case described in *(i)*, the empty network (i.e., the case of no interbank trade) constitutes the PSTE. Indeed, from Corollary 1, it follows that for any pair of banks, no link can be formed. Moreover, it is impossible to have a PSTE with trade, because if such a situation existed, we would have two banks, a lender i and a borrower j, for which, according to Corollary 3, the conditions would imply $\rho_i^0 < \rho^{\mathrm{L}} < \rho_j^{\mathrm{B}} < \rho_j^0$. This leads to a contradiction, as it violates the condition of the case.

For the case described in *(ii)*, recall the definition of the function ℓ_i in (11). Let $\tilde{\ell}_i$ denote this function restricted to the domain $D = (r^d, r^p)$, i.e., the common domain for all banks. Define the aggregate function $L : D \to \mathbb{R}$ by $L(\rho) = \sum_i \tilde{\ell}_i(\rho)$. For each bank i, consider the interval $I_i = \left[\max\{r^d, (1-\gamma q_i)\rho_i^0 - \gamma q_i\}, \rho_i^0\right]$, which is the range of ρ over which $\tilde{\ell}_i$ is constant (equal to zero). Since there exist two banks i and j such that $\rho_i^0 < (1-\gamma q_j)\rho_j^0 - \gamma q_j$, the intersection $\cap_i I_i$ of these intervals is empty. Consequently, the function L is strictly increasing on its domain because there is no point in D where all functions $\tilde{\ell}_i$ are constant simultaneously. By continuity and strict monotonicity of L, there exists ρ^* that solves $L(\rho^*) = 0$ and such ρ^* is unique. Now, it is easy to see, from Lemma 4, that taking this solution as ρ^{L} we obtain PSTE, where by Corollary 3, the partitioning of banks is uniquely determined. □

A.8. Proof of Corollary 4 (page 10)

Proof. In the proof of Theorem 1, we defined the function L, which on the domain $D = (r^d, r^p)$ can be written as $L(\rho) = \sum_i \ell_i(\rho)$. We showed that for any set of parameter values and whenever there is interbank trade, there exists a unique ρ^{L} such that $L(\rho^{\mathrm{L}}) = 0$. In this way, we define the function $\rho^{\mathrm{L}}(\gamma, r^d, r^p, q_1, \dots, q_n, T_1, \dots, T_n)$.

Since each of the ℓ_i functions, and consequently the function L, is continuous in all the parameters, the function $\rho^{\mathrm{L}}(\gamma, r^d, r^p, q_1, \dots, q_n, T_1, \dots, T_n)$ is also continuous.

Consider now Eq. (11) that defines ℓ_i. Note that both arguments of F^{-1}, that is, $\left(\frac{\rho+\gamma q_i}{1-\gamma q_i} - r^d\right)/(r^p - r^d)$ (for the low-ρ case) and $(\rho - r^d)/(r^p - r^d)$ (for the high-ρ case), are strictly decreasing in r^p and strictly decreasing in r^d. The former fact is obvious, while the latter follows from applying the domain restrictions. Indeed, in the low-ρ (first) case, $\rho < r^p(1-\gamma q_i) - \gamma q_i$ (since $\rho_i^0 \le r^p$ and $1 - \gamma q_i > 0$), so that $\frac{\rho+\gamma q_i}{1-\gamma q_i} < r^p$, and in the high-$\rho$ (third) case, $\rho < r^p$, from where the fact follows. Also, in the low-ρ case, the argument of F^{-1} is strictly increasing in γ and in q_i.

The function F_i^{-1} is strictly increasing. Thus, whenever we are in the first case of its definition, the function ℓ_i is strictly increasing in γ and q_i, and strictly decreasing in r^p and r^d. Whenever we are in the third case of its definition, ℓ_i is strictly decreasing in r^p and r^d. Moreover, in both the first and third cases, ℓ_i is strictly decreasing in T_i.

Therefore, the function L, which depends on the parameters only through the ℓ_i functions of each active bank i, is strictly decreasing in r^p and r^d, and strictly increasing in γ (since there is always at least one borrower, i.e., a bank for which the low-ρ case of ℓ_i applies). It is also weakly increasing in q_i and weakly decreasing in T_i, with strict relations for active banks.

Since we assume there are at least two active banks in the PSTE and at least one borrower, the unique solution of $L(\rho) = 0$, i.e., unique intersection point of L with the horizontal axis, will then satisfy the statement of the proposition. □

A.9. Proof of Theorem 4 (page 11)

Proof. Since all ψ_i are drawn from the independent distributions with full support $[0, 1]$, the minimum of ψ_i converges almost surely to 0, and the maximum of ψ_i converges almost surely to 1 as $n \to \infty$. With these limiting values, inequality (12) simplifies to $q < q^c$.

Let us suppose that $q < q^c$, and let $\varepsilon > 0$ be such that $\gamma q < \gamma q^c - \varepsilon$. With probability 1, there exists an N such that for $n > N$, we have

$$\min_{i \in \mathcal{N}_n} \psi_i < \varepsilon \quad \text{and} \quad \max_{i \in \mathcal{N}_n} \psi_i > 1 - \varepsilon\,.$$

Then, for large n, the difference between the right- and left-hand sides of (12) becomes

$$\begin{aligned}
&(1-\gamma q)r^d + (1-\gamma q)(r^p - r^d)\max_i \psi_i - \gamma q - r^d - (r^p - r^d)\min_i \psi_i = \\
&\quad = (r^p - r^d)\Big((1-\gamma q)\max_i \psi_i - \min_i \psi_i\Big) - \gamma q(1 + r^d) > \\
&\quad > (r^p - r^d)((1 - \gamma q^c + \varepsilon)(1-\varepsilon) - \varepsilon) - (\gamma q^c - \varepsilon)(1 + r^d) = \\
&\quad = \underbrace{(r^p - r^d)(1 - \gamma q^c) - \gamma q^c(1 + r^d)}_{=0 \text{ from the definition of } q^c} + (r^p - r^d)\big(\gamma q^c \varepsilon - \varepsilon^2 - \varepsilon\big) + \varepsilon(1 + r^d) = \\
&\quad = \varepsilon\big((r^p - r^d)(\gamma q^c - 1) + (1 + r^d)\big) - (r^p - r^d)\varepsilon^2 = \varepsilon\,\frac{(1 + r^d)^2}{1 + r^p} - (r^p - r^d)\varepsilon^2\,,
\end{aligned}$$

where the last equality can be directly verified by substituting the expression for q^c.

The last expression can be made positive by choosing a sufficiently small $\varepsilon > 0$. Hence, the left-hand side is positive, implying that interbank trade will occur in the LML.

If, instead, $q > q^c$, then for any number of banks, the difference between the right- and left- hand sides of (12) is

$$
\begin{aligned}
&(1-\gamma q)r^d + (1-\gamma q)(r^p - r^d)\max_i \psi_i - \gamma q - r^d - (r^p - r^d)\min_i \psi_i = \\
&\quad = (r^p - r^d)\Big((1-\gamma q)\max_i \psi_i - \min_i \psi_i\Big) - \gamma q(1+r^d) < \\
&\quad < (r^p - r^d)\left(\left(1 - \frac{r^p - r^d}{1+r^p}\right)\max_i \psi_i - \min_i \psi_i\right) - \frac{(r^p - r^d)(1+r^d)}{1+r^p} = \\
&\quad = \frac{(r^p - r^d)(1+r^d)}{1+r^p}\Big(\max_i \psi_i - 1\Big) - \min_i \psi_i \le 0
\end{aligned}
$$

and inequality (12) cannot be satisfied. Therefore, there is no trade for any n. □

A.10. Proof of Theorem 5 (page 11)

Proof. Consider a realization of the projected shortfall, $T_i - c_i^0$, for bank i. Function $\ell_i(\rho)$, as given in Eq. (11) describes the bank's net lending according to Lemma 4. To express this function concisely, we introduce

$$
S_i^{\mathrm{B}}(\rho) := F_i^{-1}\left(\frac{\frac{\rho+\gamma q_i}{1-\gamma q_i} - r^d}{r^p - r^d}\right) \quad \text{and} \quad S_i^{\mathrm{L}}(\rho) := F_i^{-1}\left(\frac{\rho - r^d}{r^p - r^d}\right).
$$

These terms, when evaluated at ρ^{L}, correspond to the expected interim shortfall, $T_i - c_i$, as shown in Eq. (10). Specifically, $S_i^{\mathrm{B}}(\rho^{\mathrm{L}})$ represents the shortfall for a borrower, and $S_i^{\mathrm{L}}(\rho^{\mathrm{L}})$ represents the shortfall for a lender.

Restricting functions ℓ_i to the common domain (r^d, r^p) for all banks, we consider the sum of these restricted functions, denoted by L_n, over all banks. It can be expressed as

$$
\begin{aligned}
L_n(\rho) &= \sum_{\{i:\ i \text{ is a borrower}\}} \Big(S_i^{\mathrm{B}}(\rho) + c_i^0 - T_i\Big) + \sum_{\{i:\ i \text{ is a lender}\}} \Big(S_i^{\mathrm{L}}(\rho) + c_i^0 - T_i\Big) = \\
&= \sum_{\{i: T_i - c_i^0 > S_i^{\mathrm{B}}(\rho)\}} \Big(S_i^{\mathrm{B}}(\rho) + c_i^0 - T_i\Big) + \sum_{\{i: T_i - c_i^0 < S_i^{\mathrm{L}}(\rho)\}} \Big(S_i^{\mathrm{L}}(\rho) + c_i^0 - T_i\Big),
\end{aligned}
\tag{A.10}
$$

where the first equality follows from the definition of the function ℓ_i and the fact that inactive banks contribute zero to the sum, and the second equality reflects that borrowers decrease their expected shortfall, while lenders increase theirs.

Given the homogeneity assumptions, we can remove the subscript i from S_i^{B} and S_i^{L}. For every n, when interbank trade is possible, the equilibrium expected marginal rate ρ_n^{L} is the unique solution to the equation $L_n(\rho) = 0$. Since ρ_n^{L} is bounded and the trade exists in the LML, the Bolzano-Weierstrass theorem ensures the existence of a convergent subsequence. Dividing Eq. (A.10) by n and taking the limit as $n \to \infty$, we apply the law of large numbers to obtain the following equation for the limiting ρ:

$$
\big(1 - \Phi(S^{\mathrm{B}}(\rho))\big)S^{\mathrm{B}}(\rho) - \int_{S^{\mathrm{B}}(\rho)}^{\infty} \xi \,\mathrm{d}\Phi + \Phi(S^{\mathrm{L}}(\rho))S^{\mathrm{L}}(\rho) - \int_{-\infty}^{S^{\mathrm{L}}(\rho)} \xi \,\mathrm{d}\Phi = 0,
$$

where ξ is the random variable for $T - c^0$ with CDF Φ. Rewrite this equality as

$$
\big(1 - \Phi(S^{\mathrm{B}}(\rho))\big)S^{\mathrm{B}}(\rho) + \Phi(S^{\mathrm{L}}(\rho))S^{\mathrm{L}}(\rho) = \int_{-\infty}^{S^{\mathrm{L}}(\rho)} \xi \,\mathrm{d}\Phi + \int_{S^{\mathrm{B}}(\rho)}^{\infty} \xi \,\mathrm{d}\Phi. \tag{A.11}
$$

We claim that the solution of (A.11) satisfies $S^{\mathrm{B}}(\rho) = -S^{\mathrm{L}}(\rho)$. Indeed, if this equality holds, both sides of (A.11) are equal to 0 by the symmetry of Φ.

Finally, to solve $S^{\mathrm{B}}(\rho) = -S^{\mathrm{L}}(\rho)$, we apply the symmetry of F and obtain that

$$
\frac{\frac{\rho+\gamma q}{1-\gamma q} - r^d}{r^p - r^d} = 1 - \frac{\rho - r^d}{r^p - r^d}
$$

from which we determine that the limiting ρ is as in (14). □

Generalization: symmetric shortfalls with a non-zero mean. Assume that the distribution Φ for the projected shortfalls, $T - c^0$, is symmetric, as before, but its mean $\mu \neq 0$. Proceeding as before, we derive Eq. (A.11). We now show that the value of ρ that solves

$$
S^{\mathrm{B}}(\rho) + S^{\mathrm{L}}(\rho) = 2\mu \tag{A.12}
$$

will also solve Eq. (A.11). Specifically, with this assumption, we have $\mu = \frac{S^{\mathrm{B}}(\rho)+S^{\mathrm{L}}(\rho)}{2}$. Let us define $x = \frac{S^{\mathrm{B}}(\rho)-S^{\mathrm{L}}(\rho)}{2}$. The left-hand side of (A.11) can then be written as

$$
(1 - \Phi(\mu + x))(\mu + x) + \Phi(\mu - x)(\mu - x) = 2\mu\Phi(\mu - x) = 2\mu\Phi(S^{\mathrm{L}}(\rho)),
$$

where we used the symmetry property of Φ, i.e., $\Phi(\mu - x) + \Phi(\mu + x) = 1$. The right-hand side of (A.11) can also be simplified to the same expression, as can be shown by rewriting it as

$$\mu - \int_{S^{\mathrm{L}}(\rho)}^{S^{\mathrm{B}}(\rho)} \xi \, \mathrm{d}\Phi,$$

changing the variable of integration ξ to $\xi' = \xi - \mu$, and noting that the integral of an odd function over symmetric bounds is zero.

Although a general closed-form solution for ρ^{L} in Eq. (A.12) when $\mu \neq 0$ is not available, solving the equation numerically (e.g., for normal CDF F and $\mu > 0$) yields the curve shown in Fig. 5. Since both S^{B} and S^{L} are increasing functions of ρ, ρ^{L} increases with μ.

Further generalizations, such as accounting for heterogeneous variances in the distribution of reserve prediction errors or for heterogeneous default probabilities, can be derived in a similar fashion.

A.11. *Proof of Proposition 6 (page 12)*

Proof. Following the same steps as in the proof of Proposition 5, we apply the homogeneity assumption to Eq. (A.10), conclude that there is a converging subsequence, and divide (A.10) by the total number of banks, n. Since the distributions of expected shortfalls now differ across types, we apply the law of large numbers separately to each type, and obtain

$$S^{\mathrm{B}}(\rho) \sum_k f_k \big(1 - \Phi_k(S^{\mathrm{B}}(\rho))\big) + S^{\mathrm{L}}(\rho) \sum_k f_k \Phi_k(S^{\mathrm{L}}(\rho)) =$$
$$= \sum_k f_k \left(\int_{-\infty}^{S^{\mathrm{L}}(\rho)} \xi \, \mathrm{d}\Phi_k + \int_{S^{\mathrm{B}}(\rho)}^{\infty} \xi \, \mathrm{d}\Phi_k \right). \tag{A.13}$$

It is straightforward to verify that the solution to (A.13) satisfies $S^{\mathrm{B}}(\rho) = -S^{\mathrm{L}}(\rho)$, as in the homogeneous case. Indeed, if this equality holds, then Eq. (A.11) holds for each type k. Summing these equations over all types yields Eq. (A.13). Finally, proceeding as in the proof of Proposition 5, we conclude that the equilibrium ρ^{L} is given by (14).

Whether a bank is a lender or borrower in the PSTE depends on whether its initial shortfall is above $S^{\mathrm{B}}(\rho^{\mathrm{L}})$ or below $S^{\mathrm{L}}(\rho^{\mathrm{L}})$, respectively. Therefore, in the LML, among all type-k banks, the proportion of borrowers is given by $1 - \Phi_k(S^{\mathrm{B}}(\rho^{\mathrm{L}}))$, while the proportion of lenders is given by $\Phi_k(S^{\mathrm{L}}(\rho^{\mathrm{L}}))$.

Since $q > 0$, it follows that $\rho^{\mathrm{L}} < (r^p + r^d)/2$ and $(\rho^{\mathrm{L}} + \gamma q)/(1 - \gamma q) > (r^p + r^d)/2$ (see Eq. (14)). Because F is a symmetric CDF, the definitions of S^{L} and S^{B} imply that $S^{\mathrm{L}}(\rho^{\mathrm{L}}) < 0$ and $S^{\mathrm{B}}(\rho^{\mathrm{L}}) > 0$.

Suppose that a k-type has a CDF Φ_k that is more peaked than the CDF $\Phi_{k'}$ of a k'-type. Since the expected shortfall distributions of both types are symmetric around zero, it follows from the definition of relative peakedness in footnote 22 that $\Phi_k(S^{\mathrm{L}}(\rho^{\mathrm{L}})) < \Phi_{k'}(S^{\mathrm{L}}(\rho^{\mathrm{L}}))$. This inequality implies that the equilibrium proportion of lenders is smaller among k-banks than among k'-banks. Similarly, it follows from the definition that $\Phi_k(S^{\mathrm{B}}(\rho^{\mathrm{L}})) > \Phi_{k'}(S^{\mathrm{B}}(\rho^{\mathrm{L}}))$, which means that $1 - \Phi_k(S^{\mathrm{B}}(\rho^{\mathrm{L}})) < 1 - \Phi_{k'}(S^{\mathrm{B}}(\rho^{\mathrm{L}}))$. Thus, the equilibrium proportion of borrowers is also smaller among k-banks than among k'-banks. □

Appendix B. Interest rate under competitive behavior

The notion of the pairwise stable trading equilibrium (PSTE) is sufficient to uniquely identify the amount of borrowing and lending that any bank will do. However, it does not uniquely determine interest rates. Rather, it only imposes boundary conditions on them, as illustrated by the example of two banks in Fig. 2. To refine the predictions on interest rates, we introduce a stronger equilibrium concept, the competitive PSTE.

Definition 2. Network g associated with the lending matrix $\mathbf{L}$ is a **competitive pairwise stable trading equilibrium** (competitive PSTE) if it is a PSTE and, on every existing link ij, the lending amount $\ell_{ij} > 0$ maximizes the profits of both lender i and borrower j given the interest rate r_{ij}.

In a competitive PSTE, banks act as price takers in bilateral interactions, meaning they do not exert market power over interest rates. This equilibrium may emerge from a bargaining process in which the central bank's outside option serves as a reference point.

To study the competitive PSTE, assume that in the corresponding network, bank i lends some reserves to bank j. Then, in equilibrium, the interest rate r_{ij} is determined by the first-order conditions of both banks. Using (A.8) for the lending bank i, we derive the first-order condition as follows:

$$r_{ij}(1 - \gamma q_j) = \gamma q_j + \rho_i \, . \tag{B.8}$$

Thus, the lender equates its marginal benefit (the expected repayment from bank j) with its marginal cost, which consists of the expected loss of the principal due to j's default and the increase in expected costs of dealing with the CB. Similarly, using (A.7) for the borrowing bank j, we obtain its first-order condition:

$$r_{ij} = \rho_j \, . \tag{B.9}$$

The optimal borrowing decision follows from comparing the marginal cost (the interest rate paid to i) with the marginal benefit (the reduction in expected costs of transacting with the CB). The second order conditions are satisfied, see Appendix A.1.

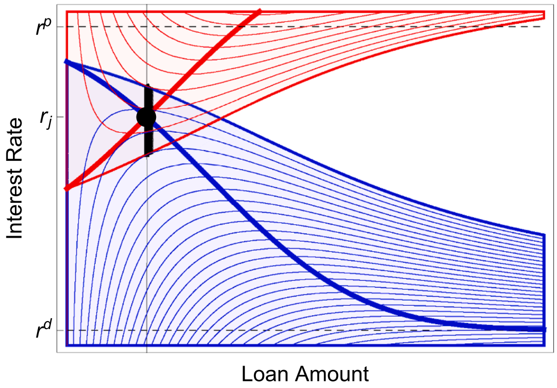

Fig. B.1. The competitive PSTE as an intersection of demand and supply curves.

Note that the interest rate that borrower j pays is independent of the lender's identity and depends only on the borrowing amount (determining ρ_j), according to (B.9). Therefore, in the competitive PSTE, each borrower faces a unique interest rate. We denote this rate as r_j. Recall, also, that as in any PSTE, the expected marginal rate of the lender is denoted as ρ^{L}.

Combining (B.8) and (B.9), we obtain that in the competitive PSTE,

$$r_j = \frac{\gamma q_j + \rho^{\mathrm{L}}}{1-\gamma q_j}\,. \tag{B.10}$$

Eq. (B.10) is consistent with (8), reflecting the alignment between competitive behavior and Pareto efficiency. Thus, as long as the contract curve for banks i and j is non-empty (or equivalently their feasibility sets intersect), there exists a unique pair (ℓ_{ij}, r_{ij}) that satisfies the first-order conditions of both lender and borrower. Moreover, as shown in the proof of Proposition 2, this point (ℓ_{ij}, r_j) lies within the feasibility regions of both the lender and the borrower. Combined with Theorem 1, this implies the following:

Corollary 5. *If there exist at least two banks i and j such that $\rho^0_i < (1-\gamma q_j)\rho^0_j - \gamma q_j$ (i.e., there is a PSTE with interbank trade), then a competitive PSTE exists. In this equilibrium, the expected marginal rate of lenders, ρ^{L}, the net positions of all banks, c_i, and the borrowing interest rates for each borrower, r_j, are uniquely determined.*

We illustrate the competitive PSTE in Fig. B.1 for two banks, i and j. The thick red and blue curves represent the loan supply and demand schedules, respectively, as derived from (B.8) and (B.9). The black point indicates the equilibrium values of ℓ and r in the competitive PSTE.

Appendix C. Extensions: conditions for intermediation

In our model in Section 2, no intermediation occurs within a period, as all equilibrium networks are bipartite. In this section, we present two modifications of the basic framework where intermediation can emerge within a period. Our goal is to demonstrate the framework's versatility and explore the consequences of altering its assumptions.

The first extension assumes that not all pairs of banks can have a link. While this is less relevant for transparent, centrally organized markets like e-MID, it may apply to opaque interbank markets or situations where institutional or historical factors segment the market.[39] Theorem 1 rely on the no-intermediation result of Lemma 3*(ii)*, whose proof assumes that any pair of banks within a chain of intermediaries can always trade directly if they wish. If this assumption does not hold, the bipartite network result cannot be established. Thus, if we assume that banks can only trade if their direct link is included in the network of *potential* links, a subset of the full network, our main results no longer hold. In Appendix C.1, we study a simple case of two separate markets with one potential intermediary.

In the second extension we allow banks to have heterogeneous beliefs about the default probabilities about the same borrower. This setting can be more relevant during periods of heightened uncertainty, when asymmetric information and/or heterogeneous monitoring cost may lead potential lenders to form heterogeneous beliefs about the default probabilities of their potential borrowers. We show in Section C.2 that heterogeneous beliefs about default probabilities may lead to intermediation. Intuitively, lenders with more optimistic views about their counterparties and who are perceived as more trustworthy by other banks, are likely to be the intermediaries. This is formalized in Propostion 7.

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[39] For example, small banks may not be able to borrow across regional markets due to greater asymmetric information about their credit risk, opacity of their assets, and insufficient borrowing size to incentivize other banks to screen them (Ashcraft et al., 2011).

C.1. Connecting two segmented markets

Assume two markets, M1 and M2, consisting of non-intersecting sets of banks $\mathcal{N}_1$ and $\mathcal{N}_2$ with no potential links between them, meaning no trade can occur between banks in different sets. Within each set, all potential links exist, so the results from Section 3 apply. Additionally, assume that there is a PSTE with interbank trading in each market. Let the equilibrium expected marginal rates of lenders be denoted as ρ_1^{L} and ρ_2^{L}.

Now, consider a new bank, indexed I, with an initial expected marginal rate ρ_I^0 and default probability q_I. This bank has potential links with any other bank. Denote the quantities traded by bank I with each market as $\ell_{I,1}$ and $\ell_{I,2}$. A positive quantity indicates lending by bank I to the market, while a negative quantity indicates borrowing.

We analyze the PSTE where all potential links are allowed, except those between any bank in M1 and any bank in M2. The equilibrium configuration occurs when: (1) the market consisting of all banks in $\mathcal{N}_1$ and bank I is in the PSTE; and (2) the market consisting of all banks in $\mathcal{N}_2$ and bank I is in the PSTE.

To proceed, define for each bank i in markets M1 and M2, the function $\ell_i(\rho)$ as in (11). Let $\tilde{\ell}_i$ represent the restriction of this function to the interval $D = (r^d, r^p)$. Define the functions $L_1(\rho) = \sum_{i\in\mathcal{N}_1} \tilde{\ell}i(\rho)$ and $L_2(\rho) = \sum_{i\in\mathcal{N}_2} \tilde{\ell}_i(\rho)$. In the PSTE for the markets without bank I, we have $L_1(\rho_1^{\mathrm{L}}) = 0$ and $L_2(\rho_2^{\mathrm{L}}) = 0$.

When bank I is added, the PSTE conditions for the two markets become:

$$L_1\left(\rho^L_{I,1}\right) + \ell_{I,1} = 0 \quad \text{and} \quad L_2\left(\rho^L_{I,2}\right) + \ell_{I,2} = 0, \tag{C.11}$$

where $\rho^{\mathrm{L}}_{I,1}$ and $\rho^{\mathrm{L}}_{I,2}$ are the equilibrium expected marginal rates of lenders in markets M1 and M2, respectively. The equilibrium reserves for bank I, denoted c_I, are defined as:

$$c_I = c_I^0 - \ell_{I,1} - \ell_{I,2}\,.$$

Rewriting the equilibrium and initial reserves in terms of the expected marginal rates, the equation becomes:

$$-F_I^{-1}\left(\frac{\rho_I^0 - r^d}{r^p - r^d}\right) + F_I^{-1}\left(\frac{\rho_I - r^d}{r^p - r^d}\right) + L_1(\rho^{\mathrm{L}}_{I,1}) + L_2(\rho^{\mathrm{L}}_{I,2}) = 0\,. \tag{C.12}$$

This equation relates three endogenous quantities: $\rho^{\mathrm{L}}_{I,1}$, $\rho^{\mathrm{L}}_{I,2}$, and ρ_I. There are several possible equilibrium network configurations involving the two markets and bank I. Depending on the configuration, additional restrictions on these quantities arise from the results of Section 3 for the sets of banks in $\mathcal{N}_1 \cup I$ and $\mathcal{N}_2 \cup I$, separately. There are four cases for bank I's trading:

Case 1. I lends to M1 and borrows from M2. Then, $\rho_I = \rho^{\mathrm{L}}_{I,1} = (\rho^{\mathrm{L}}_{I,2} + \gamma q_I)/(1-\gamma q_I)$.

Case 2. I lends to M2 and borrows from M1. Then, $\rho_I = (\rho^{\mathrm{L}}_{I,1} + \gamma q_I)/(1-\gamma q_I) = \rho^{\mathrm{L}}_{I,2}$.

Case 3. I lends to both M1 and M2. Then, $\rho_I = \rho^{\mathrm{L}}_{I,1} = \rho^{\mathrm{L}}_{I,2}$.

Case 4. I borrows from both M1 and M2. Then, $\rho_I = (\rho^{\mathrm{L}}_{I,1} + \gamma q_I)/(1-\gamma q_I) = (\rho^{\mathrm{L}}_{I,2} + \gamma q_I)/(1-\gamma q_I)$.

For each case, the additional conditions can be substituted into Eq. (C.12). Since the interbank trading PSTE exist in both markets, L_1 and L_2 are strictly increasing functions. Hence, the left side of Eq. (C.12) strictly increases in ρ_I in each case. Thus, if the equilibrium exists, it will be unique.

The necessary and sufficient conditions for the *existence* of the equilibrium can be derived in each case. We will discuss case 1 only. When bank I is a lender to M1 and a borrower from M2, then $\rho_I = \rho^{\mathrm{L}}_{I,1} = (\rho^{\mathrm{L}}_{I,2} + \gamma q_I)/(1-\gamma q_I)$. In this case, Eq. (C.12) becomes:

$$-F_I^{-1}\left(\frac{\rho_I^0 - r^d}{r^p - r^d}\right) + F_I^{-1}\left(\frac{\rho_I - r^d}{r^p - r^d}\right) + L_1(\rho_I) + L_2\big((1-\gamma q_I)\rho_I - \gamma q_I\big) = 0\,. \tag{C.13}$$

As mentioned in the main text, this equation has at most one solution. To derive the necessary and sufficient conditions for the existence of the equilibrium, note that the configuration where I lends to M1 and borrows from M2 is described by the conditions $\ell_{I,1} > 0$ and $\ell_{I,2} < 0$. From the monotonicity of functions L_1 and L_2, these conditions are equivalent to $\rho^{\mathrm{L}}_{I,1} < \rho^{\mathrm{L}}_1$ and $\rho^{\mathrm{L}}_{I,2} > \rho^{\mathrm{L}}_2$, which become:

$$\rho_I < \rho^{\mathrm{L}}_1 \quad \text{and} \quad (1-\gamma q_I)\rho_I - \gamma q_I > \rho^{\mathrm{L}}_2.$$

Thus, if a solution to (C.13) exists and belongs to the interval

$$\left(\frac{\rho^{\mathrm{L}}_2 + \gamma q_I}{1-\gamma q_I}, \rho^{\mathrm{L}}_1\right),$$

then we have found an equilibrium for Case 1. By specifying the corresponding signs of the left side of Eq. (C.13) on this interval, we derive the following two *necessary and sufficient* conditions:

$$-F_I^{-1}\left(\frac{\rho_I^0 - r^d}{r^p - r^d}\right) + F_I^{-1}\left(\frac{\frac{\rho_2^{\mathrm{L}}+\gamma q_I}{1-\gamma q_I} - r^d}{r^p - r^d}\right) + L_1\left(\frac{\rho_2^{\mathrm{L}} + \gamma q_I}{1-\gamma q_I}\right) < 0,$$

and

$$-F_I^{-1}\left(\frac{\rho_I^0 - r^d}{r^p - r^d}\right) + F_I^{-1}\left(\frac{\rho_1^{\mathrm{L}} - r^d}{r^p - r^d}\right) + L_2\big((1-\gamma q_I)\rho_1^{\mathrm{L}} - \gamma q_I\big) > 0,$$

or, writing it together:

$$\begin{aligned} &L_2\big((1-\gamma q_I)\rho_1^{\mathrm{L}} - \gamma q_I\big) + F_I^{-1}\left(\frac{\rho_1^{\mathrm{L}} - r^d}{r^p - r^d}\right) \\ &\qquad > F_I^{-1}\left(\frac{\rho_I^0 - r^d}{r^p - r^d}\right) > F_I^{-1}\left(\frac{\frac{\rho_2^{\mathrm{L}}+\gamma q_I}{1-\gamma q_I} - r^d}{r^p - r^d}\right) + L_1\left(\frac{\rho_2^{\mathrm{L}} + \gamma q_I}{1-\gamma q_I}\right). \end{aligned}$$

This condition relates the initial (without bank I) equilibrium expected marginal rates in markets 1 and 2, ρ_1^{L} and ρ_2^{L}, and the initial expected marginal rate of bank I, ρ_I^0.

The remaining three cases can be analyzed similarly. An important point is that a bank can be an intermediary in our framework when trade is exogenously restricted along some links. Moreover, the analytical characterization of such a scenario is achieved by dividing the whole set of banks into the largest possible subsets within which all links are possible and applying the previous results to every subset, as we did in (C.11). The resulting financial flows between the subsets will define the final equilibrium reserves of the banks that are at the intersection of the subsets, as for bank I in our example.

C.2. Heterogeneous beliefs

We now demonstrate how our model can incorporate the heterogeneous beliefs about the default probabilities. Denote the default probability belief of lender i about borrower j by q_{ij}, and assume that all these probabilities, as well as the loss given default, γ, are positive. PSTE in this setting will solve the following system:

$$\begin{aligned} &c_i^0 + \sum_{j\neq i}\ell_{ji} - \sum_{j\neq i}\ell_{ij} = T_i - F_i^{-1}\left(\frac{\rho_i - r^d}{r^p - r^d}\right) && \text{for all } i, \\ &\rho_j < \frac{\rho_i + \gamma q_{ij}}{1-\gamma q_{ij}} && \text{for all } i, j \text{ such that } i \neq j \text{ and } \ell_{ij} = 0, \\ &\rho_j = \frac{\rho_i + \gamma q_{ij}}{1-\gamma q_{ij}} && \text{for all } i, j \text{ such that } i \neq j \text{ and } \ell_{ij} > 0, \end{aligned} \tag{C.14}$$

where $\ell_{ij} \geq 0$ are quantities borrowed by j from i. The first equation connects the banks position after the interbank trade to its ρ_i, the expected marginal rate of transacting with the CB. The middle inequality and the last equality are the consequences of Corollaries 1 and 2, respectively, which hold for heterogeneous q_{ij} as well.

We now characterize the PSTE in this setting. It follows from Eq. (C.14), that as in the homogeneous belief case, along any link in the equilibrium, the expected marginal rate of a borrower is larger than the expected marginal rate of a lender. As before, this rules out the configurations, where a pair of banks lend to each other as well as loops of any length. However, this does not rule out intermediation as the next result establishes.

Proposition 7. *Consider the setting with heterogeneous beliefs about default probabilities and let all these beliefs be positive. If in a PSTE, there are three banks, i, j and k, such that $\ell_{ij} > 0$ and $\ell_{jk} > 0$, then:*

(i) $\rho_i < \rho_j < \rho_k$,
(ii) $\ell_{ki} = 0$,
(iii) $\gamma q_{ik} \geq \gamma q_{ij} + (1-\gamma q_{ij})\gamma q_{jk}$,
(iv) $\ell_{ik} > 0$ *if and only if* $\gamma q_{ik} = \gamma q_{ij} + (1-\gamma q_{ij})\gamma q_{jk}$.

Proof.

(i). From Eq. (C.14), we have $\rho_j = \frac{\rho_i+\gamma q_{ij}}{1-\gamma q_{ij}}$ and $\rho_k = \frac{\rho_j+\gamma q_{jk}}{1-\gamma q_{jk}}$, which is more handy to rewrite as $1+\rho_i = (1+\rho_j)(1-\gamma q_{ij})$ and $1+\rho_j = (1+\rho_k)(1-\gamma q_{jk})$, respectively. As default probabilities are positive, we then have $\rho_i < \rho_j < \rho_k$.

(ii). Suppose $\ell_{ki} > 0$. Then in addition to $\rho_i < \rho_k$ as above, we have $\rho_i = \frac{\rho_k+\gamma q_{ki}}{1-\gamma q_{ki}}$ which implies that $\rho_k < \rho_i$. This is not possible.

(iii) and (iv). From *(i)* we already know that $1+\rho_i = (1+\rho_j)(1-\gamma q_{ij})$ and $1+\rho_j = (1+\rho_k)(1-\gamma q_{jk})$, which together imply

$$1+\rho_i = (1+\rho_k)(1-\gamma q_{ij})(1-\gamma q_{jk}).$$

From Corollaries 1 and 2 applied to banks i and k, we obtain that $1+\rho_i \geq (1+\rho_k)(1-\gamma q_{ik})$ with the equality if and only if $\ell_{ik} > 0$. Combining this together and dividing both sides by $1+\rho_k$, we obtain $(1-\gamma q_{ij})(1-\gamma q_{kj}) \geq (1-\gamma q_{ik})$ with the equality if and only if $\ell_{ik} > 0$. Rewriting this, we obtain the requested result. □

Point *(iii)* provides the necessary condition on the default probabilities for intermediation to arise. This condition compares, from the perspective of lender i, the expected loss rate on the direct link, q_{ik}, with the expected loss rate on the path via j. Essentially, lender i chooses a path to lend to k (direct or via an intermediary) by minimizing the expected loss rate along the path. In the knife-edge case when the two paths are equivalent, we may have a situation where i lends both directly and through the intermediary.

The question remains whether the intermediary situation is generically possible. For the case when there are only three banks, one can see that if ρ_i^0 is sufficiently larger than ρ_k^0, and the condition from Proposition 7*(iii)* holds with a strict inequality, then there is a range for ρ_j^0 such that bank j is an intermediary in a PSTE. Therefore, intermediation becomes a rather generic property of the model with heterogeneous beliefs.

Making the beliefs about the default probabilities decreasing with the frequency of transactions between banks will further allow modeling the preferential relationships between banks, whose evidence was reported by Cocco et al. (2009) and Temizsoy et al. (2015), among others. For doing this, one would need to define a learning dynamics and thus we will leave this direction for future research.

Appendix D. Comparative statics

In this Appendix we assume that the function F_i is differentiable for each bank i. For a bank i, define the *shortfall function*, which expresses the shortfall as a function of the marginal rate ρ:

$$S_i(\rho; r^d, r^p) = F_i^{-1}\left(\frac{\rho - r^d}{r^p - r^d}\right).$$

In particular, it holds for any bank that $S_i(\rho^0) = T_i - c_i^0$.

Define the transformation of the expected marginal rate from a lender to a borrower:

$$R(\rho, q; \gamma) = \frac{\rho + \gamma q}{1 - \gamma q}.$$

The *net lending* function ℓ_i for bank i, as defined in (11), can be written as

$$\ell_i(\rho) = \begin{cases} S_i(R(\rho, q_i)) + c_i^0 - T_i & \text{if } R(\rho, q_i) < \rho_i^0 \\ 0 & \text{if } \rho \leq \rho_i^0 \leq R(\rho, q_i) \\ S_i(\rho) + c_i^0 - T_i & \text{if } \rho > \rho_i^0 \end{cases}$$

Since S_i is a strictly increasing function of ρ, by applying it to the inequalities that define the domains of the branches and using the fact that $c_i^0 - T_i = -S_i(\rho^0)$, we can rewrite the same function as follows:

$$\ell_i(\rho) = \begin{cases} S_i(R(\rho, q_i)) - S_i(\rho_i^0) & \text{if } S_i(R(\rho, q)) < S_i(\rho^0) \;\Leftrightarrow\; i \in \mathcal{B} \\ 0 & \text{if } i \in \mathcal{I} \\ S_i(\rho) - S_i(\rho_i^0) & \text{if } S_i(\rho) > S_i(\rho^0) \;\Leftrightarrow\; i \in \mathcal{L} \end{cases}$$

so that it becomes clear that the first branch is negative (shortfall decreases, i.e., $i \in \mathcal{B}$) and the third branch is positive (shortfall increases, i.e., $i \in \mathcal{L}$).

The equilibrium marginal rate ρ^{L} satisfies to $L(\rho^{\mathrm{L}}) = 0$ where the function L is the sum of all functions ℓ_i.

We study the effect of exogenous variables on the endogenous variables, given the initial c_i^0. For this purpose, let us evaluate all partial derivatives. Let f_i denote the PDF of the reserve error distribution. (For brevity, we will omit the argument of this function.)

For function $S_i(\rho; r^d, r^p)$:

$$\frac{\partial S_i}{\partial \rho} = \frac{1}{f_i} \cdot \frac{1}{r^p - r^d} > 0, \quad \frac{\partial S_i}{\partial r^d} = -\frac{1}{f_i} \frac{r^p - \rho}{(r^p - r^d)^2} < 0, \quad \frac{\partial S_i}{\partial r^p} = -\frac{1}{f_i} \cdot \frac{\rho - r^d}{(r^p - r^d)^2} < 0,$$

The signs show that shortfalls increase in ρ and decrease in r^p and r^d.

For function $R(\rho, q; \gamma)$:

$$\frac{\partial R}{\partial \rho} = \frac{1}{1 - \gamma q} > 0, \quad \frac{\partial R}{\partial q} = \frac{\gamma(1+\rho)}{(1-\gamma q)^2} > 0, \quad \frac{\partial R}{\partial \gamma} = \frac{q(1+\rho)}{(1-\gamma q)^2} > 0.$$

Thus borrowers' rate increases in ρ, its own q and γ.

Combining this, we can obtain the information about derivatives of function ℓ_i. First,

$$\frac{\partial \ell_i}{\partial \rho} = \begin{cases} \dfrac{1}{f_i} \cdot \dfrac{1}{r^p - r^d} \cdot \dfrac{1}{1 - \gamma q_i} & \text{if } i \in \mathcal{B} \\ 0 & \text{if } i \in \mathcal{I} \\ \dfrac{1}{f_i} \cdot \dfrac{1}{r^p - r^d} & \text{if } i \in \mathcal{L}. \end{cases}$$

Since $r^p > r^d$ and $\gamma q_i < 1$, it follows that ℓ_i is increasing in ρ (it is strictly increasing for active banks).

Second,

$$\frac{\partial \ell_i}{\partial r^d} = \begin{cases} -\frac{1}{f_i} \cdot \frac{r^p - R(\rho, q_i)}{(r^p - r^d)^2} & \text{if } i \in \mathcal{B} \\ 0 & \text{if } i \in \mathcal{I} \\ -\frac{1}{f_i} \frac{r^p - \rho}{(r^p - r^d)^2} & \text{if } i \in \mathcal{L} \end{cases}, \qquad \frac{\partial \ell_i}{\partial r^p} = \begin{cases} -\frac{1}{f_i} \cdot \frac{R(\rho, q_i) - r^d}{(r^p - r^d)^2} & \text{if } i \in \mathcal{B} \\ 0 & \text{if } i \in \mathcal{I} \\ -\frac{1}{f_i} \frac{\rho - r^d}{(r^p - r^d)^2} & \text{if } i \in \mathcal{L}\,. \end{cases}$$

It follows that ℓ_i is decreasing in r^d and r^p (strictly, for active banks). To see this for the borrower case, note that on the one hand $R(\rho, q_i) > \rho \geq r^d$ by its definition, and on the other hand, $R(\rho, q) \leq r^p$ for any bank. A borrower with $R(\rho, q_i) > r^p$, would be inactive in the interbank market (Fig. 3), as the CB would then provide better conditions.

Finally, we compute

$$\frac{\partial \ell_i}{\partial q_j} = \begin{cases} \frac{1}{f_i} \cdot \frac{1}{r^p - r^d} \cdot \frac{\gamma(1+\rho)}{(1-\gamma q_i)^2} & \text{if } i = j \text{ and } i \in \mathcal{B} \\ 0 & \text{otherwise}\,, \end{cases}$$

$$\frac{\partial \ell_i}{\partial \gamma} = \begin{cases} \frac{1}{f_i} \cdot \frac{1}{r^p - r^d} \cdot \frac{q_i(1+\rho)}{(1-\gamma q_i)^2} & \text{if } i \in \mathcal{B} \\ 0 & \text{otherwise}\,, \end{cases} \quad \text{and} \quad \frac{\partial \ell_i}{\partial T_j} = \begin{cases} -1 & \text{if } i = j \text{ and } i \in \mathcal{B} \cup \mathcal{L} \\ 0 & \text{otherwise}\,. \end{cases}$$

We can then calculate the derivatives of the function $L(\rho) = \sum_i \ell_i(\rho)$. These are

$$\frac{\partial L}{\partial \rho} = \frac{1}{r^p - r^d} \left(\sum_{j \in \mathcal{B}} \frac{1}{f_j} \cdot \frac{1}{1 - \gamma q_j} + \sum_{i \in \mathcal{L}} \frac{1}{f_i} \right) \geq 0$$

and whose sign is strictly positive as soon as there are buyers and sellers, i.e., $\mathcal{B} \cup \mathcal{L} \neq \emptyset$. Similarly, (and with the same condition to make inequalities strict)

$$\frac{\partial L}{\partial r^d} = -\frac{1}{(r^p - r^d)^2} \left(\sum_{j \in \mathcal{B}} \frac{1}{f_j} \cdot \left(r^p - R(\rho, q_i)\right) + \sum_{i \in \mathcal{L}} \frac{1}{f_i} \left(r^p - \rho\right) \right) \leq 0\,,$$

$$\frac{\partial L}{\partial r^p} = -\frac{1}{(r^p - r^d)^2} \left(\sum_{j \in \mathcal{B}} \frac{1}{f_j} \cdot \left(R(\rho, q_i) - r^d\right) + \sum_{i \in \mathcal{L}} \frac{1}{f_i} \left(\rho - r^d\right) \right) \leq 0\,.$$

Also we have

$$\frac{\partial L}{\partial \gamma} = \frac{1+\rho}{r^p - r^d} \sum_{j \in \mathcal{B}} \frac{1}{f_j} \cdot \frac{q_j}{(1-\gamma q_j)^2} \geq 0\,,$$

with a strict inequality in the PSTE with trade. Finally,

$$\frac{\partial L}{\partial q_j} = \begin{cases} \frac{1}{f_j} \cdot \frac{1}{r^p - r^d} \cdot \frac{\gamma(1+\rho)}{(1-\gamma q_j)^2} & \text{if } j \in \mathcal{B} \\ 0 & \text{otherwise}\,, \end{cases}$$

and

$$\frac{\partial L}{\partial T_i} = \begin{cases} -1 & \text{if } i \in \mathcal{B} \cup \mathcal{L} \\ 0 & \text{otherwise}\,. \end{cases}$$

Expected marginal interest rate of lenders. Consider now the equilibrium marginal interest rate of lenders, ρ^{L}. As already mentioned (from the proof of Theorem 1), it solves the equation $\sum_i \ell_i(\rho) = 0$, i.e., $L(\rho) = 0$. Then, applying the IFT, we can determine the derivative of ρ^{L} with respect to any variable z as

$$\frac{\mathrm{d}\rho^{\mathrm{L}}}{\mathrm{d}z} = -\frac{\partial \sum \ell_i}{\partial z} \Big/ \frac{\partial \sum \ell_i}{\partial \rho} = -\frac{\partial L}{\partial z} \Big/ \frac{\partial L}{\partial \rho}$$

and we already found that the denominator is positive in any PSTE with trade.

Therefore, we obtain

$$\frac{\mathrm{d}\rho^{\mathrm{L}}}{\mathrm{d}r^d} = -\frac{\partial L}{\partial r^d} \Big/ \frac{\partial L}{\partial \rho} > 0 \quad \text{and} \quad \frac{\mathrm{d}\rho^{\mathrm{L}}}{\mathrm{d}r^p} = -\frac{\partial L}{\partial r^p} \Big/ \frac{\partial L}{\partial \rho} > 0\,.$$

Also, we compute

$$\frac{\mathrm{d}\rho^{\mathrm{L}}}{\mathrm{d}\gamma} = -\frac{\partial L}{\partial \gamma} \Big/ \frac{\partial L}{\partial \rho} < 0\,,$$

$$\frac{\mathrm{d}\rho^{\mathrm{L}}}{\mathrm{d}q_j} = -\frac{\partial L}{\partial q_j}\Big/\frac{\partial L}{\partial \rho} = \begin{cases} -\dfrac{1}{f_j}\cdot\dfrac{1}{r^p - r^d}\cdot\dfrac{\gamma(1+\rho)}{(1-\gamma q_j)^2}\Big/\dfrac{\partial L}{\partial \rho} < 0 & \text{if } j \in \mathcal{B} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\frac{\mathrm{d}\rho^{\mathrm{L}}}{\mathrm{d}T_i} = -\frac{\partial L}{\partial T_i}\Big/\frac{\partial L}{\partial \rho} = \begin{cases} 1\Big/\dfrac{\partial L}{\partial \rho} > 0 & \text{if } i \in \mathcal{B}\cup\mathcal{L} \\ 0 & \text{otherwise}\,. \end{cases}$$

All these results are consistent with Corollary 4.

Competitive interest rates. Now let us study the effect of parameter changes to the interest rates in the competitive PSTE, given for a borrower j by $r_j = R(\rho^{\mathrm{L}}, q_j; \gamma)$. We calculate

$$\frac{\mathrm{d}r_j}{\mathrm{d}r^d} = \frac{\partial R}{\partial \rho}\cdot\frac{\partial \rho^{\mathrm{L}}}{\partial r^d} > 0\,, \quad \frac{\mathrm{d}r_j}{\mathrm{d}r^p} = \frac{\partial R}{\partial \rho}\cdot\frac{\partial \rho^{\mathrm{L}}}{\partial r^p} > 0\,.$$

To find the effect of γ, we calculate:

$$\begin{aligned} \frac{\mathrm{d}r_j}{\mathrm{d}\gamma} &= \frac{\partial R}{\partial \rho}\cdot\frac{\partial \rho^{\mathrm{L}}}{\partial \gamma} + \frac{\partial R}{\partial \gamma} = \frac{1}{1-\gamma q_j}\cdot\left(-\frac{\partial L}{\partial \gamma}\Big/\frac{\partial L}{\partial \rho}\right) + \frac{q_j(1+\rho^{\mathrm{L}})}{(1-\gamma q_j)^2} = \\ &= \frac{1+\rho^{\mathrm{L}}}{1-\gamma q_j}\cdot\left(-\frac{\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{q_k}{(1-\gamma q_k)^2}}{\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{1}{1-\gamma q_k} + \sum_{i\in\mathcal{L}}\frac{1}{f_i}} + \frac{q_j}{1-\gamma q_j}\right) = \\ &= \frac{1+\rho^{\mathrm{L}}}{(1-\gamma q_j)^2}\cdot\frac{-(1-\gamma q_j)\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{q_k}{(1-\gamma q_k)^2} + q_j\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{1}{1-\gamma q_k} + q_j\sum_{i\in\mathcal{L}}\frac{1}{f_i}}{\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{1}{1-\gamma q_k} + \sum_{i\in\mathcal{L}}\frac{1}{f_i}} = \\ &= \frac{1+\rho^{\mathrm{L}}}{(1-\gamma q_j)^2}\cdot\frac{\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{1}{(1-\gamma q_k)^2}(q_j - q_k) + q_j\sum_{i\in\mathcal{L}}\frac{1}{f_i}}{\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{1}{1-\gamma q_k} + \sum_{i\in\mathcal{L}}\frac{1}{f_i}}\,. \end{aligned}$$

It follows that when a borrower's default probability q_j is larger than or equal to that of any other borrower, the derivative is positive. However, for a borrower with a lower default probability compared to other borrowers, the effect will be reversed.

As for the effect of the default rate, we find for another borrower k, i.e., $k \neq j$, that

$$\frac{\mathrm{d}r_j}{\mathrm{d}q_k} = \frac{\partial R}{\partial \rho}\cdot\frac{\partial \rho^{\mathrm{L}}}{\partial q_k} < 0\,.$$

As for the effect of own q_j, we have

$$\begin{aligned} \frac{\mathrm{d}r_j}{\mathrm{d}q_j} &= \frac{\partial R}{\partial \rho}\cdot\frac{\partial \rho^{\mathrm{L}}}{\partial q_j} + \frac{\partial R}{\partial q_j} = \frac{1}{1-\gamma q_j}\cdot\left(-\frac{\partial L}{\partial q_j}\Big/\frac{\partial L}{\partial \rho}\right) + \frac{\gamma(1+\rho^{\mathrm{L}})}{(1-\gamma q_j)^2} = \\ &= \frac{\gamma(1+\rho^{\mathrm{L}})}{(1-\gamma q_j)^2}\cdot\left(-\frac{\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}}{\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{1}{1-\gamma q_k} + \sum_{i\in\mathcal{L}}\frac{1}{f_i}} + 1\right) = \\ &= \frac{\gamma(1+\rho^{\mathrm{L}})}{(1-\gamma q_j)^2}\cdot\frac{\sum_{k\in\mathcal{B},k\neq j}\frac{1}{f_k}\cdot\frac{1}{1-\gamma q_k} + \sum_{i\in\mathcal{L}}\frac{1}{f_i}}{\sum_{k\in\mathcal{B}}\frac{1}{f_k}\cdot\frac{1}{1-\gamma q_k} + \sum_{i\in\mathcal{L}}\frac{1}{f_i}} > 0\,. \end{aligned}$$

Finally, for any active bank k, we have that

$$\frac{\mathrm{d}r_j}{\mathrm{d}T_k} = \frac{\partial R}{\partial \rho}\cdot\frac{\partial \rho^{\mathrm{L}}}{\partial T_k} > 0\,,$$

while it is zero for any inactive bank.

Trading volume. We define trading volume in the PSTE as

$$\mathrm{Vol} = \sum_{i\in\mathcal{L}} S_i(\rho; r^d, r^p) + \sum_{i\in\mathcal{L}} c_i^0 - \sum_{i\in\mathcal{L}} T_i$$

Therefore,

$$\begin{aligned}
\frac{\mathrm{dVol}}{\mathrm{d}r^d} &= \sum_{i\in\mathcal{L}} \left\{ \frac{\partial S_i}{\partial\rho}\cdot\frac{\partial\rho^{\mathrm{L}}}{\partial r^d} + \frac{\partial S_i}{\partial r^d} \right\} = \\
&= \sum_{i\in\mathcal{L}} \left\{ \frac{1}{f_i}\cdot\frac{1}{r^p-r^d}\cdot\frac{\frac{1}{(r^p-r^d)^2}\left(\sum_{j\in\mathcal{B}}\frac{1}{f_j}(r^p-R(\rho^{\mathrm{L}},q_j))+\sum_{i\in\mathcal{L}}\frac{1}{f_i}(r^p-\rho^{\mathrm{L}})\right)}{\frac{1}{r^p-r^d}\left(\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}\right)} - \frac{1}{f_i}\cdot\frac{r^p-\rho^{\mathrm{L}}}{(r^p-r^d)^2} \right\} \\
&= \frac{1}{(r^p-r^d)^2}\sum_{i\in\mathcal{L}}\frac{1}{f_i}\left\{ \frac{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\left(r^p-R(\rho,q_j)\right)+\sum_{i\in\mathcal{L}}\frac{1}{f_i}\left(r^p-\rho^{\mathrm{L}}\right)}{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}} - r^p + \rho^{\mathrm{L}} \right\} = \\
&= \frac{1}{(r^p-r^d)^2}\sum_{i\in\mathcal{L}}\frac{1}{f_i}\left\{ \frac{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\left(r^p-R(\rho,q_j)-\frac{r^p-\rho^{\mathrm{L}}}{1-\gamma q_j}\right)}{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}} \right\} = \\
&= \frac{1}{(r^p-r^d)^2}\sum_{i\in\mathcal{L}}\frac{1}{f_i}\left\{ \frac{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{r^p-r^p\gamma q_j-\rho^{\mathrm{L}}-\gamma q_j-r^p+\rho^{\mathrm{L}}}{1-\gamma q_j}}{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}} \right\} < 0\,.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\mathrm{dVol}}{\mathrm{d}r^p} &= \sum_{i\in\mathcal{L}} \left\{ \frac{\partial S_i}{\partial\rho}\cdot\frac{\partial\rho^{\mathrm{L}}}{\partial r^p} + \frac{\partial S_i}{\partial r^p} \right\} \\
&= \sum_{i\in\mathcal{L}} \left\{ \frac{1}{f_i}\cdot\frac{1}{r^p-r^d}\cdot\frac{\frac{1}{(r^p-r^d)^2}\left(\sum_{j\in\mathcal{B}}\frac{1}{f_j}(R(\rho^{\mathrm{L}},q_j)-r^d)+\sum_{i\in\mathcal{L}}\frac{1}{f_i}(\rho^{\mathrm{L}}-r^d)\right)}{\frac{1}{r^p-r^d}\left(\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}\right)} - \frac{1}{f_i}\cdot\frac{\rho^{\mathrm{L}}-r^d}{(r^p-r^d)^2} \right\} \\
&= \frac{1}{(r^p-r^d)^2}\sum_{i\in\mathcal{L}}\frac{1}{f_i}\left\{ \frac{\sum_{j\in\mathcal{B}}\frac{1}{f_j}(R(\rho^{\mathrm{L}},q_j)-r^d)+\sum_{i\in\mathcal{L}}\frac{1}{f_i}(\rho^{\mathrm{L}}-r^d)}{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}} - \rho^{\mathrm{L}} + r^d \right\} = \\
&= \frac{1}{(r^p-r^d)^2}\sum_{i\in\mathcal{L}}\frac{1}{f_i}\left\{ \frac{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\left(R(\rho^{\mathrm{L}},q_j)-r^d-\frac{\rho^{\mathrm{L}}-r^d}{1-\gamma q_j}\right)}{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}} \right\} = \\
&= \frac{1}{(r^p-r^d)^2}\sum_{i\in\mathcal{L}}\frac{1}{f_i}\left\{ \frac{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{\rho^{\mathrm{L}}+\gamma q_j-r^d+r^d\gamma q_j-\rho^{\mathrm{L}}+r^d}{1-\gamma q_j}}{\sum_{j\in\mathcal{B}}\frac{1}{f_j}\cdot\frac{1}{1-\gamma q_j}+\sum_{i\in\mathcal{L}}\frac{1}{f_i}} \right\} > 0\,.
\end{aligned}$$

Next, we calculate

$$\frac{\mathrm{dVol}}{\mathrm{d}\gamma} = \sum_{i\in\mathcal{L}}\frac{\partial S_i}{\partial\rho}\cdot\frac{\partial\rho^{\mathrm{L}}}{\partial\gamma} < 0$$

and for any borrower j

$$\frac{\mathrm{dVol}}{\mathrm{d}q_j} = \sum_{i\in\mathcal{L}}\frac{\partial S_i}{\partial\rho}\cdot\frac{\partial\rho^{\mathrm{L}}}{\partial q_j} < 0$$

while this effect is zero for any lender or inactive j.

Finally, for any borrowing bank j, we have that

$$\frac{\mathrm{dVol}}{\mathrm{d}T_j} = \sum_{i\in\mathcal{L}}\frac{\partial S_i}{\partial\rho}\cdot\frac{\partial\rho^{\mathrm{L}}}{\partial T_j} > 0\,,$$

while for any lending bank k, we have that

$$\frac{\mathrm{dVol}}{\mathrm{d}T_k} = \sum_{i\in\mathcal{L}} \frac{\partial S_i}{\partial \rho} \cdot \frac{\partial \rho^{\mathrm{L}}}{\partial T_k} - 1 = \frac{\sum_{i\in\mathcal{L}} \frac{1}{f_i} \cdot \frac{1}{r^p - r^d}}{\frac{1}{r^p - r^d}\left(\sum_{j\in\mathcal{B}} \frac{1}{f_j} \cdot \frac{1}{1-\gamma q_j} + \sum_{i\in\mathcal{L}} \frac{1}{f_i}\right)} - 1 < 0\,.$$

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