The mediating role of artefacts in deductive reasoning

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The Mediating Role of Artefacts in Deductive Reasoning

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Abstract

We present the results of an experiment designed to investigate the mediating role of artefacts in syllogistic reasoning. The aim of the experiment is to compare a new form of representation of the premises, designed according to the Models Theory, with two "classical" representations (Euler circles and Propositions).

The results provide preliminary empirical evidence for both hypothesis of the experiment:

- the representation designed according to the principles of mental Models Theory supports the activity of subjects performing a syllogistic reasoning task better than the two "classical" representations;
- subjects' performance depends also on the specific syllogism to be solved. Each form of representation offers specific constraints and affordances for the production of the mental models that need to be manipulated to produce an answer.

Keywords: Syllogistic reasoning, Representations, Cognitive Artefacts, Mental Model Theory.

Artefacts and Human Cognition

Recent developments within Cognitive Science (Zhang & Norman, 1994; 1995) have provided empirical support to the long lasting thesis that human cognition is mediated by artefacts (tools, rules, models, representations), which are both internal and external to the mind (Vygotsky, 1978). According to these findings, human activity cannot be investigated without taking into account the mediating artefacts.

In the present paper our aim is to investigate the design of a new external representation developed according to some of the assumptions of the Model Theory and to compare it with other forms of external representations (propositions and Euler circles) in order to test i) the mediating role of artefacts in reasoning and ii) the potential advantages offered by an external representation that is in keeping with the assumptions of the Model Theory. The study focuses on a special kind of deductive reasoning i.e. syllogistic reasoning. The choice of syllogistic reasoning is due, on one hand, to the long tradition in designing representations that could support this task (Stenning, 2002) and, on the other hand, to the rich empirical evidence supporting Model Theory for such cognitive activity.

Syllogistic Reasoning

According to Johnson-Laird and Byrne (1991), syllogistic reasoning is a particular kind of deductive reasoning in which two premises containing a single quantifier (all, none and some) and describing the relation among classes of elements (for instance "As" and "Bs" for the first premise and "Bs" and "Cs" for the second premise) are combined to obtain, when possible, a valid conclusion that describes the relationships among the "As" and the "Cs". A valid conclusion describes a state of the world that is true when the states of the world described by the premises are true.

Different theories have proposed to explain human performance on such tasks (e.g. probability heuristic model, formal rules, mental models). Among these approaches, mental Models Theory is the one that has received most empirical support to date. The basic tenet of this theory states that syllogistic reasoning is a semantic (non syntactic) process based on mental representations that are structurally isomorphic to the state of the world they describe. According to the theory, syllogistic deductions are the result of a three stage process:

1. flesh out the content of the premises;
2. combine the first and the second premises;
3. modify the model of the two premises to search for counterexamples.

The theory allows one to formulate detailed predictions about subjects' performance that have received strong empirical confirmation. For instance, it has been shown that the difficulty of a particular syllogism is a function of the number of different models of the premises the subject must flesh out and take into account to formulate valid conclusions. Although mental Models Theory assumes that there can be different sources of external information one could use for building mental models (Bucciarelli & Johnson-Laird, 1999), the experiments carried out so far on syllogistic reasoning have used mainly one kind of artefact, propositions, to represent the premises of syllogisms.

Propositions, as mediating artefacts, have properties that intervene in the elaboration of the information they provide. The mental Model Theory has to some extent overlooked the role played by the specific kinds of representations the subjects are provided with. What would be the impact of different representations of the premises on subjects' performance and errors? What would be the impact of a representation of the premises designed according to mental

Models Theory in an effort to support the subjects' search for alternative models of the premises?
The experiment described below has been designed to test the impact that different representations of the premises have on the subject performance and errors in syllogistic deduction.

Experiment

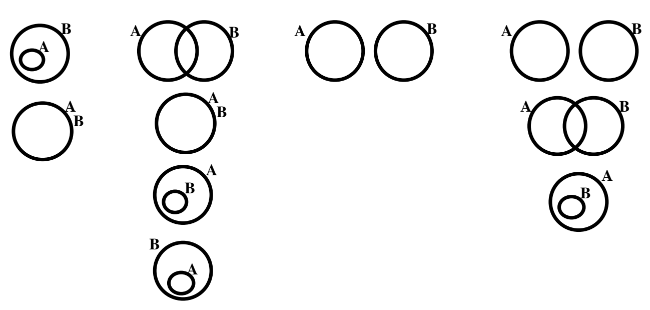
The first aim of the experiment was to provide further evidence to the hypothesis that subjects using different representations of the premises would have different performances in solving the same syllogisms (Rizzo and Palmonari, 2000). The second aim was to explore how different types of syllogisms interact with the different representations. With these aims, three different representations of the premises were used in the experiment: Propositions, Euler circles, and the new representation (Valence, henceforth) created for this experiment according to the three stage process put forward by the mental Models Theory.

First Representation: Proposition

Propositions (strings of symbols close to natural language) are the "standard" way to represent premises. In spite of this, propositions are just one of the possible representations which can be used. Propositions are particularly not well suited to support subjects' activity in any of the three stages of the process. Indeed, unlike Euler circles and Valence representations (see below), propositions do not explicitly represent the possible states described by a premise and unlike Valence representation they do not support the combination of the elements of the premises in a model and its revision.

Second Representation: Euler Circles

Euler circles, a geometrical representation named after the mathematician Leonhard Euler, represent premises using a circle as a model of a set of elements; the advantage of this representation is that it explicitly represents all the possible relations between two entities contained in a single premise (see fig. 1).



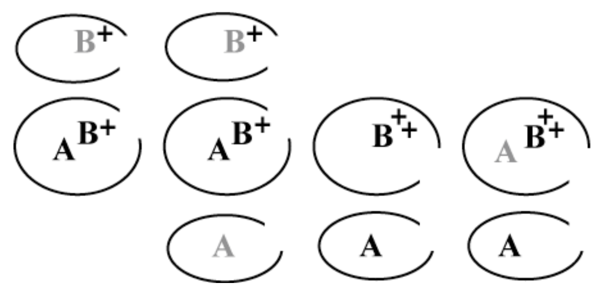
All A are B   Some A are B   No A are B   Some A are not B

Fig. 1. Representation of the four possible kinds of premises using Euler circles

Despite the explicitness of relationship representation, Euler circles do not support the combination of the premises in a single model because the representations of all the possible relations between the elements of a given premise are made up of disparate entities and not "integrated" in a single configuration or model. Instead, for each premise, a different system of 1 to 4 models is provided, each representing possible relationships in keeping with the premise. It follows that for most syllogisms the number of combinations of possible relationships between the elements of the two premises that one must consider in order to build a single solution is often very high.

Third representation: Valence

The Valence representation was designed on the basis of the three steps process put forward by mental Models Theory. Following is a description of its representation properties for each of the three steps
1) Fleshing out the states of the world. Unlike propositions, which do not explicitly represent elements which can or cannot exist, and like Euler circles, which represent all the possible states of the world described by the premise, Valence models explicitly represent (see fig. 2) in a single representation both the entities which are certain to exist (black colour) and the entities which may or may not exist (in grey).



All A are B   Some A are B   No A are B   Some A are not B

Fig. 2. The Valence representation of the 4 premises. The elements of the premise are clustered in semicircles to indicate relationships of association and separation. In the premise "Some A are B," the bold A and B indicate that the existence of associated As and Bs is certain, while the separated A and B in grey indicate the possible existence of As and Bs outside the association. The "+" and "++" symbols are used for combining premises.

2) Build a model representing the content of the two premises. A metaphor is introduced to support subjects in understanding the relationship between the "As" in the first premise and the "Cs" in the second by means of the "Bs" which can be combined according to an atomic metaphor: As depicted in fig. 3, the "Bs" in the first premise have positive valence, the "Bs" in the second have negative valence. According to the atomic principle, different valences attract each other, and equal valences repel each other. When a black "B" has a double valence ("+ +" or "- -"), it represents all of the "Bs" that exist in one of the premises. When a black "B" has just a single valence it means that in the premise there are other possible “Bs” (in grey) which may or may not exist. In this case, one cannot be certain that the black “B” represents all the “Bs” in the premise. The possibility of building a model of the two premises that do not have a counterexample depends on the way “Bs” of the first and second premise combine: when (as shown in fig. 3) it is not possible to group the “Bs” in a single stable configuration, there is no definite link between “As” and “Cs” because there isn’t any definite identity between the “Bs” in the first and the “Bs” in the second premise. In this case, the answer to the syllogism is always “no conclusion”.

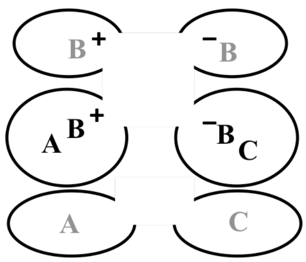


Fig. 3. “Bs” do not form a single stable configuration; there are two “Bs” with a single valence “+” and two with a single valence “-“. Thus, it can be immediately seen that there is no certain relationship between the As and the Cs: if the positive B associated with the A connects with the upper negative B it could be that No A are C; if the same positive B connects with the lower negative B it could be that All A are C. Further models are possible if we consider the grey As and Cs, but already the two possible models no A are C and All A are C are enough to deduce “no conclusion.”

On the contrary, when “Bs”, according to the atomic metaphor, can be grouped into a single stable configuration, it is possible to look at the relations among the “As” and “Cs” because the “Bs” mentioned in the first and second premises can be visually identified as a single set of “Bs” that bridge between “As” and “Cs” (fig. 4). When this relationship exists, there is known to be a defining identity among the sets of “Bs” in the first and second premise.

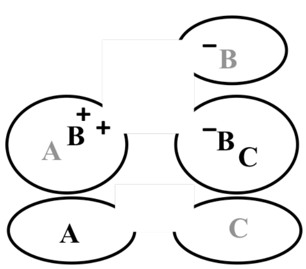


Fig. 4. According to the atomic metaphor, Bs can be grouped in a single stable configuration, there is a B with a double valence ++ and two “Bs” with a single valence -. Yet no valid conclusion can be drawn (see text).

3) Search for counterexamples. A conclusion can be drawn by analyzing the single, stable configuration resulting from the combination of the two premises. To assist in this task the Valence representation uses two additional symbols (fig. 5).

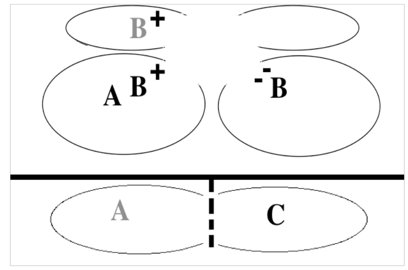


Fig. 5. Some A are B, No C are B.
Conclusion: “Some A are not C”

The black horizontal line indicates the disjunction between the elements placed above and below it, whereas the dashed line indicates a possible but not certain conjunction among the elements placed below the horizontal line. According to the atomic metaphor, “Bs” can combine in three different configurations; when, as depicted in fig. 3, they do not group into a single and stable configuration, they might be linked to different sets of “Bs”. In this case it is not possible to get any permanent state across configurations, and the correct answer is “no conclusion”, as the attempt to combine the premises immediately produces two conflicting models. When, as depicted in fig. 5, “Bs” combine in a single configuration, the relationships among “As” and “Cs” determine the conclusion. In this case, it is sometimes possible to draw immediate valid conclusions about the black “As” and “Cs”, the elements that are certain to exist, by looking at their location in the representation. To understand the relationship among black “As” and black “Cs” subjects can read the emergent configuration using the same syntax used to represent the premises. In particular, the meaning of the notation is the following:

- when they are separated by the horizontal line, there is disjunction among them;
- when both are below the horizontal line, their relationship is not certain (they are separated by the dashed line) .

The grey elements in the representation play a critical role in that, by explicitly representing entities that might or not exist, they can support subjects in deducing alternative models of the premises. Again, we believe that this is a crucial property of the representation: if, according to mental Models Theory, the difficulty in solving syllogisms is related to the numbers of models representing the state of the world described by the premises, a representation which includes the entities which might or might not exist should support subjects in taking into account all the possible models of the premises. In fig. 5, for instance, the black “A” does not represent the whole set of “As” because of the grey “A” below the line, which stands for the possible existence of some other “As” which may or may not have a relationship of identity with the black “Cs”. In this case the

valid conclusion one can draw is "Some A are not C" instead of "All A are not C".
When different models are in conflict, no valid conclusion can be drawn. In the case shown in fig. 4, the possible existence of a set of "As" and a set of "Cs" (in grey) prevents one from drawing any conclusion: all possible relationships between A and C could be true.
Thus, summarizing the differences among Propositions, Euler circles and Valence we have three representations that support in different ways the three-stage process of the mental Model Theory (Table1).

Table 1: Differences among representations in supporting syllogistic reasoning according to Models Theory

	Propos	Euler	Valence
Fleshing out the premises	NO	YES	YES
Combination of the premises	NO	NO	YES
Search for counterexamples	NO	NO	YES

Hypotheses

The design of the experiment was a 3 (Representation) X 3 (Syllogism), and we expected:
A main effect due to the kind of representation. Subjects provided with the Valence representation should show, in a trial of 24 syllogisms (the same as Rizzo and Palmonari, 2000), a better overall performance than the subjects provided with Euler circles and Propositions.
A main effect due to the Type of Syllogism. The 24 syllogisms used in the experiment were grouped into three classes, distinguished by their relationship to the processes of generation of a model and search for counterexamples indicated by Bara & Johnson Laird (1984):
One Model syllogisms (n = 5), with conclusion requiring the generation of only one model to be solved (such as All A are B / All B are C);
More Models syllogisms (n = 7) with conclusion requiring more than one model (two or three) to be solved (such as All B are A / All B are C);
No Conclusion syllogisms (n = 12) requiring the generation of at least two incoherent models (such as All A are B / All C are B).
An interaction between Representation and Syllogism. Different syllogisms require the generation of a different number of models to be solved. However, the three representations might have a different impact on the number and type of mental models generated by subjects when solving syllogisms. The subjects' performance should depend on the interaction between the provided representation and the kinds of syllogisms they face.

Subjects

26 subjects, all students in Communication Sciences at the University of Siena, took part in the experiment. Each subject was randomly assigned to one of the three experimental conditions (8 subjects for Euler circles and Proposition and 10 subjects for Valence). Each subject faced the same 24 syllogisms, which were randomly presented.

Dependent measures

The subjects' performance was evaluated by means of two dependent measures, *accuracy* (the percentage of syllogisms properly solved) and *time* (the amount of time, in seconds, employed by each subject to solve each syllogism).

Procedure

The subjects met individually with a research assistant and received a 30 minute session on syllogistic reasoning (explanation of the premises and their combination), including a supervised trial on 6 syllogisms to provide every subject with the basic knowledge needed to understand the syntax of the type of representation s/he would have to interact with. In the experimental session the subjects faced a computer screen where the syllogisms were randomly presented one at time in one of the modalities (Proposition, Euler, Valence). At the bottom of the screen there was a set of possible conclusions represented in the same modality, except for "No Conclusion," which was represented by a simple NO. The subject could select any number of conclusions s/he thought to be valid, and the time was recorded when the "Next" button was pressed.

Results

An analysis of variance was performed for each dependent variable (*accuracy* and *time*), considering as independent factors *Representation* and *Syllogism*.
A main effect due to *Representation* was found for the dependent variable *accuracy* ($F= 18.7$, $p<.001$): in particular, (see table 2) the score for subjects in the Valence condition was significantly better than the score for subjects in the Euler circles condition ($p<.001$) and in the Proposition condition ($p<.001$). In addition, the score in the Proposition condition was significantly better than in the Euler circles condition ($p<.001$).

Table 2: Summary of results for *accuracy* (left) and for *time* (right)

	% of correct answers	Std Error	**Mean Time (sec.)**	Std Error
Valence	**.79**	.683	**42**	2.4
Proposition	**.69**	.760	**81**	5.2
Euler	**.55**	1.222	**95**	6.0

A main effect for *Representation* was also found for the dependent variable *time* ($F= 39.4$, $p<.001$). Post-hoc analysis revealed a significant difference between the Valence and Euler circles conditions ($p<. 001$) as well as the difference between Valence and Propositions ($p<.001$) and Propositions and Euler circles ($p<.001$).
An analysis of variance was also performed for each dependent variable (*accuracy* and *time*), considering as an independent factor *Type of Syllogism*. The main effect of *Type of syllogism* was found to be significant with respect to both the dependent variables *accuracy* ($F=5.9$, $p<.001$) and *time* ($F=1.8$, $p<.025$). Finally, the interaction between

Representation and *Type of syllogism* was also found to be significant ($F= 1.86$, $p<.001$) for *accuracy* and ($F=1.5$, $p<.025$) for *time*.
Here for reason of space only the post-hoc analysis (Fisher test, $p<.01$) of the interaction for *accuracy* will be reported (*time* presents a close pattern of result).
Within Representation levels: For Euler Circles a significant difference was found for the class "One Model" vs "More Models" and "No Conclusion" (see Table 3) . For Valence, the significant difference found was for "No Conclusion" vs "More models" while no difference has been found for "More models" vs "One Model". Finally, for Propositional, a significant difference for "More Models" vs "One model" and "No conclusion" was found.
Within Syllogism levels: Euler had a better performance than Proposition and Valence for "One Model," while Valence had a better performance for "More Models" with respect to Proposition and Euler. Finally, Euler had a worse performance in respect to Proposition and Valence for "No Conclusion".

Table 3. Summary of results for *accuracy* for the three representations with respect to Type of Syllogism

	One Model	More Models	No Conclusion
Valence	.78	.67	.87
Proposition	.85	.42	.78
Euler	.95	.42	.45
average	.86	.50	.70

DISCUSSION

Data provided by the experiment support the main hypothesis concerning the mediating role of representations in syllogistic reasoning. Subjects using different representations of premises show different performances both in terms of *accuracy* and *time*. In particular, subjects using Valence representation, the representation explicitly designed to support all these three phases proposed by Models Theory show an overall better performance (more accuracy and less time) than subjects using Propositions (no phases supported) and Euler circles (supporting just the first phase, the fleshing out of the properties of the premises).
However, the mediating role of the different representations cannot be demonstrated solely by the type of support provided by the representations to the three phases of the syllogistic reasoning (see table 1). The results indicate, for instance, that Euler Circles, which supports the fleshing out of the properties of the premises, do not induce a better performance (in terms of accuracy) with respect to the Propositional representation. Furthermore, if one considers the dependent variable "Time", Euler Circles induce a significantly worse performance than Propositional.
If the mediating role of the representations is not a linear function of the support provided by those representations for the three phases of deduction, neither can it be shown to be a simple external mnemonic support in the deductive process elicited by representations. Indeed, if the external representations functioned as mnemonic support, the assistance provided for the deductive process should be constant independent of the content (the specific syllogisms) that subjects have to elaborate in order to generate accurate model(s) of the conclusion. This hypothesis is clearly contradicted by the results concerning the interaction *Representation* x *Type of Syllogism* which indicates that the effectiveness of each representation varies across syllogisms.
The main hypothesis put forward in the present paper is that the principal property of artefacts is their simultaneous affordance and constraint of the manipulation and recombination of their constituent elements. The data support this hypothesis: it is the form of each representation (the way in which the content is represented and the manipulation of the content afforded) which, by interacting with the process of generation and falsification of models, facilitates or makes difficult the resolution of different syllogisms.
The data concerning the post-hoc analysis and the subjects' errors allow more detailed hypotheses to be made about the roles played by the different representations in syllogistic reasoning. Since the results related to the distinction between One model – More models – No conclusion of subjects using Propositional are in line with data of mental Models Theory, only data about Valence and Euler Circles will be discussed.
In Valence the "one model" syllogisms and the "more model" syllogisms are represented in the same way and afford the same manipulations: in both cases subjects can isolate different models associated with the conclusion. Even for "one model" syllogisms (as depicted in fig. 6) it is possible to isolate up to four different models of the relationships between the "As" and the "Cs". Obviously, those models have to be integrated in order to generate the correct answer. The "All B are A – Some B are C" is, in fact, among the "One model" syllogisms the one shown to be the most difficult to solve using Valence representation. On the contrary, subjects did not generate any wrong conclusions for syllogisms such as "All A are B – No B are C" (see fig. 7) for which Valence representation "presents" just one relationship between the "As" and the "Cs". At the same time "more models" syllogisms can be solved as simply as "one model" syllogism (see, for example, the syllogism "Some A are B – No C are B" reported in Figure 5 and compare it with the Euler representation for the same syllogism reported below in Figure 8 right).
To summarize, subjects' performance with Valence representation for "one model" and "more models" syllogisms is related to the number of different conclusion models the representation allows the subject to immediately isolate (the most difficult being syllogisms which allow to isolate more than three different models). The good performance shown by subjects for "no conclusion" syllogisms can be explained again by the properties of

Valence representation, which allows, in most cases, to immediately isolate two incompatible conclusion models.
The performance of subjects using Euler Circles can be explained also by the form of the representation. As shown in Figure 8 on the left, for “one model” syllogisms, Euler Circles allows to isolate one of the models describing the relation between elements (i.e; “A” and “B”) in one of the premises and to directly map the model onto all the models representing the relationship between elements in the other premise (i.e; “B” and “C”). Once the relationship between the elements described by one premise is directly mapped onto the relationship between the elements described in the other premise, then a conclusion can be easily drawn. This strategy, strongly afforded by the representation and which appears to be optimal for “one model” syllogisms, leads subjects to errors in the cases of “more models” and “no conclusion” syllogisms. Since the appropriate strategy in these cases would be, unlike for “one model” syllogisms, to consider all the possible combinations of different models of the premises, subjects generate either a conclusion when no conclusion exists (for “no conclusion” syllogisms) or a “wrong conclusion” that does not integrate all the possible relationships between premises (for “more models” syllogisms). See for example the syllogism “Some A are B - No C are B” reported in Figure 8 right, where even though one of the premises is represented by one model the manipulation it affords produce several models that are difficult to be integrated.

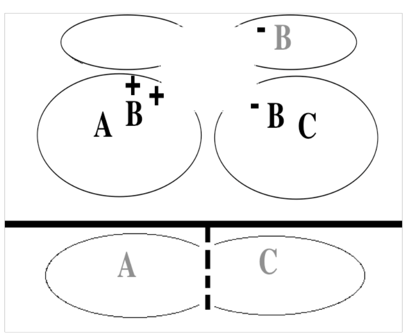


Figure 6. All B are A - Some B are C

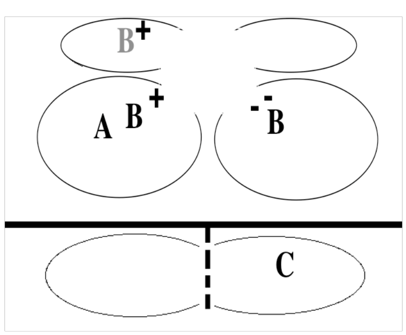


Figure 7. All A are B; No B are C

According to the Models Theory, the resolution of syllogisms involves the same basic process underlying human thought and what makes different syllogisms easier or more difficult to solve are the number of models one must generate and compare to draw the appropriate conclusion. The experiment provides empirical support to the main claim of the Model theory, yet it also supports the hypothesis that representations play a central role in activating and structuring those basic processes that the Mental Model theory proposes.

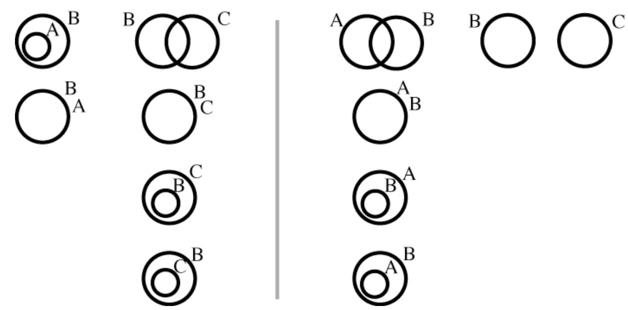


Figure 8. Left “All A are B – Some B are C”;
Right “Some A are B - No C are B”

To some extent, Model theory has overlooked the role played by the specific representations used to present the content to be manipulated. These findings indicate that each representation modifies the content subjects must manipulate in order to generate the appropriate models of conclusion. The number of mental models generated for a specific syllogism is not solely a function of its formal properties but additionally depends on the form of representation assumed by the syllogism. Thus, the critical property of representations lies in the way they represent the content (the properties of syllogisms) and afford and constrain the manipulation and recombination of their constituent elements in relation to the basic process underlying the elaboration of this content. External representations do not modify these processes; rather they modify the ways in which content can be manipulated.

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