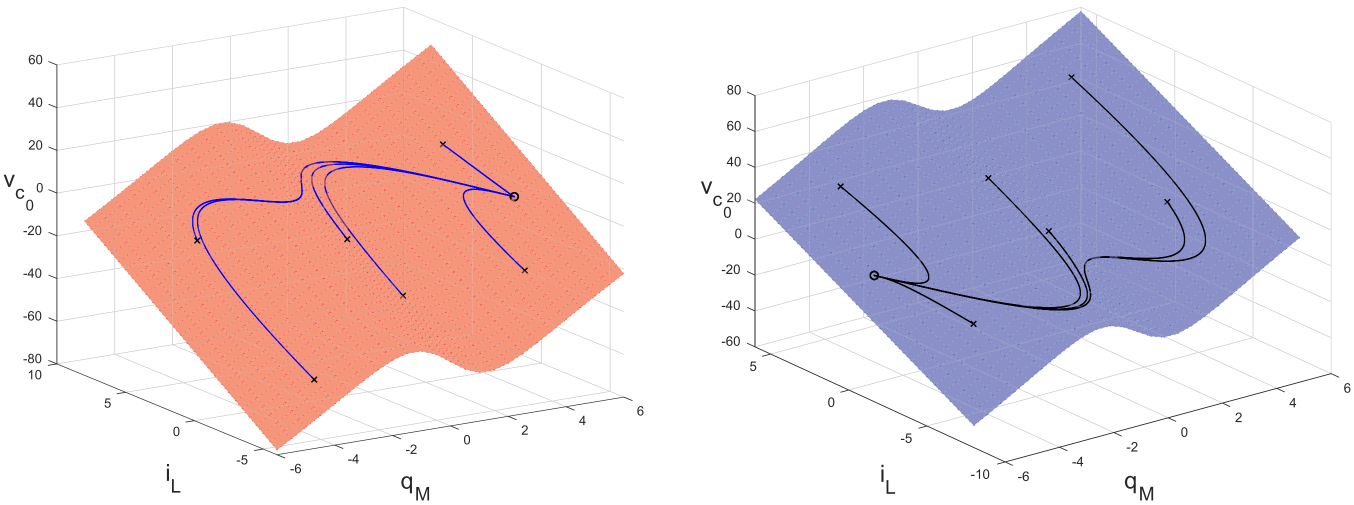


(a) (b)

Fig. 11. State space trajectories of the RLC-memristor circuit with $R = 1$, $L = 0.5$, $C_0 = 0.1$ and $a = 2$, $b = 1/3$, $c = 0.2$ in memristor characteristic (50). The initial conditions (marked with $\times$) are chosen to ensure that the trajectories lie on the invariant manifolds $\mathcal{M}_{\mathcal{I}}$. (a) Manifold with $\mathcal{I} = 2$ (the unique equilibrium point is marked with $\circ$); (b) Manifold with $\mathcal{I} = -2$ (the unique equilibrium point is marked with $\circ$).

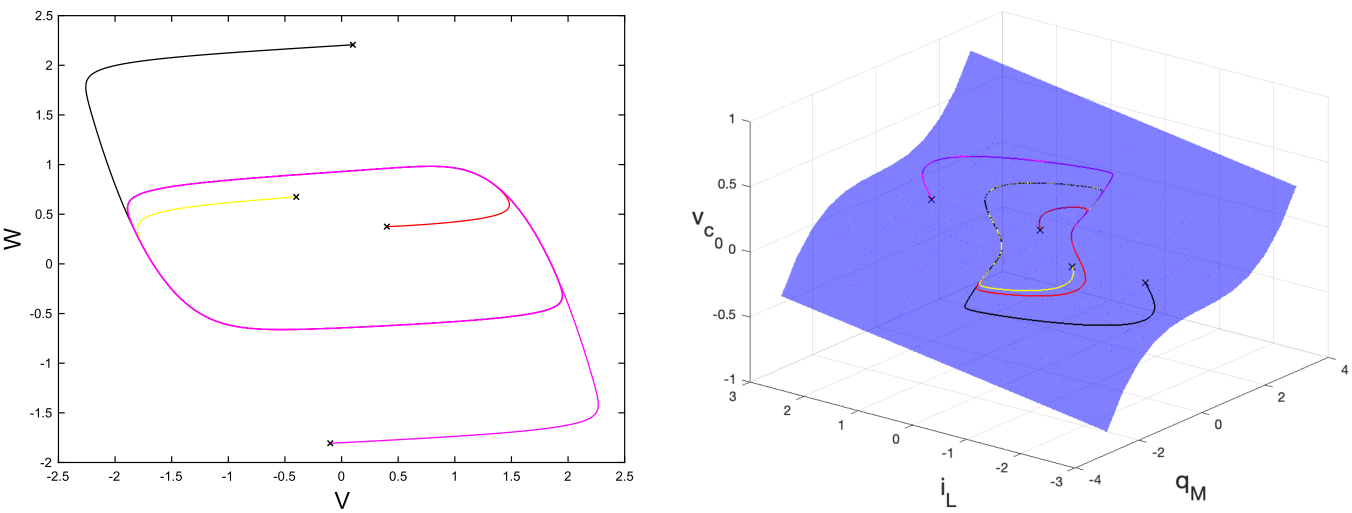
following inequalities

$$1 < \frac{a}{R} < \frac{L}{R^2 C_0}, \quad c < \frac{\dfrac{b}{R}}{\dfrac{a}{R-1}}$$

hold, then the RLC-memristor circuit is nonoscillatory and the manifold $\mathcal{M}_{\mathcal{I}}$ has three equilibrium points (two stable and one unstable) for small values of $|\mathcal{I}|$ and a unique stable equilibrium point for large values of $|\mathcal{I}|$. This scenario is illustrated for the circuit parameters $R = 1$, $L = 0.5$, $C_0 = 0.1$ and the constants $a = 2$, $b = 1/3$, $c = 0.2$ in Figs. 10 and 11, where all the trajectories show convergence toward one of the stable equilibrium points.

Finally, we observe that if we set $L = 1$, $\alpha = 1/C_0$, $\beta = 1/(RC_0)$ and define the new state variables

$$V = x_1, \quad W = -x_2 + \beta x_1 + I, \tag{52}$$



(a) (b)

Fig. 12. (a) Trajectories in the (V, W)-plane of the FitzHugh–Nagumo model with $a = 1$, $b = 1/3$, $c = 0$, $\alpha = 0.08$, $\beta = 0.064$, and $I = 0.2$. (b) Trajectories in the (v_c, i_L, q_m)-space of the RLC-memristor circuit with $a = 1$, $b = 1/3$, $c = 0$, $R = 1.25$, $L = 1$, $C_0 = 12.5$ and $\mathcal{I} = 0.16$. The initial conditions are marked with $\times$. Trajectories with the same color are related via (49) and (52).

with $I = \alpha\mathcal{I}/\beta$, then system (48) can be rewritten equivalently as

$$\begin{cases} \dot{V} = aV - b\dfrac{V^3}{1 + cV_1^2} - W + I, \\ \dot{W} = \alpha V - \beta W. \end{cases} \tag{53}$$

It can be verified that for $a = 1$, $b = 1/3$, $c = 0$ and $\alpha = 0.08$, $\beta = 0.064$, system (53) is indeed the classic FitzHugh–Nagumo model [FitzHugh, 1961; Nagumo *et al.*, 1962] which is known to display oscillatory behaviors for some values of the constant input I, as shown for $I = 0.2$ in Fig. 12(a). This implies that system (48) shows nontrivial periodic solutions on the manifold $\mathcal{M}_{\mathcal{I}}$ with $\mathcal{I} = 0.16$ when $R = 1.25$, $L = 1$, $C_0 = 12.5$ and $a = 1$, $b = 1/3$, $c = 0$, as depicted in Fig. 12(b). Hence, condition (51) cannot hold for the RLC-memristor circuit with this set of parameters, as it can be readily verified from (51) since we have $a = 1$ and $L/(RC_0) = 0.0064$.

As a final remark, we point out that this equivalence between the FitzHugh–Nagumo model and the RLC-memristor with this parameter setting has been investigated in [Innocenti *et al.*, 2022], showing that there is a one-to-one correspondence between the solutions of the FitzHugh–Nagumo model obtained by varying the constant input I and the memristor circuit solutions.

5. Conclusions

In this paper, we have derived some conditions to ensure that memristor circuits are *nonoscillatory*, i.e. they do not display oscillations and more complex attractors, thus enabling convergence toward the equilibrium points. Specifically, we have considered the class of memristor circuits composed by the interconnection of a linear time-invariant two-terminal (one port) element and an ideal memristor with a slope-bounded characteristic. The conditions exploit a well-known feature of these memristor circuits, i.e. their state space is decomposed in a continuum of invariant manifolds, and rely on two steps: (i) to derive an explicit state space representation of the circuit dynamics on each invariant manifold; (ii) to ensure that the variational equations associated to the 2-additive compound matrix of the Jacobian of all these representations are exponentially stable. The second step has been addressed by looking for a suitable Lyapunov function and exploiting the fact that the Jacobian of the representations does not depend on the considered invariant manifold. Notably, the Jacobian coincides with that of the circuit where the memristor is replaced by a nonlinear resistor having the same characteristic. This makes it possible to show that the memristor circuit is nonoscillatory if there exists a common quadratic (or homogeneous polynomial) Lyapunov function for two suitable 2-additive compound matrices, a problem which requires the solution of two LMIs. The results are illustrated via the analysis of two memristor circuits, showing how regions on the circuit parameters space can be derived where oscillations and more complex behaviors are ruled out and the solutions display convergence toward one of the infinitely many nonisolated equilibrium points.

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