Diversity in Motion: Essays on Macro-Dynamics and Heterogeneity

by

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Abstract

This thesis represents a comprehensive exploration of the intricacies of macroeconomic dynamics through the lens of heterogeneous agents. The research navigates through three pivotal chapters with a foundation built on the transformative shift toward incorporating diversity in economic agents. Chapter 1 pioneers a groundbreaking savings target mechanism, offering a nuanced understanding of consumption dynamics in response to transitory income shocks. Chapter 2 strategically positions itself at the intersection of financial phenomena and economic fluctuations, unveiling the multifaceted impact of informality on developing economies. Chapter 3 critically evaluates the consumption habits hypothesis, introducing an innovative mechanism rooted in an asymmetric income diffusion process within a network perspective. Each chapter contributes to theoretical innovation and proposes to bridge the gap between economic theory and observed empirical realities, offering valuable insights for policymakers and enriching the landscape of macroeconomic research.

Foreword

In recent years, the landscape of macroeconomic research has undergone a profound transformation, marked by a discernible shift towards integrating heterogeneous agents. The Heterogeneous Agents New-Keynesian (HANK) models are at the forefront of this transformative wave, diverging from the conventional representative agent models by introducing diversity into Dynamic Stochastic General Equilibrium (DSGE) frameworks. A seminal milestone in this transformative journey was the publication by Kaplan et al. (2018) Kaplan et al. (2018), illuminating the macroeconomic implications of diverse households on fiscal multipliers, monetary policy transmission channels, and asset allocation dynam-ics. This paradigm shift has compelled both researchers and policymakers to recognize the pivotal role of heterogeneity in understanding economic phenomena and prompted a reevaluation of forecasting methodologies for improved accuracy.

Notably, the exploration of heterogeneity in macroeconomics has deep historical roots, with early works by Krusell and Smith (1998) Krusell and Smith (1998) and Aiyagari (1994) Aiyagari (1994) laying the groundwork for the subsequent development of HANK models. Simultaneously, investigations into heterogeneity in agents' expectations, as demonstrated in works by Brock and Hommes (1997) Brock and Hommes (1997) and Lux (1995) Lux (1995), have further underscored the evolving nature of macroeconomic research.

This thesis embarks on a comprehensive exploration, delving into the intricacies that shape economic behaviors and outcomes through the lens of heterogeneous agents. Setting itself apart from presenting mere iterations of the HANK model with incremental modifications, this work systematically delves into various theoretical aspects of heterogeneity, exerting influence on the outcomes of diverse macroeconomic models. Through rigor-ous analytical and simulation analyses, this research illuminates that, even in seemingly straightforward scenarios, incorporating heterogeneous agents or mechanisms for emulating their effects is indispensable for understanding empirical evidence and stylized facts.

Chapter 1 takes center stage, meticulously examining the nuanced dynamics of consumption and proposing a groundbreaking savings target mechanism to bridge the theoretical-empirical gap. In contrast to many models introducing heterogeneity, this chapter revisits a representative agent model, employing a mechanism that replicates heterogeneity's impact on the marginal propensity to consume. The chapter addresses challenges posed by variable agent distributions in HANK models, focusing on capturing dynamic consumption responses to transitory income shocks. The savings target mechanism, grounded in prospect theory, adeptly reconciles disparities between prevailing representative agent models and empirical evidence.

Chapter 2 strategically positions itself at the intersection of financial phenomena and economic fluctuations, particularly against the backdrop of the 2008 Great Financial Crisis. It confronts the challenge of applying models designed for developed economies to comprehend the dynamics of developing economies characterized by high levels of informality. Pioneering a heterogeneous agents model, this chapter systematically explores the multifaceted impact of informality on liquidity, solvency, and output dynamics. By integrating the informality status of firms and agents' expectations, the chapter unveils a three-dimensional nonlinear map, scrutinizing the sources of instability and their threshold conditions.

Chapter 3 critically evaluates the consumption habits hypothesis, a linchpin in DSGE models for forecasting. Confronting conceptual and empirical challenges, this chapter introduces an innovative mechanism rooted in an asymmetric income diffusion process, departing from traditional consumption habits hypotheses. The endogenous mechanism establishes connections between the income processes of agents in a random network with non-constant intensity, aiming to provide a nuanced understanding of inertia effects in macroeconomic models.

The interdisciplinary nature of this research bridges gaps between micro and macroeconomic perspectives, enriching the cohesiveness of economic models. This interdisciplinary approach enhances the theoretical foundation of macro models and establishes connections between macroeconomic theory and behavioral elements, network theory, and nonlinear dynamics. As a result, this thesis positions itself at the forefront of advancing economic modeling, promoting a more comprehensive understanding of economic systems by incorporating diverse agent behaviors and interactions. The implications extend beyond the academic realm, offering practical insights for policymakers, central banks, and economists seeking to navigate the intricate landscape of economic dynamics in an increasingly complex and interconnected world.

These three chapters collectively form this thesis's foundational pillars, providing a holistic exploration of heterogeneous agent dynamics. This work significantly contributes to the ever-evolving landscape of macroeconomic dynamics research through a judicious combination of analytical and numerical methods. The fresh perspectives and innovative solutions presented herein offer valuable insights to address critical challenges in the field, grounded in recognizing the indispensable role of agent heterogeneity. As the thesis concludes, it underscores the necessity of acknowledging and incorporating diversity among economic agents, providing a nuanced understanding of the unraveling complexities of macroeconomic dynamics.

This thesis centers on the notion that the advancement of economic theory hinges on moments of creative freedom. It underscores the significance of engaging in work with the straightforward intention of introducing theoretical innovations, marking the inception of new traditions and methodologies in the field. This compilation of works seeks to provide insights into how we can creatively and rigorously approach economic phenomena. It encourages thinking beyond conventional boundaries while remaining rooted in established economic principles, paving the way for a fresh economic theory and practice perspective.

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Chapter 1

Savings Target and Consumption Dynamics

Abstract

This study proposes a savings target mechanism, incorporating elements of prospect theory, to enhance agent behavior in macroeconomic models. Specifically tailored to rectify deviations from the permanent income hypothesis in line with empirical evidence, the mechanism captured consumption dynamics in response to transitory income shifts in deterministic and stochastic settings in partial and general equilibrium. Within a DSGE framework, the model amplifies both output and consumption responses to fiscal policy shocks, aligning with observed empirical patterns. This contribution underscores the practical relevance of the savings target mechanism for qualitative and quantitative research in macroeconomics. Results found qualitatively align with HANK models consumption and fiscal multiplier dynamics.

Keywords: Consumption; DSGE; Fiscal Policy; Prospect Theory; Heterogeneous Agents.

1.1 Introduction

Incorporating heterogeneous agents into macroeconomic models has recently attracted attention for its potential insights into consumption decisions and economic forecasting. The method's focus on capturing the consumption dynamics of diverse agents makes it particularly appealing for understanding the net fiscal impact on an economy and the transmission channels of monetary policy (Kaplan et al., 2018). As highlighted by Auclert et al. (2020), the variability in the marginal propensity to consume is crucial for Heterogeneous Agents New-Keynesian (HANK) models to provide more accurate forecasts of the effects of fiscal policy.

An observed challenge is that the variable distribution of heterogeneous agents in HANK models with incomplete markets leads to deviations from the optimal consumption path based on the permanent income hypothesis (PIH). Despite this, the prevailing design of agent behavior in modern macroeconomics continues to rely on the PIH. This reliance hampers representative agent models from accurately reflecting consumption dynamics in line with empirical evidence, especially in response to fiscal policy shocks driven by agents' behavior during transient income changes (Fagereng et al., 2021).

To address this issue, this research proposes a framework centered on savings targets as a pragmatic solution to adjust consumption dynamics. The objective is to bring agent behavior closer to empirical responses to transient income shocks, thereby reconciling the disparities between macroeconomic models' responses to fiscal policy shocks and observed empirical evidence. This approach contributes to refining the microfoundation for agents' behavior and aligns with the broader agenda of incorporating the effects of HANK models within more straightforward frameworks (Cantore and Freund, 2021; Debortoli and Galí, 2021).

The proposed strategy introduces a savings target mechanism that imposes disutility when an agent's effective asset stock deviates from the predetermined goal. This prompts agents to consider this impact over time in their consumption decisions. Drawing from prospect theory (Tversky and Kahneman, 1992), this effect incorporates using a reference point to shape future decisions. Additionally, it integrates a variable intensity component to capture different responses for shocks with different sizes.

The research begins by developing and scrutinizing the concept of a savings target mechanism, illustrating how changes in the savings stock, whether in the form of a riskless asset for cases with fixed or variable intensity coeficients, impact consumption dynamics. Through an illustrative exercise involving a deterministic income shock, the consumption dynamics of agents in a partial equilibrium scenario align with empirical evidence.

The model's compatibility extends to scenarios involving uncertainty about the future trajectory of consumption, where precautionary savings interact additively with the target savings mechanism. After conducting 1000 Monte Carlo simulations, the model demonstrates compatibility with stochastic income, exhibiting behavior on average akin to the deterministic case for constant and non-constant intensity cases. In a general equilibrium setting within a closed log-linear economy, the model's results remain consistent with those observed in partial equilibrium for increases in income stemming from fiscal policy shocks.

Finally, the savings target mechanism is applied within a Dynamic Stochastic General Equilibrium (DSGE) model. This model, combining various real-market frictions typically found in the literature, compares the impact of fiscal policy shocks when the savings target mechanism is active versus the standard case. Simulation exercises reveal that the influence of fiscal policy on consumption and output aligns qualitatively with empirical evidence by approximating the fiscal multiplier of a HANK model output.

1.2 Why Savings Targets?

The economic perspective on household savings, initially pioneered by Modigliani and Brumberg (2005) and further elaborated by Friedman (1957), suggests that individuals make deliberate choices regarding saving and borrowing to attain a balanced consumption pattern over their life cycle. This concept aligns with utility functions exhibiting concavity in consumption, compelling individuals to distribute consumption over time for optimal utility realization. This principle is encapsulated in the Euler equation, linking present and future consumption decisions:

$$U^{'}(C_t) = \beta E_t U^{'}(C_{t+1})(1 + r) \tag{1.1}$$

Here, the marginal utility of consumption in period t equals the marginal utility of consumption in period $t + 1$ times an interest rate r and a discount factor β. The model with no uncertainty predicts that $E_t \{U^{'}(C_{t+1})\} = U^{'}(C_{t+1}) = U^{'}(C_t)$, implying $C_{t+1} = \beta(1+r)C_t$ under strict concavity. Consequently, marginal utility equals the marginal price, leading to a stable consumption profile. In that sense, changes in transitory income are associated with increased savings to increase consumption in the lifetime horizon.

However, empirical studies consistently challenge this theory. Authors such as Parker et al. (2013), Stephens Jr and Unayama (2011), and Johnson et al. (2006) reveal that individuals react by adjusting their consumption in response to both expected and unexpected transitory changes in income. By conducting experiments with lottery winners, Fagereng et al. (2021) found significant changes in the marginal propensities to consume after a transitory income shock. This result is corroborated by other methods such as quasi-experimental evidence (Kueng, 2018), survey instruments (Bunn et al., 2018; Fuster et al., 2021), and semi-structural methods (Ganong et al., 2020; Commault, 2022).

Without any modification, the consumption dynamics for utility functions and budget constraints that generate an Euler equation similar to Eq. (1.1) cannot account for the stylized facts described previously. Proposition 1 highlights this statement.

Proposition 1. With no uncertainty or borrowing constraints, the marginal propensity to consume associated with Eq. (1.1) when $r_t = r$ for all t and $U'(C_t) = C_t^{-\gamma}$ is given by:

$$\bar{\alpha} = 1 - (1+r)^{-1}[\beta(1+r)]^{\frac{1}{\gamma}}$$

.

For the case in which $U'(C_t) = 1/(C_t)$, we have that:

$$\bar{\alpha} = (1 - \beta)$$

Proof. See Mathematical Appendix. □

The marginal propensity to consume $\bar{\alpha}$ is determined exclusively by the relationship between the interest rate r, the discount factor β, and the relative risk aversion parameter γ. Thus, the basic model does not show any relevant change in the marginal propensity to consume in the face of a transitory income shock, as shown by empirical evidence.[1] Note that for the case in which the relative risk aversion coeficient is 1, we have the case of a log consumption function, in which changes in the interest rates do not affect the marginal propensity to consume, being determined only by the parameter β. Fig. 1.1 shows the consumption dynamics of a model based on a traditional Euler equation with empirical evidence for a transitory income shock estimated by Fagereng et al. (2021).

As seen in the graph, the dynamics of consumption after an income shock are widely different from the conventional result, which can generate severe distortions in micro-based models with a representation of consumption based on a significant substitution between present and future consumption for variations in income. Table 1.1 summarizes the moments for the aftershock for Data and Model.

	Data	Model
Mean	0.1482	0.0186
Var	0.0339	0.0000
Std. Dev.	0.1841	0.0046

Table 1.1: Moments for Data and Model. Calculated using Fagereng et al. (2021) data.

Despite these challenges, the Euler equation remains valuable for theoretical and empirical models. One potential approach is to explore deviations from optimal behavior along the life cycle. For this, I intend to change how agents make their consumption decisions in the presence of a transitory shock based on a savings target mechanism. It is necessary

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[1] As shown by Cantore and Freund (2021), even for cases when $\gamma = 1$ and r_t is not constant, the necessary values of the relative risk-aversion parameter to generate a response of MPC to match the empirical data is implausible from the empirical point of view.

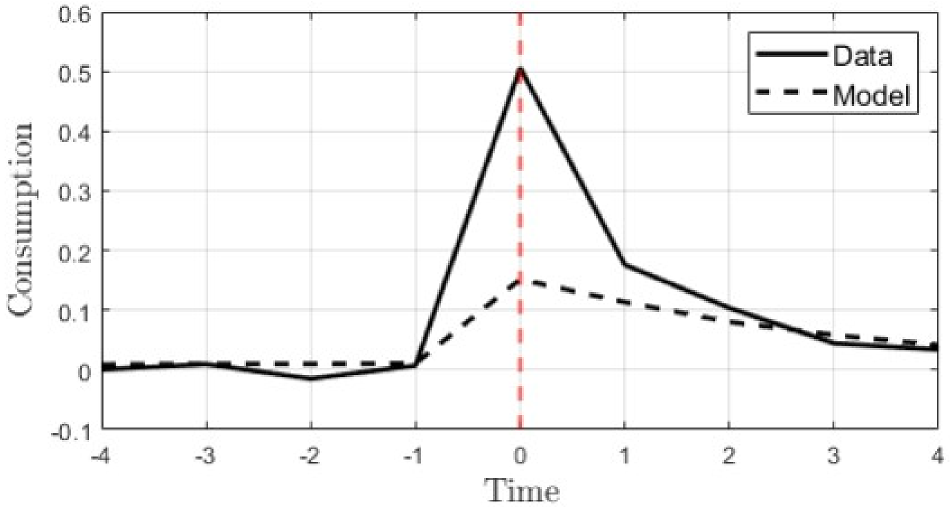

Figure 1.1: Yearly consumption response for a transitory income shock in t = 0. Data (solid line) and a standard model (dashed line). Source: Fagereng et al. (2021).

to think about the concept of savings at the individual level and how an approach based on behavioral evidence can bring models based on the Euler equation closer to empirical evidence.[2]

By following Wärneryd (1989), savings can be broadly defined as regularly setting aside resources for specific goals as medical expenses in elderly (De Nardi et al., 2010), retirement plans (Clark et al., 2017; Lee and Hanna, 2015), children education (Sherraden et al., 2013), and consumption of durable goods (Fernandez-Villaverde and Krueger, 2011). These goals change over time and depend on individuals' income, cultural experience, age, education, and family structure (Ekerdt, 2010).

Departing from the idea that goals influence the act of saving, we can rely on an essential element of prospect theory (Kahneman and Tversky, 1979, 1984) in which agents use reference points to help in their decision process given cognitive limitations (Kahneman, 2003). It is feasible to assume that agents' savings decisions consider the role of an objective that guides the agents' accumulation path. In this case, the reference level can be seen as a target agent trying to hit.

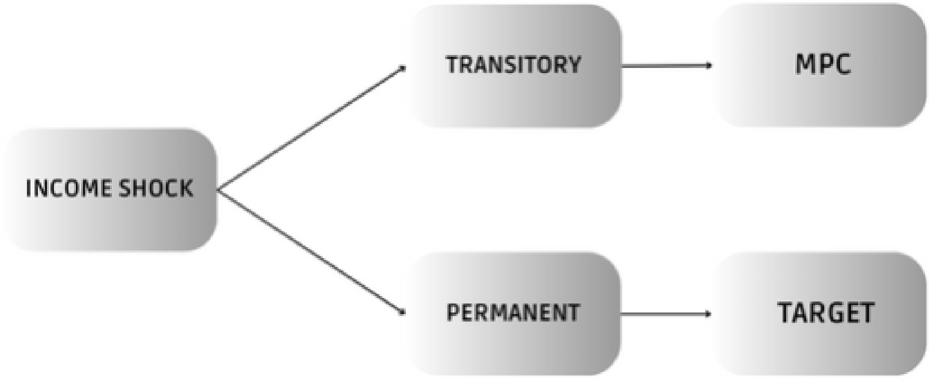

Figure 1.2: Agents' reaction to transitory and permanent income shocks.

As shown in Fig. 1.3, when an individual's savings diverge from his reference level

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[2] A similar strategy was employed by Michaillat and Saez (2021) to explore zero-lower-bound dynamics, considering the positive utility effects of wealth.

for a transitory income shock, he is incentivized to return to the equilibrium where his effective savings meet the target. This effect affects the MPC, which leads to an increase in consumption. At the same time, setting a target tied to an objective also evolves with the agent's permanent income, allowing for adjustments as new consumption goals become available, with the possibility of a lower MPC if the target's response to the permanent income change is strong enough. This perspective reconciles the empirical observation that higher-income individuals tend to exhibit a lower marginal propensity to consume (Albuquerque and Green, 2023; Drescher et al., 2020).

Given that this research aims to achieve a better response from macroeconomic models to fiscal policy, only the case of transitory shocks will be considered. However, the idea of savings targets can also be applied to cases in which the goal is variable.

1.3 Savings Target and Individual Consumption

To illustrate the mechanism responsible for changing consumption dynamics in general equilibrium, this section addresses the problem in a simple scenario to introduce the economic intuition of the savings target mechanism. Let us illustrate how a savings target can be incorporated into a utility function.

Suppose that agents choose to maximize a separable utility function strictly concave in consumption $U(C_t)$ and convex in a savings target mechanism $U(S_t)$:

$$\max_{\{C_t, A_t\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(U(C_t) - U(S_t) \right) \right\} \tag{1.2}$$

In this scenario, in addition to maximizing utility when consuming over an infinite time horizon, agents also need to consider the disutility generated by their savings in the form of a savings portfolio S_t. This savings portfolio can contain N assets with different risk profiles. For the sake of simplicity, let us assume that agents save in a single risk-free asset A_t. By assuming a quadratic functional form, $U(S_t)$ can be defined as:

$$U(S_t) = \varphi \frac{(A_t - \bar{A})^2}{2} \tag{1.3}$$

In this way, $U(S_t) > 0$ whenever the total assets A_t is different from the reference value $\bar{A}$. Given the distance from its goal, this indicates that the agent will face an increasing disutility effect. The effect of this mechanism on the agent's total utility depends on the intensity parameter φ. Initially, the case for constant intensity will be explored, followed by the case with variable intensity, and finally, the case with consumption decision under uncertainty.

1.3.1 The Case with Constant Intensity

Let us assume a constant value for parameter φ in this first case. As shown in Fig. 1.3, the effect of disutility generated by deviations of the stock of assets A_t from the reference level $\bar{A}$ is filtered by parameter φ. Given the quadratic functional form of the savings target mechanism, the disutility effect tends to grow non-linearly, given the increase in the gap between the effective value and the reference level.

We must find the relative prices that guide agents' consumption decisions over time to understand how this mechanism affects consumption dynamics. By assuming a Constant Relative Risk Aversion (CRRA) consumption function and using Eq. (1.3), we can rewrite Eq. (1.2) as:

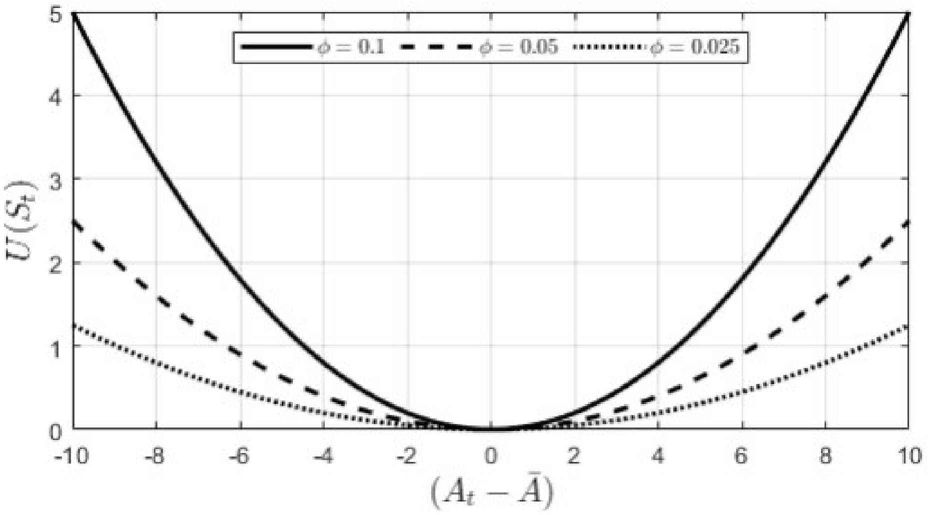

Figure 1.3: Savings target mechanism disutility for different values of parameter φ.

$$\max_{\{C_t, A_t\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \varphi \frac{(A_t - \bar{A})^2}{2} \right) \right\} \tag{1.4}$$

Where the parameter γ captures the relative risk aversion of agents. This optimal program is subject to the following budget constraint:

$$A_t + C_t = A_{t-1}(1 + r) + \bar{Y} \tag{1.5}$$

agents can decide between consuming a homogeneous good C_t or saving in the risk-free asset A_t with a return r. Initially, the income is exogenous and given by a measure of permanent income $\bar{Y}$ for all t. The First-Order Conditions (FOC) for the problem are given by:

$$\beta^t (C_t^{-\gamma} - \lambda_t) = 0 \tag{1.6}$$

$$\beta^t (\varphi(A_t - \bar{A}) - \lambda_t) + \beta^{t+1} E_t \{\lambda_{t+1}(1 + r)\} = 0 \tag{1.7}$$

After organizing Eqs. (1.6) and (1.7), we have that:

$$C_t^{-\gamma} = \lambda_t \tag{1.8}$$

$$\varphi(A_t - \bar{A}) + \lambda_t = \beta E_t \{\lambda_{t+1}(1 + r)\} \tag{1.9}$$

By using Eqs. (1.8) and (1.9), and considering that under certainty $E_t\{C_{t+1}\} = C_{t+1}$, the Euler equation associated with the problem can be obtained:

$$C_t^{-\gamma} + \varphi(A_t - \bar{A}) = \beta C_{t+1}^{-\gamma}(1 + r) \tag{1.10}$$

after reorganizing, we arrived at:

$$\frac{C_{t+1}}{C_t} = \left(\frac{\beta(1 + r)}{1 + C_t^{-\gamma} \varphi(A_t - \bar{A})} \right)^{\frac{1}{\gamma}} \tag{1.11}$$

Eq. (1.11) expresses the marginal rate of substitution of present and future consumption on the left side and the relative price on the right. The term $\left(1 + \varphi(A_{i,t} - \bar{A}_i)\right)^{-1}$ captures the effect of the savings target over the present/future consumption decisions. This result shows that choosing between present and future consumption also depends on the dynamics of asset accumulation. In sum, under savings target, if agents want to increase their savings beyond the target, they are affected by the convex cost in an increasing manner, which generates disutility and decreases future consumption as detailed in Eq. (1.11).[3]

As an exercise to understand the effect of the savings target mechanism on the dynamics of consumption, we can include auto-regressive deterministic dynamics to agents' income:

$$Y_t = (1 - \rho^y)\bar{Y} + \rho^y Y_{t-1} + Y_t^+ \tag{1.12}$$

such that ρ^y is an autoregressive parameter and Y^+ the increase on income. The response for the baseline case ($\varphi = 0$) and the case with the savings target mechanism active ($\varphi = 0.08$) for an income temporary increment $Y_t^+ = 1$ is graphically represented in Fig (1.4).[4]

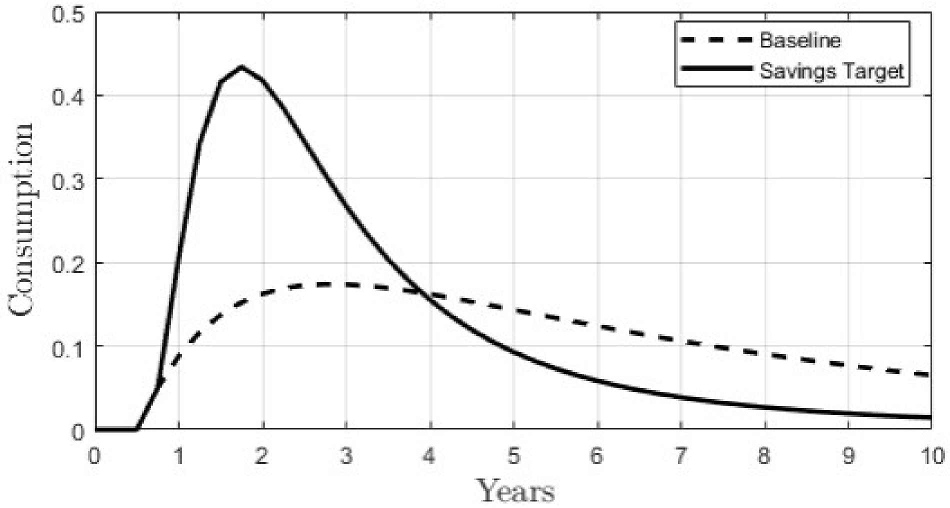

Figure 1.4: Yearly consumption response for a transitory income shock in $t = 1$ for a savings target with intensity $\varphi = 0.08$ and a baseline case with $\varphi = 0$.

As can be seen, the model with the active savings target causes a significant increase in consumption for a transitory shock in income that is qualitatively similar to the behavior of the empirical series expressed in Fig. (1.4). However, given the intensity of the effect, the peak and return to steady-state response can become not adequately match the data. For this, we can utilize a non-constant intensity to adjust the consumption response of the model to match the moments of the empirical evidence and increase the response for shocks of different sizes.

1.3.2 The Case with Non-Constant Intensity

Another important point that also deserves attention is the fact that the intensity of the shock is important in defining how the agent will respond by modifying its plans. As pointed out by Kaplan and Violante (2022), there is an asymmetry in how agents react to different income shocks. To include this effect, it is possible to count on differences in how

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[3] This result is similar to the solution presented by Cantore and Freund (2021). However, in terms of utility instead of price, as in their model, the effect appears as friction on the agent's portfolio. Beyond that, this design allows for more general interpretation and extension possibilities, as presented in the next subsection.

[4] For this exercise, parameters values are: $\beta = 0.95$, $r = 0.01$, $\rho^y = 0.8$, $\bar{A} = 0$, $\bar{Y} = 0.1$ and for the sake of simplicity we can assume that $\gamma = 1$ to work with the case in which $U(C_t) = \ln(C_t)$.

agents feel the impact of disutility due to deviations from the goal. The explanation for this effect can follow two lines:

1) The existence of transaction costs means that agents keep their savings pattern the same for every income change since there is not always a new optimal allocation available Gârleanu and Pedersen (2016); Baule (2010). For shocks of greater intensity, the transaction cost associated with changes in strategy becomes less relevant, reducing the effect's intensity.

2) Agents have a status-quo bias, in which, for small changes in income, agents prefer to ignore the effort of recalculating their savings decisions (Godefroid et al., 2022). As changes become significant, agents feel more encouraged to change their behavior and move closer to standard optimizing behavior. This effect can also be related to an inattention bias (Gabaix, 2019).

To generate different intensities that follow the size of the shock, we can use a distance metric that captures when the agent suffered a significant change in its income. This way, we can rewrite parameter φ as:

$$\varphi = \frac{\varphi_{max}}{1+\kappa(Y_t-\bar{Y})^2} \tag{1.13}$$

When we assign a maximum value to φ given by the parameter φ_{max}, we have that the intensity of the effect is reduced as the effective income Y_t moves away from the permanent income $\bar{Y}$ given the intensity of change parameter $\kappa \in (0,\infty)$. Fig. 1.5 shows this effect graphically.

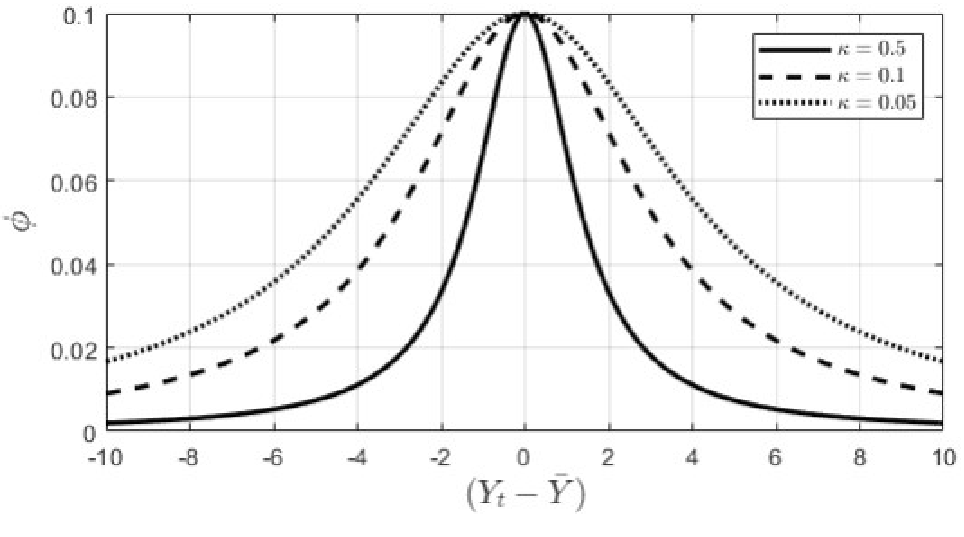

Figure 1.5: φ value for different parametrization of κ in function of the difference $Y_t - \bar{Y}$ with $\varphi_{max} = 0.1$.

As lim κ → 0, φ = φ_{max} for all t. When lim κ → ∞, φ assumes a value perfectly compatible with the effective income Y_t for all t. Fig. 2 shows graphically the relationship between φ and the difference $Y_t - \bar{Y}$ for different values of the intensity of change parameter κ.

We can revisit the consumer problem using a non-constant intensity to understand this mechanism better. By plugging Eq. (1.13) into Eq. (1.4), we have that:

$$\max_{\{C_t,A_{t+1}\}} E_t\left(\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{\varphi_{max}}{(1+\kappa(Y_t-\bar{Y})^2)}\frac{(A_t-\bar{A})^2}{2}\right)\right) \tag{1.14}$$

In this way, the response of the savings target mechanism in the utility function also responds to variations in income. Fig 1.6 displays the disutility response generated by the savings target mechanism for deviations in $(A_t - \bar{A})$ and $(Y_t - \bar{Y})$ given for parameters $\varphi = 0.1$ and $\kappa = 0.1$.

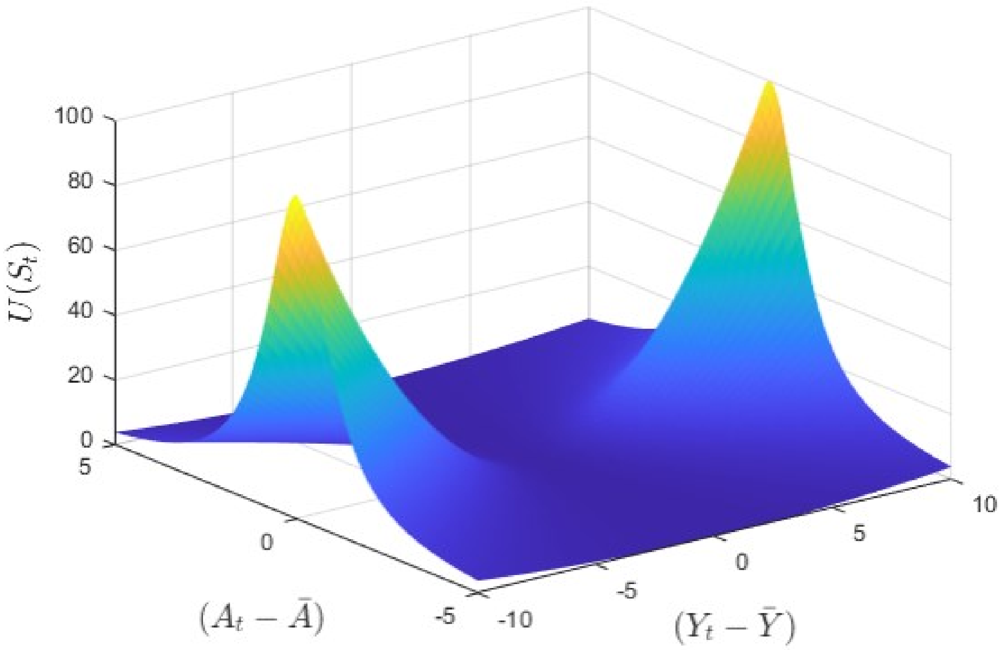

Figure 1.6: $U(S_t)$ in function of the difference $(A_t - \bar{A})$ and $(Y_t - \bar{Y})$ with $\varphi_{max} = 0.08$ and $\kappa = 1$.

Once again, by taking the FOC and solving for consumption with $E_t\{C_{t+1}\} = C_{t+1}$, we have that:

$$\frac{C_{t+1}}{C_t} = \left(\frac{\beta(1+r)}{1 + \left(C_t^{-\gamma}\varphi_{max}(A_t - \bar{A})\right)\left(1 + \kappa(Y_t - \bar{Y})^2\right)^{-1}} \right)^{\frac{1}{\gamma}} \tag{1.15}$$

Given that variations in Y_t do not enter directly into the agent's optimal program, the only difference between Eqs. (1.11) and (1.15) is the parameter φ replaced by Eq. (1.13). By inserting a deterministic shock to income as done in the case shown in Fig. (1.4), we can, once again, study the dynamics of consumption for the baseline case and the case of the savings target mechanism with non-constant intensity. For this, it follows Fig. (1.7).

The consumption dynamics are changed, and the peak response is also smoothed. The κ parameter can also be helpful to adjust the moments of the consumption series, as shown in Appendix B. If we compare different increases in Y, we can see that the difference between the cases with constant and non-constant φ becomes more evident, as shown in Fig. 1.8.

1.3.3 The Case with Stochastic Income

So far, we have done exercises using deterministic shocks to understand the model's mechanisms. However, the most common case is that variations are described in the form of random perturbations in certain variables as a way to model uncertainty. For the stochas-tic case, agents now face uncertainty regarding the trajectory of their income, which now

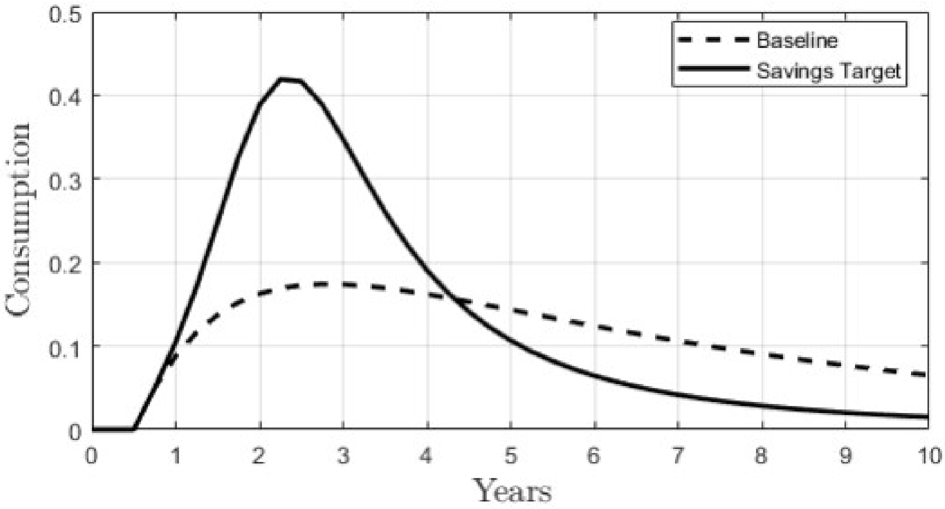

Figure 1.7: Yearly consumption response for a transitory income shock in t = 1 for $\varphi = 0.08$.

has a stochastic error component. We can rewrite the budget constraint expressed in Eq. (1.5) as:

$$A_t + C_t = A_{t-1}(1 + r) + Y_t \tag{1.16}$$

such that:

$$Y_t = \bar{Y} + \epsilon_t^Y \tag{1.17}$$

where $\bar{Y}$ is the exogenously given deterministic permanent income and $\epsilon_t^Y \sim N(0, \sigma^2)$ the stochastic error term.

For this new situation, agents can no longer consider that they know exactly what their future consumption C_{t+1} will be given their uncertainty about their future income Y_{t+1}. As this is the only source of uncertainty in this case, we can assume that the expectation about future consumption is conditioned to income. In that case, we can rewrite equation Eq. (1.15) as:

$$\frac{E\{C_{t+1}\,|\,Y_t\}}{C_t} = \left[\frac{\beta(1+r)}{\left(1 + C_t^{-\gamma}\varphi_{max}(A_t - \bar{A})\right)\left(1 + \kappa(Y_t - \bar{Y})^2\right)^{-1}}\right]^{\frac{1}{\gamma}} \tag{1.18}$$

as $E_t\{C_{t+1}\,|\,Y_t\}$ was assumed, the distribution of consumption should follows:

$$\Delta\hat{C}_{t+1} \sim N\left(E_t\left\{\hat{C}_{t+1}\right\}, \sigma^2\right) \quad \text{and} \quad -\gamma\Delta\hat{C}_{t+1} \sim N\left(-\gamma E_t\left\{\hat{C}_{t+1}\right\}, \gamma^2\sigma^2\right) \tag{1.19}$$

where $\Delta\hat{C}_{t+1} = \log(C_{t+1}) - \log(C_t)$. By using the general property for a conditional random variable with a log-normal distribution, we can find an approximation for the expected value of $\Delta\hat{C}_{t+1}$:

$$E_t\left\{\Delta\hat{C}_{t+1}\right\} = \frac{1}{\gamma}\left(\hat{r} - \hat{\beta}^{-1} - \varphi_{max}\tilde{A}_t + 2\kappa\tilde{Y}_t\right) + \frac{\gamma}{2}\sigma_t^2 \tag{1.20}$$

where $\hat{r}$ and $\hat{\beta}$ are log approximations of r and β, and $\tilde{A}_t$ and $\tilde{Y}_t$ approximations from reference levels deviations such that $\tilde{A}_t = \log(A_t) - \log(\bar{A})$ and $\tilde{Y}_t = \log(Y_t) - \log(\bar{Y})$. The second term on the right side of Eq. (1.20) captures the precautionary savings given the degree of relative risk aversion defined by γ and the standard deviation of the income

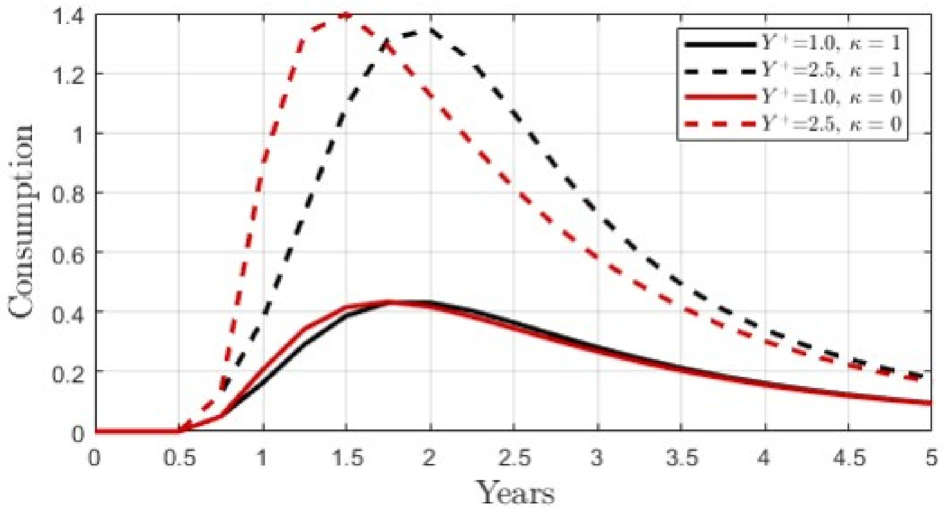

Figure 1.8: Comparison between the cases when $\kappa = 0$ and $\kappa = 1$ with $\varphi = 0.08$ for extra incomes is $Y^+ = 1$ and $Y^+ = 5$.

stochastic component given by σ^2. Fig. 1.9 follows a similar simulation for the deterministic case but considers the abovementioned elements.

After 1000 Monte Carlo Simulations, the result shows that the savings target mechanism is also compatible with uncertainty. This result is insightful as it shows the generality of the mechanism.

1.3.4 Savings Target in General Equilibrium

Finally, we will test the model in general equilibrium in a simple DSGE model to understand the effect of the savings target mechanism on consumption. The idea is that the increase in the economy's income comes from a shock in government consumption.

Environment - The economy is populated by a continuum of agents indexed by $i \in [0, 1]$, which chooses to consume a homogeneous consumption good C_t with constant relative risk aversion γ or save in a riskless public bond B_t with a return r_t[5]. The disutility of labor is expressed by L_t with ϕ being the inverse of Frish elasticity of labor supply. Agents have to consider the effect of the savings target mechanism with non-constant intensity. Households solve the following intertemporal optimization program:

$$\max_{\{C_{i,t},B_{i,t}\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{\varphi_{max}(B_{i,t} - \bar{B}_i)^2}{2(1+\kappa(Y_{i,t} - \bar{Y}_i)^2)} - \frac{L_{i,t}^{1-\phi}}{1-\phi} \right) \right\} \tag{1.21}$$

subject to:

$$C_{i,t} + B_{i,t} = B_{i,t-1}(1 + r_{t-1}) + W_t L_{i,t} - T_t \tag{1.22}$$

where W_t is the wage level and T_t a measure of lump-sum taxes.

The productive sector is divided into final and intermediate goods production. Intermediate goods are aggregated using a CES technology in a competitive final goods market, and intermediate goods are produced in an imperfect competition market using only labor as input:

$$Y_{j,t} = L_t \tag{1.23}$$

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[5] Note that the variable A from previous examples becomes B as the model assumes that it is no longer a generic asset, but public securities linked to a fiscal policy rule.

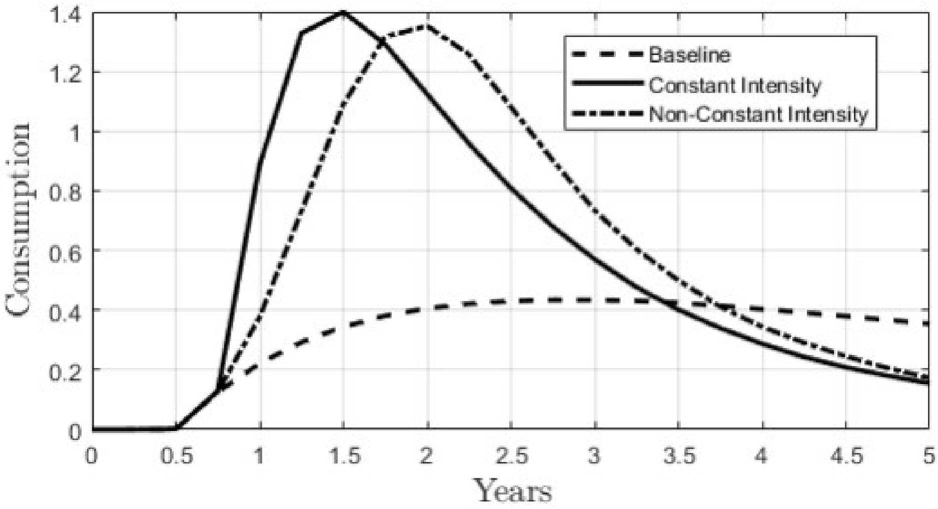

Figure 1.9: Comparison between the cases when $\kappa = 0$ and $\kappa = 1$ with $\varphi = 0.08$ fo an extra income $Y^+ = 5$. The result is the average of 1000 Monte Carlo simulations with the income variance $\sigma = 0.2$.

with the marginal cost $mc_t = L_t$. Prices determined by the Phillips curve:

$$(1+\tau)(1-\eta) + \eta\, mc_t - \Pi_t \xi(\Pi_t - 1) + \beta E_t \; \xi \frac{C_{i,t+1}}{C_{i,t}} \Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} = 0 \qquad (1.24)$$

where ξ is a Rotemberg adjustment cost parameter, η is the elasticity of substitution between intermediate goods, and τ is an optimal subsidy to induce marginal cost pricing in the steady-state.

The fiscal authority has two instruments: government consumption G_t and lump-sum taxes T_t, and are restricted by the following budget constraint:

$$B_t = G_t + (1 + r_{t-1})B_{t-1} - T_t \qquad (1.25)$$

with the following fiscal rule:

$$\frac{T_t - T_{ss}}{Y_{ss}} = \varphi^{\tau T} \frac{T_{t-1} - T_{ss}}{Y_{ss}} + \varphi^{\tau B} \frac{B_t - B_{ss}}{Y_{ss}} + \varphi^{\tau G} \frac{G_t - G_{ss}}{Y_{ss}} \qquad (1.26)$$

where the subscript ss denotes the steady-state of a given variables and $\varphi^{\tau T}$, $\varphi^{\tau B}$, and $\varphi^{\tau G}$ are fiscal instruments coeficients. Beyond that, government spending follows an autoregressive process:

$$G_t = \rho^G G_{t-1} + \epsilon_t^G \qquad (1.27)$$

Monetary authority follows a simple monetary policy rule based on the Taylor principle, with inflation π_t as a single-fold objective:

$$\frac{R_t}{R^{ss}} = \left(\frac{\Pi_t}{\Pi^{ss}}\right)^{\varphi\pi} \qquad (1.28)$$

with $\varphi\pi$ being an inflation coeficient.

The equilibrium is defined by $Y_t = C_t + G_t$, where the savings target and permanent income are denoted as A_{ss} and Y_{ss}.[6] Figure 10 illustrates the baseline case outcome, comparing constant and non-constant intensity for a government consumption shock to approximate both scenarios.[7]

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[6] Appendix C provides the log-linear version of the model.

[7] Appendix D outlines the parameter values, and simulations were performed using MATLAB with Dynare.

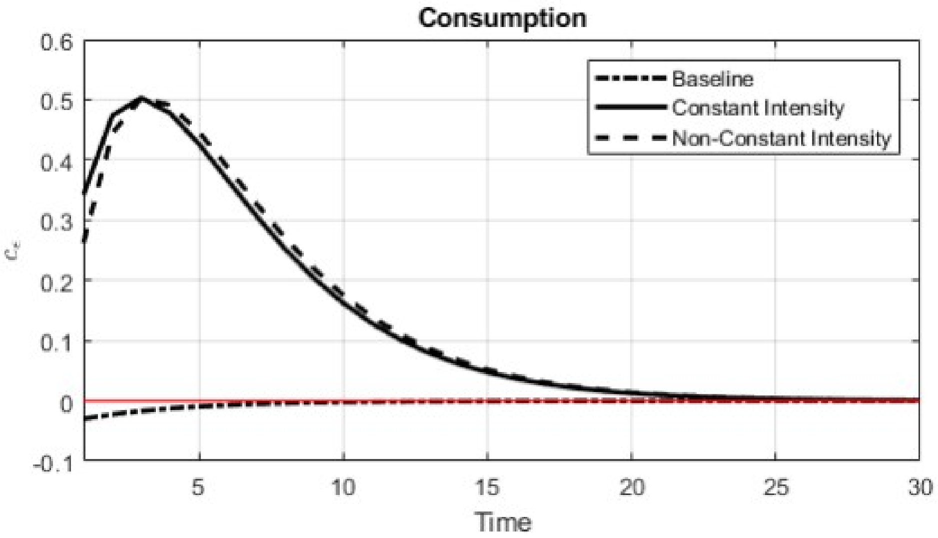

Figure 1.10: Comparison of baseline, constant, and non-constant cases for a government spending shock.

In the baseline case, the consumption response is marginally negative, returning to the steady state, reflecting the nullifying effects of fiscal policy under Ricardian equivalence. Utilizing the savings target mechanism, the consumption response becomes positive for a transitory shock, consistent with empirical evidence. Table 1.2 illustrates the differences in moments for each model.

	Baseline	Constant	Non-Constant
Mean	-0.0040	0.2022	0.1987
Var	0.0001	0.0309	0.0320
Std. Dev.	0.0074	0.1758	0.1789

Table 1.2: Moments of baseline, constant, and non-constant cases.

The non-constant intensity mechanism can approximate constant results for $\kappa = 0$, allowing adjustments in impulse-response functions, as explored in the non-constant partial equilibrium case. Calibration becomes more sensitive in general equilibrium due to the model's structure. Overall, the moments closely approximate the aftershock moments in the data presented in Table 1.1. As agents respond effectively to the variation in income caused by an expansion of government consumption, the dynamics of consumption in the presence of the savings target mechanism makes the model output closer to the data output in terms of moments.

1.4 Savings Target and Fiscal Policy in a Medium-Scale DSGE Model

This section delves into fiscal policy's impact on consumption within a broader context. Now, the analysis is extended to encompass the influence of real-market frictions on prices, wages, capital stock utilization, and investment. This broader perspective allows us to scrutinize the behavior of the savings target mechanism within a commonly used structure for a DSGE economy, building upon the work of Costa Junior (2016) with tailored adjustments in the households sector to address the savings target mechanism.[8]

1.4.1 Households

Within this framework, a proportion $\lambda \in [0,1]$ of households, called Ricardians, possess access to financial markets, enabling them to save through public bonds B and investments in capital stock equities K. The optimal program for Ricardian households involves maximizing the following utility function:

$$\max_{\{C_t^R, B_{t+1}\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{R,1-\gamma}}{1-\gamma} - \frac{\varphi_{max}(B_t - \bar{B})^2}{2(1+\kappa(Y_t^R - \bar{Y}^R)^2)} - \frac{L_t^{R,1-\phi}}{1-\phi} \right) \right\}$$

Ricardian households optimize their consumption C^R decisions over an infinite horizon, considering a constant relative risk aversion γ. In addition to consumption and saving, the model accounts for disutility penalties associated with deviations from a savings target $\bar{B}$, modulated by income deviations Y^R from a measure of permanent income $\bar{Y}^R$ through parameters φ_{max} and κ. Ricardians also contribute to the labor supply L^R, and the inverse of Frish elasticity of labor supply is denoted by ϕ. The budget constraint for Ricardian households is given by:

$$\begin{aligned} P_t(1+\tau_t^c)(C_t^R + I_t) + B_t = {} & (1+\tau_t^k)K_t R_t^k U_t - P_t K_t \left(\psi_1(U_t - 1) + \frac{\psi_2}{2}(U_t - 1)^2 \right) \\ & + B_{t-1}R_{t-1} + (1+\tau_t^l)W_t L_t^R + \lambda P_t TRANS_t \end{aligned}$$

Here, P denotes the general price level, I represents investment, U is the rate of capital utilization, ψ_1 and ψ_2 are sensitivity parameters regarding the utilization of installed capacity, R signifies the return on public bonds, and τ_t^c and τ_t^k denote taxes on spending decisions and capital income, respectively. The wage is denoted by W, and $TRANS$ represents government transfers. The capital stock K evolves according to the law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t \left(1 - \frac{\chi}{2} \left(\frac{I_{i,t}}{I_{i,t-1}} - 1 \right)^2 \right)$$

δ represents the capital depreciation rate, and χ is the sensitivity parameter concerning capital adjustment costs.

A fraction $(1-\lambda)$ of households labeled as non-Ricardians lack access to financial markets. Their optimal program is simplified, focusing solely on maximizing consumption C^{NR} while subject to a budget constraint:

$$\max_{\{C_t^{NR}\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{NR,1-\gamma}}{1-\gamma} - \frac{L_t^{NR,1-\phi}}{1-\phi} \right) \right\}$$

Subjected to:

$$P_t(1+\tau_t^c)C_t^{NR} = (1+\tau_t^l)W_t L_t^{NR} + (1-\lambda)P_t TRANS_t$$

1.4.2 Labor Union

In this economy, households contribute diverse labor inputs within a monopolistic competition market structure. Labor is aggregated by a union, employing a Dixit and Stiglitz

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[8] Refer to the appendix for the derivation of the First-Order Conditions (FOC) for households; the remainder of the model closely follows the solution presented by Costa Junior (2016).

(1977) aggregator, expressed as:

$$L_t = \left(\int_0^1 L_{i,t}^{\frac{\psi_W-1}{\psi_W}} di\right)^{\frac{\psi_W}{\psi_W-1}} \tag{1.29}$$

Where ψ_W represents the elasticity of substitution between differentiated labor skills distributed by a continuum of households of measure $i \in [0,1]$. The union maximizes its objective function, considering both the aggregated labor supply and the disutility of employing individual labor inputs:

$$\max_{L_{i,t}} W_t \left(\int_0^1 L_{i,t}^{\frac{\psi_W-1}{\psi_W}} di\right)^{\frac{\psi_W}{\psi_W-1}} - \int_0^1 W_{i,t}L_{i,t} di \tag{1.30}$$

Under Calvo (1983) staggered contracts, households can only change their nominal wage with a probability $\theta_W \in [0,1]$. When a given household chooses a wage, it must consider the probability of being unable to adjust its wage in future periods. The optimal wage is defined as:

$$W^*_{i,t} = \frac{\psi_W}{\psi_W - 1} E_t \left[\sum_{j=0}^{\infty} (\beta\theta_W)^j \left(\frac{L^{\phi}_{i,t+j}}{\lambda_{i,t+j}(1+\tau_{t+j})}\right)\right] \tag{1.31}$$

$(1-\theta_W)$ fraction of households choose the optimal wage level $W^*_{i,t} = W^*_t$, as the rest of the households remain with the same wage from the past period. The aggregated wage level will be given by:

$$W_t = (\theta_W W_{t-1}^{1-\psi_W} + (1-\theta_W) W_t^{*1-\psi_W})^{\frac{1}{1-\psi_W}} \tag{1.32}$$

1.4.3 Production

Retail Sector

Goods in the economy, indexed as $j \in [0,1]$, are exclusively produced by individual wholesalers. Imperfect substitutability among goods is modeled using the Dixit and Stiglitz (1977) aggregator. The representative retail firm optimizes profit, leading to a mark-up rule for retail goods.:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} di\right)^{\frac{\psi}{\psi-1}} \tag{1.33}$$

in this context, Y_t represents the total output of retailers in period t, $Y_{j,t}$ signifies the wholesale good indexed by j, and $\psi > 1$ characterizes the elasticity of substitution among wholesale goods. Within this framework, P_t denotes the nominal price of a retail product, while $P_{j,t}$ corresponds to the nominal price of wholesale good j. Retail firms accept the price of each wholesale good as given. The representative firm maximizes its profit function:

$$\max_{Y_{j,t}} P_t \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} di\right)^{\frac{\psi}{\psi-1}} - \int_0^1 P_{j,t}Y_{j,t} di \tag{1.34}$$

Moreover, arrive at the following mark-up rule for retail goods:

$$P_t = \left(\int_0^1 P_{j,t}^{\frac{\psi-1}{\psi}} di\right)^{\frac{\psi}{\psi-1}} \tag{1.35}$$

Wholesale Sector

The wholesale sector operates in two stages, beginning with the optimal allocation of capital and labor to minimize production costs. The Cobb-Douglas technology governs production, leading to the determination of the marginal cost. Prices are then set considering staggered contracts. Cost minimization is given by:

$$\min_{L_{j,t},K_{i,t}} = W_t L_{i,t} + R_t^k K_{i,t} \tag{1.36}$$

Subject to the following Cobb-Douglas technology:

$$Y_{j,t} = (U_t K_t)^{\alpha 1} L_t^{\alpha 2} K_t^{G\,\alpha 3} \tag{1.37}$$

where K_t^G denotes public capital. After considering the marginal price of labor and capital, the marginal cost is given by:

$$MC_{j,t} = \frac{1}{A_t K_t^{G\,\alpha 3}} \left(\frac{W_t}{1-\alpha 2}\right)^{\alpha 2} \left(\frac{R_t^k}{\alpha 1}\right)^{\alpha 1} \tag{1.38}$$

Similarly to Eq. (1.31), the optimal price defined in by firms with probability θ under Calvo (1983) staggered prices is:

$$P_t^{\star} = \frac{\psi}{\psi - 1} E_t \left(\sum_{t=0}^{\infty} (\beta\theta)^t MC_{t+1} \right) \tag{1.39}$$

With the following aggregate price level:

$$P_t = (\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{\star 1-\psi})^{\frac{1}{1-\psi}} \tag{1.40}$$

1.4.4 Fiscal Authority

Government fiscal policy involves expenditure instruments (I_t^G, G_t, $TRANS_t$) and revenue instruments (τ_t^c, τ_t^c, τ_t^c). The government's budget constraint incorporates these components and is given by:

$$B_{t+1} - B_t R_t + \tau_t^c P_t (C_t + I_t) + \tau_t^l L_t W_t + \tau_t^k (R_t^k - \delta) K_t = P_t (G_t + I_t) \tag{1.41}$$

where G_t is government consumption and I_t^G government investment. Government capital has the following law of motion:

$$K_{t+1}^G = (1-\delta) K_t^G + I_t^G \tag{1.42}$$

with δ being the same depreciation parameter as the private capital. All fiscal instruments follow the same fiscal rule:

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}}\right)^{\gamma_Z} \left(\frac{B_t Y_{ss} P_{ss}}{Y_{t-1} P_{t-1} B_{ss}}\right)^{(1-\gamma_Z)\varphi_Z} + S_t^Z \tag{1.43}$$

such that $Z = \{G, I^G, \tau_t^c, \tau_t^l, \tau_t^k, TRANS\}$ and γ_Z is the persistence of each instrument defined in Z and φ_Z its proportion over debt. The fiscal policy shock S_t^Z is given by:

$$\log S_t^Z = (1-\rho_S) \log S_{ss}^Z + \rho_A \log S_{t-1}^Z + \epsilon_t^Z \tag{1.44}$$

with the error term $\epsilon_t^Z \sim N(0, \sigma^2)$.

1.4.5 Central Bank

Monetary policy follows a Taylor rule, aiming for price stability and economic growth. The rule is expressed as follows:

$$\frac{R_t^B}{R_{ss}^B} = \left(\frac{R_{t-1}^B}{R_{ss}^B}\right)^{\gamma_R} \left(\frac{Y_t}{Y_{ss}}\right)^{\gamma_Y} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi(1-\gamma_R)} + S_t^M \tag{1.45}$$

γ_Y and γ_π represent the sensitivities of the interest rate concerning the product and inflation rate, respectively, while γ_R denotes the smoothing parameter. The monetary policy shock S_t^M is given by:

$$\log S_t^M = (1-\rho_M)\log S_{ss}^M + \rho_A \log S_{t-1}^M + \epsilon_t^M \tag{1.46}$$

with the error term $\epsilon_t^M \sim N(0,\sigma^2)$.

1.4.6 Aggregation and Equilibrium

To finalize the model, we aggregate consumption and labor from both Ricardian and non-Ricardian agents. Aggregate consumption (C_t) and labor (L_t) are expressed as weighted sums of their components. Aggregate consumption can be expressed as:

$$C_t = \lambda C_t^R + (1-\lambda)C_t^{NR}$$

By applying the same procedure, we can find aggregate labor:

$$L_t = \lambda L_t^R + (1-\lambda)L_t^{NR}$$

The equilibrium condition ensures that total output (Y_t) is the sum of consumption, investment, government consumption, and government investment:

$$Y_t = C_t + G_t + I_t + I_t^G$$

1.4.7 Numerical Simulations

Simulation exercises are valuable for comprehending consumption responses in an environment with varying frictions. We compare the model versions against data using the standard deviation consumption ratio concerning output and interest rates. This strategy is chosen due to the ability to set the exogenous shock variance for government spending at 1, facilitating the analysis of fiscal multipliers for each model. Numerical simulations were conducted using the Dynare platform in MATLAB.[9]

To influence total income, a government spending shock was employed to check the consumption behavior of agents. Fig. 1.11 shows the response of the Marginal Rate of Substitution (MRS) for both model versions, baseline and with savings target, to a government spending shock. The savings target mechanism effectively mitigated the impact of the intertemporal substitution mechanism by making the MRS not respond to the shock with the same intensity as in the baseline case. The increase in accumulated assets generates a loss of utility given the savings target restriction, leading to an optimal path characterized by more consumption compared to the baseline case.

With the savings-target mechanism operating, the relationship between interest and future consumption becomes less decisive, which leads to a lower correlation between the interest rate and movements in the MRS, as shown in table 1.3. This result is in line with the result presented by the HANK literature, which points to indirect effects as responsible for up to 2/3 of the dynamics of aggregate consumption (Kaplan et al., 2018).

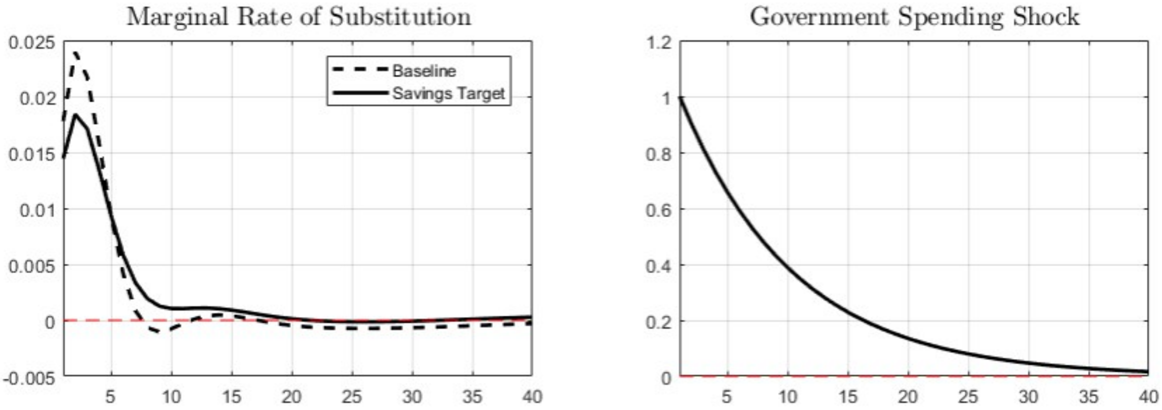

Figure 1.11: Impulse-response functions for output and consumption given a government spending shock.

	Baseline	Savings Target
Cor (MRS, R)	1	0.5296

Table 1.3: Correlation between Consumption and MRS for both models.

As can be seen in Fig. 1.12, the output and consumption response are as expected, with fiscal policy being able to positively affect output and ruling out the Ricardian equivalence. This effect happens as agents cannot fully interchange present and future consumption decisions as in the baseline model. This result is interesting as it imbues the model with the ability to display a result similar to a non-constant source of market incompleteness given the quadratic nature of the savings target function plus the non-constant intensity, once more similar to the results typically found in HANK models.

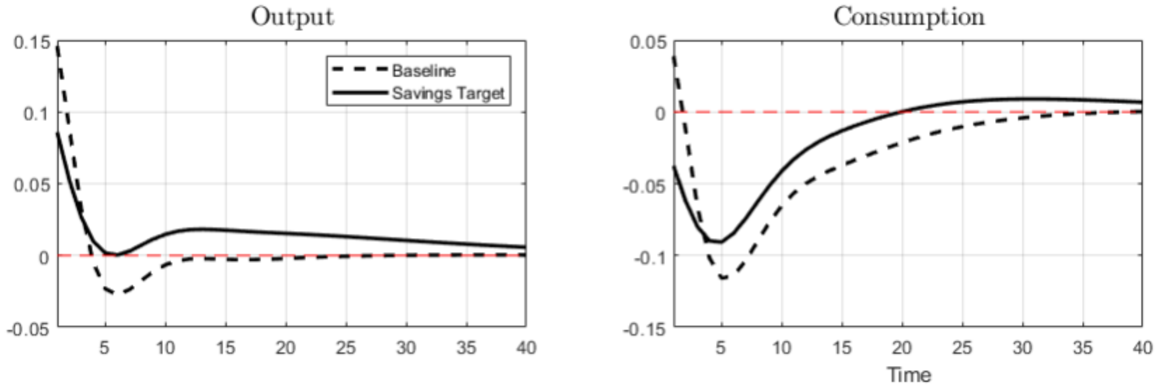

Figure 1.12: Impulse-response functions for a government spending shock.

As this paper focuses on consumption dynamics, the results only focus on this aspect of the model. The last step involves understanding the fiscal multiplier dynamics as the model affects the Ricardian equivalence. The present-value fiscal multiplier for income can be calculated as follows:

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[9] Parameter calibration follows the original model by Costa Junior (2016), except for parameters φ_{max} and κ. A comprehensive table with calibrated parameter values is provided in the appendix.

$$\text{Present Value Multiplier}(k) = \frac{E_t\left\{\sum_{j=0}^{k} \Pi_{i=0}^{k}(1+r_{t+i})^{-1}\Delta Y_{t+j}\right\}}{E_t\left\{\sum_{j=0}^{k} \Pi_{i=0}^{k}(1+r_{t+i})^{-1}\Delta G_{t+j}\right\}} \quad (1.47)$$

A similar multiplier can also be constructed to check the discounted effect of government spending over consumption. The exercise is carried out for a fiscal policy shock using government consumption as an instrument, as shown in Fig. 1.13.

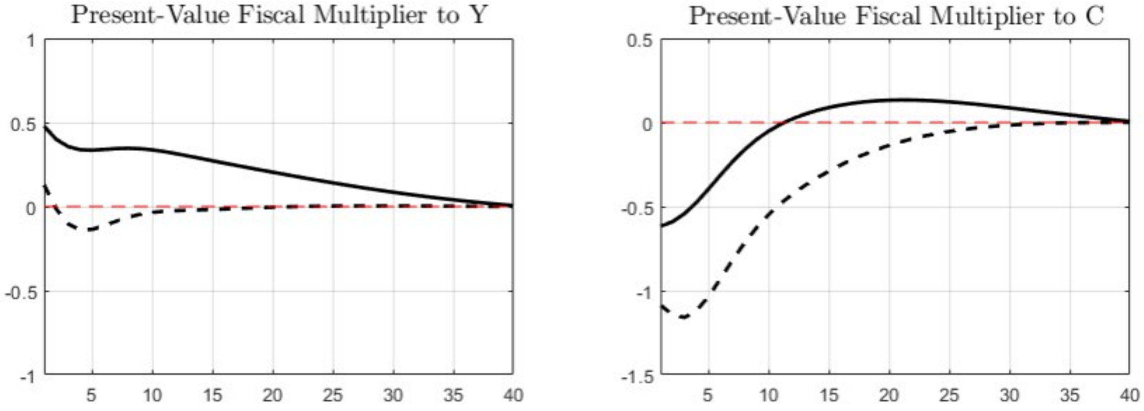

Figure 1.13: Output and consumption multipliers for a fiscal policy shock.

The positive response of aggregate consumption is quite subtle. Still, the impact on output is considerable and guarantees a positive multiplier in line with the HANK result provided by Hagedorn et al. (2019) for the Taylor rule case. The savings target mechanism can deliver a qualitative improvement in the dynamics of fiscal multipliers in a simple environment. As the loss of effect of fiscal policy depends on Ricardian equivalence, which in turn depends on the ability of agents to foresee and react to an increase in tax in the future, this effect loses effectiveness in the case in which the savings-target mechanism is active.

In practice, the savings-target mechanism captures, in the aggregate, the effect of the inability to substitute between present and future consumption, which can be explained by other channels such as expectations, non-constant discount factors, and incomplete markets. The advantage of this approach is that it exhibits satisfactory results from a simple mechanism using a conventional representative agent.

1.5 Concluding Remarks

This paper has proposed a novel approach to address the challenges posed by incorporating heterogeneous agents into macroeconomic models, particularly in the context of fiscal policy shocks. The prevailing reliance on the PIH in representative agent models has proven insuficient to capture the nuanced consumption dynamics observed empirically, especially in response to transitory income shocks driven by fiscal policy changes.

To overcome these limitations, this research introduces a savings target mechanism as a pragmatic solution to adjust consumption dynamics in the presence of heterogeneous agents. Drawing inspiration from prospect theory, the proposed mechanism incorporates a reference point that influences agents' decisions regarding consumption and savings.

Through a series of illustrative exercises, including deterministic income shocks, non-constant intensity scenarios, and stochastic income variations, the paper demonstrates the effectiveness of the savings target mechanism in aligning consumption behavior with empirical evidence. The results highlight how this mechanism influences agents' behavior, leading to more realistic responses to fiscal policy shocks and improving the overall fit with observed consumption dynamics.

The analysis includes a theoretical foundation, addressing the shortcomings of traditional utility functions based on the PIH, and provides a detailed exploration of the proposed savings target mechanism. By incorporating behavioral elements and considering the influence of individual goals and reference points, the model offers a more robust microfoundation for agents' behavior within macroeconomic frameworks.

Moreover, the research extends its application to a DSGE model, combining various real-market frictions. Simulation exercises reveal that the savings target mechanism, when active, produces qualitative alignments with empirical evidence, approximating the fiscal multiplier observed in HANK models.

In summary, this paper contributes to advancing the understanding of consumption dynamics in macroeconomic models with heterogeneous agents. By introducing the savings target mechanism, the research provides a promising avenue for reconciling theoretical models with real-world observations, enhancing the ability of macroeconomic frameworks to predict and explain economic outcomes in response to fiscal policy shocks. Incorporating mechanisms to match the empirical evidence of consumption behavior in macroeconomic models marks a significant step toward building more accurate and realistic macroeconomic models.

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Mathematical Appendix

Proof of Proposition 1

By considering that $r_t = r_{t+1}$ for all t and iterating forward the budget constraint, we obtain:

$$C_0 + \frac{1}{(1+r)}C_1 + \frac{1}{(1+r)^2}C_2 + \ldots = (1+r)W_0 + \sum_{t=0}^{\infty}\left(\frac{1}{(1+r)}\right)^t Y_t$$

After applying the relationship between c_t and c_{t+1}, we have that:

$$C_0 + \frac{1}{(1+r)}C_0[\beta(1+r)]^{\frac{1}{\gamma}} + \frac{1}{(1+r)^2}C_0[\beta(1+r)]^{\frac{2}{\gamma}} + \ldots = (1+r)W_0 + \sum_{t=0}^{\infty}\left(\frac{1}{(1+r)}\right)^t Y_t$$

By collecting the terms on the left, we arrive at the following:

$$\frac{C_0}{\bar{\alpha}} = (1+r)W_0 + \sum_{t=0}^{\infty}\left(\frac{1}{(1+r)}\right)^t Y_t$$

such that:

$$\bar{\alpha} = 1 - (1+r)^{-1}[\beta(1+r)]^{\frac{1}{\gamma}}$$

For the case when $U(C_t) = \log(C_t)$, it becomes:

$$\bar{\alpha} = (1-\beta)$$

Moments for the Non-Constant Case

Compared to the constant case, the κ parameter allows not only to adapt the response to different shocks but also to change the distribution of consumption. This mechanism can be useful when affecting the match of model variables with empirical series.

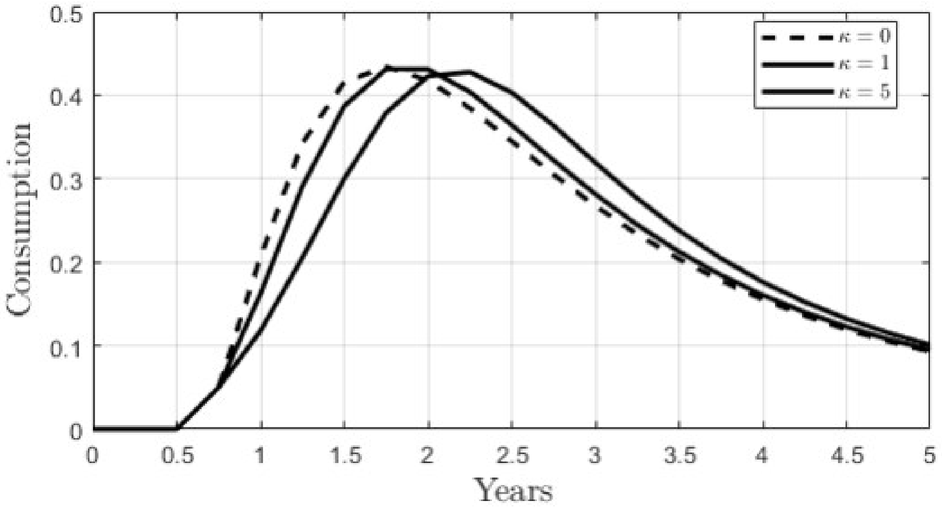

Figure 1.14: φ value for different parametrization of κ in function of the difference $Y_t - \bar{Y}$ with $\varphi_{max} = 0.1$.

Value	Mean	Standard Deviation	Skewness	Kurtosis
κ = 0	0.2091	0.1440	0.0888	1.7997
κ = 1	0.2091	0.1441	0.1003	1.8093
κ = 5	0.2088	0.1422	0.0700	1.8107

Table 1.4: Consumption Moments for Different Values of κ with $\varphi_{max} = 0.8$ and a deterministic income shock deviating in 1 unit from the permanent income level.

Furthermore, for more significant deviations in Y, this effect is even more effective in shaping the moments of the series. Fig. 1.15 and table 1.2 summarize the changes in consumption dynamics graphically and show the moments for the different cases with a deviation of 5 units from the permanent income measure.

Value	Mean	Standard Deviation	Skewness	Kurtosis
κ = 0	1.1298	1.0756	0.8035	2.2977
κ = 1	1.1423	0.9790	0.5461	1.9239
κ = 5	1.1448	0.8705	0.2936	1.7683

Table 1.5: Consumption Moments for Different Values of κ with $\varphi_{max} = 0.8$ and a deterministic income shock deviating in 5 units from the permanent income level.

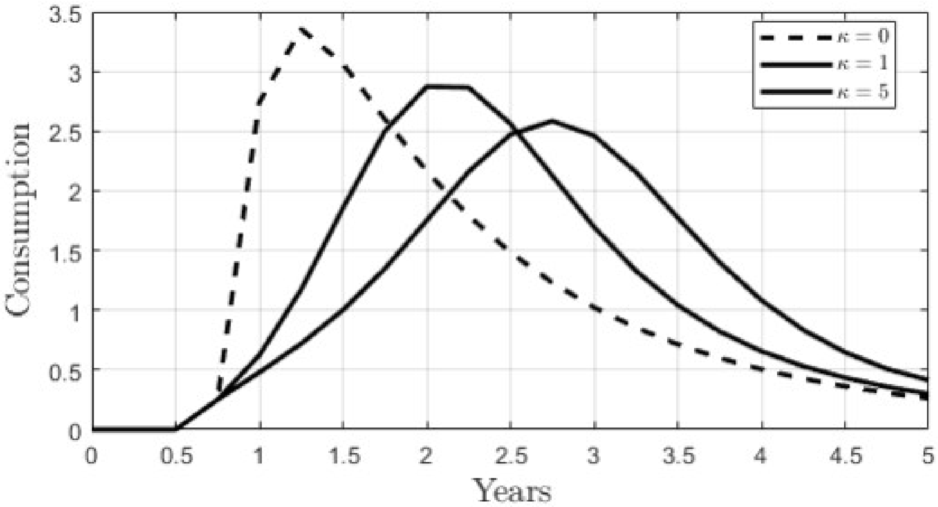

Figure 1.15: φ value for different parametrization of κ in function of the difference $Y_t - \bar{Y}$ with $\varphi_{max} = 0.1$.

Savings Target in General Equilibrium - Log-Linear Model Equations

Description	Equation
Euler equation	$C_t = E_t\hat{C}_{t+1} - \hat{r}_t - \varphi_{max}\tilde{B}_t + 2\kappa\tilde{Y}_t$
Budget constraint	$\hat{C}_t + \tilde{B} = \hat{L}_t + \hat{w}_t - \tilde{T}_t + R\tilde{B}_{,t-1}$
Labor supply	$\hat{L}_t = \varphi^{-1}(\hat{W}_t - \hat{C}_t)$
Phillips curve	$\hat{\Pi}_t = \beta E_t\hat{\Pi}_{t+1} + \eta\xi\hat{W}_t$
Gov. budget constraint	$\tilde{B}_t = R\tilde{B}_{t-1} + \tilde{G}_t - \tilde{T}_t$
Gov. spending	$\tilde{G}_t = \rho_g\tilde{G}_{t-1} + \varepsilon_{Gt}$
Fiscal rule	$\tilde{T}_t = \varphi_\tau \tau_t\tilde{T}_{t-1} + \varphi_{\tau B}\tilde{B}_t + \varphi_{\tau G}\tilde{G}_t$
Taylor rule	$\hat{R}_t = \varphi_\pi\Pi_t$
Fisher equation	$\hat{r}_t = \hat{R}_t - E_t\hat{\Pi}_{t+1}$

Table 1.6: log-linear equations of the model presented in section 3.4.

Calibration for Savings Target in General Equilibrium

Parameter	Symbol	Value
Inverse of Frish elasticity of Labor Supply	ϕ	0
Adjustment Costs Parameter	ξ	1028.571
Discount Factor	β	0.99
Fiscal Rule Coeficient (T)	$\varphi^{\tau T}$	0
Fiscal Rule Coeficient (G)	$\varphi^{\tau G}$	0.33
Fiscal Rule Coeficient (G)	$\varphi^{\tau B}$	0.1
Taylor rule Coeficient (π)	φ^{π}	1.5
Autoregressive Paramter (G)	ρ	0.9
Elasticity of substitution (Y_j)	η	6
Max Savings Target Intensity	φ_{max}	0.08
Intensity of Change	κ	0.02
Relative Risk-Aversion Parameter	γ	1

Table 1.7: Parameter values used in the simulation exercise of the model presented in section 3.4.

Household Problem for the DSGE model

This appendix contains the solution for the household sector with the modifications in the utility function following the savings target developed in section 3. The other sectors follow the original model presented in Costa Junior (2016).

Ricardian Households

$$\max_{C_t^R, B_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{R,1-\gamma}}{1-\gamma} - \frac{\varphi_{max}(B_t - \bar{B})^2}{2(1+\kappa(Y_t^R - \bar{Y}^R)^2)} - \frac{L_t^{R,1-\phi}}{1-\phi} \right)$$

Subject to:

$$P_t(1+\tau_t^c)(C_t^R + I_t) + B_t = (1+\tau_t^k)K_tR_t^kU_t - P_tK_t\left(\psi_1(U_t-1)+\frac{\psi_2}{2}(U_t-1)^2\right)$$
$$+B_{t-1}R_{t-1} + (1+\tau_t^l)W_tL_t^R + \lambda P_t TRANS_t$$

$$K_{t+1} = (1-\delta)K_t + I_t\left(1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)$$

The First-Order Conditions (FOC) associated with the problem are given by:

$$\beta^t(C_t^{R,-\gamma} - \lambda_t^R P_t(1+\tau_t^c)) = 0$$

$$\beta^t(\lambda_t^R R_t K_t(1-\tau_t^k) - \lambda_t^R P_t K_t \psi_1 \lambda_t P_t K_t \psi_2 (U_t - 1)) = 0$$

$$\beta^t\left(Q_t\left(1-\frac{\chi}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 - \chi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-1\right)\right) - \lambda_t P_t(1+\tau_t^c)\right)$$
$$+ \chi\beta^{t+1}E_t\left(Q_{t+1}\left(\frac{I_t}{I_{t-1}}\right)^2\left(\frac{I_t}{I_{t-1}}-1\right)\right) = 0$$

$$-\beta^t\lambda_t^R + \beta^{t+1}E_t\lambda_{t+1}R_{t+1} - \frac{\varphi^c_{max}(B_{t+1} - \bar{B})}{1+\kappa^c(Y_t^R - \bar{Y}^R)^2} = 0$$

$$\beta^{t+1}E_t\left(\lambda_{c,t+1}R_{t+1}U_{t+1}(1-\tau_t^k) - \beta^t\lambda_{t+1}P_{t+1}\left(\psi_1(U_t-1)+\frac{\psi_2}{2}(U_t-1)^2\right) - Q_t + Q_{t+1}(1-\delta)\right) = 0$$

After organizing, we arrive at:

$$C_t^{R,-\gamma} = \lambda_t^R P_t(1+\tau_t^c)$$

$$Q_t = \beta E_t\left((1-\delta)Q_{t+1} + \frac{1}{P_t(1+\tau^c_{t+1})}\left(C_{c,t+1}^{-\gamma}R_{i,t+1}U_{t+1}(1-\tau_{t+1}) - C_{c,t+1}^{-\gamma}\left(\psi_1(U_{t+1}-1)+\frac{\psi_2}{2}(U_{t+1}-1)^2\right)\right)\right)$$

$$\frac{R_t}{P_t} = \frac{1}{1-\tau_t^k}[\psi_1+\psi_2(U_t-1)]$$

$$C_t^{R,-\gamma} - Q_t\left[1-\frac{\chi}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 - \chi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-1\right)\right] = \chi\beta E_t\left[Q_{t+1}\left(\frac{I_t}{I_{t-1}}\right)^2\left(\frac{I_t}{I_{t-1}}-1\right)\right]$$

$$C_t^{R,-\gamma} = \beta E_t(C_{t+1}^{R,-\gamma}R_{t+1}) - \frac{\varphi^c_{max}(B_{t+1}-\bar{B})}{1+\kappa^c(Y_t^R-\bar{Y}^R)^2}$$

Non-Ricardian Households

$$\max_{C_t^{NR}} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{NR,1-\gamma}}{1-\gamma} - \frac{L_t^{NR,1-\phi}}{1-\phi} \right)$$

Subjected to:

$$P_t(1+\tau_t^c)C_t^{NR} = (1+\tau_t^l)W_t L_t^{NR} + (1-\lambda)P_t TRANS_t$$

The First-Order Condition (FOC) associated with the problem are given by:

$$\beta^t(C_t^{NR,-\gamma} - \lambda_t^{NR}P_t(1+\tau_t^c)) = 0$$

After organizing, we arrive at:

$$C_t^{NR,-\gamma} = \lambda_t^{NR}P_t(1+\tau_t^c)$$

Calibration for the DSGE Model

Parameter	Meaning	Value
γ	Relative risk aversion coeficient	2
ϕ	Marginal disutility with regard to supply of labor	1.5
α_1	elasticity of level of production in relation to capital	0.35
α_2	elasticity of level of production in relation to labor	0.6
α_2	elasticity of level of production in relation to public capital	0.05
β	Discount factor	0.985
δ	Depreciation rate	0.025
θ	Price stickiness parameter	0.75
ψ	Elasticity of substitution among intermediate goods	8
θ_W	Wage stickiness parameter	0.75
ψ_W	Elasticity of substitution between differentiated labor	21
τ^c_{ss}	Rate of tax on consumption in steady state	0.16
τ^l_{ss}	Rate of tax on income from labor in steady state	0.17
τ^k_{ss}	Rate of tax on income from capital in steady state	0.08
χ	Sensitivity of investments in relation to adjustment cost	1
Ψ_1	Sensitivity of cost of under-utilization maximum installed capacity 1	1
Ψ_2	Sensitivity of cost of under-utilization maximum installed capacity 2	$\frac{1}{\beta}-(1-\delta)$
γ_R	Interest rate persistence	0.79
γ_Y	Sensitivity of interest rate in relation to GDP	0.16
γ_π	Sensitivity of interest rate in relation to inflation	2.43
φ_{Bss}	Proportion of public debt in relation to GDP	1
γ_G	Public spending persistence	0
γ_{IG}	Public investment persistence	1
γ_{TRANS}	Transfers persistence	1
$\gamma_{\tau c}$	Persistence of tax on consumption	0
$\gamma_{\tau l}$	Persistence of tax on labor income	0
$\gamma_{\tau k}$	Persistence of tax on capital income	0
φ_G	Public spending over debt	0
φ_{IG}	Public investment over debt	-1
φ_{TRANS}	Transfers over debt	-1
$\varphi_{\tau c}$	Tax on consumption over debt	0
$\varphi_{\tau l}$	Tax on labor income over debt	0
$\varphi_{\tau k}$	Tax on capital income over debt	0
λ	Share of Ricardians	0.7
φ_{max}	Max savings target effect	0.012
κ	Savings Target intensity parameter	0.01

Chapter 2

Informality, Financial Markets, and Macroeconomic Instability

Abstract

This research aims to understand the impact of informality on the dynamics of economies, particularly concerning their equilibrium and instability. For this, a pioneering heterogeneous agents model was developed. It combines two cru-cial elements of heterogeneity: expectation formation processes and the pres-ence of formal and informal firms. Analytical and numerical results show that economies with high informality exhibit specific unstable dynamics compared to those with a more significant presence of formal firms. Additionally, an expanded model incorporating endogenous expectations and informality shows heightened instability due to feedback effects between expectation changes and access to financial markets. These results provide fresh perspectives on the relevance of considering the structural characteristics of the economies to policy design.

Keywords: Informality; Real-financial interactions; Financial Instability; Heterogeneous Expectation; Business Cycles

2.1 Introduction

The academic community's interest in financial phenomena has surged significantly, primarily driven by the profound impact of the 2008 Great Financial Crisis. This event served as a stark reminder of the central role played by the financial sector in influenc-ing economic fluctuations. As a result, policymakers presented new recommendations for mitigating systemic risk by employing formal models to aid their decisions. However, a challenge arises when applying models designed for developed economies, with their unique structural elements, to understand the dynamics of developing economies. Policies that mitigate instability may be less effective or counterproductive by ignoring a structural characteristic that qualitatively impacts an economy's dynamics.

This research aims to approach such a question by evaluating how economies characterized by a high level of informality, i.e., production of goods and services not being reported to public authorities, may exhibit specific unstable dynamics compared to those dominated by formal firms (La Porta and Shleifer, 2014). Persistent levels of high informality characterize a structural phenomenon that mainly affects developing economies (Elgin et al., 2021). The significance of my inquiry is reinforced by empirical evidence from various studies, indicating that informality impacts macroeconomics through both real and financial channels (Fernández and Meza, 2015; Granda Carvajal, 2015; Gandelman and Rasteletti, 2017; Horvath, 2018).

The literature focused on understanding the connections between real and financial markets features prominent models in two different approaches: models that consider a framework with rational expectations like Brunnermeier and Sannikov (2014), Gertler et al. (2016), and Beaudry et al. (2020), and studies departing from expectations formed through the use of heuristics (simple rules) as in Naimzada and Pireddu (2015), Flaschel et al. (2018), Cavalli et al. (2022), and Gardini et al. (2023).[1] To the best of my knowledge, this research landscape has yet to explore the role of informality within the context of these models. This situation presents a significant gap in the literature, making the exploration of informality's impact on the dynamics of economies, especially those with a high level of informality, an essential and uncharted area of investigation.

To make a substantive contribution to this literature, the present paper develops a pioneering behavioral heterogeneous agents model to capture the effects of informality over the dynamics of liquidity, solvency, and output. The distinctive hallmark of the model lies in its fusion of two crucial elements of heterogeneity. Firstly, it incorporates a well-established form of heterogeneity in expectation formation processes that follow optimistic agents that extrapolate the effective liquidity trend and pessimistic that expect a return to a measure of fundamental liquidity as in Brock and Hommes (1997). Secondly, it introduces a novel dimension of heterogeneity concerning the presence of formal and informal firms, in which formal firms have a greater production capacity and can access external financing to bring future liquidity flows to the present. At the same time, informal firms have a lower production capacity and rely on their effective liquidity only with no access to financial markets.

A three-dimensional non-linear map characterizes the model. It is compatible with persistent endogenous fluctuations from a Flip and Neimark-Sacker bifurcation. Those properties align with recent empirical findings pointing to the necessity of understanding macroeconomic dynamics by taking fluctuations as an endogenous phenomenon (Beaudry et al., 2020; Barrales-Ruiz and von Arnim, 2021). The results follow analytical methods such as equilibrium and local stability analysis as such as numerical methods such as bifurcation diagrams and phase portraits.

My findings indicate that changes in the share of the informal sector have discernible effects on the equilibrium point of the system and its stability conditions. Economies characterized by consolidated high participation of the formal sector present a higher output level in equilibrium. However, they are more prone to episodes of financial instability than those characterized by high informality given the amount of firms operating in the financial sector for specific areas in the parameter space. This phenomenon connects to the fact that the formal sector can expand production and demand more than the informal sector, leading to increased output. At the same time, by accessing credit markets, formal firms increase the destabilizing effects generated by the expectation channel that may lead to liquidity mismatch episodes. From an equilibrium point of view, there is a trade-off between higher output levels and less instability expressed by the composition of formal and informal firms.

An expanded version of the model endogenizes agents' expectations and informality. While the incorporation of endogenous beliefs about the future is a standard procedure in bounded rationality models (Franke and Westerhoff, 2017), this extension sets itself apart by simultaneously implementing two additional mechanisms that engender heterogeneity also in the levels of informality, encompassing both the real and financial dimensions. The

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[1] For a comprehensive review see Dieci and He (2018), Franke and Westerhoff (2017) and Hommes (2006).

crux of this innovative extension lies in the fact that informality changes according to variations in liquidity and production along the business cycle. This effect allows us to capture empirical facts that the formal sector can capture resources from the informal sector in expansion phases and vice-versa (Loayza and Rigolini, 2011; Colombo et al., 2016).

My analysis suggest that incorporating endogenous expectations and informality results in heightened instability. By considering the endogenous expectations only, agents start to exhibit optimistic behavior during expansion phases of the business cycle and pessimistic behavior during contractions, in line with the empirical studies such as Boeck and Zörner (2023) and similar models in this literature (Naimzada and Pireddu, 2015; Cafferata et al., 2021; Cavalli et al., 2022). On the other hand, the system exhibited minor changes regarding its dynamic behavior for endogenous informality. By combining both effects, the system presents high instability compatible with chaotic motions. This effect results from the amplification effect generated by the increase in production and demand on the real side and the number of firms accessing the financial markets on the financial side and vice-versa. In that sense, the effects of endogenous expectations become amplified, lead-ing to instability scenarios with an intensity of change between expectations in line with empirical evidence (Kukacka and Sacht, 2023).

Informality is a structural phenomenon affecting the economies' equilibrium and instability boundaries, with the capacity to generate feedback effects along the business cycles. In that sense, not only does the level of informality matter but also how the informal sector interacts with other mechanisms that increase instability or even create chaotic dynamics. By analyzing both results, I concluded that economies with higher levels of informality can be more prone to episodes of instability as the interaction between endogenous informality in real and financial sectors is a significant source of instability along the business cycle.

Indeed, the results of the benchmark and extended models offer a fresh perspective on the empirical debate surrounding the impact of informality on macroeconomic volatility. The existing literature has yielded contradictory findings, with some studies suggesting that informality is associated with increased volatility in consumption and output (Restrepo-Echavarria, 2014; Horvath, 2018). In contrast, others point to situations where informality can reduce volatility (Mitra, 2013; Mapp and Moore, 2015). According to my results, these discrepancies result from the structural differences which give rise to distinct feedback effects. The idiosyncratic capacity to shift resources between the formal and informal sectors and the capacity of the financial sector to influence firms' access to external finance in each economy explains why these results seem contradictory.

2.2 Facts on Informality

Informality can be broadly defined as a phenomenon where goods and services are produced within the market and legal framework but are not reported to public authorities for various reasons, including monetary, regulatory, or institutional factors Schneider et al. (2010). Such unreported or informal firms often face legal challenges in accessing external funds, acquiring machinery and equipment, and hiring skilled labor. According to La Porta and Shleifer (2014), informal businesses typically operate with lower productivity, limited access to technology and capital goods, and predominantly employ low-skilled workers who receive lower wages. Additionally, they often rely on cash transactions and have limited access to formal credit markets, hindering their growth and development compared to their formal counterparts.

Examining the GDP of the world's largest economies reveals that several countries experience significant informal sector participation in their economic activities. Fig. 2.1 illustrates this trend, indicating that countries like Russia and Brazil have a substantial

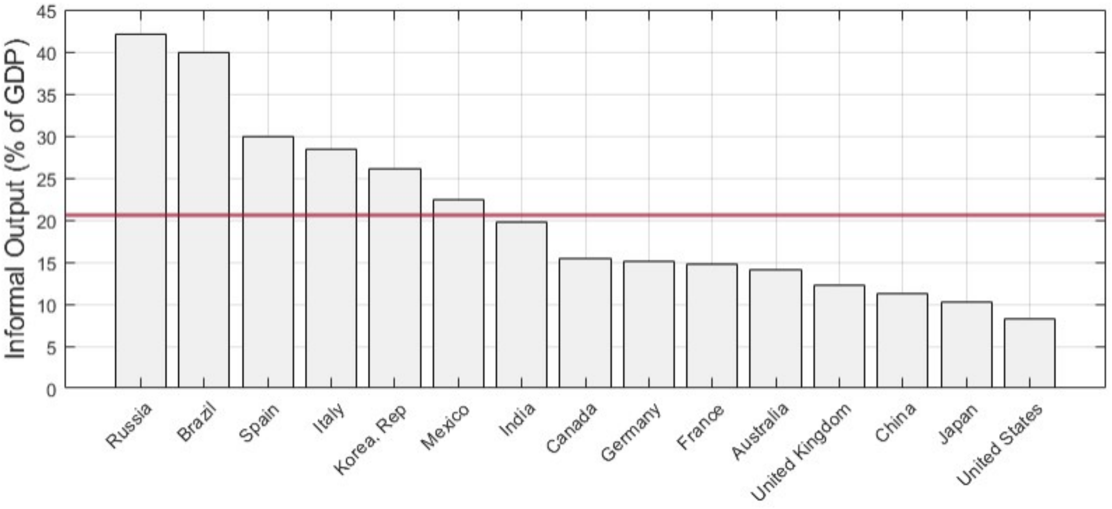

Figure 2.1: A graphical representation of the informal output of the 15 largest economies regarding % of their GDP with the red line as the average. Results provided by Medina et al. (2019) using a Multiple Indicators and Multiple Causes (MIMIC) estimation.

share of informal sector contribution, amounting to approximately 40% of their outputs.

Despite the already expressive numbers, estimates for informal output may incur an underreporting problem since informal employment contracts in formal companies can mask the actual size of the informal sector (Ulyssea, 2020). In this case, the impact of informality may be even more significant, leaving no doubt about the topic's relevance when trying to understand the dynamics of these countries. For example, despite the informal output estimated as 19.7% of India's GDP, informal unemployment for the same period follows 88.6%. Beyond that, estimates by Elgin et al. (2021) indicate that the global economy's average informal output is 33.3%, which rises to 37.3% in emerging markets, pointing to a possible relevant structural difference between developed and developing economies.

As a way of understanding the informal sector's relationship with the cyclical dynamics of economies, some stylized facts need to be revisited more carefully as they have possible relevant macroeconomic repercussions. Despite the vast expanse of literature on informality, this research will focus on three specific stylized facts at the heart of real-financial market interactions. This choice aims to prioritize characteristics and elements directly linked to the scope of this research.[2]

2.2.1 Coexistence over the business cycle

The first intriguing fact lies in the ability of the formal and informal sectors to adjust their production during the same moments of the business cycle (Loayza and Rigolini, 2011; Colombo et al., 2016). During periods of economic growth, formal firms tend to expand their operations, leading to increased demand for labor and inputs. Consequently, resources previously employed in the informal sector are attracted to the formal sector. As the informal sector typically offers lower wages, workers often find it beneficial to transition to formalized employment when such opportunities arise. Additionally, rising wages in the formal sector can make hiring labor challenging for the informal sector, pushing them to invest in more machinery for eficient production. Conversely, during economic

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[2] For a more comprehensive analysis of informality see Schneider et al. (2010), La Porta and Shleifer (2014), Ulyssea (2020), Elgin et al. (2021), and Elgin et al. (2022).

downturns, the formal sector reduces production, increasing unemployment. Displaced workers, now without formal employment, turn to the informal sector to survive, even if it means accepting lower wages and potentially poorer working conditions.

These effects have been empirically verified in various studies. For instance, Fernández and Meza (2015) demonstrated that informal employment in Mexico is counter-cyclical, meaning it moves in the opposite direction of the economic cycle and is negatively correlated with formal sector employment. Similarly, Sun (2022) employed a DSGE model to find that the informal sector helps mitigate unemployment fluctuations in the Chinese economy. Moreover, research by Elgin et al. (2022) using a database covering over 160 economies concluded that employment growth in the informal sector negatively correlates with employment growth in the formal sector.

By understanding the labor market dynamics, we gain insight into the broader production dynamics of both formal and informal sectors. In general, the operational feasibility of the informal sector depends on factors such as labor availability and production scale. This stylized fact sheds light on the viability of the parallel coexistence of the informal and formal sectors.[3]

2.2.2 Lack of external funding

Another crucial aspect is that informal firms often face significant limitations in access-ing the credit market due to a lack of legal documentation and the absence of collateral (Granda Carvajal, 2015). Empirical studies, such as those conducted by Koeda and Dabla-Norris (2008) and Gandelman and Rasteletti (2017), demonstrate a strong and significant association between informality and increased credit constraints. This constraint directly affects the sector's productivity, as access to capital goods typically necessitates external financing or internal funding capacity, which is often inconsistent with the capabilities of most informal firms. A study focusing on the Uruguayan economy (Gandelman and Rasteletti, 2017) found that financial restrictions substantially impacted investment reduction for firms operating in sectors with higher informality rates.

Beyond its impact on investment, credit constraints influence the ability to manage liquidity mismatches throughout the business cycle. The volatility of consumption in the informal sector is linked to the lack of capacity to use working capital loans to stabilize cash flow fluctuations. For instance, a study examining the effects of an unanticipated currency contraction in India (Subramaniam, 2019) documented that companies operating in sectors with a higher proportion of employees without formal contracts experienced a decrease in their labor share, with the extent of cash usage relative to credit playing a significant role. Similarly, in a study analyzing firm-level responses to the COVID-19 pandemic, Khan (2022) found that small and medium-sized credit-constrained firms were more likely to face liquidity problems, and financing constraints had an impact on their ability to adjust business operations.

By elucidating the challenges associated with credit access for informal firms, these empirical findings emphasize the critical role that credit availability plays in the productivity and resilience of the informal sector. The lack of access to credit hinders investments and affects the day-to-day operations and stability of informal businesses.

2.2.3 Consumption and output volatility

Despite serving as a mechanism that can moderate fluctuations in unemployment, the presence of a significant informal sector in an economy has been associated with increased volatility in output and consumption. By conducting a study using a DSGE model,

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[3] It is essential to acknowledge that other factors, such as prior savings or social security, may also play a significant role in shaping these labor market dynamics (Fisher and Hardy, 2022).

Restrepo-Echavarria (2014), has provided evidence supporting the notion that the informal sector's presence affects the volatility of output and consumption. Moreover, this effect is directly related to the productivity differential between the formal and informal sectors. The findings of Horvath (2018) further corroborated this result, revealing a positive relationship between the relative volatility of consumption to output and the size of the informal economy in a two-sector real business cycles model. These studies highlight the importance of considering the informal sector's impact on a country's overall economic stability and performance.

At the same time, a different result was presented by Mapp and Moore (2015). After examining the effects of the informal economy on economic volatility in the Caribbean economies, the study concluded that the growth in the informal economy had reduced consumption volatility. This finding suggests that possible structural effects regarding the economy's capacity to propagate macroeconomic instability can explain the effects' net result. Mitra (2013) found a similar result showing that reducing the financial markets' effect on the economy via informality may reduce consumption volatility. The available empirical evidence does not justify the discrepancy between these results that may vary in direction and intensity. However, the existence of a relationship between informality and volatility is robust and well-documented.

2.3 A Basic Setup for a Financial-Dual Economy

In this section, I develop a dynamic model that considers the stylized facts described earlier while enabling a comprehensive analysis of the economy's real and financial sides. The model consists of a system of difference equations, incorporating liquidity (L_t) and solvency (S_t) as the financial side of the economy, similar to Sordi and Vercelli (2012). At the same time, output (Y_t) represents the real side as in Naimzada and Pireddu (2015) and Cafferata et al. (2021). Most theoretical models addressing informality from a macroeconomic point of view departs from the same DSGE structure. My model's main innovation commits to bringing the role of informality into the discussion in a framework that acknowledges heterogeneity in the formation of expectations and endogenous dynamics. Fig. 2.2 synthesizes its transmission channels.

The economy is populated by firms indexed by $i \in [F, I]$ for the formal and informal sectors, respectively. On the financial side, the formal sector has access to financial markets and form expectations regarding future liquidity flows using extrapolative and regressive heuristics. On the other hand, informal firms rely solely on their cash flow to fulfill their financial commitments. On the real side, both sectors produce a homogeneous good and determine the quantity produced in period $t + 1$ based on the demand observed in period t. The formal sector has a higher productive capacity, given the possibility of purchasing inputs in periods of growth, using eficient machinery, and hiring skilled labor to extend production. Given the lack of fixed capital and low-skilled labor, the informal sector operates with a lower productive capacity.

2.3.1 Financial Side

As a way to construct the equations that compose the financial side of the economy, we can initially define formal firms' solvency in $t + 1$ as a measure of the evolution of their net worth:

$$S^F_{t+1} = \sum_{h=0}^{T} \frac{E_t[L^F_{t+1+h}]}{(1+r)^h} \tag{2.1}$$

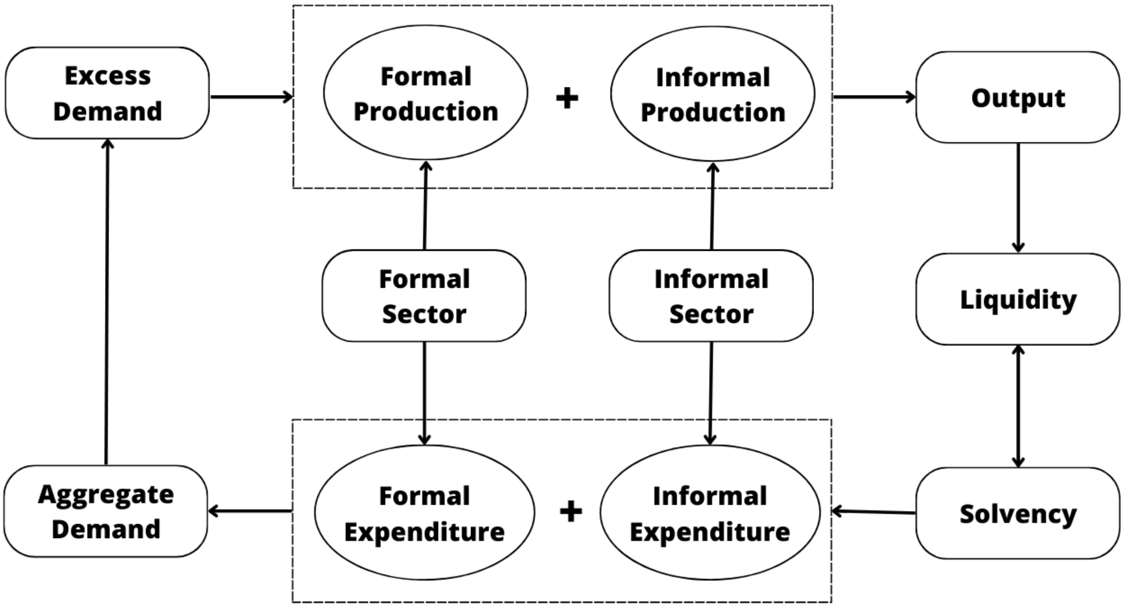

Figure 2.2: Diagram that captures the main connection of the model.

with $E_t[l_{i,t+1+h}]$ representing formal firms' expected flows of liquidity given a time horizon $T \in N_+$ discounted by a nominal interest rate $r \in (0, \infty)$. At the same time, the informal firms have their future solvency described by:

$$S^I_{t+1} = L^I_t \tag{2.2}$$

Informal firms cannot access external financing and cannot bring future liquidity flows to period t, restricted to operate only with their contemporaneous liquidity. This follows the empirical evidence pointing to credit constraints in the informal sector (Koeda and Dabla-Norris, 2008; Gandelman and Rasteletti, 2017).

By following the literature on heterogeneous expectations in financial markets (Brock and Hommes, 1997; Agliari et al., 2018; Cavalli et al., 2022), it is possible to distinguish between different heuristic that guides agents' expectation formation processes. In this case, agents can behave as extrapolative, which expects an increase in the current trend, or fundamentalist, which expects a reversal of the current trend to some fundamental value. Thus, the expected liquidity can be defined as:

$$E_t[L^F_{t+1}] = L^F_t + \rho(L^F_t - \bar{L}) \tag{2.3}$$

with parameter $\rho \in (-1, 1)$ reflecting the direction and intensity in which agents respond to discrepancies between effective liquidity L_t and a measure of "fundamental" liquidity $\bar{L}$ that also characterizes agents' liquidity safety margin. If $\rho > 0$, agents will behave like extrapolators and expect a continuous change in future liquidity in the direction of the actual trend. If $\rho < 0$, agents will act like fundamentalists and expect a return to the fundamental level of liquidity. At the same time, the distance $|\rho - 0|$ determines the intensity at which agents respond to the deviations.

As a way to combine different heuristics from a set of agents, Eq. (2.3) can be decomposed as:

$$E_t[L^F_{t+1}] = \omega[L^F_t + \rho^e(L^F_t - \bar{L})] + (1 - \omega)[L^F_t + \rho^r(L^F_t - \bar{L})] \tag{2.4}$$

$\omega \in [0, 1]$ is a weight parameter that captures the share of extrapolators and $(1 - \omega)$ the share of fundamentalists. Moreover, $\rho^e \in (0, 1)$ and $\rho^r \in (-1, 0)$ are specific intensity coeficients for extrapolators and fundamentalists, respectively. After some manipulation, (2.4) becomes:

$$E_t^F[L_{t+1}] = L_t^F + \rho^{er}(L_t^F - \bar{L}) \tag{2.5}$$

such that

$$\rho_t^{er} = \omega\rho^e + (1-\omega)\rho^r \tag{2.6}$$

During periods of tranquillity, it is reasonable to assume that agents expect their liquidity conditions to remain almost constant in the near future. As a result, their expectations can be approximated by the fundamental level of liquidity $\bar{L}$. So that, Eq. (2.1) becomes:

$$S_t^F = \frac{\bar{L}}{a} \tag{2.7}$$

where

$$a = \frac{r(1+r)^T}{(1+r)^{T+1}-1} \leq 1$$

with a grouping the effect of the interest rate r in a given time horizon T. The present value of solvency is discounted by a. At the same time, if the agents start to observe discrepancies between their effective liquidity and their safety margin, they will respond by increasing or decreasing their expenditures. In that case, by considering a time horizon h, Eq. (2.5) extends to:

$$E_t[L_{t+h}^F] = L_t^F + (1+\rho^{er})^h(L_t - \bar{L}) \qquad h = 0, 1, 2, ..., T \tag{2.8}$$

Understanding the effect of both components on agents' future solvency is now possible. By substituting Eq. (2.8) into Eq. (2.1), we have that:

$$S_{t+1}^F = \sum_{h=1}^{T+1} \frac{\bar{L} + (1+\rho^{er})^h(L_t - \bar{L})}{(1+r)^h}$$

$$= (1+\rho)^h(L_t^F - \bar{L}) \sum^{T+1} \frac{1+\rho^{er}}{1+r}^{h-1} + \bar{L} \sum_{h=0}^{T} \frac{1}{1+r}^{h}$$

Grouping discount and expectation parameters in a and β after iterating, we have that:

$$S_{t+1}^F = \beta(L_t^F - \bar{L}) + \frac{\bar{L}}{a} \tag{2.9}$$

such that

$$\beta = (1+\rho^{er})\frac{(1+r)^{T+1} - (1+\rho^{er})^{T+1}}{(1+r)^T(r-\rho^{er})} \tag{2.10}$$

where β captures the formal firms' response of solvency to liquidity, which means that solvency will be more responsive to changes in liquidity in an economy populated by long-memory agents.

By substituting Eq. (2.7) into Eq. (2.9), we become at:

$$S_{t+1} = \beta L_t^F + (1-a\beta)S_t^F \tag{2.11}$$

By combining equations (2.11) and (2.2) we obtain the total solvency of the economy as:

$$S_{t+1} = S^F_{t+1} + S^I_{t+1} \tag{2.12}$$

$$= \beta L^F_t + (1 - a\beta)S^F_t + L^I_t \tag{2.13}$$

Firms' liquidity in $t + 1$ will be defined by liquidity in t plus a variation captured by the difference between their liquidity in t and a safety margin described μ:

$$L^i_{t+1} = L^i_t + \alpha\,(S^i_t - \mu^i) \tag{2.14}$$

with $\alpha \in (0, 1)$ being a parameter that captures the reaction speed of agents to the gap between the solvency and the safety margin. In that scenario, firms operate with a safety net worth margin to deal with the risk of bankruptcy. Firms only increase their financial exposure to achieve high returns if $S^i_t > \mu^i$. Conversely, firms prioritize their financial survivability when $S^i_t < \mu^i$.

The aggregate version of Eq. (2.14) can be obtained by combining the liquidity for both sectors as:

$$L_{t+1} = L^F_{t+1} + L^I_{t+1} \tag{2.15}$$

$$= L^F_t + \alpha\,(S^F_t - \mu^F) + L^I_t + \alpha\,(S^I_t - \mu^I) \tag{2.16}$$

The share of the formal sector in the financial side of the economy can be defined by the parameter $\lambda \in [0, 1]$ as:

$$\lambda = \frac{L^F}{L} = \frac{S^F}{S} \quad \text{for all } t \tag{2.17}$$

2.3.2 Real Side

The production dynamics of formal and informal firms can be described by following Blanchard and Wyplosz (1981). A measure of production variation $g(x_t) : R \rightarrow R$, with x_t being a variable that guides the production decisions of firms. By taking Y_{t+1} as firms output in $t + 1$, the dynamics of production will follow:

$$Y_{t+1} = Y_t + g(x_t) \tag{2.18}$$

where changes in the production decisions will be driven by movements in x_t. Since capital and labor are not being modeled explicitly, a production specification with an S-shaped functional form is appropriate, as it allows for both a ceiling and floor that indirectly captures production constraints. For this purpose, the hyperbolic tangent function can indeed be utilized:

$$g(x_t) = \eta^i \tanh(x_t) \tag{2.19}$$

where parameter $\eta^i \in (0, +\infty)$ determines the extension of the output variation limits as a production ceiling and floor and can assume different values for formal and informal firms. The idea is that when the economy is operating with high use of installed capacity, companies find it increasingly dificult to increase production given the costs associated with inputs shortages, increasing labor costs, or the absence of immediately available capital goods. On the other hand, in a regime of low capacity utilization, the existence of a floor is justified by the execution of previously agreed contracts and a share of autonomous production.

The ceiling and floor structure are different between formal and informal sectors. While informal companies operate without capital goods, the formal sector uses its machinery

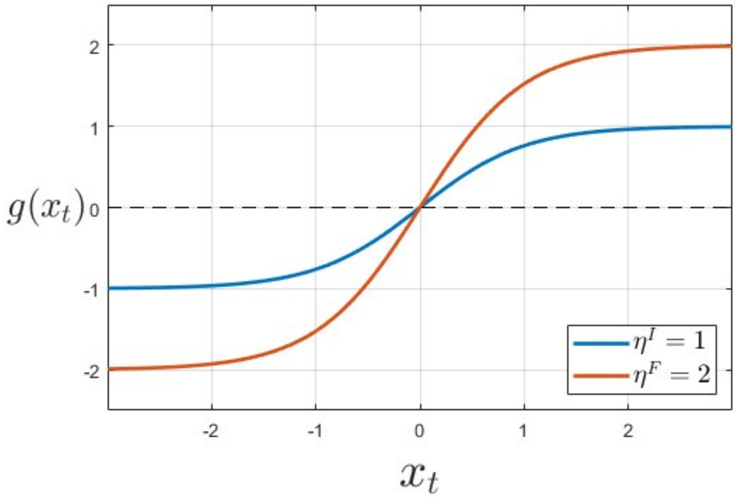

Figure 2.3: A S-shaped specification described by a hyperbolic tangent function with different ceilings and floor specifications.

below maximum capacity to increase production in response to excess demand. Beyond that, formal companies can recruit the labor force from informal businesses in moments of high growth when a production expansion is necessary. The production ceiling is higher in the formal than in the informal sector. At the same time, the informal sector softens part of the drop in output by receiving part of the workforce that has left the formal sector and needs to continue working, justifying the existence of a higher floor in the informal sector. From this perspective, it is possible to define η^F and η^I as the production amplitude for the formal and informal sector respectively with $\eta^F > \eta^I$. Fig. 2.3 shows graphically how a given single input x_t changes the production response $g(x_t)$ for η^F and η^I.

To guide agents' production decisions, it is essential to precisely define what determines the feasible quantity of goods to be produced. Given that this model focuses on short-term interactions, changes in production are solely determined by firms' response to demand. In this case, x_t can be represented by an excess demand E_t measure that can be written as:

$$g(E_t) = \eta^i \tanh(E_t) \tag{2.20}$$

such that E_t is the difference between output Y_t and total demand Z_t:

$$E_t = Z_t - Y_t \tag{2.21}$$

The adjustment in production for the next period is influenced by the deviation between expenditure and production in the current period. If demand falls short of production, indicating negative excess demand, firms may decrease production in response to the imbalance and vice-versa. In a closed economy without government, total demand Z_t is equal to:

$$Z_t = C_t^F + C_t^I + I_t^F \tag{2.22}$$

where C_t^F and C_t^I is the consumption for formal and informal sectors, respectively, and I_t is the formal sector investment. This definition follows the evidence showing that the informal sector faces severe investment constraints (Gandelman and Rasteletti, 2017). A more realistic definition of equation (2.22) could allow for a component I^I. However, for the moment, such a simplification is in line with a stylized dual economy as it significantly simplifies algebra. Therefore, the informal sector is restricted to consumption only. Total expenditures can be defined as a function of Y_t and S_t:

$$Z_t = Z_0 + \underbrace{(\varphi^c Y_t - \psi^c S_t)}_{C_t^F + C_t^I} + \underbrace{(\varphi^k Y_t^F - \psi^k S_t^F)}_{I_t^F} \tag{2.23}$$

$Z_0 > 0$ groups the autonomous demand components of both sectors. Moreover, the parameters φ and ψ capture the intensity at which output and solvency will affect the total expenditure. The parameter $\varphi^c \in (0, 1)$ captures the positive relationship between output and total expenditure. A higher value of φ^c indicates that as Y_t increases, consumption responds through an induced demand channel. In contrast, the parameter $\psi^c \in (0, 1)$ measures the magnitude of the negative impact of solvency on total expenditure. This effect is explained by the fact that an increase in solvency means that agents are not consuming as much as they could. Parameters φ^k and ψ^k capture the induced effects of investment and work similarly to their consumption counterparts. The main difference is that it is solely connected to the formal sector output and solvency, as the informal sector does not make investment expenditures. As a way to guarantee the economic meaning of Eq. (2.23) and avoid negative output, the total induced effects of consumption in investment is defined in the parameters combinations $[(\varphi^c + \varphi^k), (\psi^c + \psi^k)] \in (0, 1)$. By plugging Eq. (2.23) into Eq. (2.21), aggregate excess demand is equal to:

$$E_t = Z_0 + (\varphi^c Y_t - \psi^c S_t) + (\varphi^k Y_t^F - \psi^k S_t^F) - Y_t \tag{2.24}$$

Using Eq. (2.24), it is possible to define total excess demand as a signal that guides firms' production decisions in aggregate. Eq. (2.20) becomes:

$$g^i(E_t) = \eta^i \tanh Z_0 + (\varphi^c Y_t - \psi^c S_t) + (\varphi^k Y_t^F - \psi^k S_t^F) - Y_t \tag{2.25}$$

Indexing Eq. (2.25), we can define the law of motion of output as the sum of the production of formal and informal sectors:

$$\begin{aligned} Y_{t+1} &= Y_{t+1}^F + Y_{t+1}^I && (2.26) \\ &= Y_t^F + \eta^F \tanh(E_t) + Y_t^I + \eta^I \tanh(E_t) && (2.27) \end{aligned}$$

such that the constant participation of the formal sector in the total output for all t is given by $\theta \in [0, 1]$ such as:

$$\theta = \frac{Y^F}{Y} \quad \text{for all } t \tag{2.28}$$

Besides output's response to financial conditions through consumption and investment channels, a second connection between the real and financial sectors is a cash flow mechanism that accounts for endogenous changes in agents' margin of safety. When demand increases, firms experience enhanced financial stability as their cash flow rises. As a result, the desired safety margin μ^i now has an endogenous component and changes following the business cycle dynamics. By using Eq. (2.25) as a measure of cash flow, Eq. (2.14) becomes:

$$L_{t+1}^i = L_t^i + \alpha S_t^i - \mu_0^i + \bar{\mu}\eta^i \tanh Z_0 + \varphi^c Y_t - \psi^c S_t + \varphi^k Y_t^F - \psi^k S_t^F - Y_t \tag{2.29}$$

where the parameter $\bar{\mu} \in (0, 1)$ denotes the extent to which the financial dimension responds to fluctuations in output. As E_t increases, the cash-flow effect increases firms' liquidity as it represents an increase in firm revenue given a high economic activity and vice-versa.

2.3.3 Three-Dimensional Map

To understand the dynamics of the real and financial sides, it is possible to define a three-dimensional map to explain the interdependent movements between the real and financial sides while considering formal and informal sectors. As a first step, it is possible to use Eqs. (2.17) and (2.28) to substitute the variables indexed by [F, I] for its aggregate form weighted by the shares of each sector. Recall that parameters λ and $\theta \in [0,1]$ represent the share of the formal sector in the financial and real side, respectively. Its counterparts $(1-\lambda)$ and $(1-\theta)$ stand as the shares for the informal sector. In that vein, Eqs. (2.13), (2.27), and (2.29) become:

$$
\begin{aligned}
L_{t+1} &= \lambda L_t - \alpha\lambda S_t - \mu_0^F + \bar{\mu}\eta^F \tanh(E_t) + (1-\lambda)L_t - \alpha(1-\lambda)S_t - \mu_0^I + \bar{\mu}\eta^I \tanh(E_t) \\
S_{t+1} &= \beta L_t^F + (1-a\beta)S_t^F + L_t^I \\
Y_{t+1} &= Y_t^F + \eta^F \tanh(E_t) + Y_t^I + \eta^I \tanh(E_t)
\end{aligned}
$$

where

$$E_t = Z_0 + (\varphi^c Y_t - \psi^c S_t) + (\varphi^k \theta Y_t - \psi^k \lambda S_t) - Y_t$$

rearranging, the three-dimensional map M1 can be defined as:

$$
M1: \begin{cases}
L_{t+1} = L_t - \alpha(S_t - (\mu^F + \mu^I) + \bar{\mu}(\eta^F + \eta^I)\tanh E_t) \\
S_{t+1} = \lambda(\beta L_t + (1-a\beta)S_t)\theta + (1-\lambda)L_t \\
Y_{t+1} = Y_t + (\eta^F + \eta^I)\tanh E_t
\end{cases}
$$

such that:

$$E_t = Z_0 + Y_t(\varphi^c + \varphi^k\theta - 1) - S_t(\psi^c + \psi^k\lambda)$$

The interconnections of the economy represented by map M1 can be summarized in the following points:

- Production Decisions: Firms decide their production strategy for the upcoming period based on the existing excess demand. If the production in period t was lower than the demand in the same period, firms increase the production for the period t + 1 and vice-versa.
- Cash Flow and Financial Slack: The output obtained from the production strategy results in changes in the companies' cash flow, affecting their solvency situation and spending capacity.
- Expectations and Spending Decisions: Formal firms project their expectations regarding future cash flows to guide their expenditure decisions. Conversely, the informal sector relies on contemporaneous liquidity due to limited access to formal credit markets.
- Demand for Goods and Services: Based on the solvency situation of the agents described in step (3) and how the effectual output induces expenditure decisions, agents may demand more or less goods and services. As the informal sector spends its income only on consumption goods, the formal sector can also spend on investment goods.

Thus, it is possible to state and the existence and uniqueness of fixed points in the system.

Proposition 2. The map M1 has a unique fixed point (S^{*}, L^{*}, Y^{*}) that satisfies:

$$
\begin{aligned}
S^{*} &= \mu_0^F + \mu_0^I \\
L^{*} &= \frac{(\mu_0^F + \mu_0^I)[1 - (1 - a\beta)\lambda]}{[\beta\lambda + (1 - \lambda)]} \\
Y^{*} &= \frac{Z_0 - (\psi^c + \psi^k\lambda)(\mu_0^F + \mu_0^I)}{[1 - (\varphi^c + \varphi^k\theta)]}
\end{aligned}
$$

Proof. See Mathematical Appendix. ☐

On the financial side, the equilibrium solvency S^{*} is given by the sum of the safety margins $\mu_0^F + \mu_0^I$ that each sector wishes to operate. In that sense, firms' decisions about their safety margins regarding spending decisions will define how solvent the economy will be in the aggregate in a situation of equilibrium. At the same time, equilibrium liquidity depends on safety margins, expectations, and time discount parameters (β, a), and the share of the formal sector in the financial side λ. As the equilibrium solvency is defined exclusively by the safety margins, the share of the formal sector on the financial side affects the equilibrium liquidity as:

$$\frac{\partial L^{*}}{\partial\lambda} = \frac{\beta(\mu_0^F + \mu_0^I)(a - 1)}{[\beta\lambda + (1 - \lambda)]^2} \leq 0 \tag{2.30}$$

in this context, an increase in the share of the formal sector on the financial side results in a negative or neutral impact given the discount factor $a \leq 1$ impacting the safety margins. A discount factor indicates that accessing the financial markets incurs a cost. As a consequence, the equilibrium liquidity cannot increase with a growing formal sector, as the discount factor must pay interest in equilibrium, and for a given $\beta > 1$, an increase in λ will also increase the intensity of this effect. In a zero-lower bound situation, i.e., $r = 0$, the equilibrium liquidity behaves neutrally to changes in the share of the formal sector on the financial side. In equilibrium, part of the liquidity is diverted from firms to the financial sector, which indicates a negative derivative for any situation.

As for the real side, the equilibrium output Y^{*} depends positively on the autonomous demand component Z_0 and negatively on S^{*}, with this relationship mediated by the parameters that induce demand $(\varphi^c, \psi^c, \varphi^k, \psi^k)$ and the share of the formal sector in the financial and real side of the economy (λ, θ). Regarding the role of λ, the equation above describes how the equilibrium output is affected by the share of the formal sector on the financial side of the economy:

$$\frac{\partial Y^{*}}{\partial\lambda} = -\frac{\psi^k(\mu_0^F + \mu_0^I)}{1 - (\varphi^c + \varphi^k\theta)} < 0 \tag{2.31}$$

an increase in the share of the formal sector in the financial side means a decrease in the equilibrium output as it increases the negative wealth effect connected to the safety margins that reduce the demand. For a given equilibrium output, autonomous demand is compensated by a compatible safety margin. If the formal sector grows, the negative effect of ψ^k grows as formal sector investment starts to have greater participation in the output. By definition, $(\varphi^c + \varphi^k) \in (0, 1)$ means that this result is guaranteed for any parameter combination in Y^{*}.

As for θ, we can check its impact on Y^{*} as:

$$\frac{\partial Y^{*}}{\partial\theta} = \frac{\varphi^k(Z_0 - (\psi^c + \lambda\psi^k)(\mu_0^F + \mu_0^I))}{[1 - (\varphi^c + \varphi^k\theta)]^2} \gtreqless 0 \tag{2.32}$$

the share of the formal sector in the real side may increase, decrease, or do not affect the equilibrium output given the difference between the autonomous demand Z_0 and the total wealth effect of the safety margins $(\psi^{c}+\psi^{k})(\mu^{F}_{0}+\mu_{0})$. An increase in θ means an increase in the induced income effect given an increase in investment spending, but it equally affects the parameterized safety margins. In that case, if $Z_{0}-(\psi^{c}+\psi^{k})(\mu^{F}_{0}+\mu^{I}_{0})>0$, the impact is positive and vice-versa (if equal, there are no changes in Y^{*}), with parameter θ also increasing the strength of this effect. Despite the theoretical possibility, the scenario where autonomous demand exceeds the safety margins is more plausible for real economies.

Regarding the unique fixed point of map M1, it is possible to state and prove its local stability properties.

Proposition 3. The fixed point (L^{*}, S^{*}, Y^{*}) is locally asymptotically stable in the following region of the parameter space:

i) $(\eta^{F}+\eta^{I})\{\lambda[\alpha(\beta(1-\varphi^{c}-\varphi^{k}\theta)+\varphi^{c}+\varphi^{k}\theta-1)]+\alpha(1-\varphi^{c}-\varphi^{k}\theta\}>0$
ii) $A_{1}+\lambda[2\alpha(\beta-1)-4a\beta+4]+2\alpha+4>0$
iii) $[(\eta^{F}+\eta^{I})(\varphi^{c}+\varphi^{k}\theta-1)-\alpha(a\beta-1)+2](B_{1}+\alpha)+B2-(B_{1}+2)^{2}>0$

such that

$$A_{1}=(\eta^{F}+\eta^{I})\{\lambda^{2}[2\alpha\bar{\mu}\psi^{k}(1-\beta)]+\lambda[\alpha(\beta(\varphi^{c}+\varphi^{k}\theta-2\bar{\mu}\psi^{c}-1)-\varphi^{c}-\varphi^{k}\theta+2\bar{\mu}(\psi^{c}-\psi^{k})+1)+a\beta(2-2\varphi^{c}-2\varphi^{k}\theta)+2\varphi^{c}+2\varphi^{k}\theta-1]+\alpha(\varphi^{c}+\varphi^{k}\theta-2\bar{\mu}\psi^{c}-1)+2\varphi^{c}+2\varphi^{k}\theta-2\}$$

$$B_{1}=\{(\eta^{F}+\eta^{I})\{\lambda^{2}[\alpha\bar{\mu}\psi^{k}(1-\beta)]+\lambda[\alpha(\beta(\varphi^{c}+\varphi^{k}\theta-\bar{\mu}\psi^{c}-1)+1-\varphi^{c}+\bar{\mu}(\psi^{c}-\psi^{k})-\psi^{k}\theta)+a\beta(1-\varphi^{c}-\varphi^{k}\theta)+\varphi^{c}+\varphi^{k}\theta-1]+\alpha[\varphi^{c}+\bar{\mu}(\psi^{k}\theta-\psi^{c})-1]\}+\lambda[\beta(\alpha-a)-\alpha+1)$$

$$B_{2}=(\eta^{F}+\eta^{I})\{\lambda^{2}[\alpha\bar{\mu}\psi^{k}(\beta-1)]+\lambda[\alpha\bar{\mu}(\psi^{c}+\psi^{k}+\psi^{c}\beta)+a\beta(\varphi^{c}+\varphi^{k}\theta-1)+1-\varphi^{k}\theta]+1+\varphi^{c}-\varphi^{k}\theta-\alpha\bar{\mu}\psi^{c}\}\lambda[\beta(2a-\alpha)+\alpha-2]-\alpha$$

Proof. See Mathematical Appendix. $\square$

The violation of condition (i) only is associated with an unstable system following a fold bifurcation. Given the model's parameter space, this condition cannot be violated. On the other hand, the violation of conditions (ii) and (iii) is possible. By violating condition (ii) only, the fixed point (L^{*}, S^{*}, Y^{*}) loses stability following a Flip bifurcation. In this situation, the system may present a typical unstable behavior characterized by the emergence of a stable period-2 cycle as the new limit is set. It is also possible that the period-2 cycle loses its stability as the inequality condition are increasingly violated, generating a scenario of cascade bifurcations that typically leads to chaotic limit sets for a given interval in the parameter space. On the other hand, a violation of condition (iii) only leads to instability that follows a Neimark-Sacker bifurcation. In that situation, the fixed point loses its stability with the possibility of the existence of an equilibrium set, given the coexistence of an attracting invariant closed curve with the unstable fixed point. In this case, the orbits can be seen as points moving around the curve periodically or quasi-periodically (Lines et al., 2020). Nevertheless, when both condition (ii) and condition (iii) are violated, the system can display complex and unpredictable behavior, often involving a mixture of stable and unstable dynamics, periodic orbits, and possibly chaotic regions.

2.3.4 Numerical Exercises

The local stability analysis of the fixed point of map M1 is quite challenging, especially given that we are dealing with a reasonably nonlinear model involving many parameters. While Proposition 3 indicates if the model is compatible with endogenous business cycles, it

Financial		Real	
Parameter	Value	Parameter	Value
T	1	μ	0.5
r	0.05	Z_0	55
ρ^e	0.5	φ^c	0.4
ρ^k	0.5	ψ^c	0.4
ω	0.5	φ^k	0.2
α	0.5	ψ^k	0.2
μ_0^F	5	η^F	1.5
μ_0^I	5	η^I	1
λ	0.5	θ	0.5

Table 2.1: Benchmark parameters values for numerical simulations.

is necessary to provide more economic insights. To better understand the model's behavior and economic implications, assigning specific values to a set of parameters and observing the resulting variations in another set of parameters of interest is beneficial. As a way of illustrating the stability conditions numerically, it is possible to define a benchmark set of parameter values according to Table 1

For the reaction speed parameters between variables such as α, μ, ρ^e, and ρ^r, a closed value between 0 and 1 was chosen to avoid explosive trajectories. For the ceiling and floor values, the formal sector has a capacity 50% higher than the informal sector, where the economy's total capacity to change production in each period is up to 2.5% following the average changes in GDP for world economy (International Monetary Fund, 2021). Parameter T is a measure of time of expectations that was normalized to one, and the interest rate accounts for 5% as an average value between advanced and developing economies. An average value in the parametric space was chosen for most parameters to avoid results compatible with extreme cases only. The equilibrium output was defined as an index with $Y^{*} = 100$, with Z_0 following a % of GDP value with other equilibrium variables following Y^{*}.

Figure 2.4 illustrates the stability conditions defined in α as a function of the parameters λ and θ for a situation where all other parameters follow their benchmark values. The parameter α was selected as a dependent variable in this exercise because it is sensitive and can easily present changes in stability conditions given variations in λ and θ. An increase in the share of the formal sector on the financial side only means an increase in the possibility of the economy's present instability regarding the stability condition that leads to a Neimark-Sacker bifurcation. In that case, an increase in the reaction speed of the gap between solvency and the safety margins drives instability, and this effect is amplified as the share of firms accessing the financial markets grows. Liquidity becomes more unstable when agents try to change their financial position faster, and this effect is amplified via expectation mechanisms affecting solvency. On the other hand, an increase in the share of the formal sector on the real side does not affect the boundary regarding a Neimark-Sacker bifurcation but reduces the chance of a Flip bifurcation to occur.

By considering the situation when the share of agents with extrapolative expectations is higher when $\omega = 0.75$ while other parameters follow their benchmark values, fig. 2.5 shows that an increase in the share of the formal sector on the financial side will also increase the probability of violating conditions (ii) and (iii) simultaneously. As agents with extrapolative expectations become dominant, the response of formal firms' solvency to variations in the liquidity will increase, generating a second instability channel as the effect present in Fig. 2.4 is stills operating. For the share of the formal sector on the real side, an increase in θ means a reduction in the possibility of violating condition (ii). As

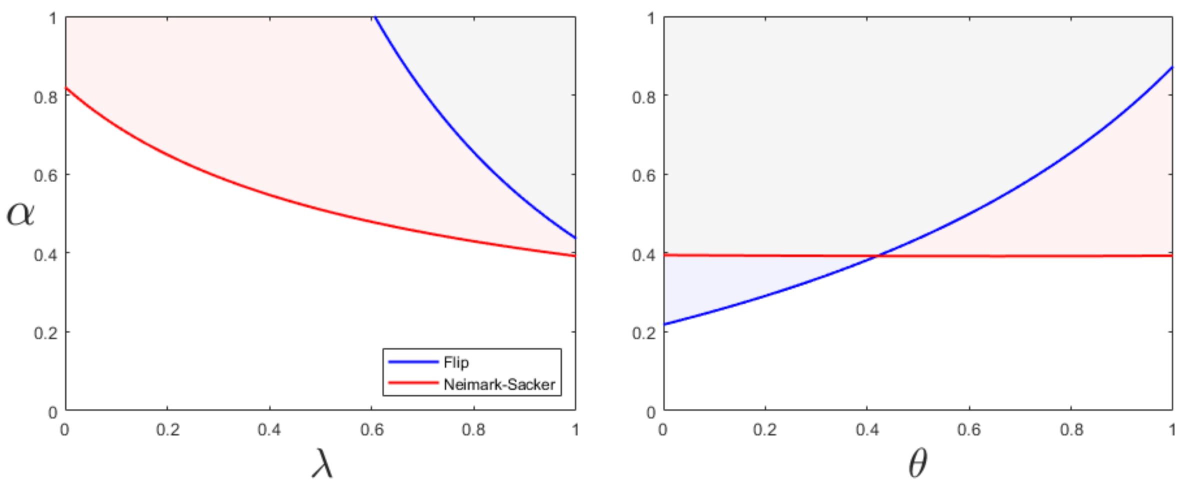

Figure 2.4: Stability boundaries for map M for benchmark values and ω = 0.75. The blue area represents an unstable region characterized by violating the stability condition (ii).

θ increases, the income effect will increase and boost the output. This movement relieves the gap between solvency and safety margins via a cash-flow effect.

On the other hand, when agents with regressive expectations dominate such that ω = 0.25 and all other parameters follow their benchmark values, the possibility of violating condition (ii) is not present anymore as can be seen in Fig. 2.6. At the same time, the bifurcation boundaries that mark the possibility of violating condition (iii) remain similar to the first case with minor changes in frontier values. As agents behave as fundamentalists, the effect of expectations over liquidity is the opposite of the last case. In that sense, the expectations, in this situation, diminish the chance of instability.

An increase in λ and θ has different and even antagonistic effects depending on the studied stability condition and other parameter values. To understand the overall result of these changes, Fig. 2.7 presents a singular bifurcation diagram assuming λ = θ. In this specific case, the participation of the real sector in the economy will be the same for both the financial and real sides. This way, it is possible to understand the result of an increased presence of the formal sector in the economy.

By setting $\mu_0^F = \mu_0^I = 0$ as a way of making the comparison more precise, on the financial side, an increase in the share of the formal sector in the economy, for the numerical values computed, generates an unstable behavior after a Neimark-Sacker bifurcation. This result shows a change in the pattern of instability given the increasing participation of the expectations variables accompanying increments in the λ parameter. While a more significant presence of the informal sector is characterized by stability with L = S, increases in the share of the formal sector lead the economy to exhibit endogenous fluctuations while creating a growing mismatch between solvency and liquidity.

On the real side, an increase in the share of the formal sector leads to a trade-off between a higher output level and increasing instability. As the formal sector share increases, more firms can invest by accessing external financing, thus ensuring a greater equilibrium GDP. At the same time, the destabilizing effects arising from expectation channels become more robust as more firms access financial markets, increasing the chance of the economy facing episodes of instability transmitted from the financial to the real side.

The results presented in this section corroborate the that informality can generate qualitatively different dynamics. It was possible to demonstrate, analytically and numerically, that informality changes both the fixed point of the system and its stability conditions. In this case, economies with a higher level of formalization, in equilibrium, tend to have

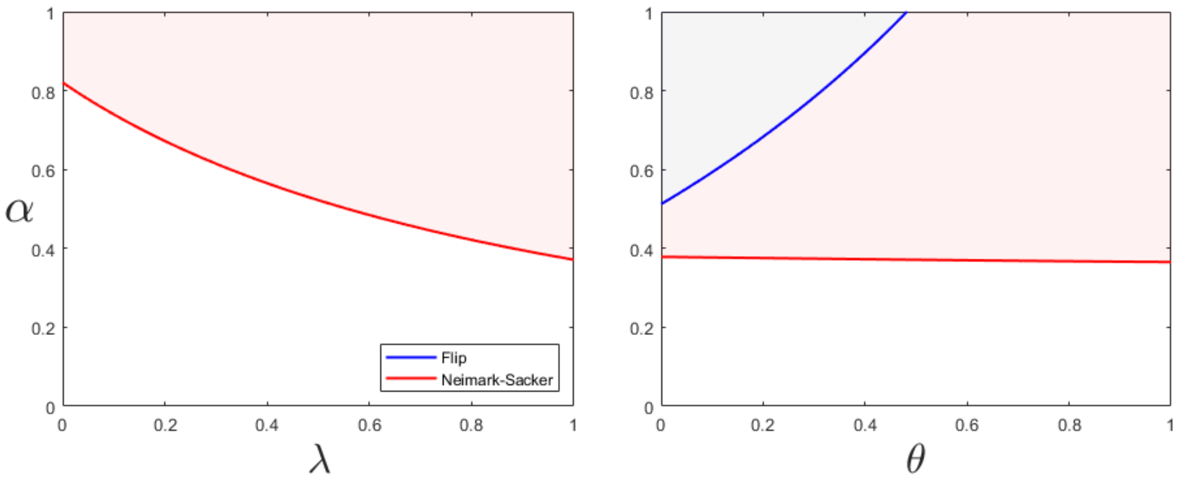

Figure 2.5: Stability boundaries for map M for benchmark values. The white area corresponds to a region of system stability. The red area represents an unstable region characterized by violating the stability condition (ii). The grey area represents a violation of conditions (ii) and (iii) simultaneously.

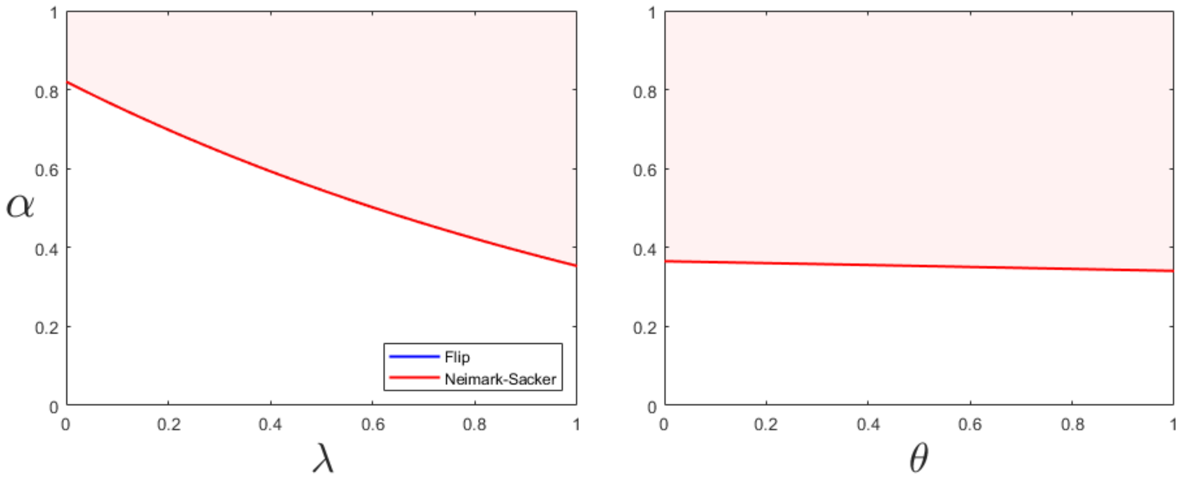

Figure 2.6: Stability boundaries for map M for benchmark values and ω = 0.25.

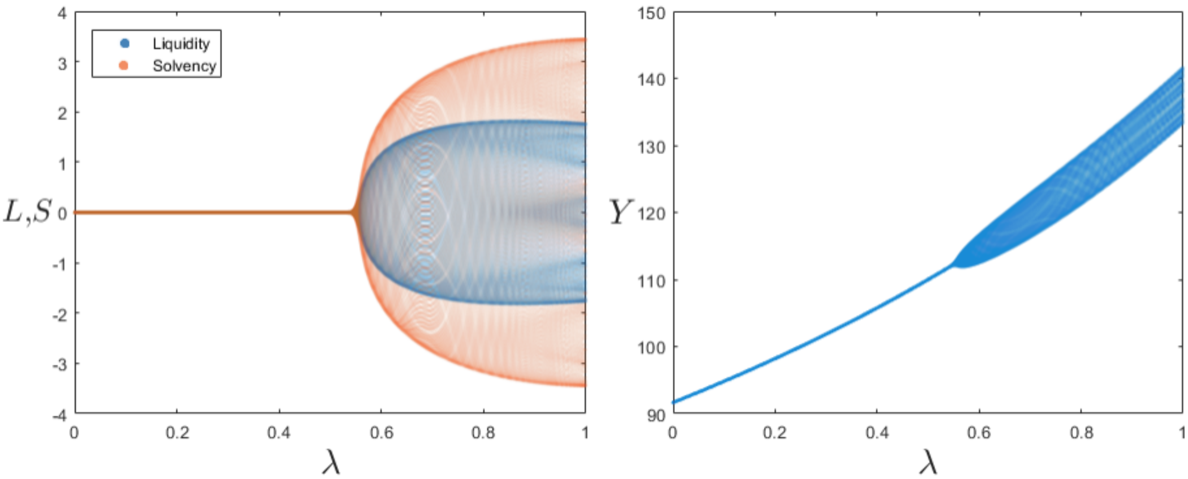

Figure 2.7: Bifurcation diagrams for Liquidity and Solvency on the left and output on the right regarding variations of λ with $\theta = \lambda$ with $\mu_0^F = \mu_0^I = 0$.

higher output with a greater chance of going through episodes of instability since the number of firms operating in the financial sector with access to the credit market is more significant, increasing thus the possibility of transmitting instabilities generated by agents' expectations to the economy as a whole.

2.4 An Extension with Endogenous Heterogeneity

So far, the population share divided between extrapolators/fundamentalists was assumed to be exogenous. Analogously, the composition of formal/informal firms also followed the same strategy. As a way of extending the results of the benchmark model, it is possible to add dynamics to the heterogeneity previously presented statically. In this case, this section aims to add changes regarding the expectation formation mechanism and the shares of formal and informal sectors in the real and financial sides of the economy. A logit-type mechanism that follows the seminal works of Manski and McFadden (1981) and Anderson et al. (1992) will be used to guide such changes.

As described by Franke and Westerhoff (2017), that kind of mechanism allows for changes in a variable periodically given a probability ζ to changes in the direction of the state A and $(1 - \zeta)$ the probability of changes regarding the state B. The probabilities vary in time according to x^+_t and x^-_t. By using a general form of the mechanism defined in Brock and Hommes (1997), ζ will change according to:

$$\zeta(x_t^+, x_t^-) = \frac{\exp\{\gamma x_t^+\}}{\exp\{\gamma x_t^+\} + \exp\{\gamma x_t^-\}} \tag{2.33}$$

where the parameter $\gamma \in [0, \infty)$ captures the intensity of change between the two states given the pair (x_t^+, x_t^-). The probability will be nearly equal for values of γ closer to 0. When $\gamma \to \infty$, the value of ω_t tends to 0 or 1, meaning that state A or B will be absolute.

2.4.1 Endogenous Expectations

The first application of the mechanism presented is a way to make endogenous changes in the heuristics that guide the expectation formation processes of the agents. This is a recurrent strategy employed in heterogeneous agent models to capture different expectation formation processes that change endogenously. This strategy followed Brock and Hommes

(1997) and was posteriorly applied in several papers interested in understanding the dynamics of real-financial markets as Naimzada and Pireddu (2015), Dieci and Westerhoff (2016), Agliari et al. (2018), Cafferata et al. (2021) and Cavalli et al. (2022). The central idea is that agents use some perception of the economy's performance to choose between two different heuristics.

By recovering the share of extrapolators captured by the parameter ω presented in Eq. (2.4), it is possible to transform it by using Eq. (2.33):

$$\omega(x_t^e, x_t^r) = \frac{\exp\{\gamma^\omega x_t^e\}}{\exp\{\gamma^\omega x_t^e\} + \exp\{\gamma^\omega x_t^r\}} \tag{2.34}$$

parameter ω now is a function of x_t^e and x_t^r that represent a variables that can affect the heuristic choice of agents. The parameter $\gamma^\omega \in [0, \infty)$ captures the intensity of choice between the mood states. In this scenario, agents can extrapolate the prevailing trend or opt for a less optimistic approach by changing their expectation given the inputs they receive from the pair (x_t^e, x_t^r).

For this model, agents will observe output variations and shape their expectations by using them to understand the state of the economy:

$$x_t^e = \Delta Y$$
$$x_t^r = -\Delta Y$$

for moments where $\Delta Y > 0$, extrapolative expectations dominate, and we observe that $\omega(x_t^e, x_t^r)$ belongs to the interval $(0.5, 1]$. In this situation, agents become more optimistic regarding the future and extrapolate the actual trend (Boeck and Zörner, 2023). Conversely, when $\Delta Y < 0$, regressive expectations dominate and $\omega(x_t^e, x_t^r)$ belongs to the interval $[0, 0.5)$ with more agents assuming a pessimistic behavior. In the steady state, $\Delta Y = 0$ as $Y_{t+1} = Y_t$, and both heuristics are equally distributed, meaning that agents are not incentivized to change their behavior. The variation ΔY can be found by simply taking the finite difference of the parameterized version of Eq. (2.27):

$$\Delta Y = (\eta^F + \eta^I)\tanh[Z_0 + Y_t(\varphi^c + \varphi^k\theta - 1) - S_t(\psi^c + \psi^k\lambda) \tag{2.35}$$

By substituting Eq. (2.35) in the combination of Eqs. (2.6) and (2.34), we have that:

$$\rho_t^{er} = \frac{\rho^e \exp[\gamma^\omega g(E_t)]^2 - \rho^r}{1 + \exp[\gamma^\omega g(E_t)]^2} \tag{2.36}$$

Finally, after substituting Eq. (2.36) in Eq. (2.10), we finally arrive at:

$$\beta_t = (1 + \rho_t^{er}) \frac{(1 + r)^{T+1} - (1 + \rho_t^{er})^{T+1}}{(1 + r)^T (r - \rho_t^{er})} \tag{2.37}$$

Parameter β now depends on the business cycle situation, given the endogenous changes in the composition of the heuristics that guide expectations.

2.4.2 Endogenous Informality

At the same time, it is possible to use a similar mechanism to understand how allowing for endogenous changes in the share of the formal sector affects the dynamics of the model. The formal sector captures part of the informal sector's resources during economic expansion and vice versa. The idea is that the share of the formal sector acts in a pro-cyclical way, covering the stylized fact that points to the ability of the formal sector to expand production by using the resources of the informal sector. On the other hand, the informal sector can also smooth the drop in employment and income in downturn moments (Loayza and Rigolini, 2011; Colombo et al., 2016).

By recovering Eqs. (2.17) and (2.28), we can rewrite the shares of the formal sector in the financial and real sides as endogenous variables that follow a switch mechanism as in Eq. (2.33). In that sense, we have that:

$$\lambda_t = \frac{\exp\{\gamma^{\lambda} x_t^{+l}\}}{\exp\{\gamma^{\lambda} x_t^{+l}\} + \exp\{\gamma^{\lambda} x_t^{-l}\}} \tag{2.38}$$

$$\theta_t = \frac{\exp\{\gamma^{\theta} x_t^{+y}\}}{\exp\{\gamma^{\theta} x_t^{+y}\} + \exp\{\gamma^{\theta} x_t^{-y}\}} \tag{2.39}$$

the parameters γ^{λ} and γ^{θ} will capture the intensity of change in the composition of shares.

For the case of the financial side described by Eq. (2.38), the participation of the formal sector in the financial side will depend on the variation of liquidity, which indicates that, to the extent that financial obligations are not met, firms are excluded from accessing the credit markets, assuming a similar position as informal firms regarding access to external financing. Analogously to the procedure done for the expectations switch in Eq. (2.36), we have that:

$$x_t^{+l} = \Delta L$$
$$x_t^{-l} = -\Delta L$$

with ΔL being the variation rate of L given a finite difference. For moments where $\Delta L > 0$, more firms can access the financial markets by having more available liquidity, and λ_t belongs to the interval $(0.5, 1]$. At the same time, when $\Delta L < 0$, more firms become unable to fulfill their financial commitments and become excluded from the financial markets, and λ_t belongs to the interval $[0, 0.5)$. In the steady state, as $L_t = L_{t+1}$, and the composition is equally distributed. We can obtain the variation of liquidity by taking the finite difference of the parameterized version of Eq. (2.16):

$$\Delta L = -\alpha\{S_t - (\mu_0^F + \mu_0^I) + \bar{\mu}(\eta^F + \eta^I)\tanh[Z_0 + Y_t(\varphi^c + \varphi^k\theta_t - 1) - S_t(\psi^c + \psi^k\lambda_t)] \tag{2.40}$$

By combining Eqs. (2.40) and (2.38) we get:

$$\lambda_t = \frac{\exp\{\gamma^{\lambda}\Delta L\}}{\exp\{\gamma^{\lambda}\Delta L\} + \exp\{\gamma^{\lambda}(-\Delta L)\}} \tag{2.41}$$

For the case of the variable θ_t, the procedure is similar to that used to obtain Eq. (2.41). In this case, the share of the formal sector on the real side will change accordingly to movements in the variation of output:

$$x_t^{+y} = \Delta Y$$
$$x_t^{-y} = -\Delta Y$$

for moments where $\Delta Y > 0$, formal firms start to capture resources from the informal sector to expand production, and θ_t belongs to the interval $(0.5, 1]$. On the other hand, when $\Delta Y < 0$, formal firms slow down their pace of production, given resources to the informal sector. In that sense, θ_t belongs to the interval $[0, 0.5)$. In the steady state, as $Y_t = Y_{t+1}$, and the share of formal and informal firms is equivalent. After recovering Eq. (2.35), we can rewrite Eq. (2.39) as:

$$\theta_t = \frac{\exp\{\gamma^{\theta} g(E_t)\}}{\exp\{\gamma^{\theta} g(E_t)\} + \exp\{\gamma^{\theta}(-g(E_t))\}} \tag{2.42}$$

2.4.3 Three-Dimensional Map

By using Eqs. (2.37), (2.41), and (2.42), we can rewrite the map M1 with endogenous changes in the expectation formation processes of the formal sector (ω_t) and also in the shares of formal and informal sectors in the financial (λ_t) and real (θ_t) sides of the economy:

$$M2: \begin{cases} L_{t+1} &= L_t - \alpha(S_t - (\mu_0^F + \mu_0^I) + \bar{\mu}(\eta^F + \eta^I)\tanh E_t) \\ S_{t+1} &= \lambda_t(\beta_t L_t + (1 - a\beta_t)S_t) + (1 - \lambda_t)L_t \\ Y_{t+1} &= Y_t + (\eta^F + \eta^I)\tanh E_t \end{cases}$$

such that:

$$E_t = Z_0 + Y_t(\varphi^c + \varphi^k\theta_t - 1) - S_t(\psi^c + \psi^k\lambda_t)$$

Proposition 4. The Map M2 has the same single fixed point (L^*, S^*, Y^*) of Map M1 that satisfies Proposition (2). It is locally asymptotically stable when conditions (ii) and (iii) defined in Proposition (3) hold. Changes in parameters that determine the violation of condition (ii) lead to a Flip bifurcation. A condition (iii) violation is associated with a Neimark-Sacker bifurcation, as in Map M1.

Proof. See Mathematical Appendix. □

2.4.4 Numerical Exercises

The previous section presented a numerical exercise to illustrate the role of sectoral shares in defining the equilibrium of each dynamic equation and how shares affect stability conditions. As seen in Proposition (4), the M2 map shows the same unique fixed point and stability conditions analogous to the M1 map, meaning the analysis in section 3 is still valid. On the other hand, the M2 map has particularities regarding the system's behavior when the local stability conditions are not fulfilled.

Thus, I perform a exercise where the parameters will follow the exact calibration reported in Table 1, except for alpha, which will be explored in different contexts. The intensity parameters γ^ω, γ^λ, and γ^θ have unity as a benchmark value as estimated by Kukacka and Sacht (2023).[4] Now that the parameters that capture sectoral participation behave endogenously, a possible exercise is to check how the presence of the switchers affects the dynamics of the model in an unstable situation. Following the stability analysis in section 3, Fig. 2.8 reports a bifurcation diagram taking α as the critical parameter.

Fig. 2.8, in Column (a), corresponds to the scenario $(\gamma^\rho, \gamma^\lambda, \gamma^\theta) = (0, 0, 0)$ so that the system behaves as in the map M1. On the other hand, Column (b) presents the variables with endogenous dynamics with an intensity of change $(\gamma^\rho, \gamma^\lambda, \gamma^\theta) = (1, 1, 1)$. In Column (a), it is possible to perceive an unstable behavior that follows a Neimark-Sacker bifurcation, as pointed out in the study of the stability conditions presented in section 3. However, in Column (b), we see that with the endogenous variations provided by the switches described in Eqs. (2.36), (2.41), and (2.42), the system presents a different dynamic and possibly compatible with chaotic motion.

The presence of endogenous shares affects the dynamics of the model. It is possible to understand how the endogenous shares γ_t, λ_t, and θ_t are connected. As a way to better understand the effect of each switch on the dynamics of the model, Fig. 2.9 presents two versions of the phase space for the M2 map, with graph (a) the model only under the effect of expectations switching and graph (b) only the effect of informality switching.

With endogenous changes in the heuristics that form agents' expectations, extrapolators become more numerous during upswings in the economic cycle, leading to more agents

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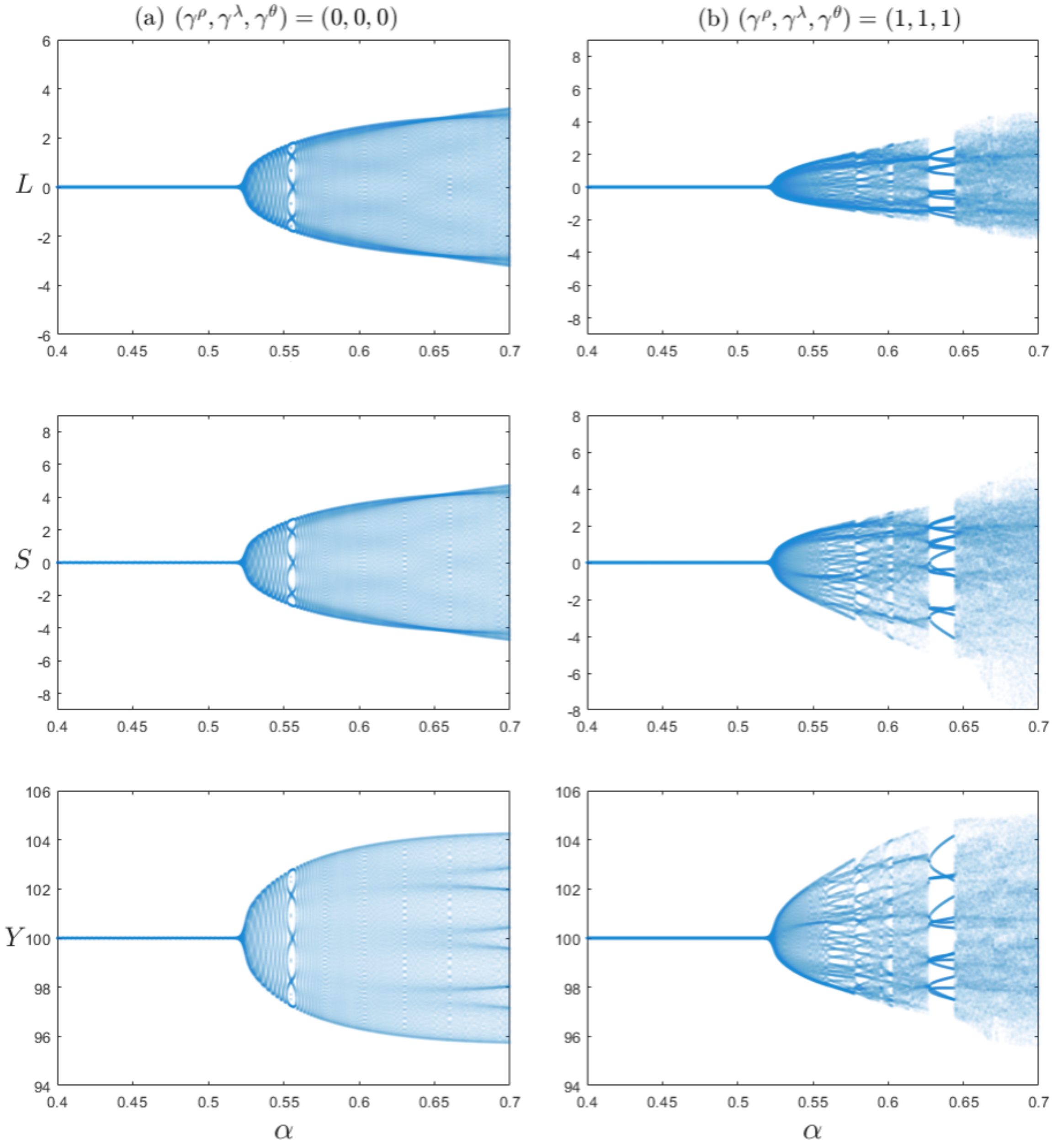
[4] even though this empirical study estimates the intensity of choice between the heuristics only, as a way of unifying the analysis, I will assume the same value for all parameters of intensity of the switchers as a starting point.



(a) $(\gamma^{\rho}, \gamma^{\lambda}, \gamma^{\theta}) = (0, 0, 0)$

(b) $(\gamma^{\rho}, \gamma^{\lambda}, \gamma^{\theta}) = (1, 1, 1)$

Figure 2.8: Bifurcation diagrams for (L, S, Y) regarding variations in α. Graphs (a) and (b) differ for different fixed intensity parameters. T

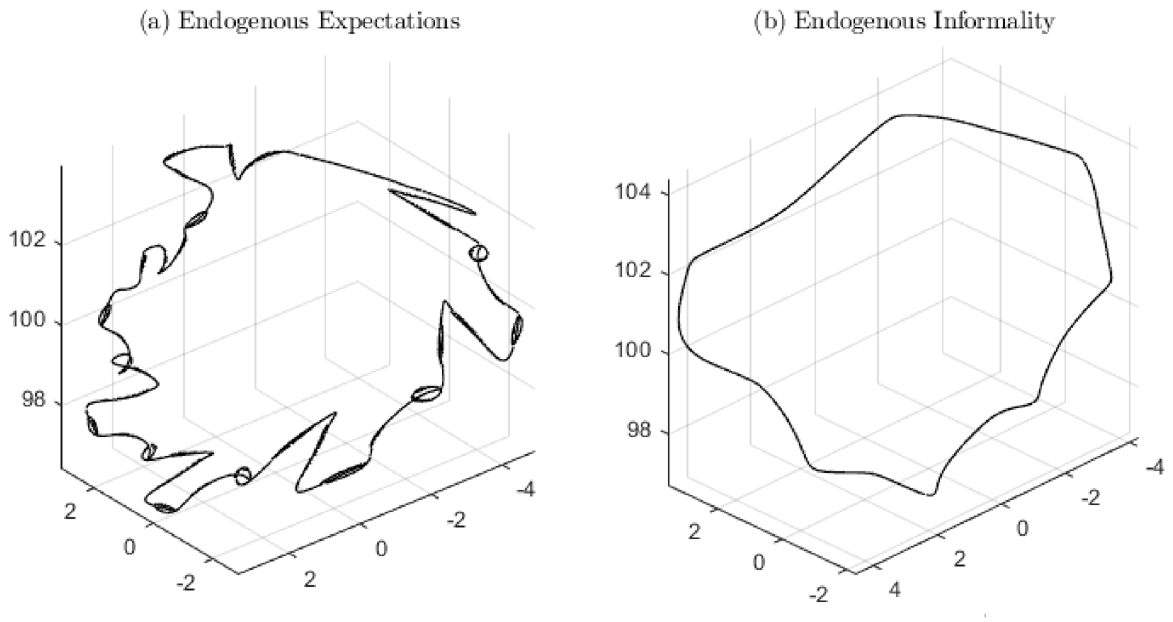


Figure 2.9: Phase plots for map M2 with (a) as $(\gamma^{\rho}, \gamma^{\lambda}, \gamma^{\theta}) = (1, 0, 0)$ and (b) as $(\gamma^{\rho}, \gamma^{\lambda}, \gamma^{\theta}) = (0, 1, 1)$

expecting more significant liquidity flows in the future. In this way, spending decisions and prospects for future solvency are affected. At the same time, in periods of downturn, most agents become fundamentalists and expect liquidity to return to fundamental value. These movements accentuate the expansion of spending in periods of euphoria, leading the economy to depend more on future results to meet its financial obligations. On the other hand, in moments of downturn, agents drag down their spending and production decisions even more as they expect a return to the baseline scenario.

When the participation of sectors changes endogenously, the capacity of the formal sector to capture resources from the informal sector is expanded in periods of growth, which allows for increasing production and also an increase in spending as more firms can access financial markets. On the other hand, output has its capacity for growth reduced as informal firms become dominant in periods of downturn in economic activity. At the same time, the number of firms accessing the credit market is reduced, which also reduces the effect of expectations on economic activity.

By considering both effects at the same time, the Graph (b) in Fig. 2.10 shows the phase portrait for the map M2 as $(\gamma^{\rho}, \gamma^{\lambda}, \gamma^{\theta}) = (1, 1, 1)$. In that case, the system exhibit a chaotic behavior characterized by a "cloud" of different values for each variable in the phase space (L, S, Y). It is also possible to note that the difference between Graph (b) in Fig. 2.9 and Graph (a) in Fig. 2.10 is not expressive, which points to possible effects of the endogenous informality that amplifies the instability generated by the endogenous expectations. In periods of expansion of economic activity, while endogenous expectations increase agents' projections regarding future liquidity, it also increases the share of formal firms, which ensures that more agents access the financial markets and discount future flows of liquidity as an extrapolator. At the same time, the production input also grows with increasing demand, leading to increased production and cash flow. Thus, the interaction of both mechanisms leads to an even more unstable system behavior as they amplify each other.

This result shows that the interaction between the two switchers can generate chaotic dynamics, while, individually, the switchers change the dynamics without generating a chaotic attractor. In this situation, the model becomes sensitive to the initial conditions,

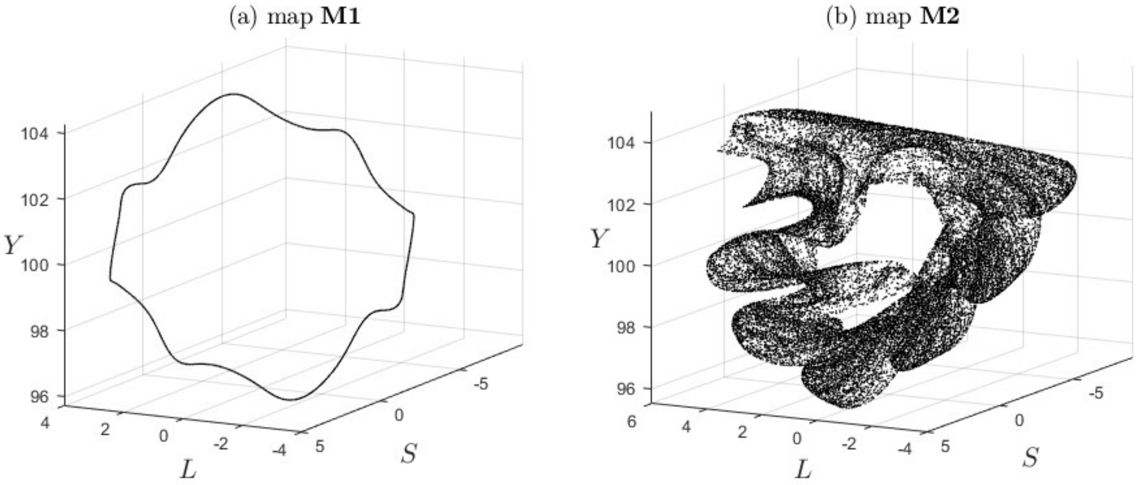


Figure 2.10: Phase plot for map M1 in Graph (a) and M2 with $(\gamma^{\rho}, \gamma^{\lambda}, \gamma^{\theta}) = (1, 1, 1)$ in Graph (b)

i.e., a minimal change in the initial values can lead to entirely different trajectories. In this scenario, fluctuations become aperiodic, and the system's behavior becomes highly unpredictable.

In a general and concise way, the steps below summarize how each switch affects the dynamics of the model:

- Endogenous Expectations: Firms' expectations about future liquidity flows become endogenous. Formal firms adjust their expectations based on the state of the economy, becoming more optimistic during ascending phases of the cycle and vice versa. In that sense, extrapolating the trend in periods of expansion becomes more aggressive as firms experience increased liquidity via the cash flow effect. At the same time, in periods of downturn, firms become more pessimistic and expect a return to fundamental liquidity.
- Endogenous Financial Informality: Increased liquidity allows more firms to access credit markets, leading to more firms projecting their expectations over future liquidity flows. This effect creates fluctuations on the credit demand that can increase the possibility of a liquidity mismatch as companies loses the access to external credit as aggregate liquidity goes down.
- Endogenous Real Informality: In a growth phase of the business cycle, formal firms can increasingly capture resources from the informal sector as the share of formal firms increases. This expansion affects both the production and spending capacity of the economy since formal firms can engage in investment expenditures reflected by parameters φ^k and ψ^k, affecting the production decisions via excess demand.

The results presented in this section complement the results obtained in the last section by showing that allowing for endogenous heterogeneity can lead to complex dynamics. As the last section shows the equilibrium points and the boundaries that lead to instability, this section shows how the model already in a situation of instability can present cyclical and even chaotic dynamics given the feedback effects generated by the endogenous expectations and informality.

2.5 Concluding Remarks

I explored the crucial role of informality in shaping the dynamics of economies, particularly in the context of structural high informality. The research findings reveal that economies characterized by a high level of informality may exhibit distinct unstable dynamics compared to those dominated by formal firms. Incorporating informality into macroeconomic models has highlighted the trade-offs between higher output levels and financial instability. By developing a pioneering heterogeneous agents model, I demonstrated how the inter-play between formal and informal firms and endogenous expectations influences macroeconomic outcomes. The results indicate that changes in the share of the informal sector have discernible effects on the equilibrium point of the system and its stability conditions. Furthermore, the extended model, which endogenizes agents' expectations and informality, emphasizes the amplification effect generated by the increase in production and demand on the real side and the number of firms accessing financial markets on the financial side. The research provides valuable insights into the empirical debate surrounding the im-pact of informality on macroeconomic volatility. The contradictory findings in the existing literature are reconciled by acknowledging the structural differences and feedback effects arising from informality. This paper contributes to the literature by exploring an essential and uncharted area of investigation. It demonstrates the significance of understanding the dynamics of economies characterized by high levels of informality. An available avenue for future work is to move toward empirical validation of the model. To achieve this, a crucial step is to ensure that the mechanisms that make informality endogenous allow for a share different from 0.5 in the steady-state. Furthermore, endogenizing safety and productivity margins can reveal interesting dynamics for studying balance and stability.

Informality is not merely an ancillary feature of developing economies; rather, it constitutes a fundamental structural characteristic that significantly impacts the equilibrium and instability boundaries. Policymakers and researchers must consider these unique dynamics when formulating strategies to mitigate systemic risk and promote economic stability in economies with high informality levels. By better understanding the complex relationship between informality, expectations, and financial interactions, we can foster more effective and tailored policies to navigate the challenges posed by financial fluctuations and promote sustainable economic growth.

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Mathematical Appendix

Proof of Proposition 1

In the Steady-state, we can consider that $L_t = L_{t+1}$, $S_t = S_{t+1}$ and $Y_t = Y_{t+1}$.The dynamic system can be seen as:

$$L = L - \alpha \left\{ S - (\mu_0^F + \mu_0^I) + \bar{\mu}(\eta^F + \eta^I)\tanh[Z_0 + Y(\varphi^c + \varphi^k\theta - 1) - S(\psi^c + \psi^k\lambda)] \right\}$$

$$S = \lambda[\beta L + (1 - a\beta)S] + (1 - \lambda)L$$

$$Y = Y + (\eta^F + \eta^I)\tanh[Z_0 + Y(\varphi^c + \varphi^k\theta - 1) - S(\psi^c + \psi^k\lambda)]$$

In equilibrium, we have that $Y_t = Y_{t+1}$, then $\eta\tanh E_t = 0$. By Proposition (1), we know that $g(0) = 0 = \eta\tanh 0$, so it is possible to find the equilibrium output by considering $E_t = 0$:

$$Y^* = \frac{Z_0 - S(\psi^c + \psi^k\lambda)}{1 - (\varphi^c + \varphi^k\theta)}$$

with $\tanh E_t = 0$, the first equation reduces to:

$$L = L - \alpha[S - (\mu_0^F + \mu_0^I)]$$

and the equilibrium solvency is defined by:

$$S^* = \mu_0^F + \mu_0^I$$

we can find the equilibrium liquidity after manipulating the equation 2:

$$L^* = \frac{S^*[1 - (1 - a\beta)\lambda]}{\beta\lambda + (1 - \lambda)}$$

after substituting S for S^* in Y^* and L^*, it follows that there is a unique fixed point (L^*, S^*, Y^*) defined by:

$$S^* = \mu_0^F + \mu_0^I$$

$$L^* = \frac{(\mu_0^F + \mu_0^I)[1 - (1 - a\beta)\lambda]}{\beta\lambda + (1 - \lambda)}$$

$$Y^* = \frac{Z_0 - (\mu_0^F + \mu_0^I)(\psi^c + \psi^k\lambda)}{1 - (\varphi^c + \varphi^k\theta)}$$

Proof of Proposition 2

The Jacobian matrix of the map (M), evaluated at the equilibrium, is given by:

$$J^E = \begin{bmatrix} 1 & \alpha(\psi^c + \psi^k\lambda)\bar{\mu}(\eta^F + \eta^I) - 1 & -\alpha\bar{\mu}(\eta^F + \eta^I)(\varphi^c + \varphi^k\theta - 1) \\ \beta\lambda + (1 - \lambda) & (1 - a\beta)\lambda & 0 \\ 0 & -(\psi^c + \psi^k\lambda)(\eta^F + \eta^I) & 1 + (\eta^F + \eta^I)(\varphi^c + \varphi^k\theta - 1) \end{bmatrix}$$

with the following characteristic equation:

$$\delta^3 + C_1\delta^2 + C_2\delta + C_3 = 0$$

The four conditions below can be used to check system stability (Lines et al., 2020):

i) $1 + C_1 + C_2 + C_3 > 0$

ii) $1 - C_1 + C_2 - C_3 > 0$

iii) $1 - C_2 + C_1C_3 - C_3^2 > 0$

where:

$$C_1 = -tr(J) = \lambda(a\beta - 1) - (\eta^F + \eta^I)((\varphi^c + \varphi^k\theta - 1) - 2$$

$$C_2 = \sum_{i=1}^{3} M_{i,i} = (\eta^F + \eta^I)\{\lambda^2[\alpha\bar{\mu}\psi^k(1 - \beta) - a\beta\varphi^c]\lambda[\alpha\bar{\mu}(\psi^c(1 - \beta) - \psi^k] + a\beta(1 - \varphi^k\theta) - 1 + \varphi^k\theta\} + \lambda\{[\beta\alpha(1 - a)] - \alpha + 2\} + \alpha + 1$$

$$C_3 = -det(J) = (\eta^F + \eta^I)\{\lambda^2[\alpha\bar{\mu}\psi^k(\beta - 1)] + \lambda[\alpha(\beta(1 - \varphi^c + \bar{\mu}\psi^c - \varphi^k\theta) + \bar{\mu}(\psi^k - \psi^c) + \varphi^c + \varphi^k\theta - 1) + \beta a(\varphi^c + \varphi^k\theta - 1) - \varphi^c - \varphi^k\theta + 1] + \alpha[\bar{\mu}\psi^c - \varphi^c - \varphi^k\theta + 1]\} + \lambda[\alpha(1 - \beta) + a\beta - 1] - \alpha$$

By substituting C_1, C_2, and C_3, the stability conditions become:

$$1 + C_1 + C_2 + C_3 = (\eta^F + \eta^I)\{\lambda[\alpha(\beta(1 - \varphi^c - \varphi^k\theta) + \varphi^c + \varphi^k\theta - 1)] + \alpha(1 - \varphi^c - \varphi^k\theta\} > 0$$

$$1 - C_1 + C_2 - C_3 = (\eta^F + \eta^I)\{\lambda^2[2\alpha\bar{\mu}\psi^k(1 - \beta)] + \lambda[\alpha(\beta(\varphi^c + \varphi^k\theta - 2\bar{\mu}\psi^c - 1) - \varphi^c - \varphi^k\theta + 2\bar{\mu}(\psi^c - \psi^k) + 1) + a\beta(2 - 2\varphi^c - 2\varphi^k\theta) + 2\varphi^c + 2\varphi^k\theta - 1] + \alpha(\varphi^c + \varphi^k\theta - 2\bar{\mu}\psi^c - 1) + 2\varphi^c + 2\varphi^k\theta - 2\} + \lambda[2\alpha(\beta - 1) - 4a\beta + 4] + 2\alpha + 4 > 0$$

$$1 - C_2 + C_1 \cdot C_3 - C_3^2 = \{(\eta^F + \eta^I)(\varphi^c + \varphi^k\theta - 1) - \alpha(a\beta - 1) + 2\}\{(\eta^F + \eta^I)\{\lambda^2[\alpha\bar{\mu}\psi^k(1 - \beta)] + \lambda[\alpha(\beta(\varphi^c + \varphi^k\theta - \bar{\mu}\psi^c - 1) + 1 - \varphi^c + \bar{\mu}(\psi^c - \psi^k) - \psi^k\theta) + a\beta(1 - \varphi^c - \varphi^k\theta) + \varphi^c + \varphi^k\theta - 1] + \alpha[\varphi^c + \bar{\mu}(\psi^k\theta - \psi^c) - 1]\} + \lambda[\beta(\alpha - a) - \alpha + 1) + \alpha]\}(\eta^F + \eta^I)\{\lambda^2[\alpha\bar{\mu}\psi^k(\beta - 1)] + \lambda[\alpha\bar{\mu}(\psi^c + \psi^k + \psi^c\beta) + a\beta(\varphi^c + \varphi^k\theta - 1) + 1 - \varphi^k\theta] + 1 + \varphi^c - \varphi^k\theta - \alpha\bar{\mu}\psi^c\}\lambda[\beta(2a - \alpha) + \alpha - 2] - \alpha - \{(\eta^F + \eta^I)\{\lambda^2[\alpha\bar{\mu}\psi^k(1 - \beta)] + \lambda[\alpha(\beta(\varphi^c + \varphi^k\theta - \bar{\mu}\psi^c - 1) + 1 - \varphi^c + \bar{\mu}(\psi^c - \psi^k) - \psi^k\theta) + a\beta(1 - \varphi^c - \varphi^k\theta) + \varphi^c + \varphi^k\theta - 1] + \alpha[\varphi^c + \bar{\mu}(\psi^k\theta - \psi^c) - 1]\} + \lambda[\beta(\alpha - a) - \alpha + 1) + 2\}^2 > 0$$

By definition, condition (i) is always satisfied. For any combination in the parameter space that violates only condition (ii), a Flip bifurcation occurs. Beyond that, a Neimark-Sacker bifurcation is associated with violating condition (iii) only. By using auxiliary variables, the conditions can be seen as:

i) $(\eta^F + \eta^I)\{\lambda[\alpha(\beta(1 - \varphi^c - \varphi^k\theta) + \varphi^c + \varphi^k\theta - 1)] + \alpha(1 - \varphi^c - \varphi^k\theta\} > 0$
ii) $A_1 + \lambda[2\alpha(\beta - 1) - 4a\beta + 4] + 2\alpha + 4 > 0$
iii) $[(\eta^F + \eta^I)(\varphi^c + \varphi^k\theta - 1) - \alpha(a\beta - 1) + 2](B_1 + \alpha) + B2 - (B_1 + 2)^2 > 0$

such that

$$A_1 = (\eta^F + \eta^I)\{\lambda^2[2\alpha\bar{\mu}\psi^k(1 - \beta)] + \lambda[\alpha(\beta(\varphi^c + \varphi^k\theta - 2\bar{\mu}\psi^c - 1) - \varphi^c - \varphi^k\theta + 2\bar{\mu}(\psi^c - \psi^k) + 1) + a\beta(2 - 2\varphi^c - 2\varphi^k\theta) + 2\varphi^c + 2\varphi^k\theta - 1] + \alpha(\varphi^c + \varphi^k\theta - 2\bar{\mu}\psi^c - 1) + 2\varphi^c + 2\varphi^k\theta - 2\}$$

$$B_1 = \{(\eta^F + \eta^I)\{\lambda^2[\alpha\bar{\mu}\psi^k(1 - \beta)] + \lambda[\alpha(\beta(\varphi^c + \varphi^k\theta - \bar{\mu}\psi^c - 1) + 1 - \varphi^c + \bar{\mu}(\psi^c - \psi^k) - \psi^k\theta) + a\beta(1 - \varphi^c - \varphi^k\theta) + \varphi^c + \varphi^k\theta - 1] + \alpha[\varphi^c + \bar{\mu}(\psi^k\theta - \psi^c) - 1]\} + \lambda[\beta(\alpha - a) - \alpha + 1)$$

$$B_2 = (\eta^F + \eta^I)\{\lambda^2[\alpha\bar{\mu}\psi^k(\beta - 1)] + \lambda[\alpha\bar{\mu}(\psi^c + \psi^k + \psi^c\beta) + a\beta(\varphi^c + \varphi^k\theta - 1) + 1 - \varphi^k\theta] + 1 + \varphi^c - \varphi^k\theta - \alpha\bar{\mu}\psi^c\}\lambda[\beta(2a - \alpha) + \alpha - 2] - \alpha$$

Proof of Proposition 3

Proposition 4 can be easily demonstrated by observing that $\Delta L = 0$ and $\Delta Y = 0$ when $L_t = L_{t+1} = L^{\star}$ and $Y_t = Y_{t+1} = Y^{\star}$. As a result, the maps M1 and M2 have the same fixed point $(L^{\star}, S^{\star}, Y^{\star})$. Regarding the stability conditions, the Jacobin matrices of both maps are equivalent as $(L_t, S_t, Y_t) = (L^{\star}, S^{\star}, Y^{\star})$. Therefore, the local stability analysis entails repeating the same steps employed in proving Proposition 3.

Chapter 3

Time to Ditch Old Habits: Network Effect as a Source of Aggregate Consumption Inertia

Abstract

This paper introduces a novel model to elucidate the emergence of a hump-shaped aggregate consumption pattern without relying on the consumption habit hypothesis or any other microeconomic inertial mechanism. Combining jumps in the marginal propensity to consume with the possibility of interaction in an Erdos-Rényi network, the model features agents responsive to income shocks regarding their consumption decisions that can influence their neighborhood income. The model demonstrates the capacity of network effects to generate inertia in aggregate consumption, providing an alternative perspective to the consumption habit hypothesis compatible with a hump-shaped behavior. Findings suggest that understanding diffusion within a network can offer valuable insights into the observed inertia in aggregate hump-shaped behavior, bridging gaps between micro and macroeconomics.

Keywords: Informality; Real-financial interactions; Financial Instability; Heterogeneous Expectation; Business Cycles

3.1 Introduction

Understanding aggregate consumption dynamics is a fundamental pursuit in macroeconomics, with implications for economic policy formulation and the comprehension of over-all macroeconomic dynamics. Over the years, Dynamic Stochastic General Equilibrium (DSGE) models featuring consumption habits have captured and explained the aggregate consumption dynamics of the economies. However, as new models become available and the empirical evidence continually challenges these models, the quest for more refined frameworks becomes imperative.

This study proposes a new hypothesis to explain the nature of the inertia in aggregate variables by employing a network perspective. One of the problems is the incompatibility of consumption hypothesis habits with the new generation of DSGE models with heterogeneous agents, as it affects the dynamics of the intertemporal marginal propensity to consume (iMPC), a key element of Heterogeneous Agents New-Keynesian (HANK) models (Auclert et al., 2020). At the same time, consumption habits presented serious empirical incongruences like discrepancies between micro and macro estimations and lack of robustness for different data sources, countries, and time frames (Havranek et al., 2017).

To solve this problem, I propose a novel source of inertia in aggregate consumption that does not rely on the consumption habits hypothesis or any other microeconomic inertial mechanism. I advocate that the inertia presented in aggregate results derives from an income diffusion process that affects individual agents through indirect effects. To explore this point, I developed a model with agents interacting in a network. Agents' behavior is compatible with changes in their consumption pattern given transitory income shocks as described by Fagereng et al. (2021). This strategy makes the behavior of the iMPC of the model similar to a HANK model.

At the same time, agents are distributed randomly in an Erdos-Rényi network with a fixed connection probability. In this context, agents' income and consumption depend on their peers' spending decisions in line with the evidence provided by Ozdagli and Weber (2017) and Barigozzi et al. (2012). The intensity of the network effect for each agent is nonconstant and follows the dynamics of their exogenous income.

The results showed that the network effect generates a significant qualitative change in aggregate consumption dynamics, approaching a hump-shaped behavior indicated by empirical evidence. While agents increase their consumption given an increase in their income, individuals in their neighborhood receive an indirect increase in income, spreading the individual effect of the income shock throughout the network. The greater the network effect on changes in agents' incomes, the later the peak of agents' consumption, as it depends on the income to be spread throughout the network.

This research delivers critical insight into how macroeconomic models can accommodate networks to explain inertial phenomena without frictions and shortcuts. Ultimately, this research seeks to enhance our understanding of the factors influencing aggregate consumption behavior, providing a foundation for more comprehensive and realistic economic models. At the same time, HANK-type models already rely on a structure of heterogeneous agents that can accommodate network dynamics to complement or replace New Keynesian frictions traditionally used in macroeconomics.

3.2 Individual Jumps and Aggregate Hump

3.2.1 The Old Habit of Resorting to Habits

The development of the DSGE methodology was a reaction to the inadequacy of RBC models in capturing empirical evidence. While initially rooted in the model proposed by (Kydland and Prescott, 1982), DSGE models evolved to incorporate mechanisms addressing market imperfections examined by New-Keynesian theorists like Rotemberg (1982) and Blanchard and Kiyotaki (1987). Noteworthy enhancements, such as the price inertia mechanism introduced by Calvo (1983), marked the transition from RBC to prominent DSGE models.

Despite the trajectory of evolution, DSGE models increasingly embraced frictions as tools to enhance predictive capacity, with some mechanisms serving as effective "filters" for data adjustments while maintaining limited microeconomic commitment. One such instance is the consumption habit hypothesis, which captures the stylized empirical fact of a bell/hump-shaped response of macro variables to policy shocks (Fuhrer, 2000; Del Negro et al., 2007). While successful in practice, a meta-analysis by Havranek et al. (2017) revealed empirical challenges in identifying the consumption habit parameter, highlighting the method's sensitivity to various factors such as data sources, countries, time frames, and the level of analysis (micro or macroeconomic).

This empirical challenge underscores the practical nature of New-Keynesian frictions, as noted by Gabaix (2019), who aptly remarked that "habits are basically just a device to generate stickiness." However, the question arises about the validity of these frictions, considering the evolving landscape of macroeconomic modeling. Macro models, originally designed to address specific puzzles and challenges, often face clashes between old solutions and new developments.

One such clash emerged with the advent of the New-Keynesian Heterogeneous Agents (HANK) model, which combines the heterogeneous agents approach of Krusell and Smith (1997) with New-Keynesian frictions (Kaplan et al., 2018; Auclert et al., 2018; Seidl and Seyrich, 2022). This model emphasizes income and wealth inequality between agents, introducing the intertemporal Marginal Propensity to Consume (iMPC) as a critical factor affecting fiscal multipliers, monetary policy transmission, and the amplification of aggregate shocks (Kaplan et al., 2018).

The iMPC in the HANK model suggests that agents respond with increased consumption to transitory income shocks. This result finds support through various identification strategies, including quasi-experimental evidence (Kueng, 2018; Fagereng et al., 2021), survey instruments (Bunn et al., 2018; Fuster et al., 2021), and semistructural methods (Ganong et al., 2020; Commault, 2022). Importantly, this challenges the permanent income hypothesis implicit in DSGE models, revealing potential pitfalls in characterizing consumption behavior over time for transitory shocks.

Attempting to reconcile "micro jumps" in individual iMPC with the "macro humps" observed in aggregate variables, Auclert et al. (2020) uncovered an inconsistency between the consumption habits hypothesis and the microeconomic behavior in HANK models. This inconsistency becomes apparent when recognizing that the consumption habits hypothesis imposes lagged consumption adjustments, contradicting the abrupt changes observed in individual iMPC for the case of transitory income shocks.[1]

3.2.2 It Calls "Macro Humps" for a Reason

According to the HANK literature, approximately two-thirds of the shift in consumption behavior following a monetary policy shock can be attributed to indirect effects arising from changes in income (Kaplan et al., 2018). These effects underscore the prevalence of "Keynesian" effects in determining consumption dynamics (Auclert et al., 2018). The concept of a "multiplier effect," where initial spending decisions trigger a chain reaction among agents, dates back to Keynes (1964). This concept gains complexity in a heterogeneous agent environment, as indirect effects manifest through interacting agents.

I posit that the hump-shaped format of aggregate consumption behavior results from chains in agents' spending decisions and consequent income flows. Even with identical agent descriptions, network positions relative to peers can lead to varying moments of decreasing utility due to the concavity of the consumption function and different income dynamics. While not anticipated by Auclert et al. (2020), recognizing micro jumps becomes crucial in explaining macro humps as genuine macroeconomic effects. A direct link between income dynamics and consumption decisions emerges when considering agents' consumption behavior in response to temporary income changes.

Viewing the indirect effects from a network perspective, we can conceptualize them as the propagation of an initial impulse throughout the network. For instance, Ozdagli and Weber (2017) found that production networks serve as essential mechanisms for monetary policy propagation. Heterogeneity in agent responses to shocks implies that the effects do not unfold simultaneously. As shown by Barigozzi et al. (2012), household consumption expenditures are not statistically independent due to high agent heterogeneity. Moreover, La'O and Tahbaz-Salehi (2022) illustrates that monetary policy can affect different sectors disparately within a network.

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[1] A more in-depth discussion of the dynamics of iMPC in response to transient shocks and possible incompatibilities can be found in detail in Kaplan and Violante (2022).

Considering that various networks coexist, it is possible to understand that the total indirect effect pointed out by the HANK literature can be related to the net impact of different network effects. The empirical evidence provided by Ghassibe (2021) indicates that at least 30% of the effect of a monetary policy shock on US aggregate consumption arises from amplification through input-output networks. Similarly, Flynn et al. (2022) reveals that, in a network context, fiscal policy can achieve significant positive net effects by targeting agents with high MPC, yielding a 30% indirect impact on household consumption decisions.

3.3 A Simple Network Model

In this section, I introduce a novel model designed to elucidate the emergence of a hump-shaped aggregate consumption pattern resulting from the diffusion of an income shock within a network. The model captures the simultaneous occurrence of micro jumps and macro humps without presuming consumption habits or inattention in agents' behavior. To enhance clarity, I summarize the different transmission channels of indirect effects (wages, portfolio reallocation, or wealth redistribution, for example) directly in the diffusion of an exogenous shock to agents' income.

Individually, agents respond to income increases with heightened consumption tendencies during temporary income shocks, reflecting micro jumps. Simultaneously, a share of agents' income relies on a network effect based on their neighbors' consumption. The interplay among agents generates varied consumption patterns over time, even among seemingly identical agents individually, driving the macro hump response.

3.3.1 Agents

The economy comprises a continuum of agents indexed by $i \in [0, 1]$, maximizing the following utility function:

$$\max_{\{C_{i,t}, A_{i,t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t \ln(C_{i,t}) - \varphi_s \frac{(A_{i,t} - \bar{A}_i)^2}{2} \tag{3.1}$$

Agents decide between consuming a homogeneous good $C_{i,t}$ today or saving in a risk-free asset $A_{i,t}$. Agents incur a convex cost for deviations from their savings target $\bar{A}_i$, with intensity captured by φ_s. This mechanism allows for jumps in the iMPC for transitory income shocks similar to Cantore and Freund (2021). It is also possible to use a nonconstant intensity mechanism for this effect. Still, as this paper intends to understand the inertia generated by a network, the model will follow the simplest mechanisms to create jumps.

To solve their optimal program, agents are subject to a budget constraint given by:

$$A_{i,t} + C_{i,t} = A_{i,t-1}(1 + r_{t-1}) + Y_{i,t} \tag{3.2}$$

where $Y_{i,t}$ is their income, and r is the risk-free asset interest rate. Agents' income consists of a exogenous component $Y^E_{i,t}$ and a network component $Y^N_{i,t}$:

$$Y_{i,t} = Y^E_{i,t} + \gamma_t Y^N_{j,t} \tag{3.3}$$

where γ_t captures the intensity of the network effect. The exogenous component follows an autoregressive process of order one with an autoregressive parameter $\rho \in (0, 1)$ and a stochastic error term $\varepsilon_{i,t+1} \sim N(0, \sigma)$:

$$Y^E_{i,t} = \rho Y^E_{i,t-1} + (1 - \rho)\bar{Y}_i + \varepsilon_{i,t} \tag{3.4}$$

where $\bar{Y}$ represents permanent income. The network effect intensity changes with variations in the exogenous income $Y^E_{i,t}$ and is bounded by maximum and minimum parameters γ_{max} and γ_{min}:

$$\gamma\left(Y^E_{i,t-1}\right) = \frac{\gamma_{max}\exp\left(Y^E_{i,t-1}\right)^2 + \gamma_{min}}{1 + \exp\left(Y^E_{i,t-1}\right)^2} \tag{3.5}$$

This mechanism captures the idea that agents adjust their interaction with the network based on changes in their exogenous income. As agents face an increase in their income, they feel more comfortable expanding their relations in the network. Another way of characterizing the problem would be to change the network topology in each period based on a measure of the predisposition of agents to increase or decrease their connections however, the intensity mechanism described by Eq. (3.5) guarantees a similar effect, as it characterizes an increase in intensity of the connections, even if the number of links is constant for all t.[2]

The network component of income is defined as:

$$Y^N_{j,t} = \int_0^1 (C_{j,t}A_{i,j})\,dj \tag{3.6}$$

To make the argument as straightforward as possible, the income from the network effects of each agent depends on the consumption decisions of the agents connected to it. An agent's expenses are directly perceived as the income of his neighbors. This effect is mediated by an adjacent matrix $A_{i,j}$ that captures the topology of a given network.

3.3.2 Network Topology

Agents interact in an Erdos-Rényi (ER) network, expressed in a graph $G(N, p(ER))$ connecting nodes i and j with exogenous probability $p(ER)$, generating a random structure. The following adjacency matrix represents the network structure:

$$A_{ij} = \begin{cases} 1, & \text{with probability } p(ER), \text{ if } i = j \\ 0, & \text{with probability } (1 - p(ER)), \text{ if } i = j \\ 0, & \text{if } i = j \text{ (no self-loops)} \end{cases} \tag{3.7}$$

Fig. 3.1 visually presents the connections in an ER network generated randomly. The income dynamics depend on the spending decisions of agents j in the neighborhood of each agent i defined randomly in the network. Agents that are less connected and further away from agents with many connections receive a minor income effect.

The dynamics of the economy's aggregate income, when considering the network effect, can be expressed as:

$$Y_{t+1} = \int_0^1 \left(\rho Y^E_{i,t} + (1-\rho)\bar{Y}_i + \frac{\gamma_{max}\exp(Y^E_{i,t-1})^2 + \gamma_{min}}{1 + \exp(Y^E_{i,t-1})^2}\int_0^1 (C_{j,t}A_{i,j})\,dj + \varepsilon_{i,t+1}\right) di \tag{3.8}$$

As the agents were defined on a continuum of measure one and the network is characterized by a finite number of nodes N, an important step to make the individual description of the agents and the network topology compatible is to discretize the continuum of agents. As agents were defined homogeneously, the continuum $i \in [0,1]$ can be discretized into equal intervals $n \in N$:

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[2] The derivation of the First-Order Conditions (FOC) and the intensity mechanism can be found in the mathematical appendix.

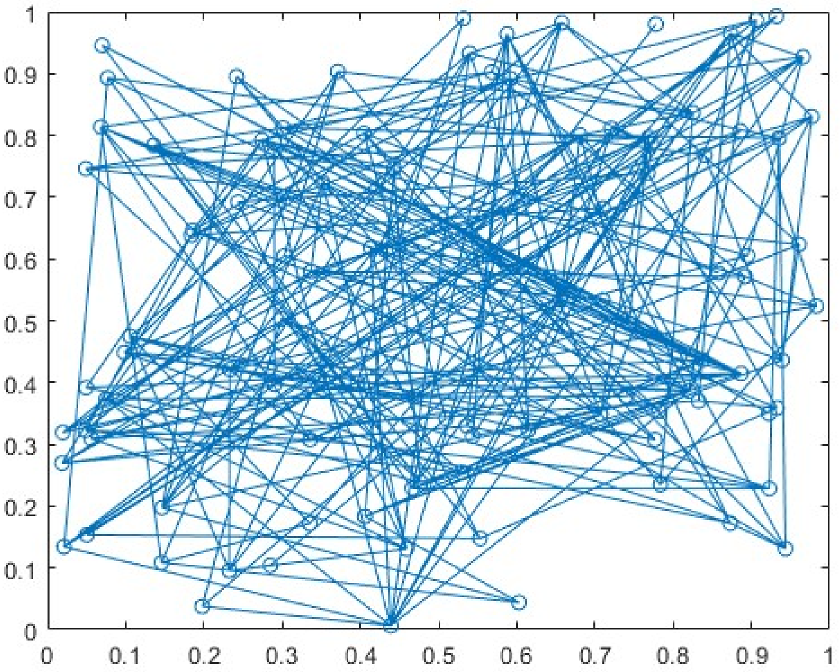

Figure 3.1: Erdos-Rényi network topology for $p(ER) = 0.05$ and $N = 100$.

$$\Delta n = \frac{b-a}{N} \quad \text{s.t.} \quad [n_i, n_{i+1}] = [a + (i-1)\Delta n,\ a + i\Delta n] \quad \text{for } i = 1, 2, \ldots, N \tag{3.9}$$

since $[a, b] = [0, 1]$, we have that:

$$n_i = (i-1)\Delta n \quad \text{for } i = 1, 2, \ldots, N+1 \tag{3.10}$$

where n_i represents the center or a reference point for the i-th subinterval. In that sense, we can rewrite Eq. (3.8) as:

$$Y_{t+1} = \sum_{i=1}^{N} \left[\rho Y_{i,t}^{E} + (1-\rho)\bar{Y}_i + \frac{\gamma_{max}\exp(Y_{i,t-1}^{E})^2 + \gamma_{min}}{1 + \exp(Y_{i,t-1}^{E})^2} \sum_{j=1}^{N} (C_{j,t} A_{i,j}) + \varepsilon_{i,t+1} \right] \tag{3.11}$$

The simplicity of the ER network topology aligns with the model's focus on the diffusion effect within a network. Agents' spending decisions contribute to their neighbors' income, which, in turn, increases their spending and continues to affect the network. This dynamic can be better observed in the results of the numerical simulations presented in the next section.

3.4 Numerical Simulations

The simulations involve a network with $N = 100$ agents over a time horizon of $T = 100$ periods. Results are considered from an individual point of view for a single round of simulations, and the aggregate results for multiple simulations are used using a Monte Carlo approach for 1000 rounds of simulations. Table 1 presents the parameter calibration for the exercises.

Parameter	Value	Description
β	0.99	Discount Factor
r	0.01	Interest Rate
φ_s	0.08	Savings Target Parameter
$\bar{A}$	0	Assets Reference Level
$\bar{Y}$	0	Permanent Income
ρ	0.8	Autoregressive Parameter
γ_{max}	0.5	Max Network Intensity
γ_{min}	-0.5	Min Network Intensity
$p(ER)$	0.05	Probability of Connection

Table 3.1: Parameter Values

The probability $p(ER)$ is set to achieve an average connection per agent following Engel et al. (2021). Parameters γ_{max} and γ_{min} calibration values follow Ghassibe (2021). Parameters β and r are consistent with the typical values found in the literature. ρ is calibrated to match the average Marginal Propensity to Consume (MPC) for an exogenous income shock Fagereng et al. (2021). $\bar{Y}$ and $\bar{A}$ maintain a 0 steady-state value for income.

3.4.1 Results

Initially, it is possible to consider the dynamics of each agent's income evolution from an individual point of view. As individual dynamics tend to get closer as the number of simulations increases, initially, we can only consider the result of a single simulation as a way of illustrating the agents' behavior. Fig. 3.2 illustrates the dynamics of income diffusion, where each agent (represented by different colors) receives an initial shock of 5 units to their income. From period two onwards, the position randomly assigned to each agent in the network means that agents have different patterns of change in their available income depending on the consumption behavior of other agents, explaining the differences between each curve representing a single agent income dynamics. For example, the red circle shows the moment where some age agents with less connections faces an immediately income decrease, while the green circle shows the agent with highest income having an income decrease 3 periods after. With several dynamics being displayed at the same time, Fig. 3.2 enables us to see the dispersion between agents' incomes generated by the network effect.

Similarly, Fig. 3.3 showcases the dependency of consumption on agents' positions in the network. In response to a temporary income variation, agents tend to increase consumption, with agents receiving network-induced income increases exhibiting prolonged and peaked consumption patterns. As can be seen, agents with weak connections, represented by the lowest curves, achieve their consumption peak before, while agents receiving a large amount of income present delays in reaching their consumption peak. For example, the agent represented by the lowest line in purple has a consumption peak in period 4. In contrast, the highest consumption agents represented by the top blue and yellow lines only have their consumption peak in period 8. Beyond that, agents with the highest consumption dynamics demand more time to return to a steady state and vice-versa. Combining individual consumption trajectories with different delayed peak responses drives the inertia in aggregate consumption.

For individual cases, Monte-Carlo exercises would prevent the visualization of different income dynamics between agents; however, for the aggregate case, the combination of multiple simulations strengthens the results given the stochastic elements of the model. The aggregate results presented are the average of 1000 Monte Carlo simulations. Fig. 3.4

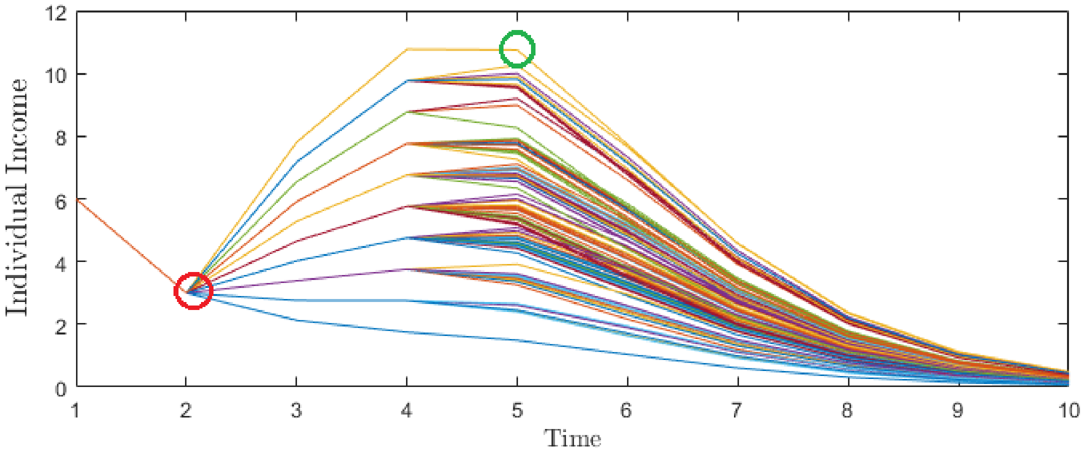

Figure 3.2: Individual Income Dynamics for an Income Shock.

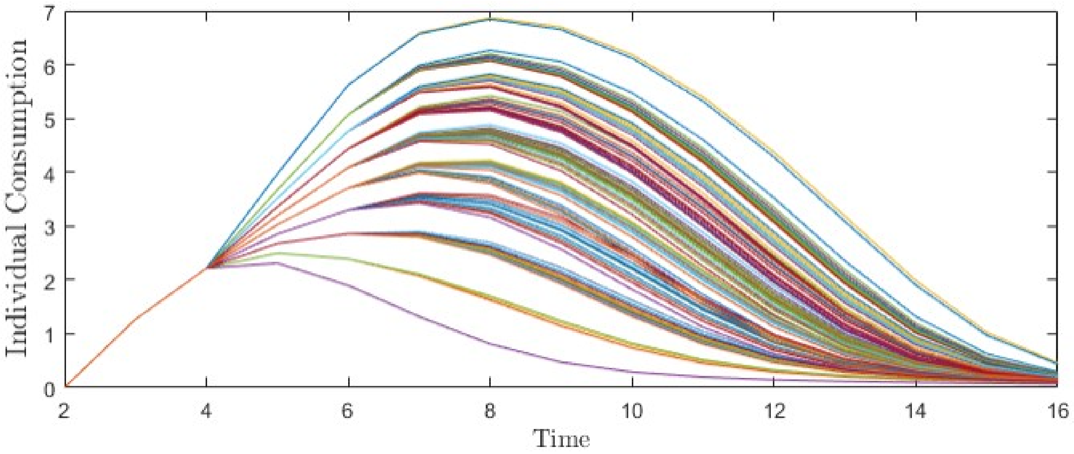

Figure 3.3: Individual Consumption Dynamics for an Income Shock.

shows the evolution of total income on the left and shock and network income on the right.

The gray area represents the possible variations within the Monte Carlo exercise, with the solid red line being the average of the results. The income dynamics have a critical qualitative change from periods two to three. At this moment, the indirect effect generated by interactions in the network increases the agents' total income. In the second graph, it is possible to visualize this transition when the network effect exceeds the effect of the shock on income.

Fig. 3.5 displays total consumption and total asset dynamics. The gray area captures the uncertainty introduced by the stochastic nature of the network. When the network effect influences consumption, the distribution of consumption expenses changes over time, contributing to the observed hump-shaped behavior. As can be seen, the peak height depends on the random effect of the distribution of agents in the network. However, the peak always occurs in the same period, which indicates the model's ability to exhibit inertia in consumption peaks. Note that even in the benchmark case, there is a small delay caused by the savings target mechanism. This happens because the quadratic nature of the disutility generated by accumulating assets beyond the target and the smoothing of consumption given the curvature of the consumption function generates an initial change

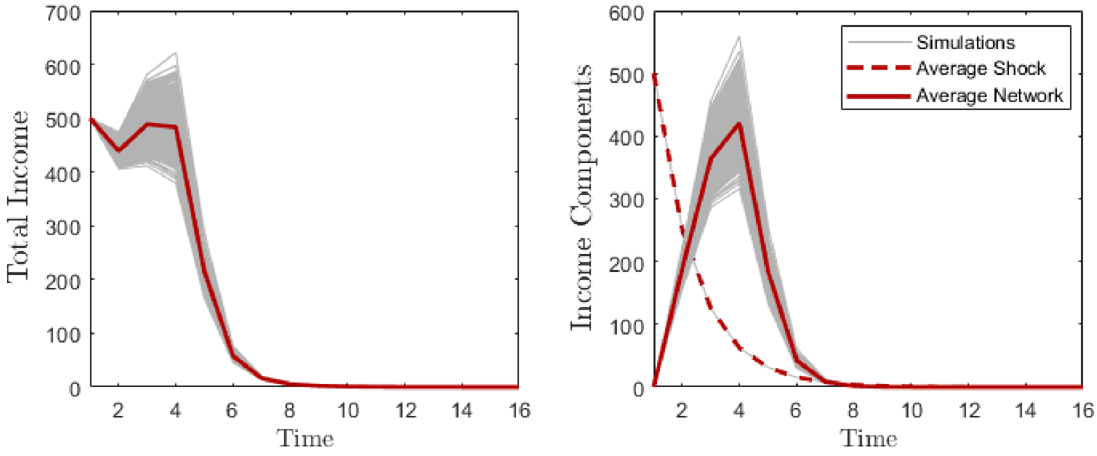

Figure 3.4: Total Income and Income Components Dynamics for an Income Shock.

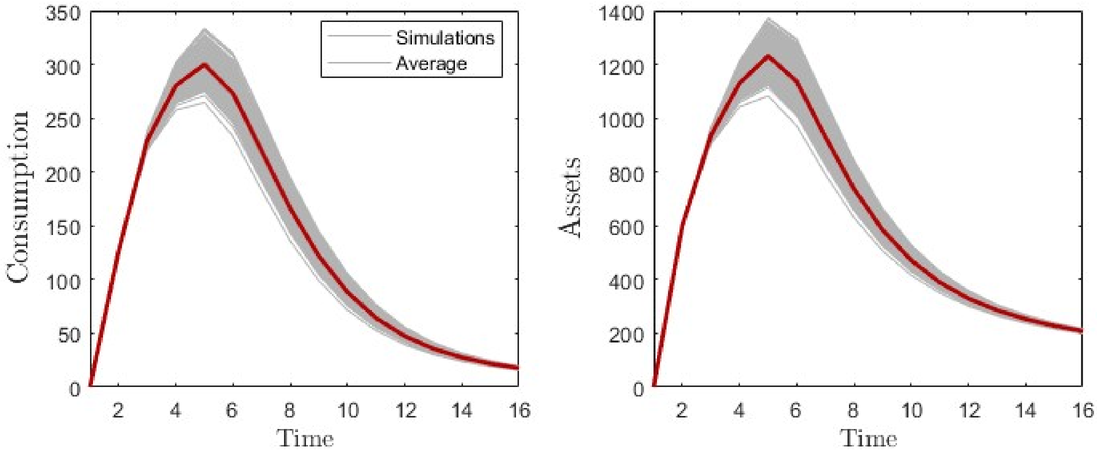

Figure 3.5: Total Consumption and Total Assets Dynamics for an Income Shock.

in the peak. Instead of returning to the steady state immediately, the agents have an extra round of consumption to compensate for the disutility of the excess assets.

It is possible to compare the case in which the entire variation of income depends only on the shock with the case in which the network effect is responsible for 30% of the impact. In Fig. 3.6, we can observe that the distribution of consumption over the same time interval starts to assume a delayed behavior in the direction of evidence for the case where the network effect is active. It is also possible to compare the dynamics of consumption considering the network effect with the benchmark case with no network effect, i.e., defining $\gamma_t = 0$ for all t.

As this is a simple model, every network effect is the indirect effect of an income shock. To demonstrate a more extreme case, Fig. 3.7 shows the difference between the benchmark case and the case where the variation in income is given by 70% by the network effect that equals the total share of indirect effects over consumption found in HANK literature. In this case, an initial diffusion period is necessary to increase consumption before period 2. This effect makes explicit the capacity of the network effect to push the dynamics of aggregate consumption in the direction of a hump-shaped format.

While the model maintains simplicity and assumes identical initial income shocks, it

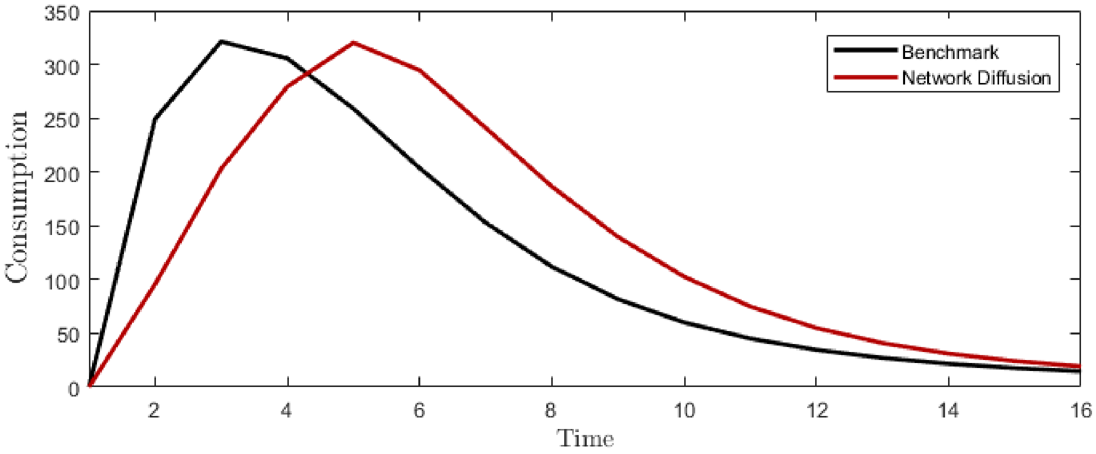

Figure 3.6: Total Consumption Dynamics for an Income Shock. Benchmark and Network cases with 30% of total income provided by the network effect.

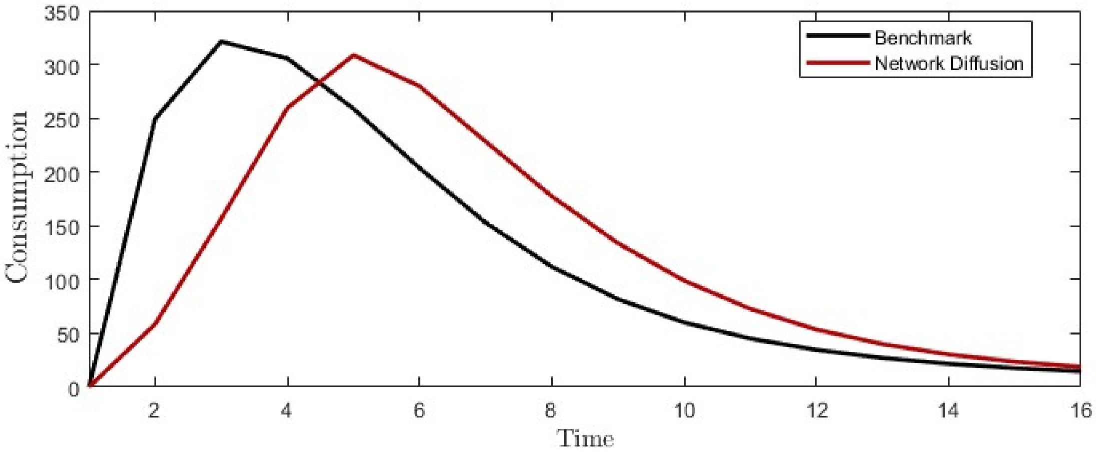

Figure 3.7: Total Consumption Dynamics for an Income Shock. Benchmark and Network cases with 70% of total income provided by the network effect.

demonstrates the potential of diffusion effects within a network in explaining the inertia observed in aggregate hump-shaped behavior. Though abstracting from certain complexities, the simulations contribute to the hypothesis by showcasing that even when considering the network effect, the model reproduces the expected aggregated hump-shaped pattern.

One could ask about the direct comparison between a model with consumer habits and the network effect presented. The incompatibility between jumps in the MPC and the consumption hypothesis (Auclert et al., 2020) is an imminent dificulty in comparing the two models. The effect that characterizes the hump-shaped behavior of aggregate consumption in the presented model effectively depends on jumps in the iMPC. As shown in Fig. 3.8, the network mechanism must have a reliable representation of the iMPC behavior for transient changes in income to generate significant changes in indirect income.

This result also shows that the characterization of jumps in the iMPC recently highlighted by the HANK models allows for a better understanding of the income dynamics of agents across networks. The attempt to observe indirect/network effects considering a direct substitution mechanism leads to a significant loss of the model's ability to represent

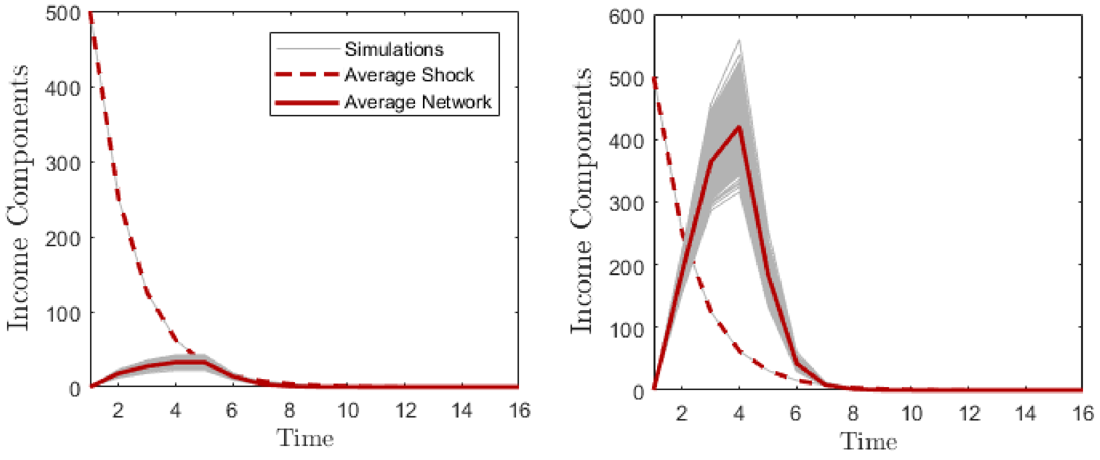

Figure 3.8: Total Consumption Dynamics for an Income Shock. Graph (a) represents the model with a full substitution mechanism, and graph (b) represents the model with the savings target mechanism.

income diffusion within the network.

At the same time, how could we know that this network mechanism is considered effective as the consumption habits hypothesis regarding the behavior of aggregate consumption? By comparing both strategies by their capacity to generate changes in aggregate consumption peak, we can see that the results provided by Kano and Nason (2014) showed that for a habit coeficient equal to 0.65, a value close to what is usually estimated in DSGE models, the model with consumption habits causes a delay in the aggregate consumption peak of 3 periods. The network effect causes a delay of 2 periods regarding the benchmark with savings target, creating a one-period peak delay. In general, the network effect can generate a hump-shaped behavior in aggregate consumption similar to what is delivered by consumption habits. Even though this is just a preliminary comparison, the intensity of the inertia generated by both effects is compatible.

3.5 Concluding Remarks

In this study, I developed a novel model to investigate the emergence of a hump-shaped aggregate consumption pattern resulting from the diffusion of an income shock within a network. The model captures the nuanced dynamics of micro jumps and macro humps without relying on assumptions about consumption habits or inattention in agents' behavior.

The model's key features include agents making decisions regarding consumption and savings based on a utility function with a savings target mechanism. Income is influenced by an exogenous component and a network effect, where the latter's intensity adapts to changes in the agents' exogenous income. The network topology is represented by an Erdos-Rényi (ER) network chosen for its ability to generate random connections among homogeneously defined agents.

Numerical simulations were conducted to explore the dynamics of income and consumption in response to an initial shock. The results revealed a rich interplay of individual responses within the network, leading to diverse income and consumption patterns. Better-connected agents experienced more substantial income increases, impacting not only their consumption but also contributing to the network-induced income of their neighbors.

The aggregate dynamics showed a qualitative change as the network effect began to exceed the impact of the initial shock on income. This transition indicates the substantial influence of the network on the overall outcomes. The simulations also illustrated the variability of different exercises introduced by the stochastic nature of the network.

Comparisons between scenarios with varying shares of the total effect provided by the network underscored the network's capacity to drive aggregate consumption dynamics. Even in a simple model, the diffusion effect within a network played a crucial role in shaping the hump-shaped behavior observed in aggregate consumption. Beyond that, the model could also demonstrate that the micro jumps in agents iMPC are essential in explaining the macro-humps following a network perspective.

While the model maintained simplicity by assuming identical initial income trajectories, it provided valuable insights into the potential of diffusion effects within a network to explain the inertia observed in aggregate hump-shaped behavior. Acknowledging heterogeneity among agents in real-world scenarios is expected to amplify the asymmetry in shock diffusion, further reinforcing the model's implications.

Future research could extend the model to incorporate additional layers of complexity, like heterogeneity in agents' portfolios, and explore a monetary policy shock by including a Taylor rule. Given that HANK models already work with heterogeneous agents, an important step would be to incorporate networks into these models to understand the diffusion of the indirect effects between the agents. As the results demonstrated by this research, network diffusion can be a possible alternative to the shortcuts proposed by New-Keynesian frictions and avoid problems like the incompatibility of "micro jumps" and "macro humps."

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Mathematical Appendix

First Order Conditions and Marginal Rate of Substitution

The First-Order Conditions (FOC) for the problem are given by:

$$\beta^t \frac{1}{C_{i,t}} - \lambda_{i,t} = 0 \tag{3.12}$$

$$-\beta^t \varphi \left(A_{i,t} - \bar{A}_i \right) + \lambda_{i,t} + \beta^{t+1} E_t \{\lambda_{i,t+1} (1 + r_t)\} = 0 \tag{3.13}$$

After organizing Eqs. (3.12) and (3.13), we have that:

$$\frac{1}{C_{i,t}} = \lambda_{i,t} \tag{3.14}$$

$$\lambda_{i,t} + \varphi \left(A_{i,t} - \bar{A}_i \right) = \beta E_t \{\lambda_{i,t+1} (1 + r_t)\} \tag{3.15}$$

By using Eqs. (3.14) and (3.15), and considering perfect foresight such that $E_t [C_{i,t+1}] = C_{i,t+1}$, the Euler equation associated with the problem can be obtained:

$$\frac{1}{C_{i,t}} + \varphi \left(A_{i,t} - \bar{A}_i \right) = \beta (1 + r_t) \left(\frac{1}{C_{i,t+1}} \right) \tag{3.16}$$

After reorganizing, we arrive at the marginal rate of substitution:

$$\frac{C_{i,t+1}}{C_{i,t}} \overset{!}{=} \frac{\beta (1 + r_t)}{1 + \varphi \left(A_{i,t} - \bar{A}_i \right)} \tag{3.17}$$

Network Intensity Mechanism

The network effect intensity will be driven by a logit-type mechanism that follows Manski and McFadden (1981) and Anderson et al. (1992). As described by Franke and Westerhoff (2018), this mechanism allows for changes in a variable accordingly to a probability ζ of driving changes towards state A and $(1 - \zeta)$ the probability of changes regarding the state B. The probabilities vary in time according to x_t^+ and x_t^-. By using a general form of the mechanism defined in Brock and Hommes (1997), ζ will change according to:

$$\zeta \left(x_t^+, x_t^- \right) = \frac{\exp\{\omega x_t^+\}}{\exp\{\omega x_t^+\} + \exp\{\omega x_t^-\}} \tag{3.18}$$

where the parameter $\omega \in [0, \infty)$ captures the intensity of change between the two states given the pair (x_t^+, x_t^-). The probability will be nearly equal for values of ω closer to 0. When $\omega \rightarrow \infty$, the value of ω_t tends to 0 or 1, meaning that state A or B will be absolute. By considering $\omega = 1$ and that the network effect have a maximum and minimum intensity γ_{max} and γ_{min}, we can define the intensity as:

$$\gamma \left(x_t^+, x_t^- \right) = \gamma_{max} \zeta \left(x_t^+, x_t^- \right) + \gamma_{min} \left(1 - \zeta \left(x_t^+, x_t^- \right) \right) \tag{3.19}$$

By assuming that x_t^+ and x_t^- are symmetric, we can use Eq. (3.4) to redefine (3.18) as:

$$\zeta(Y_{i,t-1}^E) = \frac{\exp\{Y_{i,t-1}^E\}}{\exp\{Y_{i,t-1}^E\} + \exp\{-Y_{i,t-1}^E\}} \tag{3.20}$$

After combining Eqs. (3.19) and (3.20) and reorganizing, we have that:

$$\gamma \left(Y_{i,t-1}^E \right) = \frac{\gamma_{max} \exp\left(Y_{i,t-1}^E \right)^2 - \gamma_{min}}{1 + \exp\left(Y_{i,t-1}^E \right)^2} \tag{3.21}$$

Afterword

This thesis has navigated the intricate landscape of macroeconomic dynamics through the lens of heterogeneous agents, contributing to the ongoing paradigm shift in macroeconomic research. The emergence of Heterogeneous Agents New-Keynesian (HANK) models marked a turning point, challenging the traditional representative agent models and emphasizing the importance of diversity in understanding economic phenomena. The journey embarked upon in this thesis sought not only to explore the implications of heterogeneous agents but also to provide innovative solutions to critical challenges in the field.

Chapter 1 of the thesis addressed the nuanced dynamics of consumption, proposing a groundbreaking savings target mechanism rooted in prospect theory. This chapter bridged the theoretical-empirical gap by revisiting a representative agent model and incorporating a mechanism emulating the impact of heterogeneity on the marginal propensity to consume. Through analytical and simulation analyses, it demonstrated the indispensability of incorporating a realistic intertemporal marginal propensity to consume, offering a fresh perspective on the empirical debate surrounding agents' behavior and the impact of transitory income shocks and the impacts of fiscal policy.

Chapter 2 delved into the intersection of financial phenomena and economic fluctuations, particularly in the context of economies characterized by high levels of informality. By developing a heterogeneous agents model that captured the effects of informality on liquidity, solvency, and output dynamics, this chapter provided a pioneering exploration into an essential and uncharted area of investigation. The findings highlighted the trade-off between higher output levels and increased instability in economies with consolidated high formal sector participation. Beyond that, this chapter also explained the puzzle regarding the instability of economies with high informality during the business cycles.

Chapter 3 departed from traditional consumption habits hypotheses in DSGE models and proposed a new hypothesis rooted in a network perspective. By introducing an income diffusion process within a network of interacting agents, this chapter explained the nature of inertia in aggregate consumption without relying on microeconomic inertial mechanisms. The model demonstrated a hump-shaped behavior in aggregate consumption dynamics, aligning with empirical evidence and offering a novel perspective on factors influencing aggregate consumption behavior.

These three chapters collectively form this thesis's foundational pillars, contributing to the ever-evolving landscape of macroeconomic dynamics research. The analytical and numerical methods employed aspire to offer not just incremental modifications to existing models but innovative solutions to reconcile disparities between theoretical frameworks and empirical evidence. Incorporating heterogeneous agents and novel mechanisms has enhanced our understanding of economic behaviors and outcomes, providing valuable insights for policymakers and researchers alike.

In essence, this thesis contributes to the ongoing dialogue surrounding the importance of heterogeneity in macroeconomics. It reinforces the idea that acknowledging and incorporating diversity among economic agents is not merely a trend but a necessity for comprehending the complexities of economic phenomena. As the heterogeneity agenda gains prominence, the thesis underscores the significance of reassessing conventional approaches and embracing fresh perspectives. Through this comprehensive exploration, the thesis aspires to stimulate further research, fostering a deeper understanding of the dynamic interplay between heterogeneous agents and macroeconomic outcomes.