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ORIGINAL ARTICLE

Mean-variance investing with factor tilting

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Abstract

Factor analysis proposes an alternative approach to standard portfolio theory: the latter is optimisation based, while the former is estimation based. Also, in standard portfolio theory, returns are only explained by the portfolio volatility factor, while factor analysis proposes a multiplicity of factors, which the managers can choose from to tilt their portfolios. In attempting to reconcile these alternative worlds, we propose a penalised utility function, incorporating both the Markowitzian risk-return trade-off and the manager's preferences towards factors, and discriminating among losses and gains relative to a reference asset. The penalisation affects the optimisation process, favouring the selection of portfolios with less variance and more tilted towards the chosen risk factors. Penalty levels set by the manager generalise the traditional notion of risk aversion. We test our model by building an investment portfolio based on a combination of asset classes and selected investing factors, focussed on the eurozone. To identify the optimal portfolio, we adopt a set of three metaheuristic optimisation algorithms: the fitness function stochastic maximization using genetic algorithms, differential evolution algorithm for global optimisation, and the particle swarm optimisation, and dynamically choose the best solution. In this way, we can improve the Markowitzian optimisation by tilting the asset allocation with managers' expectations and desired exposures towards designated factors.

Keywords Factor investing · Asset allocation · Portfolio optimisation · Utility functions · Behavioural risk aversion

JEL Classification G11 · G40

Their main research interests cover Portfolio Theory, Factor Investing, and Behavioural Finance.

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Multifactor strategy and related literature

The diversification is probably the most researched aim of the asset management industry, and some researchers (cf. Ilmanen and Kizer 2012) have shown that the correlation between factors is much lower than that between investment categories, as extensively explained in previous literature.[1] The authors compared factors to the ingredients that are used to prepare food. The idea is that an investment category can be unravelled into a number of factors that influence return and risk. This line of research has grown as a consequence of the crisis which has shaken the market: Asian Financial Crisis (1997), Dot-Com Crash (2000–2001), Subprime Financial Crisis (2007–2008), the European Debt Crisis (2010–2011) and Negative interest rates (2014–2017). Factor investing is a systematic approach to strategically investing in certain sectors of the financial markets, which obtain higher returns over longer periods than in other asset classes. The interest of academics and practitioners has increased so much that the expression "a factor zoo" (Cochrane 2011) has been invented. This means that a relevant number of papers over the last three decades have shown many different factors. The consequence is that factor investing is losing its feature of being a niche investment product. In fact, this investment style has been adopted not only by some of the world's largest institutional investors, but also by retail investors, who now have access to hundreds of factor products. Even if many researchers (cf. Hsu et al. 2015; Dimson et al. 2017) have identified from 250 to 316 factors, generally many practitioners like to use five factors: market, value, small cap, momentum and low beta factors. The authors assert that we should follow three simple rules to establish the robustness of factors. First, the factor must be validated in numerous research papers published in high level journals. Second, the effect should endure for a significant long period and it should be statistically significant. Third, the effect should resist the definition of factor strategy.

The selection of factors is important because we can be attracted by different features. Berkin and Swedroe (2016) come to our help by classifying five criteria linked to a potential factor. They affirm that a factor must be persistent over time, pervasive across markets, robust to the different definitions, intuitive to common sense and investable at acceptable cost. Even though a large range of factors has been introduced and updated (macroeconomic factors, statistical factors and fundamental factors), the most used today are value, size, momentum, low volatility, quality and liquidity. Fama and French (1992), Fama and French (1993) demonstrated that some components of the market can have better returns and that not all stock performance is explained by other factors, next to market risk. Jegadeesh and Titman (1993) introduced the feature later known as the "momentum effect" showing that equity with higher performances in the past would also be likely future winners. Recently, Fama and French (2015) proposed a new five-factor asset pricing model. They suggested augmenting their three-factor model with two additional factors, namely

[1] This study is the final result of a joint effort; however, we can attribute "Multifactor strategy and related literature" section to Claudio Boido and sections "Portfolio selection with factors", "Data set", "Empirical results", and "Conclusion" to Antonio Fasano.

robust minus weak profitability (RMW) and conservative minus aggressive investments (CMA). This new five-factor model significantly raises the bar for new anomalies. Even if Fama-French increased the number of factors compared to their previous model, the new five-factor model is unable to explain the momentum premium and continues to ignore it. Beyond the ordinary market factor, the five factors used are (a) RMW, the difference between the returns on diversified portfolios of stocks with robust and weak profitability, (b) CMA, the difference between the returns on diversified portfolios of the stocks of low and high investment firms, which, respectively, we call conservative and aggressive (c) the Size factor, SMB, the average of the small stock portfolio returns minus the average of the big stock portfolio returns, (d) the value factor HML, the average of the high B/M (book/market value) portfolio returns minus the average of the low B/M.

Another factor raising a growing interest is the investor sentiment, which has been studied by Baker and Wurgler (2006). They studied the link between sentiment and factors, such as, size, volatility, dividend yield, growth and profitability. In fact, when sentiment is low, they highlight higher returns for the stocks with the following features: small, young, high-volatility, unprofitable, non-dividend-paying extreme growth and distressed. Their claim is that investors tend to avoid these stocks if their sentiment is low.

Other authors (cf. Harvey et al. 2016 and Pukthuanthong and Roll 2014) advise some criteria for qualifying factors. In addition, Hsu et al. (2015) suggest three steps to understand the robustness of a factor premium: (1) economic underpinnings and persistence in numerous research papers; (2) the stability of the factor effect and the statistical significant level in most countries and (3) the factor effect should survive "reasonable perturbations" in the definitions of the factor strategy.

The academic term "factor investing" is often spelled in business contexts as smart beta. It is a strategy between passive and active management and it aims at a better risk/return trade-off, using factor properties. It is mainly used among investment practitioners and it is similar to a "marketing trick" to push the sale of factor-based investment funds. A further way of addressing these investment approaches is risk premia investing. All these terms can have a unique interpretation key, that is investors try to identify different sources of return and then their goal is to make the correct factor weighting. There is a new tendency to discover smart betas and to position them in the innovative financial products market. This choice helps the active managers to offer superior returns and to justify the grade of activism with higher performance fees. Some researchers define smart beta products as a disruptive innovation in asset management because the managers create new factors which do not remain stable over a period of time.

After being applied for a long time, and enjoying popularity among both academics and practitioners, in the current phase, factor investing is moving from the revolution of its inception to a sensible evolution driven by changes in capital markets and investors' awareness. We can identify three main evolutionary trends. First, we witness the application of the concept to new contexts beyond typical investment portfolios. Secondly, new alternative data sources and new technologies to process them make it possible to refine the scouting techniques and identify further explicative factors. Thirdly, just as markets and models describing them have changed, so have the investors. On one side, their increased financial education allows them to embrace new active investment views; on the other, they want to align their investment objectives with their social concerns. (cf. Melas 2021)

With respect to the broader scope of factor investing, it is worth mentioning the recent study of Henke et al. (2020). The authors identify a new thematic area, which they subsequently named "credit factor investing" [see also their previous analysis in Heckel et al. (2019)]. They pinpoint five factors, "Value", "Equity Momentum", "Carry", "Quality", and "Size", which better characterise bond portfolios. Specifically, the carry is the return of a future written on a bond, under the assumption of constant prices. With this approach, the authors prove the existence of factor premia in the bond market with a twofold structure: that is, factors depending on option-adjusted spread do not give significant excess return benefits for high yield bonds, while investment grade premia are positively affected by all factors identified. They also show that multifactor strategies, if correctly implemented, offer more value than single factor ones. Henke et al. (2020) continue the study with a more pragmatic tack, by introducing benchmarks to measure the performance improvement and find that factor signals can be realistically exploited to obtain a positive information ratio. Again there is a difference between investment and speculative grade, the latter receiving the most benefit from the factor strategy.

Regarding the role of technological innovation, we should focus, in particular, on recent advances in machine learning, which had a positive spillover in the investment science. The technicalities concerning ML are beyond the scope of this study, anyway we recall that ML can be considered as an applied branch of artificial intelligence, specifically, it makes use of probability theory to learn from data in order to make predictions. At its core, traditional factor analysis consists of regression-based inference. On the contrary, predictive patterns achieved through ML are model-free, in that the researcher does not need to understand which is the underlying model governing the process. By defining utility functions, enhanced in terms of empirical risk minimization, they can identify the optimal approach to decision making under uncertainty. (see Murphy 2022). These qualifying features are both a strength and a weakness for these techniques, and they result in traditional model-based approaches not being entirely replaceable. In a typical financial application, both inference and prediction add value to the research output. Indeed, an academic like a practitioner need to know not only what will happen, but also why it will happen, whether they are studying portfolio returns or diseases.

By means of machine learning, Lanza et al. (2020) rebuild Fama-French five-factor model, integrating the original factors with company scores based on socially responsible indicators (of which we will say more ahead). In the language of ML, the scores act as features used to classify portfolios. According to the authors, the use of machine learning improves the risk-return profiles of the portfolios, when compared to the traditional Fama-French model.

A further important reference for the field is given by Coqueret and Guida (2020). Rather than posing as a single application study, this work is intended as a comprehensive review of state-of-the-art methods, with applications, regarding the use of ML in factor investing. It stresses a diversity of perspectives when moving from traditional factor analyses to ML-based analyses. For example, the authors discuss the differences in terms of computability and performance between linear factors and firm characteristics and the different techniques to tackle the complexity of big data sets.

Machine learning is a relatively new topic in finance, and its applications to factor investing are still in their infancy. We believe that the very nature of factor investing, in particular for its fundamental data component, requires a computational effort in mining the optimal features which could be addressed by learn-from-data techniques offered by ML. Wherefore, we can only expect a boost in these methods in the years to come.

A further quest for change originates from the demand side of financial products and services. Nowadays, most consumers practice sustainability-driven behaviour, in accordance with their personal beliefs and to boost their moral self-esteem (cf. Trudel 2019). These emotional connections are germane to the purchasing patterns and preferences of the economic agent consuming financial products. It is well documented how investors recognize the value in products and institutions looking as sustainable or undertaking sustainable initiatives. In this regard, Cunha et al. (2021) make a comprehensive review of the literature on sustainable finance and investment (SFI), trying to identify in a rigorous way which are the differences between traditional finance and SFI. They conclude that, despite the substantial academic output, there is still an "under-theorization" of the SFI concept, which makes it difficult to measure the economic impact of sustainability.

As regards the scope of this study, SFI is not only relevant for factor investing but is recently evolving as a valuable applied niche for the concept. To clarify why and how these fields are interconnected, it is paramount to note that a critical instrument to address sustainability concerns consists in the development of sustainability indices, acting as performance benchmarks or explanatory variables in analytical models. At present, there is a sheer number of indices available to measure sustainability at various levels, for example, global or regional, sector or industry-specific, and provided by government agencies and private companies; in particular, established financial data providers are now specialising in sustainability indices. In a critical review of methodologies for the construction and computation of sustainability indices, Kwatra et al. (2020) note that over 500 sustainability indices have been identified. As it follows, the importance of SFI and its benchmarks lies in that this broad base of financial indices can act as a direct input for factor models. More conventional macro and fundamental factors are recently integrated with sustainability factors, hence building the foundations for a sustainable factor investing. By applying the principles of factor investing, Fan and Michalski (2020) find evidence that, for an Australian equity sample, by tilting portfolios towards more sustainable allocations, there is an increase in the Sharpe ratio and the crash risk profile. The latter is the portfolio risk given a market crash, which adds to the normal market risk (see for example Zhu et al. 2020). The authors focus in particular on the global financial crisis that occurred from 2008 to 2009. However, they stress that a naïve construction of a sustainable portfolio will not produce any positive outcome, instead the ethical stretch needs to be combined in a joint framework with quality or momentum factors.

Given their broad scope of application, the sustainable indices are normally grouped into three standard categories: Environment (including pollution, climate, natural resource conservation), Social (dealing with workplace conditions, unethical corporate practices and other stakeholder issues) and Governance (including accountability, diversity and gender issues). They are addressed for short as ESG. Naffa and Fain (2022) apply the standard Fama-MacBeth regression, which is a technique to measure the explanatory power of Fama-French factors, to a portfolios based entirely on ESG factors, and find no evidence that Fama-French model can be beaten in terms of alpha generation. To this end, they identify five categories for rating portfolios, ESG "leaders", "followers", "loungers", "laggards" and "not rated". A further test is then run to check if the ESG factors can be used to integrate the five Fama-French factors, and again they find no positive evidence. Therefore, the authors conclude that their results confirm the Fama neutrality argument.

ESG factors are also analysed according to the fund manager's investment strategies. Carlsson Hauff and Nilsson (2022) investigate how the different strategies affect investors' satisfaction and perceived fund quality. To this end, the authors identify an inclusion, exclusion and engagement strategy. The exclusion strategy is, historically, the first noteworthy approach to the notion of responsible investing, and consists in avoiding investments in those companies or sectors perceived as harmful or unethical, such as the weapons or tobacco industry. On the contrary, the inclusion strategy substitutes a positive screen for a negative one, therefore selected companies have to meet minimum ESG standards. Finally, the in-last strategy managers, in their quality of shareholders, seek to actively engage the companies to achieve specific ESG objectives. By means of a web survey, administered to 261 participants, the authors document a strong preference for the inclusion strategy. With a further survey, administered to 437 participants, they show that funds' in-house sustainability experts positively affect the perceived fund quality. Surveys are conducted in Sweden, this is relevant since the country has a long history of mutual funds and the Swedish have familiarity with choosing them.

An innovative work, Bril et al. (2022), tries to analyse the intersection of the technological innovations, occasionally disruptive, that we have mentioned above, with the sustainability concerns. While it is a theoretical study, not culminating into actionable models, it is worth mentioning for the fresh perspective it brings to the subject matter. The authors propose three ways to connect sustainable finance and technological innovation, which they stylise with ESG *and/through/as* technology The "ESG and technology" combination implies that thanks to recent innovations companies have obtained an increased ability to go green, without making their business unprofitable and with fewer costs, or simply in ways not achievable before. For example, we could dramatically reduce packaging-related waste with QR codes, linking them to product information or ingredients, perhaps improving the consumer experience through videos. "ESG through technology" hints at the advances in machine learning and artificial intelligence, opening new opportunities for investors to integrate ESG in portfolio construction. Recent innovations in technology make it possible to obtain a level of refinement in factor identification unachievable with traditional models. At the same time, technology provides new sources of sophisticated and unstructured data, such as social network data, or mobile phone data. If wisely (and ethically) used, these sources can give a huge boost to current models. "ESG as technology" proposes to consider the technology as the fourth dimension of sustainability, therefore reshaped as ESGT. In fact, with the inception of applied artificial intelligence, with the emergence of remote work, and the other disruptive innovations, technology is going to pose new ethical challenges to match as for their urgency and human impact the ESG concerns. Embracing this perspective means investment professionals and academics need to add the "T"-factor among their factor investing tools as well.

The remainder of the paper is organised as follows. Section 2 summarises the theoretical background and the proposed factor model, using a behavioural approach. Section 3 details the data used for the empirical analysis. Section 4 reports and discusses the empirical results. Section 5 provides a summary and some concluding remarks.

Portfolio selection with factors

Given the sheer number of factor studies available, portfolio managers have a multitude of factors to choose from, which can affect their portfolio returns. The numerous macroeconomic, fundamental, and statistical factors are derived from estimation studies, which validate their explanatory powers with regard to portfolio returns. Also, these recent regression-based approaches prescind the more traditional mean-variance optimisation framework and one might argue that, despite its strong theoretical background, the standard Markowitz optimisation is limited in trying to explain portfolio returns only by means of one market factor: the historical portfolio volatility.

It is however clear that the asset allocation process could benefit from combining estimation and optimisation (cf. Kim et al. 2017). A manager should be able to tilt their portfolio on the basis of their factor expectations, without giving up to proven optimisation procedures. To this end, we hereafter propose a factor-enhanced optimisation, leveraging a specialised mean-variance utility function with penalties to optimal solutions deviating from factor constraints set by managers.

In order to introduce our unified optimisation model, we recall the main tenets of the Markowitzian framework, where investors are risk averse, and therefore, given a desired and feasible level of portfolio return, the rational investor selects the portfolio with the lowest portfolio variance from among all those generating that expected return. Formally, this is equivalent to solve the problem:

$$
\begin{aligned}
&\min_{w} \quad \sigma_R^2 = w'\mathbf{S}w \\
&\text{sub} \\
&\qquad w'\bar{r} = \mu, \;\; \mu \in \mathbb{R} \\
&\qquad w'\mathbf{1} = 1
\end{aligned}
\tag{1}
$$

where σ_R^2 is the variance of the portfolio return R; given r_i and w_i, denoting resp. the return and weight of the i-th security, w is the weight vector $(w_i)_i$; $\bar{r}$ is the expected return vector $(\bar{r}_i)_i$, with $\bar{r}_i := \mathbb{E}r_i$; $\mathbf{S}$ is the variance-covariance matrix $(\sigma_{ij})_{ij}$, with σ_{ij} denoting the covariance of the return of i-th and j-th security; the scalar μ denotes the investor desired expected portfolio return.

Without short-sale constraints (i.e. allowing $w_i < 0$), the problem (1) has a simple analytical solution:

$$w^* = \mathbf{S}^{-1} \frac{(\mu c - b)\bar{r} + (a - \mu b)\mathbf{1}}{ac - b^2} \tag{2}$$

where

$$\begin{aligned} a &= \bar{r}'\mathbf{S}^{-1}\bar{r} \\ b &= \bar{r}'\mathbf{S}^{-1}\mathbf{1} \\ c &= \mathbf{1}'\mathbf{S}^{-1}\mathbf{1} \end{aligned}$$

which are all scalars.

In order for a solution to exist, we need to be able to find the inverse matrix $\mathbf{S}^{-1}$. This is possible if $\mathbf{S}$ is non-singular and this essentially requires that none of the security returns are perfectly correlated and that there is no riskless security (implying zero correlations and variance). Given the solution, w^*, the related minimum variance is

$$w^{*\prime}\mathbf{S}w^* = \frac{\mu^2 c - 2\mu b + a}{ac - b^2}$$

Problem (1) can be generalised, in the broader context of the Expected Utility theory, using the *mean-variance* utility (cf. Nakamura 2015):

$$U(R) := \mathbb{E}R - \tau\sigma_R^2 \tag{3}$$

where $\tau > 0$ denotes the investor risk aversion. Model (3) can be considered an approximation for the case when the investor utility is quadratic (cf. Collins and Gbur 1991). The related optimisation problem is now

$$\max_w \quad \mathbb{E}w'\bar{r} - \tau w'\mathbf{S}w \tag{4}$$

The utility function in (3) can be enhanced with a factor component, measuring investor willingness to track, up to a certain degree, one or more factors (cf. Bergeron et al. 2018):

$$U_f(R) := \mathbb{E}R - \tau\sigma_R^2 - \lambda D \tag{5}$$

Here, D measures the deviation of portfolio returns from a single benchmark factor, or a bundle of factors, relevant to the investor and $\lambda > 0$ is the degree of deviation aversion. For factor bundles, we assign factor weights much like an equity index. The weight sign indicates a negative/positive exposure to the underlying risk factor.

The maximisation program for model (5) is

$$\max_w \quad \mathbb{E}w'\bar{r} - \tau w'\mathbf{S}w - \lambda T(w) \tag{6}$$

where $T(w)$ measures the factor deviation in terms of tracking error of the portfolio, identified by the weight vector w, from the factor index, taken as a benchmark, this is equivalent to $\mathsf{var}(\mathsf{R}w' - F)$, where R is the $T \times N$ return matrix and F is the factor index.

Model (6) can be further extended with behavioural decision-making additions. To build up the behavioural framework, we assume the investor identifies a reference point and makes up their preferences with regard to the reference value, considering lower outcomes as losses and greater ones as gains. Also, we assume the investor has the Markowitzian risk aversion only in case of gains, while they are risk seeker in case of losses.

The reference point notion is originally introduced by Kahneman and Tversky (1979) in the context of their prospect theory of decision making. In the original use, it is a generic psychologically measure affecting the perception of gains and losses. Here, we imagine the investor chooses a reference asset or market index to which they compare the absolute portfolio return. We also add volatility to the menu, so that the investor will perceive the overall portfolio risk in terms of the volatility of reference assets.

Formally the new expected utility, hereafter named behavioural-factor utility, is

$$V(R) := \begin{cases} \mathbb{E}R - \mathbb{E}B - \tau(\sigma_R^2 - \sigma_B^2) - \lambda D, & \text{if } \mathbb{E}R \geq \mathbb{E}B \\ \mathbb{E}R - \mathbb{E}B + \tau(\sigma_R^2 - \sigma_B^2) + \lambda D, & \text{if } \mathbb{E}R < \mathbb{E}B \end{cases} \tag{7}$$

where B is the reference asset or index return.

Model (7) captures the “catching up with the Joneses” preferences, as in Chan and Kogan (2001), where the reference point is the minimum social standard, and which makes the investor risk seeker, when they lie below their aspiration threshold. This model is optimised with

$$\max_{w} \ \mathbb{E}w'\bar{r} - \mathbb{E}B - s\tau(w'\mathbf{S}w - \sigma_B^2) - \lambda T(w) \tag{8}$$

where

$$s = \operatorname{sgn}(\mathbb{E}w'\bar{r} - \mathbb{E}B)$$

$\operatorname{sgn}(x)$ is the sign function, that is, $\frac{x}{|x|}$ for $x \neq 0$, and 0 otherwise.

Data set

To discuss our methodology from Sect. 2, we consider an asset allocation problem consisting of standard asset classes:

- Domestic Equity,
- Foreign Equity (Developed Markets),
- Foreign Equity (Emerging Markets),
- Domestic Bonds,
- Sovereign Debt,
- Money Market Instruments,
- Commodities.

Our study adopts a European perspective, and therefore, the assets above are respectively proxied with the MSCI EMU Total Return Index, the S&P 500 Total Return Index, the S&P BRIC 40 Total Return Index, the EURO STOXX 50 Corporate Bond Total Return Index, the Markit iBoxx Eurozone Sovereigns Quality Weighted Index, the EURIBOR one-week spot rate and the S&P GSCI Total Return Index.

Markit iBoxx Eurozone Sovereigns Quality Weighted Index is designed to reflect the quality of sovereign debt by allocating higher weights to countries with solid fundamentals and reducing weights to those with weak fundamentals.

The GSCI is a global commodity index with high exposure to the energy sector when compared to similar indices.

Data used are sourced from Refinitiv (former Thomson Reuters). For equity-based data, we follow (Faff 2003) and use total return indices, measuring market performance with the income from constituent dividend payments. Price data are denominated in euros.

We also use two macro-factors: the euro area GDP and inflation, more specifically the Eurozone GDP and the Eurozone CPI denominated in euros.

As times series have unequal depth, they are cut to the shortest one, that is the EURO STOXX 50 Corporate Bond Total Return Index , which begins from 31 December 2010. The overall common date range is 2010–12–31/2022–09–30. Financial data frequency is aligned to quarterly macroeconomic data.

The use of total returns, discussed above, is also intended to make the model output aligned with investable portfolios. Indeed, for the sake of comparability with similar studies, we adopt standard asset classes and standard proxies for them, but the theoretical framework is agnostic to asset and factor selection. Clearly, a practitioner willing to implement our theoretical model would find it convenient to track the indices via ETFs or index funds and likewise would select the asset bundle specific to their investment environment and the factors consistent with their risk views.

Historical analysis

Table 1 presents the expected returns and standard deviations of asset classes and factors, and Table 2 shows the related correlations. Table 1 shows the overwhelmingly dominant risk-return profile of US equities when compared to other equity indices. We also observe the negative commodities performance, which makes them unfit for long portfolios.

To obtain more details on historical performances, Figs. 1 and 2 show historical returns of equity indices and bond indices, respectively. With the exception of the global crisis and the pandemic, we observe the most extreme values with regard to emerging market equity returns. However, as it is evident from Table 1, this is not compensated by returns in excess of developed markets. European stock market has been particularly affected by the COVID recession, when compared to the more resilient US market, and a similar weakness is shown by the global commodities. In general, all European indices are affected by debt crises of the euro zone, e.g. 2011 and 2015

Table 1 Expected returns, standard deviations for classes and factors

Asset or factor	Expected return (%)	SD (%)
EMU equities	5.47	17.90
US equities	13.57	14.96
BRIC equities	1.92	19.24
EMU corporate bonds	1.76	4.37
EMU government bonds	1.82	5.31
EMU money market	– 0.55	0.91
Commodities	– 0.56	27.02
Eurozone GDP	1.13	5.20
Eurozone CPI	1.95	1.82

Figure 3 further compares historical returns of our commodity index and the money market index, and Fig. 4 compares annual growth of GDP with the Consumer Price Index in the eurozone. We observe the money index is always positive before 2015 and negative since then (with exception of the last available quarter). GDP and CPI appear usually correlated, except in the 12-months span starting the third quarter 2011.

While it is too early to find robust and conclusive evidence in the literature, it does not go unnoticed that all indices plunge at the end of the first quarter of 2022 as a likely consequence of the Russian invasion of Ukraine, with the notable exception of commodities and, to a lesser degree, the CPI as regards factors. Given the geopolitical role played by energy, in interpreting the huge commodity spike following the inception of the war (see Q1 2022 in Fig. 3), it bears recalling that the GSCI index has a large exposure to the energy commodities; currently a 54% weight is given to this sector. The visual clues are in line with Fang and Shao (2022), who propose an index to measure the geopolitical risk of the Russia-Ukraine conflict and find that the latter has determined a significant volatility increase in commodity markets.

To assess the empirical distributions of asset classes, Fig. 5 shows the violin plots for each of them. These plots add to both sides of a standard box plot (white), the rotated kernel density of the distribution (azure). To make the comparison of the shapes meaningful, in the case of the money market, the plot is zoomed using a factor of ten for related returns.

With regard to equities, we observe that U.S. returns present the most stable distribution, without relevant extreme values. Bonds emphasise a bimodality, particular with respect to treasuries. Commodities returns exhibit a similar bimodal shape and with significant negative extreme values.

Empirical results

We start by identifying the optimal asset allocation for a given target return, assuming there are no-short sale constraints. Table 3 shows the optimal weights, setting an annual target return which is three times the average asset class return,

Table 2 Correlations among classes and factors

Asset or factor	\	a	b	c	d	e	f	g	h	i
EMU equities	a	1.00	0.79	0.75	0.55	0.11	− 0.13	0.48	− 0.08	− 0.08
U.S. equities	b	0.79	1.00	0.58	0.51	0.15	− 0.06	0.52	− 0.09	− 0.09
BRIC equities	c	0.75	0.58	1.00	0.44	0.08	− 0.20	0.37	− 0.07	− 0.11
EMU corporate bonds	d	0.55	0.51	0.44	1.00	0.77	0.20	0.11	− 0.11	− 0.45
EMU government bonds	e	0.11	0.15	0.08	0.77	1.00	0.22	− 0.28	− 0.05	− 0.62
EMU money market	f	− 0.13	− 0.06	− 0.20	0.20	0.22	1.00	− 0.14	− 0.04	− 0.00
Commodities	g	0.48	0.52	0.37	0.11	− 0.28	− 0.14	1.00	0.08	0.33
Eurozone GDP	h	− 0.08	− 0.09	− 0.07	− 0.11	− 0.05	− 0.04	0.08	1.00	− 0.07
Eurozone CPI	i	− 0.08	− 0.09	− 0.11	− 0.45	− 0.62	− 0.00	0.33	− 0.07	1.00

Labels ‘a’ through ‘b’ denote the asset and factor names in the leftmost columns. Thus, for example, the value 0.79 at the intersection of ‘a’ and ‘b’ represents the correlation between European equities and US equities

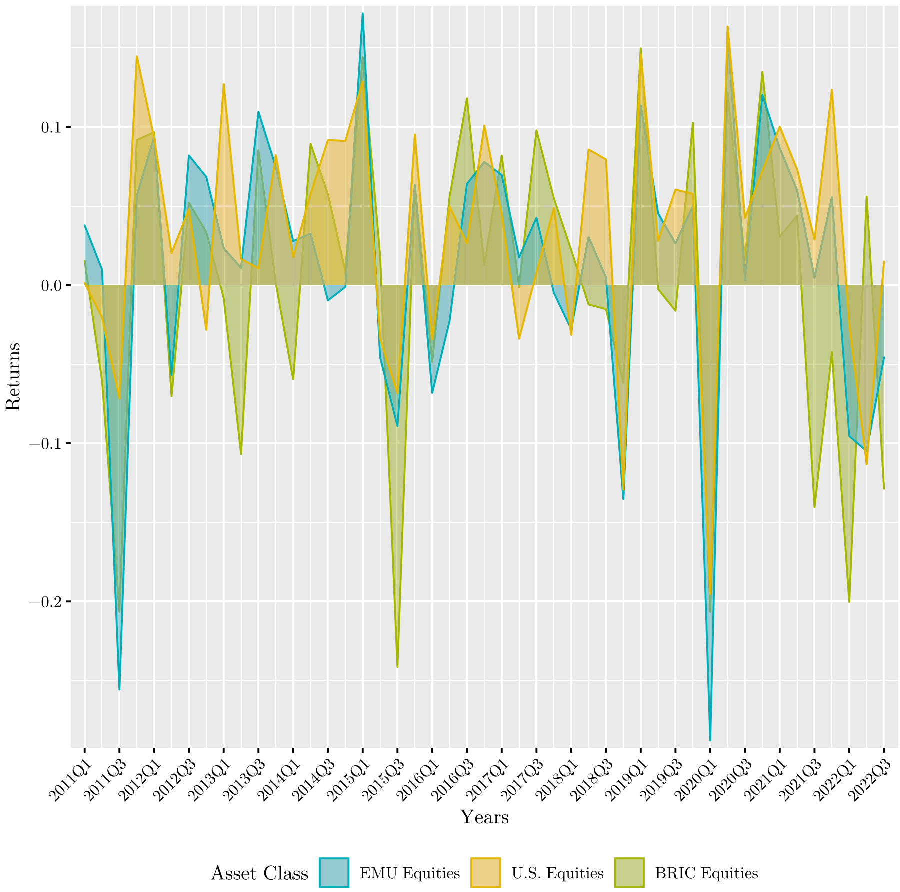

Fig. 1 MSCI EMU Total Return Index, S&P 500 Total Return Index, and S&P BRIC 40 Total Return Index: comparison of returns

that is approx. 10%. As noted by Jagannathan and Ma (2002), the lack of short-sale constraints involves taking extreme long and short positions. According to Green and Hollifield (1992), extreme values are due to the dominance of a single factor in the covariance structure of returns, and the consequent high correlation between naively diversified portfolios.

The same optimisation is implemented in Table 4, but excluding those portfolios implying short-selling ($w_i < 0$). Both solutions (with and without short-selling) bear a risk below the average; however, we see that the latter, by reducing the opportunity set, implies a slightly higher standard deviation.

The optimisation procedure is presented graphically in Fig. 6, where we plot the portfolio envelope with short-selling. The plot also presents the cases (labelled with the letters ‘a’ through ‘g’) where all wealth is invested in a single asset class. This is useful to catch graphically the relative risk-return profile of each asset class proxy.

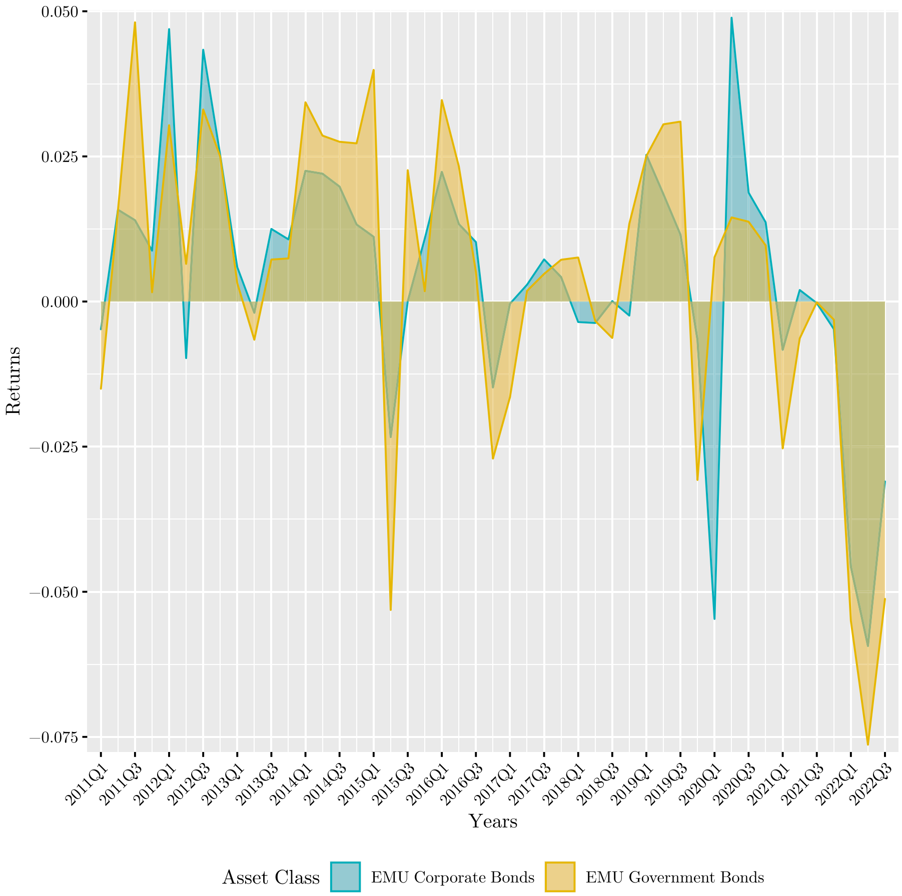

Fig. 2 EURO STOXX 50 Corporate Bond Total Return Index and Markit iBoxx Eurozone Sovereigns Quality Weighted Index: comparison of returns

A well-known problem of mean-variance analysis is it being highly sensitive to small changes to its inputs, the return sample moments; as a consequence, sampling errors can lead to significant misallocations, cf. (Ang 2014). Hence, the pun at the theory of mean–variance optimised portfolios as being indeed "estimation-error maximizers" portfolios, coined by Michaud (1989). The phenomenon is particularly evident when portfolio assets are close substitutes for one another (cf. Kritzman 2006).

To avoid this problem, we resort to robust statistics methods, which can significantly improve the estimates of the M–V optimisation inputs. Specifically, Maronna and Zamar (2002) location and scatter orthogonalised Gnanadesikan-Kettenring (OGK) estimator is used.

Figure 7 compares the frontier based on the standard mean–variance sample estimates and on the OGK estimators, emphasising substantial differences.

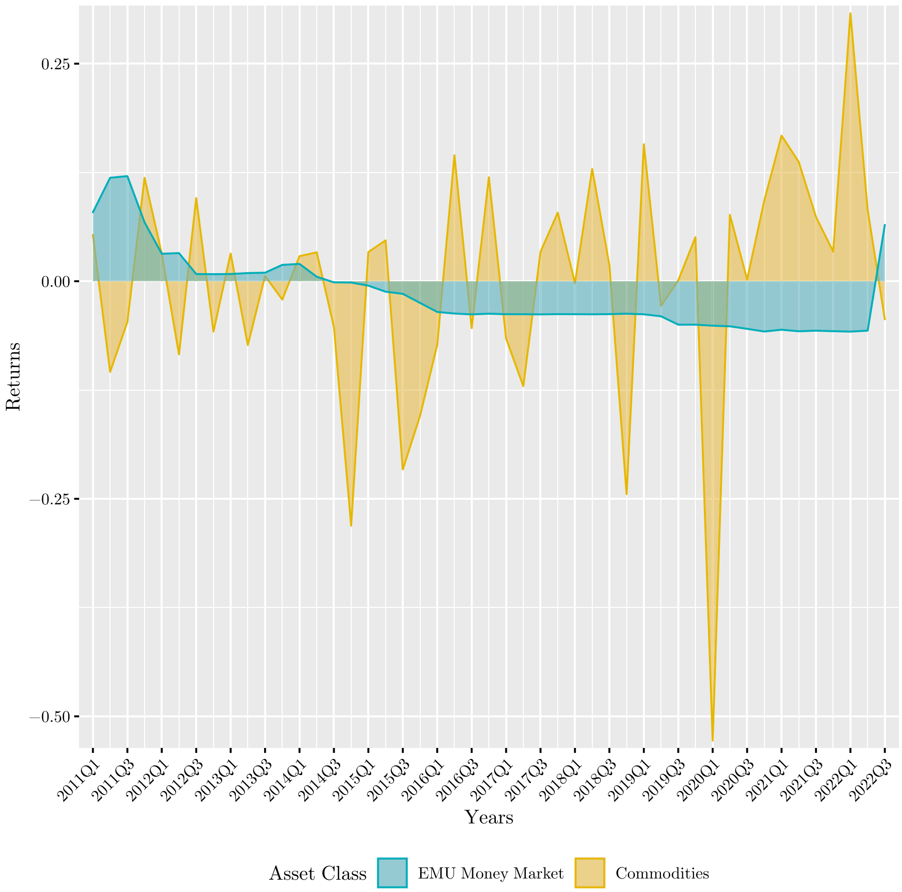

Fig. 3 S&P GSCI Total Return Index and EURIBOR 1-week spot rate: comparison of returns. The latter are multiplied by a factor of ten

Before applying the model (7), we make a visual assessment of the functional form proposed in a two-dimensional choice context. With two assets, for each return level, there exists a unique generating weight vector and so a unique utility level. Therefore, it is straightforward to plot the utility levels as the portfolio returns change. To this end, we choose to select between the Markit iBoxx Eurozone Sovereigns Quality Weighted Index and MSCI EMU Total Return Index, and we set the reference point with respect to the corporate bonds, assuming the investor targets twice the corporate bond yield (expected return). The resulting chart is plotted in Fig. 8, setting $\tau = 5, \lambda = 0$.

Because, here, the reference asset does not belong to the portfolio, we identify a portfolio giving the same expected return as the investor's reference point and set its variance as the reference variance. This is to say that, if u is the investor's reference point, the reference variance is $w_u' \mathbf{S} w_u$, where w_u is the two-asset portfolio weight vector such that $w_u' \bar{r} = u$. In this way, we avoid a discontinuity at u in the curve, due

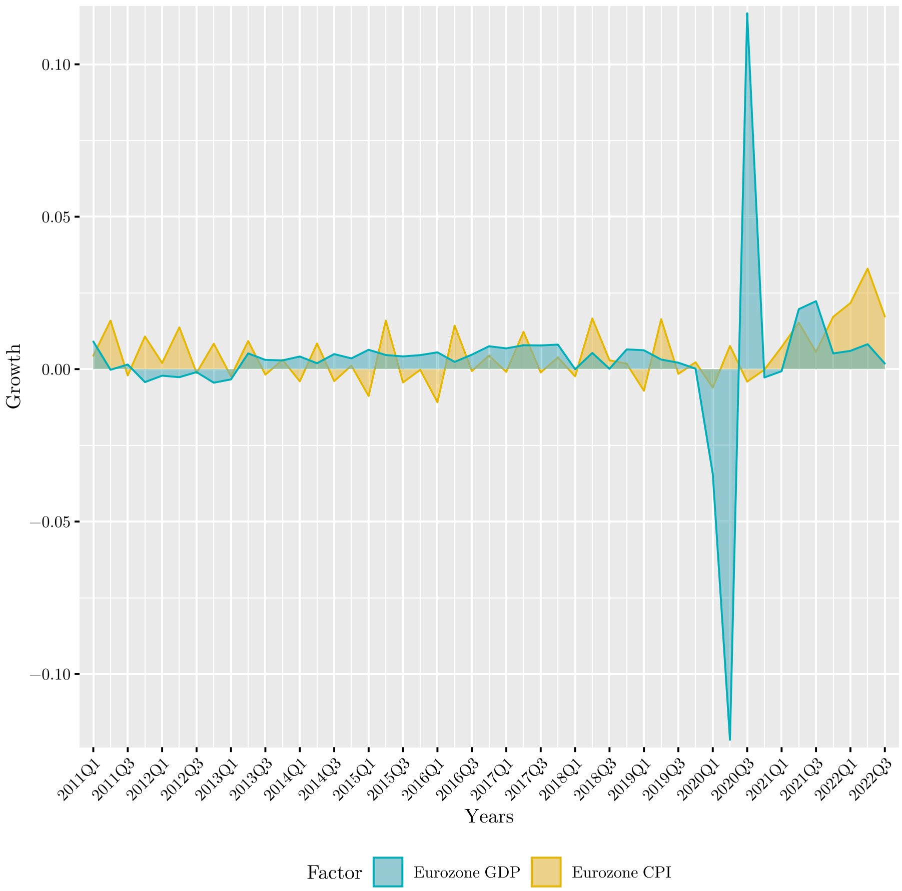

Fig. 4 Eurozone GDP and Eurozone CPI. Annual growth

to taking a reference point outside the envelope. This is not necessary when implementing the optimisation procedures. The vertical bar in Fig. 8 is set to identify the reference point. We observe a change of the curvature connected with the change of investor's risk attitude for returns below the reference one (perceived as losses) and above (perceived as gains). In the first instance, the investor is risk seeking, in the second is risk averse. This behaviour is consistent with the S-shaped value function of prospect theory. If we pretend the portfolio volatility is much higher, so is the S-effect, because the risk-return trade-off increases the need to take risks to overcome the losses. Similarly, we observe a more noticeable curvature in the plot, if the τ-aversion increases, because the variance effect is amplified.

When implementing the optimisation, the parameters τ and λ act as penalty levels, which capture the decision maker risk aversion and preference for the underlying portfolio factors. There is, in fact, no closed-form solution for the factor model (8); the maximisation is only possible by means of numerical optimisation,

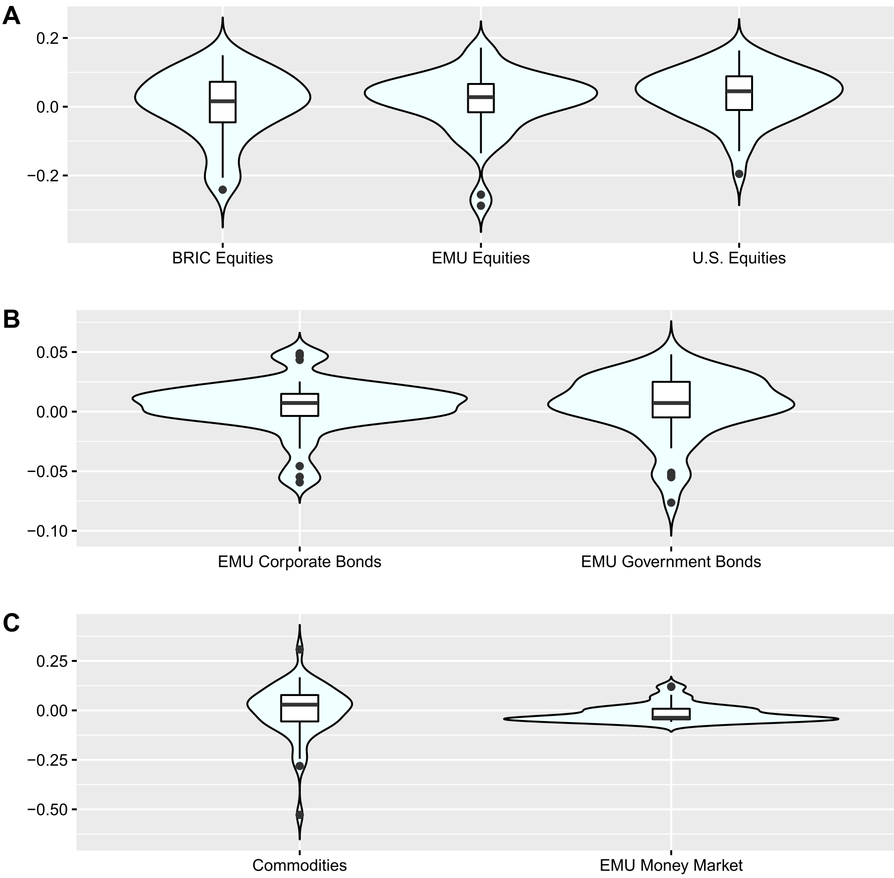

Fig. 5 Violin plots of asset class returns. To improve the visualisation of the otherwise too tiny shape, EMU Money Market returns are multiplied by a factor of ten. Violin plots consist of box-and-whiskers plots of return distributions (white-filled) and related kernel densities (azure-filled), rotated and added on both sides of the box plot

which, to a degree depending on the set level of τ and λ, will prefer portfolios with less variance and more factor tilting and discard the others.

The best results were found by applying evolutionary algorithms or metaheuristic optimisation algorithms. In particular, three approaches were used in parallel and the best result (highest utility value) automatically chosen. These are as follows:

– Fitness function stochastic maximization, using genetic algorithms, cf. (Sivanandam and Deepa 2007);
– Differential evolution algorithm for global optimisation by Price et al. (2006) and Ardia et al. (2011);
– Zambrano-Bigiarini et al. (2013) Particle Swarm Optimisation.

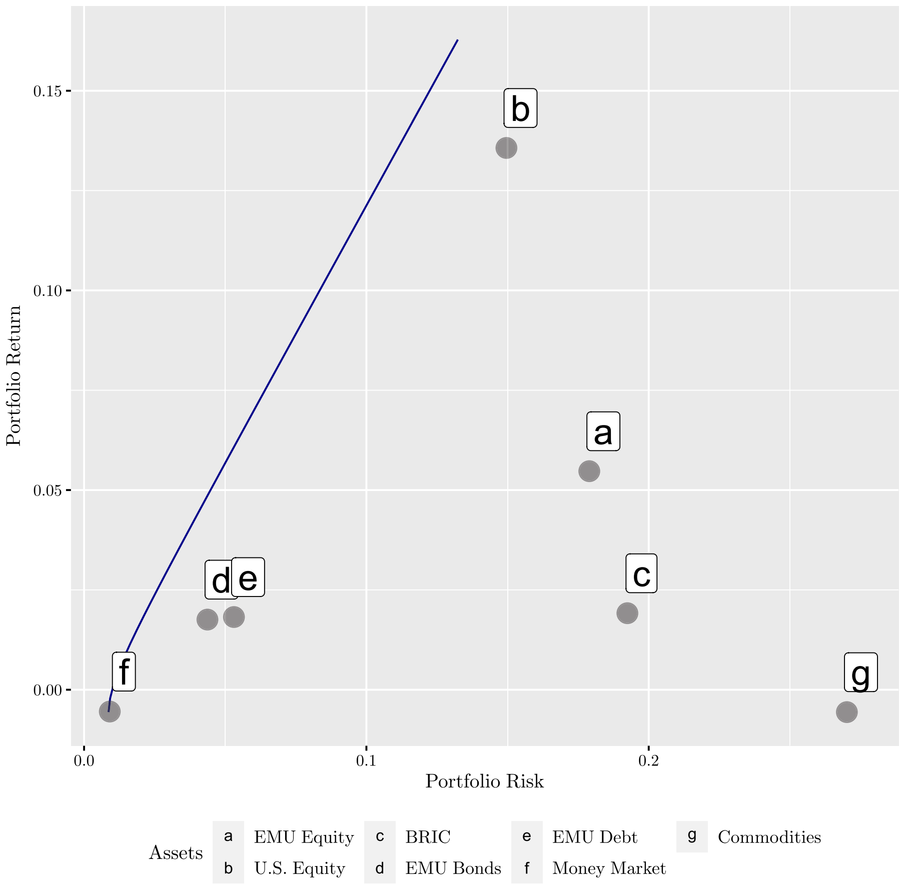

Fig. 6 Portfolio envelope consisting of global commodities, Euro zone shares, government and bonds, and BRIC countries, with risk/return of individual asset classes. Full Names: **a** EMU equities, **b** US equities, **c** BRIC equities, **d** = EMU corporate bonds, **e** EMU government bonds, **f** EMU money market, **g** Commodities

Table 3 Traditional asset class allocation for a 10% target return without short-sale constraints

Asset class	Weight (%)
EMU equities	– 34.48
US equities	85.23
BRIC equities	– 7.18
EMU corporate bonds	47.65
EMU government bonds	– 12.26
EMU money market	33.74
Commodities	– 12.70
Expected return	10.04
Standard deviation	8.38

Table 4 Traditional asset class allocation for a 10% target return with short-sale constraints

Asset class	Weight (%)
EMU equities	0.00
US equities	69.97
BRIC equities	0.00
EMU corporate bonds	0.00
EMU government bonds	30.03
EMU money market	0.00
Commodities	0.00
Expected return	10.04
Standard deviation	10.82

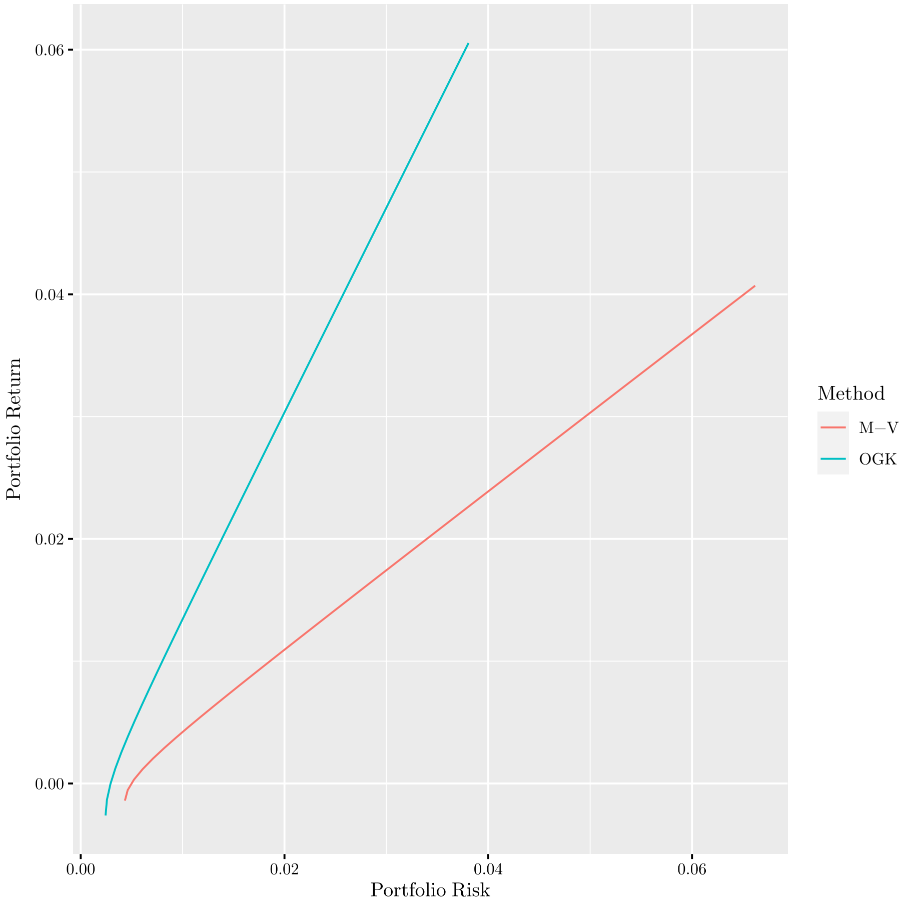

Fig. 7 Portfolio envelope consisting of global commodities, Euro zone shares, government and bonds, and BRIC countries, calculated with the standard mean–variance (M–V) ‘plug-in’ method, based on sample estimates, and with the robust estimation based on the Orthogonalised Gnanadesikan-Kettenring method (OGK)

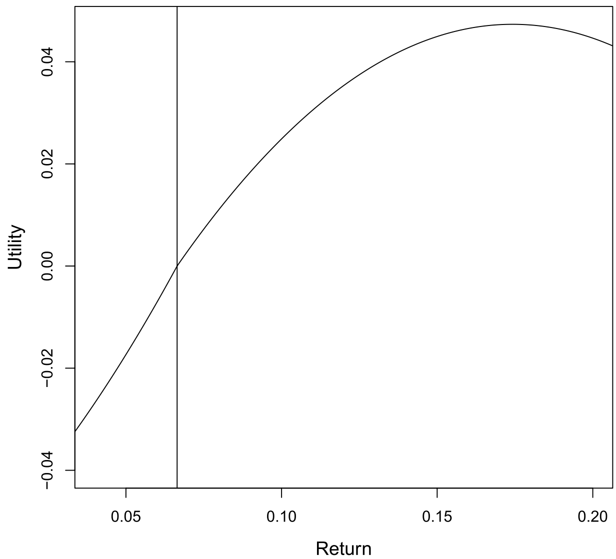

Fig. 8 Behavioural-factor utility with respect to Markit iBoxx Eurozone Sovereigns Quality Weighted Index and MSCI EMU Total Return Index, using as the reference point twice the corporate bond yield expected return. The vertical bar identifies the reference point. To avoid a discontinuity, we identify a portfolio giving the same expected return as the investor's reference point and set its variance as the reference variance

Results were implemented in R language and the following libraries were employed: "nloptr", "GA", "RcppDE", "hydroPSO".

In Table 5, we identify the optimal portfolio, using the *plain* model from (4) and using OGK estimators; while in Table 6, the full model is implemented, assuming that the management sets the macro-factor exposures to eurozone GDP/CPI as 50%/-100%. Also, we assume the investor risk-tolerance parameters to be $\tau = 15, \lambda = 10$, and we set as their reference asset class the corporate bonds, targeting twice the expected return of this class. We keep the no-short sales constraints, that is we require $w_i \geq 0$.

The plain utility model can be regarded as a refined version of the mean-variance model, whose output is given in Table 4, where we have direct control on the level of risk taken and let the model find for us the best risk-return profile, rather than just the minimising variance. In this case, the output shows a noticeable difference in terms of allocated weights. The portfolio generated is more diversified and has a better risk-adjusted performance (as we will properly document later). The diversification is affected by the risk-aversion parameter τ. Typically, to keep the utility maximised, the higher aversion to risk the higher the diversification level of the portfolio found by the optimisation algorithm.

The output Table 6 reflects the addition of the factor component to the investment utility and the other behavioural features proposed by (7). In this case, while we kept the same risk attitude τ, the generated solution further enlarges the invested assets, by tilting the allocation towards bonds. This happens because the macro-factors are

Table 5 Utility-based asset class allocation without factor exposures

Asset or factor	Weights (%)
EMU equities	21.07
US equities	56.52
BRIC equities	0.00
EMU corporate bonds	0.00
EMU government bonds	24.43
EMU money market	0.00
Commodities	0.00
Portfolio return	15.24
Portfolio standard deviation	6.09

Risk aversion $\tau = 15$

Table 6 Asset allocation with factor exposures

Asset or factor	Weights (%)
EMU equities	15.94
US equities	35.16
BRIC equities	0.00
EMU corporate bonds	18.80
EMU government bonds	30.17
EMU money market	0.00
Commodities	0.00
Eurozone GDP	50.00
Eurozone CPI	− 100.00
Portfolio return	10.99
Portfolio standard deviation	4.02

Eurozone GDP = 50%, Eurozone CPI = -100%, $\tau = 15$, $\lambda = 10$, with corporate bonds as the reference asset class

tied to the European economy, such as the portfolio bond indices, and because of the correlation of this asset class with the factors. Interestingly, the European tilt, while complying to managers' macro scenarios and perceived risk, does not affect considerably the estimated portfolio risk or return.

To make sense of the different models presented, in Table 7, we report each generated portfolio expected return, standard deviation and Sharpe ratio. For comparison, we add also the equally weighted portfolio. The riskfree rate used to compute the Sharpe ratio is based on the Bloomberg Euro Generic Government Bond 3 Month Index, which consists of generic Euro government bill and bond rates.

It should be stressed that the comparison is intended to give the reader a tangible perception of the location, in the risk-return space, of each portfolio generated. Indeed, the solutions proposed in this study do not mean to beat, in terms of efficiency or prospective returns, the traditional methods, but rather to adapt the asset allocation to the manager's (investor's) view regarding the factors they select, given

Table 7 A comparison of different optimising solutions

	Expected return (%)	SD (%)	Sharpe ratio
Equally weighted portfolio	3.35	9.58	0.38
Mean–variance with short sales	10.04	8.38	1.23
Mean–variance without short sales	10.04	10.82	0.95
M–V utility	10.99	4.02	2.80
BF utility	15.24	6.09	2.55

In the equally weighted portfolio, the same proportion of wealth is invested in each asset class. Mean–variance—with and without short sales—refers to the Markowitz optimisation procedure. The M–V utility is the utility-based version of the Markowitz optimisation, without short-selling. The BF utility includes the factor component and the behavioural enhancements

our alternative way to weigh the perceived losses. That being the case, we note that the utility-based procedures are more effective in terms of the Sharpe ratio obtained. This can be explained because, at their core, these methods look for the best portfolio return net of the risk, and this is also the underlying philosophy of the Sharpe ratio, which in fact characterises as a risk-adjusted performance measure.

Conclusion

This study implements a novel approach to factor investing. Traditionally, factor analysis explains asset returns by means of common macroeconomic or firm-characteristics factors. By its own nature, this typically involves statistical inference, where factors act as explanatory variables, returns as explained variables, and factor loadings are calculated as regression coefficients. Therefore, the traditional factor investing approach, where managers tilt their portfolios towards factors they want to be exposed to, contrasts with the mean–variance framework, where there is only one factor (the market factor) and managers' strategy consists in optimising the portfolios' risk-return profile.

To settle this dilemma, in this study, we combined factor investing, derived from estimation studies, with portfolio optimisation. Specifically, in order to enhance the Markowitzian optimisation with factors, we used a mean–variance utility distinguishing investors' risk attitude for losses and for gains (relative to a reference asset), and we added a penalty to deviations from manager chosen factors. This utilitarian approach is resemblant of the prospect theory in behavioural finance.

We applied this approach to a multiasset portfolio, consisting of equity, bonds and commodities. Given there is no possibility of analytical closed-form solution to the problem posed, the results were obtained by means of concurrent implementation of metaheuristic optimisation algorithms (i.e. genetic algorithms and evolution algorithms). Given the nature of the utility functions employed, the generated portfolios show good risk-adjusted performances, here measured with Sharpe ratios. Contrasting the results obtained with and without factor additions, we observe systematically different portfolios, in particular the procedure tilts the asset allocation, embodying managers underlying risk factors, without a significant efficiency loss in terms of reward to volatility.

Therefore, managers are significantly empowered with the possibility of incorporating their expectations about future factor evolutions into the portfolio optimisation problem.

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Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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