

On Ellsberg's commitment to dealing with the uncertainty of the real world

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This paper aims to explore what kind of ontological presuppositions can be ascribed to Daniel Ellsberg's understanding of decision-making in the real world. Contrary to what can be inferred from the famous 1961 *Quarterly Journal of Economics* article introducing the Ellsberg Paradox, an analysis of the then classified research that Ellsberg conducted at the RAND Corporation in the early 1960s shows that his interest in decision-making as a field of applied research led him to emphasise the state-space uncertainty of an unprecedented state of affairs. Ellsberg seemed determined to assert that the kind of uncertainty he was examining—the ability of a chain of command to cope with the event of a surprise nuclear attack in the unprecedented environment of a cold war—was of the highest severity. The paper argues that in such a context the analogy between Ellsberg and George Shackle is striking. The undesirable ontological implication of the assumption of a complete list of possible events evidenced by Shackle—that it is incompatible with surprise—is shared by Ellsberg.

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1. Introduction

An important branch of current decision theory has used Daniel Ellsberg's (1961b) notion of ambiguity to claim that the kind of uncertainty that has been treated in the mainstream since Leonard Savage's ([1954] 1972) axiomatisation of subjective expected utility theory is not comprehensive enough (Gilboa *et al.*, 2008). Ellsberg showed that the behavioural distinction between risk and uncertainty, negated within Savage's approach, can indeed emerge even in very simple thought experiments involving drawing coloured balls from urns. Actual laboratory experiments have largely confirmed that the standard so-called Bayesian line of reasoning—that decision-makers can always be

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represented as if having precise subjective probabilities that guide their decisions—fails in ambiguous decision contexts (Camerer and Weber, 1992).

However, this has led to studies that, while much broader in scope than usual, mostly conflate uncertainty into ambiguity: the uncertainty in question—though now clearly distinguished from risk—still concerns the occurrence of states of the world in an environment where the state space is given, and known to the decision-maker at the time of the decision (Gilboa and Marinacci, 2013). Moreover, the notion of ambiguity that originated this so-called ambiguity literature is essentially epistemic, in that it reflects the decision-maker's attitude towards an environment that can, in principle, be perfectly known. In both Ellsberg's original discussion of the risk–uncertainty dichotomy and the ensuing decision-theoretic literature, there appears to be no ontological commitment to a view of the real world characterised as an environment that cannot be perfectly known in advance because of the way it is structured. For the most part, the ambiguity literature has followed the Bayesian mainstream in dealing not with an uncertain environment, but with an environment about which the individual is uncertain.

In a recent paper, however, I have argued that an analysis of at-the-time classified research that Ellsberg conducted at the RAND Corporation in the early 1960s shows that his interest in decision-making as a field of applied research was of a very different kind (Zappia, 2021). In particular, in examining the ability of a chain of command to cope with the event of a surprise nuclear attack by United States adversaries, Ellsberg (1960) did not use the conventional framework of state uncertainty. Rather, he emphasized the impossibility of identifying a complete set of states, and focused on the uncertainty associated with a unique state of affairs. Ellsberg seemed determined to assert that the kind of uncertainty he was examining—originated by the unprecedented environment of a cold war—was of the utmost severity. The real events that condition the decision-making process in such cases can only be interpreted as irreducible to risk, because they involve not only what state of the world will manifest itself (state uncertainty), but the much more troublesome issues associated with possible unanticipated states (state-space uncertainty).

The distinction between state and state-space uncertainty is of particular relevance in the post Keynesian literature, which has argued that only the latter can adequately characterize 'true' uncertainty. The reference to a non-predetermined list of states of the world is a defining element of fundamental uncertainty (Dequech, 2011), and it has been used in particular to highlight the enduring significance of George Shackle's attempt to provide a non-probabilistic approach to decision theory under uncertainty. Shackle's (1949, 1955) main assumption—formulated in the same years in which Savage was proposing the opposite, soon to become mainstream, view that the list of states of the world can be assumed to be complete—was that an exhaustive set of rival hypotheses is meaningful only if a 'residual hypothesis' is considered by the decision-maker. A 'crucial' decision is one in which complete information does not exist at the time of the decision, precisely because an exhaustive list of mutually exclusive states of the world does not exist. Shackle's ontological commitment to an open worldview, intended as an understanding of the world as non-deterministic and productive of novelty, was epitomised by the claim that 'surprise' must find a place in a theory of decision-making, not because of 'counter-expected' events, but because of the 'unexpected', that is, events 'never been formulated in the individual's imagination'

(Shackle, 1955, p. 57). We shall see that this important point was well understood by Ellsberg.

From a general perspective, ontological uncertainty refers to uncertainty arising from the inherent characteristics of the world under study, while epistemological uncertainty is characterized as uncertainty stemming from the individuals' limited abilities to comprehend the investigated world (O'Donnell, 2013). Focusing on the restricted domain of decision theory, the purpose of this paper is to explore what kind of ontological presuppositions can be ascribed to Ellsberg's understanding of real-world decision-making. Section 2 reviews the objections raised by Ellsberg against the Bayesian approach, which are usually interpreted as an argument concerning epistemological uncertainty. Section 3 examines Ellsberg's attitude toward Shackle's claim that only a non-probabilistic approach can deal with uncertainty, an attitude that reveals a more consonant view with Shackle's than is usually imagined. Expanding on remarks already made in Zappia (2021), Section 4 shows that Ellsberg's criticism of the Bayesian view can be linked to a specific episode of his activity as a military analyst and therefore has an ontological component. Section 5 argues that, contrary to what is usually inferred from the famous 1961 article that introduced the notion of ambiguity, the undesirable ontological implication of assuming a complete list of possible events that Shackle pointed out—an assumption that makes mainstream decision theory incompatible with surprise—was shared by Ellsberg. The paper ends with final remarks.

2. Ambiguity in decision theory and the 'Ellsberg Paradox'

In decision theory a crucial distinction is made between risk and uncertainty. According to Frank Knight ([1921] 1964), there are events for which it is possible to identify a group of analogous cases, and thus to derive the probability of the occurrence of the single event. For other events, especially those characterized by being essentially unique, such as the future returns of a new business activity, this is not possible. In such cases, there are no objective, quantitatively determined, probabilities, and the best one can do is to make some generic 'estimate' of the probability of occurrence of the events that might affect the outcome of a given activity. Knight ([1921] 1964, p. 20) therefore distinguished between (probabilistically) 'measurable risk' and (probabilistically) 'unmeasurable uncertainty'.¹

Although Keynes ([1921] 1973) did not discuss the risk–uncertainty distinction in the manner of Knight, he proceeded in a similar way in his study of the probabilistic judgements inherent in the ordinary reasoning of individuals—the reasoning by which opinions or beliefs are expressed—which he felt had been neglected by the probability scholars of his time. Keynes advanced the argument that it is necessary to distinguish between cases in which probabilities can be measured in precise numbers and cases

¹ The existence of identifiable classes of more or less homogeneous trials that are reasonably consistent with the mathematical concept of independently repeated events, and therefore with a probability measure associated to 'a fairly well defined recurrent event' (Lucas [1977] 1981, p. 223), has been a widely used assumption in economics. On these grounds, Lucas distinguished between what rational expectations models can deal with, 'situations of risk in Knight's terminology', and what they cannot, uncertainty. Under uncertainty, he argued, 'economic reasoning will be of no value'. However, the practice of economists has not been one to limit the use of probability in this way, as demonstrated by the use of subjective expected utility in rational expectations models (Blume and Easley, 2008).

in which this is not possible. As he alluded to this distinction when he claimed that uncertainty was an essential feature of his argument in the *General Theory* (Keynes, 1937, pp. 213–4), the analogy with Knight is such that Keynes and Knight are often considered joint fathers of the risk–uncertainty distinction (Faulkner *et al.*, 2021). Peculiar to Keynes, however, was the claim that in uncertain contexts probabilities could nevertheless be rationalized as ‘non-numerical’ probabilities, an aspect to which we return below because it was noted by Ellsberg.

An interpretation of probability deemed to be compatible with uncertainty emerged in the works of Frank Ramsey ([1931] 1980) and Bruno de Finetti ([1937] 1980). This interpretation was consolidated by the axiomatisation provided by Leonard Savage ([1954] 1972), which expressed the so-called Bayesian approach to decision-making. On the basis of a subjective view of probability, these authors were able to accommodate situations in which decision-makers were seen by Knight and Keynes as unable to assign precise probabilities. Therefore, they used for decisions under uncertainty the same framework that John von Neumann and Oskar Morgenstern ([1947] 2004) had devised for decisions under risk, with expected utility generalized to subjective expected utility. In contrast to the views of Knight and Keynes, Bayesian scholars considered irrelevant the claim that there may be gaps in knowledge leading to the decision-maker’s inability to assign precise probabilities to uncertain events. They contended that these probabilities could be inferred from actions in all cases. Consequently, uncertainty began to be regarded as reducible to risk.²

In general terms, Bayesian epistemology has two main features. Firstly, although subjective probabilities are personal, they must not lead to logical contradictions. This is a synchronic condition: if personal probabilities are inconsistent in the sense that they do not satisfy the usual properties of a probabilistic measure, this inconsistency must be eliminated in some way. However, the elimination of the contradiction is derived from reasoning that does not appeal to the realisation of events outside the beliefs of individuals. To this end, it is sufficient to provide mathematical proof, as outlined by de Finetti ([1937] 1980, pp. 62–4), that a set of inconsistent probabilistic assessments can result in a certain loss if utilized to bet in a gambling situation. This condition is typically characterized by the impossibility of the bettor being the subject of a ‘Dutch book’, which is defined as ‘a book made against him by a cunning bettor’ (Ramsey, [1931] 1980, p. 41).³ Secondly, a diachronic condition must be

² Here and in what follows a variant of Luce and Raiffa’s (1957) classic taxonomy of situations of certainty, risk and uncertainty is employed, as proposed by Bradley and Drechsler (2014, p. 1231). If we understand the severity of uncertainty as the difficulty that individuals have in making a judgement about the prospects they face—a feature that depends on the amount of judgement-relevant information that is available to them—we can classify the levels of severity as follows. In order of increasing severity: (a) certainty, when the value of the judgement is given or known; (b) mild uncertainty, when individuals have sufficient information to make a precise judgement; (c) severe uncertainty, when they only have enough information to make a partial or imprecise judgement; (d) complete ignorance, when they have no judgement-relevant information. A recurring theme of concern throughout the paper is the different severities of uncertainty experienced by individuals.

³ It is important to note, as suggested by an anonymous referee, that the Ramsey–de Finetti–Savage approach focuses on the degree of belief that can be inferred from the individuals’ betting behaviour, provided that they have no moral or religious commitment to either participate in or avoid gambling. To avoid taking into account the attraction or repugnance toward betting, Ramsey ([1931] 1980, p. 41) based his subjective probability approach on an ‘ethically neutral’ event, identified by imagining a choice between two bets offering the same outcome in case of a win, one conditioned on the occurrence of the event and the other on its non-occurrence. If the individual is indifferent between the two bets, the event is assigned a subjective probability of 1/2, which does not reflect anything unrelated to choice.

respected: personal probabilities must be consistently revised as new facts become known. When new evidence emerges in the form of an objective fact that was previously unknown, the calculation of probability must be revised in accordance with Bayes' rule of conditionalization. Consequently, posterior probabilities possess a subjective character that is, let us say, less personal than prior probabilities. It is this condition of dynamic consistency that allows personal probabilities to be reduced to probabilities that are somehow objective, as it can be demonstrated that under certain conditions there is a tendency of posterior probabilities to converge as observed evidence increases (Schupbach, 2022).

However, if we concentrate on the rationale of the Bayesian approach as it has been specifically used in decision theory, it is necessary to focus on how these two consistency conditions apply to decision-making. And this means focusing mainly on the condition for synchronic consistency, since it is the set of assumptions from which the probability priors are derived in the Bayesian approach that has been challenged in decision theory. On such grounds, it may be useful to follow Runde's (2001) characterization of what consistency implies in the Bayesian reasoning of the Ramsey–de Finetti–Savage tradition. In particular, two basic assumptions are illustrative of the peculiar interpretation of the underlying economic reality that the Bayesian approach endorses. As noted by Runde, the descriptive power of the Bayesian model as a representation of economic agents and their choices under uncertainty is indeed linked to two 'ontological presuppositions' about decision-makers: decision-makers are assumed (a) to have real-valued degrees of belief and (b) to always be willing to take bets at odds that correspond to these degrees of belief.

Runde's point is important because the common understanding is that the subjective approach to probability of Ramsey and de Finetti is more general than previous approaches, such as those of Knight and Keynes, precisely because it does not assume anything specific about what is objective and what is not. Indeed, the subjective approach requires no specific claim about the existence or non-existence of objective probabilities. De Finetti's radical anti-realistic stance is the best confirmation of this: the great advantage to be ascribed to subjectivism over other probability interpretations is that it gives a definition of probability that is completely free from ontology. For de Finetti, objective probabilities simply do not exist (Galavotti, 1989).⁴

However, a more thorough examination of the two ontological presuppositions concerning the capabilities of decision-makers previously mentioned reveals that the subjective approach also expresses a specific view of the relationships between individuals' beliefs and the external world. This is truer if it is specified, as the two general conditions of Bayesian epistemology require in the interpretation of mainstream decision theory, that real-valued degrees of belief can be represented by precise numerical probabilities in any given instance. For a probabilistic logic of consistency to represent an individual's degrees of belief, it is assumed that the individual's degrees of belief are

⁴ In general, probability is considered objective in the sense that it expresses a claim about the world, predicated on empirical investigation and independent of the subject who employs it. However, the definition of objective probability depends on the approach to probability theory. For Knight, probability is derived from observation of frequencies, and is thus objective, whereas for Keynes it is the logic of probabilistic reasoning that gives objectivity to a probability, a view elaborated further by Carnap (1950). Finally, for subjectivists, objective probability can only exist as the limiting probability value to which the personal opinions of different individuals converge when confronted with successive pieces of evidence through the application of Bayes' rule.

consistent when the constraints they convey are jointly satisfied within a single probability distribution.⁵ As first suggested by Ellsberg, this is questionable, not only for descriptive but also for normative reasons.

Ellsberg (1961b, p. 645, italics in original) was explicit about his aim to revive the distinction between risk and uncertainty, to object to the implication of Savage's axiomatization of decision-making that 'for a "rational" man, *all* uncertainties can be reduced to risks'. The interest raised by Ellsberg's contribution among decision theorists mostly stayed in his ability to reformulate Knight's and Keynes' argument against the use of precise probabilities under uncertainty. In essence, both Knight and Keynes had claimed that in many situations where ordinary language has a probabilistic connotation, the measure that can be used to assess degrees of belief is either not a probability (Knight's 'estimates') or a non-conventional probability (Keynes' 'non-numerical' probabilities).

However, in order to claim this—that many economically relevant situations cannot be studied on the assumption that there are classes of exhaustive and exclusive outcomes or homogeneous trials, from which either *a priori* or frequency probability can be derived—both Knight and Keynes had appealed to intuition. To be sure, Keynes' approach to probability was intended to provide insights into how probability can be applied to conduct, as detailed in Chapter XXVI of his *Treatise on Probability* ([1921] 1973). But he regarded probability relations as logical relations, the rational inference of (possibly imprecise) degrees of belief from available evidence. The way in which Ellsberg developed his argument, on the contrary, shows that he intended to find a hearing among contemporary Bayesians, who had rejected the intuitive approach to probability and replaced it with a behavioural approach, with degrees of belief elicited from action, as suggested by the pragmatist orientation of both Ramsey and de Finetti.⁶

Ellsberg (1961b) presented a behavioural confirmation of the fact that not all uncertainties are perceived by decision-makers as risks by means of a kind of thought experiment concerning the drawing of coloured balls from urns. In his famous urn examples, bets on the drawings are used to elicit probabilities, in line with the Ramsey–de Finetti–Savage rules for determining *a priori* subjective probabilities from choices. In order to do so, he selected a very restrictive environment, in which a complete list of states was available.⁷ Nevertheless, in a series of tests he conducted with colleagues at

⁵ This approach is commonly referred to as 'probabilism' because it posits that degrees of belief must be represented by a single, additive probability distribution. Consequently, the notion of consistency is restricted to that of probabilities in a quantitative sense. Schupbach (2022, pp. 28–9) shows that a less demanding notion of consistency can be fulfilled by using a set of probability distributions to represent 'imprecise confidences' about events. The level of precision of beliefs presupposed by probabilism is therefore a 'strict', albeit dominant, version of the Bayesian approach.

⁶ As one anonymous referee pointed out, it is also important to emphasize that the concept of consistency, as employed by decision theorists in the footsteps of Ramsey and de Finetti, pertains to the outcomes of an individual's choices when making decisions. Consequently, it is solely in this sense that specific degrees of belief can be regarded as more or less logically consistent.

⁷ Ellsberg's most telling example concerned two urns, the first containing 100 balls, either red or black in an unknown proportion, and the second containing 50 red and 50 black balls. Ellsberg examined a sample of decision-makers when faced with the choice of betting on the draw of a red or black ball from either the first or the second urn. He found that when confronted with each urn separately, individuals tended to be indifferent between betting on red or black. However, when asked to choose from which urn they would prefer to bet that a red (or a black) ball would be drawn, most respondents revealed a preference for the second urn rather than the first. Ellsberg observed that this contradicts the assignment of a sharp additive probability distribution necessary to apply Savage's maximization of subjective expected utility as a criterion for decision-making. Indeed, while the decision-makers' indifference between betting on the red or black drawn from urn I or II examined separately means that their subjective probabilities, p_i , are such that

RAND and at various universities, it turned out that even in these very simple situations many of them—decision-makers who were ‘not only sophisticated but reasonable’ (Ellsberg, 1961b, p. 656)—did not behave as if they were endowed with a precise, additive probability measure. Moreover, there were a significant number of people who did not regret their decisions even after realising that they were acting in violation of Savage’s axioms of rationality. In Ellsberg’s words, the observed violations of Savage’s axioms were mostly ‘deliberate’ and therefore, Ellsberg concluded, must have normative significance. It was not that a substantial number of decision-makers made mistakes, as Savage ([1954] 1972, pp. 101–3) had claimed in his confrontation with Allais. They were simply not confident enough to bet in every situation at prices derived from their degrees of belief, so they were deliberately violating the axioms.

On this basis, Ellsberg proposed to move away from the simple dichotomy between risk and uncertainty. He (1961b, p. 657) thought it necessary to introduce a new notion, that of ambiguity of information, that is, ‘a quality depending on the amount, type, reliability and “unanimity” of information, and giving rise to one’s degree of “confidence” in an estimate of relative likelihood’. It is worth emphasising that Ellsberg was well aware that he was not talking about a general kind of uncertainty, as he made clear in the follow-up work on the topic that constituted his doctoral thesis submitted to Harvard University in the spring of 1962. The subject matter of his research was ‘circumstances of decision-making in which information is scanty, marked by gaps, obscure and vague, or on the contrary plentiful and precise but highly contradictory’. However, ‘decision-making under uncertainty, even in Frank Knight’s sense which distinguishes “uncertainty” from “risk”, maybe considered a broader problem, which includes decision-making under “ambiguity” as a special extreme case’ (Ellsberg, [1962] 2001), pp. 1–2).⁸

In his thesis, Ellsberg also noted that Keynes had anticipated his argument about confidence in a probability assessment, something that he claims he realized only after the publication of his 1961 seminal article. Strikingly, similar urn examples had been already examined by Keynes ([1921] 1973, pp. 82–3) in his *Treatise* with the aim to point out that the evidential basis on which certain probabilistic conclusions rest must be taken into account. Ellsberg did not refer to Keynes’ urns but noted that ‘Keynes gives detailed and convincing arguments why the notion of “weight” cannot satisfactorily be expressed in terms of probabilities, “probable error”, or the shape of a probability distribution’. The substantial equivalence between Keynes’ weight of argument and his own notion of ambiguity was therefore (belatedly) acknowledged (Ellsberg, [1962] 2001, p. 11).⁹

However, Ellsberg’s main aim was to show that there is behavioural evidence that there are uncertainties that cannot be treated as risks. As this result emerged as a

$p(\text{red}_I) = p(\text{black}_I) = 1/2$ and $p(\text{red}_{II}) = p(\text{black}_{II}) = 1/2$, if they choose to bet on the red (or black) from the second urn, they reveal $p(\text{red}_{II}) > p(\text{red}_I)$ (or $p(\text{black}_{II}) > p(\text{black}_I)$). Ellsberg termed the first urn as ambiguous with respect to the second, risky but unambiguous one, and concluded that a significant number of individuals had shown to prefer to face a risky, but unambiguous situation rather than an ambiguous one.

⁸ Following the taxonomy introduced in footnote 2, ambiguity emerges when the individual, being uncertain about which state of the world will occur among a complete list of states, lacks the information necessary to assign a precise probability to each state of the world. The more complex issue of state-space uncertainty is therefore not addressed in this context.

⁹ As is well known, Keynes ([1921] 1973, p. 345) distinguished between a judgement of probability and the confidence with which that judgement is held, with confidence associated to the ‘degree of completeness of information upon which a probability is based’, which he called the weight of an argument.

consequence of the paradoxical choices associated with his urn examples of the 1961 article, and as the much more encompassing analysis of the 1962 thesis was not published at the time, his entire contribution has been summarised in the ensuing literature simply as a paradoxical result, the so-called ‘Ellsberg Paradox’. Consequently, Ellsberg’s ambiguity has been classified as distinct from the mild uncertainty that Savage reduced to risk, yet not as severe as fundamental uncertainty, specifically because it does not address the issue of state-space uncertainty. It is important to recognise, though, that Ellsberg’s motivations were much broader, as a closer examination of his work from the early 1960s makes clear.

3. Ellsberg on Shackle

In order to comprehend Ellsberg’s constructive aim, it may be useful to summarise his comments on Shackle’s non-probabilistic approach. These comments illustrate both the conventional understanding of Ellsberg’s contribution, which, as we have just seen, is associated with ambiguity in most of the literature on decision-making, and the lesser-known interest he had in a different, more relevant kind of uncertainty, to which we shall return in the next section of the paper.

In his 1961 article, Ellsberg offered a scathing critique of Shackle’s peculiar approach. Ellsberg distanced himself from Shackle’s extreme stance, as if to demonstrate his intention to be critical of Savage but committed to probabilistic reasoning (Ellsberg, 1961b, p. 645). Ellsberg argued against Shackle’s suggestion that a representation of uncertainty in terms of numerical probabilities should be rejected in any situation of critical importance to the decision-maker, defined as a decision whose outcome depends on a single choice, regardless of the underlying stochastic environment. Indeed, Shackle had argued that rich statistical information about the frequency of a particular event should be considered irrelevant to decision-makers who are asked to make a decision conditional on a single trial when the outcome of that single trial is of critical importance to them. The rejection of the numerical expression of probability—the phenomenon in which Ellsberg was interested—was highlighted by Ellsberg as premature in Shackle’s case. Ellsberg’s argument was consistent with the perspective of a subjective probabilist: in response to Shackle’s assertion that the knowledge of the results of an aleatory mechanism in multiple trials provides insufficient information about the next trial, Ellsberg replied that he would be happy to invite him to accept a side bet on it at the odds suggested by the aleatory mechanism.¹⁰

However, as noted in Ellsberg’s 1962, Basili and Zappia (2010) dissertation acknowledges a much more important role for Shackle’s drastic rejection of probabilistic reasoning. There, Ellsberg argued that the fact that statistical information cannot be ignored when it is readily available does not mean, as the subjectivists meant, that every decision can be made using the same probabilistic attitude that statistical information

¹⁰ Shackle (1955, p. 8) had argued that 600 throws of a die, showing 100 times an ace, are meaningful to the possibility that out of 6,000 throws an ace will show about 1,000 times, but that the same information is not of help to a decision-maker whose action’s outcome is conditioned on the single 601st trial. Ellsberg (1961b, p. 645) commented: ‘If this example were not typical of a number of Shackle’s works, it would seem almost unfair to cite it, since it appears so transparently inconsistent with commonly-observed behaviour’. It should be noted, though, that Charles Peirce ([1878] 1923, pp. 69–70), the American founder of pragmatism who had a strong influence on Ramsey, expressed the same cautious attitude as Shackle when discussing the significance of frequencies in assigning a probability to the next, single trial, particularly when that trial holds crucial relevance for a decision-maker (I thank an anonymous referee for this observation).

makes possible. When Ramsey suggested that postulating certain constraints on betting patterns would ensure that numerical, subjective probabilities over all events could be meaningfully inferred from choices, he encouraged a development of his ideas to the effect that 'such constraints effectively *define* reasonable behaviour under *all* conditions of uncertainty'. On this basis, Bayesian theorists solved the problem of decision-making as if for a 'reasonable man' all "uncertainties", in Knightian terms, could be expressed numerically as "risks". But if one wants to deal with a wider "spectrum of uncertainties", ranging from the unequivocal "risks" of familiar production decisions [...] to the amorphous anticipations of long-range Research and Development', things are different (Ellsberg, [1962] 2001, pp. 4–5).

Indeed, Ellsberg noted, 'when ambiguity is extreme ... by any of his indices: relevant information sparse, or obviously unreliable and contradictory; wide differences in the expressed expectations of different individuals; low confidence in available estimates', Shackle's 'somber reflections' seem 'too ominously relevant to the very circumstances upon which this study focuses to be dismissed' (Ellsberg, [1962] 2001, pp. 16–17). Here Ellsberg quoted, this time very favourably, Shackle (1955, p. 74) when he argued: 'I conceive the nature of expectation itself as an act of creative imagination and not of "rational" calculation, for calculation is impossible when data are incomplete, and in the face of ignorance, rationality is a mere pretence'.¹¹

This change in attitude toward Shackle is confirmed in the discussion of how to replace the inadequate solution axiomatised by Savage and endorsed by Bayesian theorists, that is, when Ellsberg focused on alternative decision rules. As noted above, Ellsberg ascribed a normative content to his own critique. As a result, he was interested in suggestions for decision-makers who act while acknowledging that the maximisation of subjective expected utility is not a viable solution to their decision problems. Ellsberg's proposed solution was neither to demand 'too-ambitious answers'—as the Bayesians did—nor to reject the guidance a decision-maker might expect from a theory of 'reasonable' versus 'rational' behaviour to be studied.

He proposed a taxonomy of criteria related to parameters that express, on the one hand, the decision-makers' confidence in their probabilistic assessments, and, on the other hand, their attitude to the decision problem, reflecting the relative influence of favourable versus unfavourable outcomes of their actions. As for the first aspect, he acknowledged the influence of Keynes' *Treatise* on the imprecise probability approach of Bernard Koopman (1940) and Irving Good (1952) that he intended to support under ambiguity.¹² As for the second, he referred to Leonid Hurwicz's proposal to consider an optimism/pessimism parameter to ponder the best and the worst possible outcomes of a decision under ignorance (later known as the α parameter in Arrow and Hurwicz's (1972) criterion for decision-making under complete uncertainty).¹³ Ellsberg ([1962]

¹¹ A referee noted that the 'catchall hypothesis' employed in contemporary epistemology to permit the testing of hypotheses when it is acknowledged that those under consideration may not be exhaustive (Salmon, 1990) suggests an analogy with Shackle's residual hypothesis that is rarely mentioned. Unfortunately, it is not possible to elaborate on this in the present context.

¹² It is worth noting the link that Ellsberg sought to establish with the peculiar position advocated by Keynes, Koopman and Good. This was motivated by his search for an alternative approach to rational decision-making, one that acknowledged the vagueness and limited reliability of probabilistic judgements.

¹³ If ignorance is defined in behavioural terms as a situation in which the decision-maker is indifferent between betting on any two events as long as one is not included in the other, this indifference captures the fact that the decision-maker has no means of assessing whether one event is more likely than the other. Cohen and Jaffray (1980) show that the Arrow–Hurwicz criterion is the only decision criterion that is susceptible

2001, p. 163) noted that, if the issue of optimism/pessimism is taken into account, as he thought important to do when there is ‘considerable’ ignorance in the decision environment, Hurwicz’s suggestion is only a special case of Shackle’s (1955, pp. 47–8) solution of concentrating on only two ‘focus outcomes’ for each possible action. While he argued that Shackle seemed to focus too much on ignorance, he singled out Shackle’s theory as one that was mostly relevant in the discussion of issues that he considered to be of prominent importance for decision-making.¹⁴

In conclusion, Ellsberg’s attitude toward Shackle was much more appreciative than is usually understood. Moreover, given that Shackle’s main point is his refusal to consider a decision environment in which the list of states of the world is complete, the recognition that Ellsberg makes in his more thoroughly motivated critique of the subjectivist view of his doctoral thesis deserves to be taken into account. As will be demonstrated in the following section, this admission may have been motivated by his personal experience in the field as a military analyst.

4. Uncertainty in the real world¹⁵

It is well known that Ellsberg did not pursue further economic research in his career. While the 1961 paper is constantly cited as a seminal contribution to decision theory, there is no reference in the literature to Ellsberg’s ideas other than his paradoxical results with the urns. Notably, the 1962 dissertation was circulated only among colleagues and remained undisclosed until its belated publication in 2001.¹⁶ In fact, while a Junior Fellow in the Society of Fellows at Harvard from 1957, he began visiting the RAND Corporation, a non-profit research organization that conducted both basic research and classified analyses for the US Department of Defense. Subsequently, in the late 1959, he accepted a permanent position at RAND. As a strategic analyst at RAND, he became a consultant to the US Department of Defense.¹⁷

Thus, in the same years that he was studying decision-making in the abstract, Ellsberg was involved in its concrete application to problems such as the strategic interaction between superpowers. Some of this research at RAND reflected his early interest in game theory, and in particular his refusal to apply uncritically to every strategic interaction equilibrium notion such as von Neumann and Morgenstern’s max-min (Ellsberg, 1956). He studied conflict and bargaining, with particular reference to the kind of

to model choice in such situations. Decision-makers using this criterion would reveal that they are facing a situation of complete ignorance.

¹⁴ A similar remark about Shackle had been made by Arrow (1951) in his review of the state of decision theory in the early 1950s, just before Savage’s approach became mainstream. But the fact that Ellsberg picked it up ten years later is symptomatic, because decision theorists, precisely at Arrow’s suggestion, had already definitively discarded the notion of potential surprise on which Shackle tried to build a non-probabilistic approach (Zappia, 2014).

¹⁵ This section draws on Zappia (2021). Ellsberg’s own recollections of the episodes we briefly summarize are presented in Ellsberg (2017).

¹⁶ The *Economic Journal* reviewer of Ellsberg’s book, a renowned decision theorist, concedes: ‘Ellsberg’s discussions are possibly too non-technical and preliminary to add much to the new literature, but it is simply breathtaking to note ... how many, many years ahead of his time this man truly was’ (Mukerji, 2003, p. F188).

¹⁷ It was in a different role, as Special Assistant to the US Assistant Secretary of Defense since 1964, that he later became involved in the Vietnam War. After leaking the so-called Pentagon Papers to the press in 1971—a set of documents proving that the US Government had expanded its military operations in Vietnam without approval from the Congress and had misled the public about its true intentions during the war—he became a political activist and writer, never returning to academia (Ellsberg, 2002). A brief assessment of Ellsberg’s life and work after his recent death is provided in Moscati and Zappia (2024).

deterrence policy a superpower should pursue to avoid the opponent finding it convenient to attack first. Some of this work was published (Ellsberg, [1959] 1968, 1961c) and is well known, especially in the political science literature. But it is his then classified research for the US government that is of interest here.

After joining the Economics Department at RAND, Ellsberg chose to specialize in the command and control of the US military's nuclear retaliatory forces. The specific problem to be addressed was the decision of whether and when to use US nuclear forces against the Soviet Union in the event of a first strike by the adversary, with particular attention to the effectiveness of the chain of command after the attack (Ellsberg, 2002, pp. 32–3). At RAND, the highly influential report by Alfred Wohlstetter *et al.* (1956) on the protection of the US Air Force against a nuclear attack had set the tone for the discussion: the report suggested that US defences were vulnerable and needed drastic changes.

In a document dated August 1960, for RAND use only, Ellsberg assessed the command and control problems associated with 'post-attack capabilities'. The actual characteristics of the command and control system were examined: the strategic objective formulated by the centre must be implemented through a chain of command, a process involving plans, operational procedures and communications, that is, a complex pattern of authority and a network of information flows. After acknowledging the long tradition of describing commanders in the field as operating 'within the "fog of war"'—with information on which they must rely to make decisions that is typically 'incomplete, ambiguous, of varying and uncertain reliability' (Ellsberg, 1960, p. 6)—the paper focused on the ability of the chain of command to actually enforce the decisions taken by commanders. The vulnerability of communications and the effective level of operational control were examined in the light of the new event of a nuclear war.¹⁸

Ellsberg (1960, pp. 33–4) placed emphasis on a decision context that was that of a never-experienced-before thermonuclear war, for which he believed there was every reason to expect that 'important aspects of it ... had not been anticipated, nor appropriate responses pre-planned'. The situation would be one characterised by 'ambiguity in the evidence available to the subordinate units as to whether the time has come to exercise its authority ... the more ambiguous this evidence the more likely that, out of hundreds of subordinate units, some one will experience some day a false alarm that decides it to "Go"'.

In his advice to the military, Ellsberg strongly emphasised the need for a strategy based on flexible, adaptive behaviour, rather than reliance on what was usually a series of 'prehostilities contingency plans', however detailed they might be. His concern was that those in the military who advocated a "dehumanized" response scheme' typically tended to neglect that 'the general problem is that of dealing with *surprise*'. As a result, he made it clear that the focus should not be on the occurrence of an event deemed 'unlikely', but on the 'possibility of happenings which were considered "impossible" or were undreamt-of'. In order to be as convincing as possible, Ellsberg provided a long list of what he called 'current potential surprises'. Potential surprises could arise from the

¹⁸ The issue here is no longer that of deterrence, which was the usual focus of these studies and which was popularized by Herman Kahn (1960). In addition to examining the measures that should have been taken to deter the adversary from striking first, Ellsberg addressed the question of whether or not the ability to enforce those measures was effective in the event of a first strike by the adversary.

enemy's capabilities and tactics, technological developments concerning the military environment, the perception of available actions and their relevant consequences '*at the moment of decision*', and the actual performance of the organization in carrying out a decision. The relevance of all these possible events could, of course, be estimated, but Ellsberg argued that the actual situation in the confrontation between the superpowers was such that 'one can state almost axiomatically: *In the first thermonuclear war, surprises are to be expected*' (Ellsberg, 1960, pp. 47–9, all the italics in original).

The Wohlstetter Report (1956) had dealt with the idea of a surprise attack by the enemy, so a general issue at RAND in those years was how to respond to surprise. But in his study, Ellsberg emphasised the specific surprise associated with what we would call an unforeseen contingency, the inability to anticipate a future state of the world. It goes without saying that the use of the notion of surprise is very similar to that made by Shackle to criticize a probabilistic approach to uncertainty. Ellsberg's aim was to make apparent that an actual decision problem of such a relevance could not be analysed by assuming of a complete list of states of the world.

This admission by Ellsberg—that the uncertain environment of actual decision-making can be more severe than the uncertainty associated with his notion of ambiguity—was not marginal. In his role as a consultant to the US Government, he continued to emphasize the seriousness of the uncertainty at stake, as a second recently declassified document shows. In notes written in May 1961 for a top-secret policy document to be issued by the President of the United States—a document named Basic National Security Policy, to be circulated only within the Defence Department—Ellsberg examined the ability of the US to achieve its war aims in the event of a 'central war'. Deterrence remained the primary objective to be achieved, but Ellsberg suggested that an analysis of the ability to respond to a 'surprise attack' by the enemy forcing the US into a central war could not be excluded from an assessment of national security policy. The capacity of the US armed forces to extend deterrence into the wartime period, he argued, could only be maintained by ensuring that the ability to deny the enemy's military superiority would persist after the enemy had attacked. But this depended on the ability to adapt the US strategic response to circumstances that might be 'unforeseen, ambiguous or both'. After examining a range of situations in which the chain of command might fail to minimize the 'likelihood of accidents, unauthorized actions or unintended nuclear exchanges', Ellsberg (unsigned notes 1961a, pp. 16–7) admitted that it was impossible to provide an 'exhaustive' list of possibilities and concluded: "surprises" in the form of wholly unforeseen circumstances are likely'. The necessity to retain a 'capability for flexible response' was reaffirmed as 'most critical and most rewarding'.

5. Ellsberg's ontological commitment

We saw earlier that beyond the simple Knightian dichotomy between risk and uncertainty, Ellsberg proposed to consider different degrees of uncertainty, which are incorporated into his notion of ambiguity. Essentially, he held to a view that distinguishes between mild uncertainty—where uncertainty is analogous to risk, as in Savage and the Bayesian approach—and severe uncertainty—where the dimension of severity relates to the difficulty individuals have in making a precise judgement about the prospects they face. The more severe the uncertainty, the less meaningful is to assume that it can be reduced to risk (Bradley and Drechsler, 2014).

We have also seen that Ellsberg was prepared to explore the domain of severe uncertainty to an extent that usually goes unnoticed by readers of his single well-known contribution, the 1961 *Quarterly Journal of Economics* article. Ellsberg's attempt to give a broader meaning to his thought experiment with the urns is clear in his dissertation: situations of 'extreme ambiguity' cannot be represented by a single, definite probability distribution since this would 'totally obscure differences in clarity of underlying information and opinion'. This is particularly true in 'those very fields where pre-decision analysis is at its highest premium', that is, where

uncertainty is the central fact: uncertainty as to the future environment, as to the eventual performance of alternative programs launched today, *even as to the operative national objectives in that environment and time and hence the future evaluation of performance.* (Ellsberg, [1962] 2001, p. 16, my italics)

Having reviewed Ellsberg's classified work at RAND, we can explicitly link his understanding of uncertainty in his Harvard dissertation to his work as a military analyst. As we saw in the previous section, when asked to analyse the reliability of the military chain of command under enemy attack, Ellsberg pointed out that the problem would be how to deal with state-space uncertainty. What characterised Ellsberg's concern when dealing with an instance of actual decision-making was the impossibility of formulating a complete list of the states of the world at the moment of decision.

The fact that a list of possible states of the world is not predetermined is one of the main characteristics of situations referred to as fundamental uncertainty. Indeed, this is usually considered to be the demarcation point between fundamental uncertainty and other types of severe uncertainty, the most studied of which is ambiguity (Dequech, 2011). Under ambiguity, as much as under fundamental uncertainty, there is a lack of knowledge that prevents decision-makers from formulating a single, definite probability distribution. Ellsberg ([1962] 2001, p. 182) suggested interpreting the situation as if, among a number of probability distributions that the decision-makers consider 'reasonably acceptable', none is identifiable as the single one used by a Bayesian decision-maker. The ambiguity of the environment makes it impossible to confidently select one of these probabilities as the only acceptable one, in contrast to the assumption about the capabilities of Bayesian decision-makers.

The question, however, is whether this is sufficient to deal with fundamental uncertainty proper. Critics of the literature on decision-making under ambiguity recognize that the uncertainty dealt with in most of the recent literature inspired by Ellsberg is not reducible to risk, since it admits that a precise subjective probability distribution is not known by the decision-maker. However, they also claim that under ambiguity the decision-maker is still assumed to neglect the type of uncertainty generated by the possibility of non-predetermined change, identified therefore as fundamental uncertainty. In Dequech's (2011, p. 624) taxonomy of uncertainty, for instance, ambiguity is a form of strong uncertainty, but is not as strong as fundamental uncertainty.

Our reconstruction so far suggests a more nuanced interpretation of what Ellsberg was aiming at in his critical analysis. We must first note that by severing the link between degrees of belief and their probability representation by a unique, additive probability measure, Ellsberg clearly objected to the first ontological presupposition of the Bayesian approach, as identified by Runde. Moreover, he did so in a manner very similar to Keynes' critique of (frequency) probability, in the *Treatise*, and its use for 'Benthamite calculus', in the *General Theory* and after (Keynes, 1937, pp. 213–4).

Ellsberg's ambiguity refers to situations in which the confidence in probability assessments is dubious—as Keynes indicated may usually happen in the stock market, as discussed in Chapter 12 of the *General Theory* when referring to the weight of argument. However, it is also intended to represent a reality in which the completeness of information is lacking by definition—as Keynes discussed in Chapter XXVI of the *Treatise*, where he urged that weight be considered alongside with probability in any attempt to replace the maximisation of expected utility with an alternative decision rule. As Faulkner *et al.* (2017) have recently reminded us, the way in which Keynes introduced and used the weight of argument—understood as the degree of completeness of relevant information—indicates that he was concerned with fundamental uncertainty. A low weight is used to indicate a situation where highly incomplete evidence may mean that relevant influences on the outcome of today's actions are simply unknown, rather than difficult to assess probabilistically.

Moreover, Ellsberg noted that the effectiveness of the Bayesian approach hinges on what Runde identifies as its second ontological presupposition, that decision-makers must be prepared to act on their beliefs in a specific way. Bayesian decision-makers are assumed to be always willing to bet on the other side of a bet that reveals their beliefs, and to do so at the same price they are willing to pay to accept the bet on an event or a proposition, that is, to act as the bookie instead of the bettor at the same price, regardless of the odds.

In his dissertation, Ellsberg strongly objected to this second assumption, claiming that he simply did not see why the consistency requirement formulated by Bayesians through the so-called Dutch book condition should be considered a principle of rationality. Rather, he argued that there are obviously plenty of situations in which it is 'reasonable to prefer not to bet than to take either side of the bet at given odds'. 'Reluctance to gamble', he claimed, is a feature of reality in that it 'seems intuitively to correspond to recognizable behaviour-patterns associated with circumstances in which opinions are exceptionally "indefinite"' (Ellsberg, [1962] 2001, p. 84). Once the first ontological presupposition of the Bayesians has been challenged, the second also reveals its doubtfulness. The connection with Keynes' *Treatise* and the intuitive approach to probability of Koopman and Good is so obvious that Ellsberg explicitly mentioned it, as already noted.

From a decision-theoretic viewpoint, Ellsberg concluded, the question is therefore whether it is possible to assert meaningfully 'that a reasonable individual regards one event "not less probable than" another in every case of comparison, let alone the possibility of assigning numerical probabilities to each event'. While it is

plausible and certainly convenient to assume that fairly precise numerical probabilities can be assigned to *some* events ... the question is whether it is realistic to suppose this to be possible for *all* relevant events, even given the careful, deliberated highly-calculated decision-making we are assuming. (Ellsberg, [1962] 2001, p. 113, italics in original)

There is therefore a trade-off between what is analytically 'convenient' and what is 'realistic', a trade-off that Ellsberg did not want to obscure when he suggested how decisions should be made. This leads us to a second consideration.

The ontological implications of Ellsberg's analysis on the field are worth examining even on theoretical grounds. Having discovered that his own method of dealing with ambiguous situations was consistent with a view of severe uncertainty such as Keynes', Ellsberg could not avoid mentioning the crucial importance of the undesirable

ontological implications of assuming that a complete list of states is known to decision-makers, a point that Shackle had made in Keynes' footsteps.¹⁹ Ellsberg did not come to this conclusion in his discussion of what ambiguity means for a theory of choice. Rather, his understanding emerged from the study of a practical problem, the extremely relevant problem of how to respond to the outbreak of nuclear war. We have seen that while as a decision theorist Ellsberg was studying the normative relevance of ambiguity in the abstract, as a military analyst he was forced to consider that the ontological presuppositions of Bayesian decision-making were simply untenable.

In this respect, his reflections on Shackle are revealing. Certainly, he never suggested that true uncertainty could be equated with complete ignorance of the future, as Shackle often seemed to imply. For Ellsberg, there is probabilistic knowledge even if the list of states of the world is incomplete. However, the importance he attached to Shackle's analysis in his dissertation suggests that his interest in fundamental uncertainty became as important as that in ambiguity.

6. Concluding remarks

Fundamental uncertainty is widely regarded as a key distinguishing element of heterodox thinking (Lawson, 2006). Although it can also be characterised by reference to other elements, such as the inability to unambiguously identify the consequences associated with individual actions or how the consequences are evaluated, its main characterisation in decision theory has been in terms of the impossibility of defining a complete list of possible states of the world. In real decision problems of relevance—such as the long-term investments that concerned Keynes in Chapter 12 of his *General Theory* or the 'long-range Research and Development' mentioned by Ellsberg ([1962] 2001)—it is difficult to assume that one can construct a space of states whose elements are exhaustive of the possible realisations of events that condition the outcomes of actions. This leads to a problem that decision theorists have labelled 'unforeseen contingencies': decision-makers who are aware of their inability to fully foresee features of the world that may condition the outcome of their actions act in a context of state-space uncertainty.

It is often recalled that Savage himself distinguished between a 'small world', for which he suggested applying the decision theory that later became mainstream, and a 'grand world', which he felt could not be satisfactorily theorised in the same way. In this sense he suggested a 'look before you leap' decision strategy for problems definable as small worlds, but admitted that the appropriate strategy for grand worlds was best summarised by the saying 'cross the bridge when you come to it' (Savage, [1954] 1972, p. 16). Indeed, we have seen that this was exactly Ellsberg's point when he urged the military not to rely on contingency plans, no matter how detailed, intended for a 'dehumanized' response, because the actual environment in which the plans would be enforced does not permit such prescriptions without discussion of the surprises that will emerge (a characteristic of the actual environment that he found inescapable).

Ellsberg supported the idea that any implicit assumptions taken for granted in the formulation of an axiom system must be re-examined as the scope of application of

¹⁹ The conundrum of Shackle's inability to read the connection between Keynes' *Treatise* and the rest of his work on uncertainty, a connection that Ellsberg ([1962], 2001) found obvious, is discussed in Runde (2000).

the theory is extended. As one of the most prominent critics of the current state of decision theory has observed, ‘modern Bayesianism has fallen headlong into the errors he [Savage] sought to evade by proceeding as though his descriptive theory of subjective probability can be reinterpreted as a prescriptive theory of logical probability without any need for further foundational discussion’ (Binmore, 2017, p. 260).

However, in both the case of Savage and the ensuing Bayesian approach and that of the critics who have studied decisions under ambiguity, the question has always been posed as an epistemic problem. For the more radical critics, such as those in the post Keynesian tradition, the question is also an ontological one, if not exclusively so. Paul Davidson, who also noted that Savage himself admitted that the specification of all possible states when ‘carried to its logical extreme ... is utterly ridiculous’ (Davidson, 1991, p. 136), pointed out that in a post Keynesian ‘grand world’, decision-makers recognise that they are ignorant regarding the future because of what he classifies as ‘non-ergodic conditions’. The economic world is unknowable in principle because, *à la* Shackle, it is yet to be created.

The main point of contention between post Keynesians and mainstream economists concerns therefore the importance to be attached to unforeseen contingencies. On the one hand, the open-ended and non-deterministic nature of the world is placed at the centre of economic analysis, while on the other hand, decision theorists inspired by Savage—even those who have been critical of his overly strict Bayesian followers—see the ‘grand world’ as analytically intractable, and have therefore confined themselves, and the aim of economics in general, to the analysis of ‘small’ worlds, which they see as the only reasonable way to deal with the problem.

However, something is changing, and it has the flavour of a return to the insights introduced by Keynes, Knight and Shackle, as recently recalled by Svetlova (2021). Several studies on unawareness have adopted themes introduced in the analyses of Knight, Keynes and Shackle to make room for fundamental uncertainty, although references to these analyses remain very limited. Even among decision theorists, a form of cognitive unease is becoming more widespread, as they have long accepted an approach that denies the existence of an irreducible component real-world uncertainty (Gilboa, 2025). For all these studies, Ellsberg’s analysis of ambiguity constitutes an enduring starting point. What is still missing from the literature, however, is recognition that Ellsberg himself had already moved decisively in this direction.

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