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1. Introduction

The 1950s witnessed the advent of the so-called Bayesian mainstream in economics. The long history of how to deal with decision-making under conditions of risk and uncertainty began early with Daniel Bernoulli's (1738 [1954]) proposal to replace the use of the mathematical expectation of returns, which was the conventional method of valuing winnings in games of chance, with the mathematical expectation of the utility of returns. However, the Bernoullian criterion for choice, that the action to be chosen must be that which promises the maximum expected utility, was rejected by marginalist scholars such as William Stanley Jevons and Alfred Marshall. While both Jevons and Marshall endorsed the notion of decreasing marginal utility that had motivated Bernoulli's analysis, they disputed the use of the expected utility method, arguing that it was not possible to disentangle the pleasure (or displeasure) derived from gambling from the utility associated with probabilistic outcomes. They concurred that the extension of utility maximization from choice under certainty to choice under risk was therefore unwarranted, and their argument remained largely untouched even during the ordinalist advances of marginalism in the first part of the XX century (Blaug 1997, 317-18).

These objections were eventually overcome on the basis of the work of John von Neumann and Oskar Morgenstern. Although primarily concerned with a different topic, the study of strategic interactions between individuals, von Neumann and Morgenstern (1947) proposed an axiomatic framework in which, under the assumption that the probabilities of outcome-relevant events were given, the structure of the choice problem could be generalized. The scheme that had already been developed in economics for choices under conditions of certainty was eventually shown to be applicable to choices involving risk.

However, it was not until the reformulation of the expected utility hypothesis by the mathematical statistician Leonard Savage that it was possible to extend expected utility theory from the analysis of decisions under risk to uncertainty. In his seminal work on the approach, *The Foundations of Statistics*, Savage (1954) demonstrated that the representation theorem proved by von Neumann and Morgenstern for objectively known probabilities of payoff-relevant events could be extended to subjectively held probabilities.

Savage had begun to work on this problem shortly after he and Milton Friedman had proposed a version of von Neumann and Morgenstern's work that was accessible to the majority of economists of the time, who were still largely unfamiliar with the axiomatic method. Friedman and Savage (1948) explained in what sense the traditional objections to the Bernoulli's norm of behavior could finally be overcome. Individuals

obeying the set of axioms proposed by von Neumann and Morgenstern, which could be interpreted simply as an extension of those already used for certainty, would act as if they were maximizing a utility function, albeit a peculiar one, even when making decisions in lottery-like risky contexts. Friedman and Savage showed that this could be achieved without any contradiction with the hypothesis of diminishing marginal utility: the modern notions of risk-averse behavior and its counterpart, risk-seeking behavior, were introduced as an explanatory device.

Nevertheless, the distinction between risk and uncertainty had remained a firmly established tenet of economic theory during this period. Frank Knight (1921) had argued that the pure profit of entrepreneurial activity depended on the entrepreneur operating under conditions in which at most "estimates" of future events could be formulated, but not numerical probabilities. John Maynard Keynes himself (1921 [1973a]), who had allowed a broader scope for probabilistic analysis than Knight by introducing a notion of epistemic probability, had intended his probabilities to admit only a partial ordering. By underscoring to the importance of the "weight of the argument" conferred by evidence to a probabilistic judgement – which he later used to give content to the notion of confidence in his *General Theory* – Keynes had declared Bernoulli's expected utility maximization inapplicable to situations in which probabilities could not be assumed to be numerically precise, because when making decision the "degree of completeness of information" might be low (Keynes 1921 [1973a], Chapter 26). And while one of Keynes's main objectives in his *Treatise on Probability* was to explore the potential for a calculus of "non-numerical probabilities" in scenarios where precise prior probabilities could not be defined, his approach did not materialize in the specification of an alternative decision rule. Additionally, his allusion in the *General Theory* to the "animal spirits" of entrepreneurs, although confined to unusual market conditions, appeared to be at odds with any decision rule (Keynes 1936 [1973b], 161-62).

In light of the aforementioned considerations, Savage concentrated his efforts on identifying an axiomatic structure that would facilitate the examination of decisions under uncertainty in a manner analogous to that employed for decisions under risk. To represent the degrees of belief of an individual operating in an uncertain environment, i.e. with (objectively) unknown probabilities, he had found inspiration in the work of Bruno de Finetti. In his first presentation of the new research − strongly endorsed by the group of mathematical economists at the Cowles Commission for Research in Economics with whom he collaborated extensively during that period (Zappia 2024) − Savage (1950, 183-84) ascribed to de Finetti the role of the main innovator in probability theory, asserting that de Finetti had been alone in opposing the prevailing tradition among statisticians to admit only probabilities derived from observed frequencies (see Chapter 6).

If one were to approach the question of decision-making using de Finetti's subjective approach to probability, Savage (1950, 184) claimed, it was possible to begin from "plausible assumptions about the behavior of a 'reasonable' individual faced with uncertainty about future events," and then derive

numerical values to associate with these events, "which from the purely mathematical point of view are probabilities." In light of the von Neumann-Morgenstern theory of utility, it would be possible to indicate "how the individual should act in the face of uncertainty," given these personal probabilities. As anticipated in a communication to the Paris Conference on Risk in May 1952 (Savage 1953), the result was that expected utility theory could be applicable to both risky and uncertain circumstances, as Savage demonstrated conclusively in his 1954 volume.

Although Savage had initially moved in the vein inaugurated by de Finetti, in his 1954 volume he recognized that Frank Ramsey's contribution should be considered at least as important. It was Ramsey, not de Finetti, who "explicitly studied decision" (Savage 1954, 7), and the revival of "interest in Bernoulli's ideas of utility in the technical sense of ... a function that controls decisions among acts" had started with Ramsey's essays on probability in his *The Foundation of Mathematics and Other Logical Essays*, "which in spirit closely resembles the first [theoretical] five chapters of this book" (Savage 1954, 96). But Savage had lacked this awareness in earlier works, in which Ramsey was never cited: indeed, Ramsey's work "though now much appreciated, seemed to have had relatively little influence" (Savage 1954, 96)

The sudden turnaround in the fortunes of Ramsey's essay on probability is evidenced by the unanimous support of mathematical economists at Cowles. In 1954, its most prominent figure and former director, Jacob Marschak, declared that the unfortunate neglect in decision theory of the "necessarily subjective nature of the probabilities" was over (Marschak 1954, 59). Now built around Savage's axiomatization of subjective expected utility, decision theory could be incorporated into any economic model and all uncertainties treated simply as probabilistic risks. The "Bernoulli Norm," demonstrated to be valid for risk by von Neumann and Morgenstern, had finally been generalized thanks to what Marschak called the "Ramsey Norm." Indeed, Savage had only demonstrated the viability of Ramsey's intuition about how to derive the subjective probabilities "characterizing a consistent decision-maker's behavior" (Marschak 1954, 69).

By the mid-1950s, the normative analysis of individual decisions had reached a point where it could be said to be complete, thereby inaugurating the long phase of the Bayesian mainstream, still going strong. In this context, Ramsey came to be regarded as both the forerunner and the main initiator of a specific method of analysis.

2. The early reception of "Truth and Probability"

Although Ramsey is today recognized as one of the two founders of the subjectivist approach in economics together with de Finetti, the process of recognition of this role is peculiar. As is well known, the publication of his foundational essay "Truth and Probability," written in 1926, did not occur until after his sudden

death, in the 1931 volume edited by Richard Braithwaite that we saw cited by Savage. But Ramsey's recognition among decision theorists and economists occurs only in the early 1950s. In particular, neither von Neumann and Morgenstern (1947) nor Friedman and Savage (1948) had mentioned him. The first accurate reference to Ramsey was that of Kenneth Arrow (1951, 420), in a review of the state of the art of decision under uncertainty where he stated that Ramsey originated a "new stage" in the description of uncertainty, one in which "a priori probabilities are derived from behavior postulates." Indeed, "it was first observed by Ramsey that, by starting from an ordinal theory of risk-bearing and adding a few reasonable assumptions, one could derive a utility function such that the individual could be said to behave in such a way as to maximize the expected value of his utility" (Arrow 1951, 423-24).[1]

Even at the 1952 Paris Conference on risk in econometrics, although almost all the major decision theorists of the time were present, apart from a single reference by Samuelson, only Jacob Marschak had dealt with Ramsey's contribution. In his previous works on rational choice theory, Marschak (1950) had sought axioms to justify the extension of logic to the field of decision-making. But, following von Neumann and Morgenstern, in order to derive a utility function representing the subjective nature of preferences over uncertain prospects he had worked with objectively known probabilities. Commenting on Savage's presentation in Paris, which features the first presentation of Savage's axiomatic justification of both subjective preferences and subjective probabilities, Marschak pointed out that the fundamental step to generalize Bernoulli's expected utility maximization in this direction was due to Ramsey. Ramsey's subjective approach was similar to the one independently pursued by de Finetti, and definitively elaborated by Savage, but it was the one "easiest to present to economists" (Marschak 1953, 37). By deriving probability and utility simultaneously without any reference to objective aspects of choice, Ramsey had made it possible to implement Bernoulli's norm in a more general way than von Neumann and Morgenstern, hence Marschak's use of the label "Ramsey Norm" in his 1954 presentation of the new theoretical approach.[2]

It is difficult to understand why Ramsey's work was neglected in economics for over twenty years. His sudden death had caused consternation throughout Cambridge, underlined by an obituary by Keynes, in which Keynes pointed to Ramsey's (1927; 1928) essays on taxation and growth published in the *Economic Journal* as fundamental contributions to the development of a mathematical approach to economics. But at the time of the publication of "Truth and Probability" attention to Ramsey's work came more from the

[1] It can be seen as indicative of the sudden turn of events that Arrow had only cursorily mentioned Ramsey in the initial version of his review, which was published as a working paper of the Cowles Commission in November 1950. A similarly concise reference was also made by Little (1950).

[2] Even in the aforementioned communication to the Paris Conference in May 1952, Savage (1953) acknowledged the influence of de Finetti and von Neumann and Morgenstern, but did not mention Ramsey.

logical-mathematical and philosophical fronts than from the economic front. Keynes may have played a role in this development given that, in his reminiscence of Ramsey, he did not place as much value on "Truth and Probability" as he did on the *Economic Journal*'s essays, indicating that the part of the *Foundations of Mathematics* dealing with probability was "fragmentary and not completely satisfactory" (Keynes 1933, 338).[3]

Moreover, Bertrand Russell's reviews of *Foundations of Mathematics* in *Mind* and in *Philosophy* were very positive about the mathematical contributions, but concluded with a dismissive tone about the essay on probability, which he found "less penetrating than his work on mathematical logic" (Russell 1931, 481). Russell (1932, 86) went so far as to claim that the attempt to identify probability with degrees of partial belief was "unsatisfactory for reasons of which he [Ramsey] himself later became aware."[4] And the more innovative English probabilistic tradition remained anchored in Keynes's logical approach to probability, with Harold Jeffreys quoting Ramsey as no more than an epigone of Thomas Bayes. In Jeffreys's view, Ramsey's subjectivist view of probability added nothing to the logical approach when compared to the frequentist tradition he sought to overcome.[5]

In addition, when addressing the issue of induction, both Russell and Jeffreys, like Keynes, found Ramsey's approach to be unconvincing. We shall come back to this issue later, but it is worth noting that to Ramsey, who had argued that only a human logic was apt to deal with induction and that inferences were necessarily done according to habits that proved to be useful, Keynes (1933 [1972], 339) objected that "it is not getting to the bottom of the principle of induction merely to say that it is a useful habit." Similarly, Russell (1931, 481) argued that Ramsey's view of induction hardly did justice to the problem, asserting that "induction as a habit exists among animals and savages, and is a very frequent source of error." Jeffreys, resembling Keynes, believed that logical probability was inherent in the problem of induction, whereas

[3] We do not discuss here the initial failure of the economic profession to recognise the importance of these two essays, on which see Duarte (2009), Chapter 14 and Chapter 15.

[4] Russell was referring to the 1929 note "Probability and Partial Belief" which in fact moves from the following statement by Ramsey (1931b [1990], 95): "The defect of my paper on probability was that it took partial belief as a psychological phenomenon to be defined and measured by a psychologist. But this sort of psychology goes a very little way and would be quite unacceptable in a developed science." Actually, the content of the note did not make clear what the meaning of this recantation might have been, nor did Russell (1931, 482) offer a precise interpretation of it, claiming that the note was "so condensed to be obscure." However, Galavotti suggests there was no recantation by Ramsey, but an evolution towards a specific interpretation of chance (see Chapter 6).

[5] Before focusing on the frequentist Ronald Fischer's objections to the assumption that statistical analysis should always start from an a priori probability, Jeffreys (1934, 12-13) noted that "Ramsey's rules of consistency may be held to assume an ideal man who always makes his decisions for action correctly, according to the value of their consequences." But then he concluded claiming: "I do not understand the statement that the prior probability is subjective."

Ramsey had argued that a probabilistic logic of choice was insufficient to justify the principle of induction. But of course Ramsey could not reply to his critics.[6]

A more substantial attention to Ramsey's work came from the philosophical front, with Ludwig Wittgenstein attributing a significant part of the reconsideration of the content of his *Tractatus Logico-Philosophicus* to the criticism of the young interlocutor (Misak 2000, 364-6, but see also Chapter 1). But Wittgenstein waited till the preface to his 1953 *Philosophical Investigations* to credit Ramsey in print, so it was mainly A. J. Ayer who defended Ramsey, in particular promoting an explicit version of Ramsey's theory of causation (Misak 2000, 428). Keynes himself, after his somewhat ambiguous treatment of Ramsey's criticism of his *Treatise on Probability* (on which see later), included excerpts from Ramsey's philosophical arguments in the reprint of his review of *Foundation of Mathematics* in *Essays on Biography*. Reproduced from *Foundations of Mathematics*, these excerpts might have been interpreted as suggesting that Ramsey's philosophical considerations warranted greater attention than his contribution to probability theory. However, even on the philosophical front, the potential for a subjective philosophy of probability was neglected.

The subjectivist perspective on probability advocated by Ramsey thus remained isolated, squeezed between the dominant frequentist approach that was producing rapid advances in statistical practice and a relatively new English tradition too committed to logicism to recognise an autonomous status to subjectivism. De Finetti, who would become the major living proponent of the subjectivist approach, recognized this state of affairs well in an essay he wrote in Italian in 1938, entitled "Probabilisti di Cambridge."

De Finetti offered support to what he saw as a revival of an epistemic approach to probability blurred by the empiricist perspective of frequency probability. His critique of the objective perspective implicit in the logic of probability endorsed by Keynes in his foundational study was downplayed given that "disagreement regarding the subjective or objective character of probability does not rule out, except for different shades of meaning and points of view consequently entailed, a substantial identity of views on a deeper

[6] As is well known, the Humean tradition posits that, in circumstances where demonstrative inferences are rendered unfeasible due to incomplete or uncertain knowledge, conclusions attain validity solely to a degree of probability. Conclusions derived from such reasoning bear a certain degree of relation to the initial premises, albeit with a lower degree of certainty. Consequently, probabilistic reasoning can be regarded as providing a justification for induction as a scientific method. This perspective was articulated by Keynes in his *Treatise* and subsequently defended by Jeffreys. However, as we shall see in what follows, Ramsey countered this by asserting that the objective of probability theory should be limited to defining the rules for coherent thought, emphasizing the necessity for a distinct logical system to address induction.

philosophical problem: that of induction" (de Finetti 1938 [1985], 354).[7] De Finetti also backed Jeffreys's idea that probability theory was an instrument of logical reasoning and not the alleged description of certain categories of facts assumed by the frequentists. But even he was not yet familiar with Ramsey's work.[8]

Among probabilists, Ernest Nagel may have been the first to acknowledge the significance of Ramsey's interpretation of probability. He deemed it as a significant development of the classical conception of probability into "a measure of the extent to which a man is prepared to act on a belief," but concluded that Ramsey's proposal, which he mentioned as being shared by de Finetti, was based on "a dubious psychological theory" (Nagel 1939, 41, 46).[9] Nevertheless, even among probabilists, it was not until 1950 that a significant recognition of Ramsey's specific contribution to the field emerged. Irving Good – a peculiar figure among Bayesian statisticians because he supported the subjective approach while denying that subjective probabilities should be regarded as precise as commonly assumed – singled out Ramsey's theory as "axiomatic, not entirely objective, neither statistical nor dependent only on degrees of belief, numerical," and distinguished it from Jeffrey's "axiomatic, objective, non-statistical, numerical (essentially)" and Keynes's "axiomatic, objective, non-statistical, non-numerical (in general)" (Good 1950, 8-10).

3. On measuring probability and utility

As pointed out by Marschak and later confirmed by Savage, the interest of economists in "Truth and Probability" was due to Ramsey's ability to measure utility and probability simultaneously, and to do so without reference to objective elements. Ramsey proposed an empirical method for comparing differences in utility and for measuring degrees of belief, which he thought it could not be meaningfully achieved by an analysis based on introspection. He argued that individuals' judgments about the strength of their own

[7] As noted by Gillies and Ietto-Gillies (1987, 203), at the time, the real contrast was between the degrees of belief theories of de Finetti and Keynes on the one hand, and Richard von Mises's frequency theory on the other.

[8] De Finetti's first quotation of Ramsey appears to be from a 1952 paper in Italian commenting on the proceedings of the aforementioned Paris Conference on Risk, in which he had participated. De Finetti (1952, 694) cited Ramsey, along with Bernoulli, as the forerunners of "probabilistic behaviour" in the sense of von Neumann and Morgenstern. He did this by misnaming Ramsey as Ramsay, arguably because Marschak had made the same mistake in the above-mentioned commentary on Savage. This suggests that the quotation was indirect and not derived from a reading of the original text of "Truth and Probability."

[9] Apparently it was this reference by Nagel that prompted Savage to read first de Finetti and then Ramsey. At least this is how Savage recounted his first interest in subjective probability theory – that Nagel's volume had "two or three references to people called subjectivists" – in correspondence with de Finetti (L. J. Savage to B. de Finetti, March 3, 1961, Leonard Jimmie Savage Papers, Manuscripts and Archives Department of Yale University Library, Box 7, Folder 192)

beliefs rested on what they think they would decide in practical situations: preferences and expectations were to be interpreted as dispositions to make choices.

The process of eliciting probability and utility proceeds in successive steps.[10] First, Ramsey (1931a [1990], 73) identified an "ethically neutral" event, an event in respect of which the act of betting reveals subjective probabilities of 1/2. To render irrelevant for analysis the elements of pleasure or displeasure associated with gambling that had worried both Jevons and Marshall, he compared two symmetric gambles. Ramsey argued that if a first bet conditional on an event E such that it offers α, if E realizes, and offers β, if non-E realizes, is considered by a person indifferent to a second bet that offers β, if E, and offers α, if non-E, then the subjective probability assigned by that person to that event is p=1/2.

Secondly, bets conditional on the ethically neutral event E make it possible to reconstruct the utilities associated with the consequences: if the individual is indifferent between the bet that offers α, if E realizes, and δ, if non-E, and the bet that offers β, if E realizes, and γ, if non-E, then, by application of the Bernoulli Norm, the expected utilities of the bets must be equal: $(1/2)u(\alpha)+(1/2)u(\delta)=(1/2)u(\beta)+(1/2)u(\gamma)$ and thus $u(\alpha)-u(\beta)=u(\gamma)-u(\delta)$. Given an order of preference on the outcomes, the indifference between the bets signals an equality between differences in utilities, therefore making it possible to assign numbers to the utilities of these bets. Assuming that in the set of outcomes there is an outcome α that guarantees the maximum utility, and an outcome β that gives the minimum utility, to which $u(\alpha)=1$ and $u(\beta)=0$ can be attributed by normalization, it is immediate to see that the utilities become numerical, as von Neumann and Morgenstern later proposed.

Finally, once the numerical utilities have been derived, it is possible to deduce the probabilities of any event other than the one considered ethically neutral. For example if the individual is indifferent between the bet that offers α, if F realises, and offers β, if non-F, and a certain value γ whether F realizes or not, it must hold that $pu(\alpha)+(1-p)u(\beta)=u(\gamma)$, so that $p= u(\gamma)-u(\beta)/u(\alpha)-u(\beta)$. If α and β are the consequences with maximum and minimum utility, $p=u(\gamma)$.

Ramsey was not the first to imagine a context in which observed choices allow the subjective components implicit in choice to be derived, as would become usual in economics for utilities of a bundle of goods after Samuelson (1938). For lotteries with given probabilities, Bernoulli's (1738 [1954], 24) examination of the different utilities that a poor person and a rich person would attribute to a lottery ticket found accidentally in the street implicitly addressed this issue. In this analysis, the utilities were derived as the price at which the two individuals would be willing to sell the ticket. However, to argue in favor of the maximization of

[10] For simplicity, we abstract here from a number of technical details, for which see Davidson and Suppes (1956). The representation of Ramsey's argument that we shall follow is that put forth by Newman (1987). As later became usual in decision theory, we concentrate on the probability of events rather than propositions concerning events, as both Keynes and Ramsey were used to do.

expected utility, Bernoulli had assumed the utility function to be given, proposing that it should increase at a decreasing rate, as suggested by the observation that the marginal utility of money should be decreasing.

The axiomatic structure that justified this procedure was eventually established by von Neumann and Morgenstern. In fact, Ramsey's 1926 procedure for measuring utilities was analogous to that introduced by von Neumann and Morgenstern in 1944, but it was more general, since Ramsey had made no reference to known probabilities generated by random mechanisms such as lotteries.[11] As popularized by Savage among decision theorists, risky alternatives were no longer lotteries but gambles (called acts by Savage), ordered by individuals using their personal probabilities on the events determining the actual outcomes to calculate the expected utilities of gambles. [12]

Ramsey (1931a [1990], 74) had also indicated which set of axioms was necessary to guarantee generality to his "system of measuring," noting in particular that an assumption of additivity applied to the ordering of bets would be sufficient to apply the typical properties of a probability function to the degrees of beliefs derived from choices. The crucial role of this assumption – very much similar to de Finetti's (1930, 2-3; 1937 [1964], 100-101) fourth axiom, arguing that inequalities between the probabilities of events are preserved in logical sums – would later be acknowledged by von Neumann and Morgenstern's commentators such as Marschak and Samuelson, who introduced the so-called independence axiom, and eventually adapted by Savage to what he called the "sure-thing principle" for decisions under uncertainty (Moscati 2016).[13]

4. Probability and the logic of consistency

[11] Interestingly, in his recollections of the genesis of *Theory of Games and Economic Behavior*, Morgenstern (1976) makes specific reference to when von Neumann had realized that it was possible to measure utility by deriving it from lottery choices. According to Morgenstern, Von Neumann was surprised that what seemed to him an obvious method of measurement had never been seen by economists. Evidently, it had seemed at least as obvious to Ramsey.

[12] The fact that this approach had been essentially ignored for over twenty years amazed even Savage. In correspondence with de Finetti, who had not given Ramsey any particular prominence in one of his works on the historical reconstruction of the approach, Savage wrote: "As soon as I saw your paper [in preparation for Savage (1950)] I knew very clearly how to blend utility and probability into one theory, and that was before I had seen Ramsey's papers … but when I saw them, it was clear to me that he anteceded me in the whole matter. His work is of the first quality intellectually, and if it is a little sketchy mathematically, it leaves no serious mathematical question … I suspect that if you reread this two brief essays that you too would feel, and wish to expess admiration" (L. J. Savage to B. de Finetti, March 3, 1961, Leonard Jimmie Savage Papers, Manuscripts and Archives Department of Yale University Library, Box 7, Folder 192).

[13] A classic explanation of how Ramsey proceeded from a set of axioms to a representation theorem, showing that a person satisfying the axioms would act as if maximizing expected utility, is provided by Sahlin (1990).

Ramsey's analysis began with a critical examination of the probability theory proposed by Keynes. For Ramsey, the Keynesian idea that there was a "rational" way to derive from evidence a degree of belief in a proposition (or in an event, as later became conventional in probability analysis) was devoid of substantive meaning. Keynes had relied on introspection and a presumed ability of a person to derive degrees of belief by self-interrogation, but he had not really been able to propose a method for assigning probabilities to events. Ramsey's formal structure, on the other hand, was based on the idea that examining a person's betting behavior would provide the method for deriving the probabilities assigned to events.[14]

Ramsey wanted to replace Keynes's intuitionist approach with a pragmatic one, in line with the philosophy of Charles Peirce, which would later be linked in decision theory to Percy Bridgman's (1927) operationalism and his key idea that the meaning of a scientific concept is defined by the set of concrete operations through which the concept is identified. Utility and probability, therefore, could only be seen as numerical entities by detailing the particular set of operations required to calculate them. And this would involve studying choices made in situations imagined to be empirically relevant, so that one could say what it actually means to have a certain numerical belief in an event, rather than simply saying that it is rational to believe in that event with that numerical belief. Decision-making took on a central role in Ramsey's approach from the very definition of subjective probability. The application of probability to conduct was no longer a chapter following the theoretical construct of probabilistic analysis. This was in contrast to Keynes's approach, who had confined his considerations about economic decision-making to Chapter 26 of his *Treatise*.

Thus, Ramsey (1931a [1990], 70) started by assuming that the maximization of expected values was a "law of psychology" governing the behavior of decision-makers on which to ground the analysis, defining degrees of beliefs in a way that "presupposes the use of mathematical expectation." As Jeffreys observed, this placed him firmly within the tradition of Thomas Bayes, whose definition of probability was based on a similar assumption regarding the calculation of the expected values of a bet. Indeed, although he did not cite Bayes in his paper, the Bayesian philosophy that was to become the dominant approach in decision theory after Savage was explicitly stated by Ramsey (1931a [1990], 72): "Let us begin by supposing that our subject has no doubts about anything, but certain opinions about all propositions. Then we can say that he will always choose the course of action which will lead in his opinion to the greatest sum of good."

The second assumption put forth by Ramsey was about the degree of consistency that could be attributed to probabilities that were no longer derived from frequencies, or from rules of deductive reasoning, but were subjective in the sense of being personal to the decision-maker. Again, Ramsey made reference to what he deemed to be common practice in the context of betting. Subjective degrees of belief had to obey

[14] A similar suggestion had been made by Émile Borel (1924) in his review of Keynes's *Treatise*. It is unclear whether Ramsey was aware of Borel's review.

the laws of a probabilistic measure because "if anyone's mental condition violated these laws … he could have a book made against him by a cunning better and would then stand to lose in any event" (Ramsey 1931a [1990], 78). This so-called Dutch book logic allowed the conclusion to be drawn that, under the conditions specified, the numbers assigned to the degrees of belief elicited from choices would satisfy the properties of a probability measure, understood as a function satisfying the mathematical calculus of probability (Hájek 2009).

What Ramsey wanted to present, as he himself emphasized, was a "logic of consistency" of probabilistic reasoning. While working within the same epistemic framework as Keynes, Ramsey did not propose a substantive rationality of probabilistic reasoning. In pursuing his goal of constructing a theory of probability as an inductive argument, Keynes was interested in identifying principles of inductive rationality which would imply that different individuals sharing the same evidence would be constrained by these principles to agree on definite probability judgements. In contrast, Ramsey considered it reasonable to admit that two individuals, presented with the same evidence, might assign different probabilities to the same event. In a sense, he regarded prior probabilities to be as subjective as preferences. Despite the initial perplexities, including those from Cambridge mentioned above, after the consolidation of the subjectivist approach in the 1950s, Ramsey's logic of consistency enlightened the whole development of economics, while Keynes's logical approach became a minority view confined to discussions of the philosophical foundations of probability (Kyburg and Smokler 1964).

Moreover, by anticipating the developments in expected utility theory of the 1950s, Ramsey could not but be credited with pioneering a fundamental change in the interpretation of rationality in economics. A traditional theory of rational choice, designed to explain why and how economic agents make decisions, was replaced by one based on the idea that to be rational simply meant to be consistent, and that the representation of consistent choices could be derived from a list of axioms about preferences (Giocoli 2003). After Savage, with the final extension of rationality to the realm of individuals' mental states, this could also happen in the more general domain of choice under uncertainty. In their systematization of the new approach to decision making, Luce and Raiffa (1957, 304) noted that the main element of Savage's theory was that "no concept of objective probability is assumed, rather a subjective probability measures arises as a consequence of his axioms." But as Savage himself had admitted, it was essentially a matter of refining Ramsey's intuition.

Ramsey's ability to take the role of rational behavior in economics to the extreme was also evident in the essays on taxation and growth praised by Keynes. In this sense, there was a normative component inherent in Ramsey's reasoning. As we have seen, the conditions under which numerical utilities and probabilities could be derived were ideal conditions. In rejecting Keynes's presupposition that "we can decide by introspection the nature of belief, and measure its degree," Ramsey (1931a [1990], 67) argued that "in

many cases, I think, our judgment about the strength of our belief is really about how we should act in hypothetical circumstances." The extent to which this claim should be taken literally as a commitment to a normative approach has been questioned at least since D. H. Mellor's assessment (1990, xviii), who warned that Ramsey's claim would be misread if intended as prescriptive, because his was a psychological theory providing simply a useful approximation to the truth. But the conventional reading seemed to have among its supporters Keynes himself, when he argued that Ramsey's main contribution had been to put formal logic "into good order" and to define "its highly limited scope" (Keynes 1933 [1972], 339).[15]

Similarly, the conditions Ramsey imagined for a tax structure on goods that could avoid distortionary effects (Ramsey 1927) or those that indicated how an optimal saving rate should take into account both the elasticity of marginal utility and the productivity of capital (Ramsey 1928), could be interpreted as normative conditions, goals of a policy maker or a social planner (see Duarte 2009, Chapter 14 and Chapter 15). Ramsey's work in pure economics was not based on axioms, but the extreme abstraction of the mathematical reasoning, using tools that were unknown to most economists at the time but later became commonplace, favored its normative interpretation. In an early recognition of Ramsey's role in economics, Samuelson (1943) mentioned his paper on optimal saving as dealing with a "rational maximizing individual" and proving "without logical inconsistency" results about the long-run tendency of the interest rate that authors such as Schumpeter and Knight may have hinted at, but were unable to prove.[16]

5. The logic of truth

It is challenging to ascertain whether Ramsey would have approved of this interpretation of his work, which essentially positions him as a pioneer both in placing the study of uncertainty within a risk-related framework and in ascribing a predominantly normative character to it. After "Truth and Probability," Ramsey had begun work on a book which would be a substantial extension and improvement of his theory, and which would have included a thorough analysis of the concept of truth. For sure, it would have included a clearer analysis of the relation between subjective probability and chance. A number of notes

[15] Whether Ramsey's idealized arguments were different from the typical idealization in utility theory, providing a kind of false ideal, is still open to debate (see Duarte and Misak 2022, 52).

[16] Ramsey himself seemed to suggest a normative interpretation of his result in the 1928 paper. In a letter to Keynes, to whom he had submitted his paper on saving as Editor of the *Economic Journal*, Ramsey argued: "Although the matter is terribly oversimplified, the equations must arise in any attempt to apply utilitarianism to saving and so far as I know they've never been treated before. The difficulty is to find simple results of sufficient generality to be interesting and yet not obvious" (reported in Duarte 2009, 465). Gaspard (2014) underlines the line of continuity that can be drawn between "Truth and Probability" and the economic essays in this respect.

written in his last years testify to a clear development in progress, but leave many questions unanswered (Misak 2020; Chapter 6).

Certainly, the final part of "Truth and Probability," which has been completely neglected in mainstream readings of his work, acknowledged that there were still many open questions, starting with the very treatment of the concept of probability. Ramsey was aware that his way of treating probabilities as consistent degrees of belief was nevertheless incomplete. We saw that in order to reject the rational degrees of belief *à la* Keynes, he wanted to allow for differences of opinion between rational-as-consistent individuals. But what then justifies differences of (a priori) opinion between different individuals if these opinions are based on the same evidence? Ramsey concluded that if "formal logic, mathematics and the calculus of probability are all concerned simply to ensure that our beliefs are not self-contradictory," this was "obviously not enough." Indeed, he claimed, "we want our beliefs consistent not merely with one another but also with the facts" (Ramsey 1931a [1990], 87).

To address this issue, Ramsey turned from the "lesser, formal logic" of consistent beliefs to the "larger, human logic" of inductive reasoning. He argued that the "human mind works essentially according to general rules or habits," but admitted that he found it difficult to adequately discuss the "large and vague question" concerning "useful habits." He therefore concluded by advocating a "kind of pragmatism," that is, judging mental habits by "whether they work, … by whether the opinions they lead to are more often true than those which alternative habits would led to" (Ramsey 1931a [1990], 93-94).

There is not much more to be found in the final section of "Truth and Probability," which was left unfinished also because of his sudden death. Nor were the scattered notes that became available in the following years of much help in this respect. There seems to be a connection between Ramsey's normatively driven work in probability and economics and the appeal to human logic at the end of "Truth and Probability" and in further notes on theories and scientific laws (Ramsey 1931c). Here the question of the extent to which individual beliefs lead to successful actions turns into an analysis of the reliability of scientific propositions on the basis their capacity to deliver beliefs leading to effective actions (Gaspard 2014, 149-150). But while Ramsey's pragmatist account of beliefs became increasingly appreciated among decision theorists, who found in it the philosophical bases for their turn to operationalism, his pragmatist account of the truth value of beliefs remained in its embryonic stage, discussed mostly among philosophers (Mellor 1990; Misak 2020).

This twofold nature of Ramsey's contribution was, of course, recognized by Keynes. As already noted, Keynes's response to Ramsey was ambiguous, and has indeed given rise to different interpretations and conflicting assessments (testifying to a still lively debate, see O'Donnell 2021, Gerrard 2023, and Chapter 1). Keynes could not have foreseen that Ramsey would end up as the founder of the (still) dominant approach to economic decision-making. But he saw that Ramsey's axiomatization was of the utmost importance.

Indeed, Keynes's main point of appreciation in his obituary was that Ramsey had succeeded in showing that the calculus of probabilities "simply amounts to a set of rules for ensuring that the system of degrees of belief which we hold shall be a consistent system." As a result, Keynes added, "Ramsey may have been pointing the way to the next field of study when formal logic has been put into good and its highly limited scope properly defined." This laudatory comment explains Keynes's apparent recantation of his own theory: "So far I yield to Ramsey − I think he is right" (Keynes 1933 [1972], 339).

However, Keynes's comments were two-sided: his appreciation of the brilliant result achieved by Ramsey in his discussion of "formal logic," possibly showing the strictures of his own logical theory of probability, was coupled with the great caution he showed in assessing the overall significance of Ramsey's enterprise. The main problem to which Keynes had tried, perhaps unsuccessfully, to provide a solution with his logical, i.e. rational, degrees of belief did not find a solution in Ramsey. Prior probabilities were now explained, according to Ramsey, in terms of "the action to which asserting it would lead." But it remained to be understood what might justify an individual opinion in the face of another, if any prior probability was in principle admissible as rational only because consistent in itself.

Ramsey's human logic was clearly intended to provide an answer to this crucial question. Separating the logic of consistency from the logic of truth, Ramsey was denying, against Keynes, that the issue of induction could find a solution within probability theory. But this "larger" logic only suggested that some (probabilistic) inductions were better than others because they were considered more successful than others. This was the essence of Keynes's apparent rejection of Ramsey's explanation of induction, which we already mentioned: the claim that induction merely is "a useful mental habit," Keynes found it unsatisfactory (Keynes 1933 [1972], 339).

As already noted, Keynes's attitude was ambiguous to the extent that it still gives birth to contrasting assessments. But a conceivable explanation of Keynes's attitude seems to be, as suggested by Jochen Runde, that while Keynes saw that Ramsey's human logic aimed to provide an account of what a decision maker ought to believe – and therefore agreed with Ramsey that human logic was distinct from both formal logic and descriptive psychology, admitting that "the application of these ideas to the logic of probability is very fruitful" (Keynes 1933 [1972], 338) – he ultimately wanted "a more prominent role for reason, in judging what we ought to believe, than Ramsey would allow" (Runde 1994, 111).

Indeed, Keynes's comment on Ramsey's approach can be interpreted in the context of his remarks on the unfortunate detour that logic took at Cambridge, to which Ramsey, in his view, had initially contributed. For Keynes, "the gradual perfection of the formal treatment at the hands of himself [Russell], of Wittgenstein and of Ramsey had been, however, gradually to empty it [the formal logic] of content and to reduce it more and more to mere dry bones, until finally it seemed to exclude not only all experience, but most of the principles, usually reckoned logical, of reasonable thought" (Keynes 1933 [1972], 338). In comparison with

these developments, Ramsey's turn to pragmatism was favorably noted by Keynes. While Ramsey's logic of consistency could indeed be seen as a fundamental contribution to Russellian logic, Ramsey himself had recognized that it needed a sounder justification other than the simple assurance that beliefs were not self-contradictory. Keynes therefore acknowledged that Ramsey's logic of truth, designed to test the consistency of beliefs with "the facts," might be seen as Ramsey's departure from the strictness of Russellian logic. But with his dry comment on how to get to the "bottom of induction," Keynes showed to consider Ramsey's proposed human logic as inadequate, insufficiently developed before his premature death.[17]

Before concluding, it should also be noted that it is difficult to discern any significant influence of Ramsey on Keynes's technical view of probability. Apart from the absence of references to Ramsey in his major work, which on the contrary contains two crucial references to the *Treatise*, this is evidenced by Keynes's correspondence with Hugh Townshend after the *General Theory*. While discussing with Townshend the characteristics of decision-making under uncertainty, Keynes (1938 [1979]) was still pondering whether it might not be possible to develop a doctrine of mathematical expectations free from the assumption that probabilities are always numerically defined, as he had explored in Chapter 26 of his *Treatise*. Had Keynes yielded to Ramsey on the possibility of deriving point probabilities from action in every case, as the strict subjectivist perspective requires, he would no longer have referred to non-numerical probabilities as a relevant objection to the received analysis of decision-making under uncertainty, unlike what he actually did (Zappia 2015).[18]

[17] Furthermore, the discrepancy between the two approaches was exacerbated by the fact that Ramsey's rationale for the efficacy of certain habits was dependent upon the observation of the frequencies with which a specific action was either successful or not. This might have been seen by Keynes as a re-evaluation of frequentism that was at odds with his own perspective. Indeed, Ramsey's understanding of chance was not one of unreserved rejection, something that evidences a significant difference in the subjectivist camp from de Finetti's approach. However, Ramsey's rejection of frequentism appeared to be unequivocal in his notes after "Truth and Probability" (see Chapter 6). Ramsey saw the importance of bridging degrees of belief and frequencies, and he did it by an appeal to the notion of success. He also held that science needs some notion of objective probability. But the philosophical and mathematical notes of the years 1928-1929 show that he was developing an original way to make sense of objective probability (chance and probability in physics) in terms of degrees of belief (Galavotti 2011).

[18] It can be argued, moreover, that for Keynes the solution could be found in a different, generalized logic of consistency. This interpretation was first endorsed by Good (1950, 14), who rejected Jeffreys's (1948) claim in the preface to the second edition of his *Theory of Probability*, that Keynes had withdrawn the suggestion of a partial ordering of probabilities in his comments on Ramsey. Good (1950, 10) argued that, although Keynes may have admitted in his comments on Ramsey that he no longer adhered to his epistemic-but-objective version of the theory of probability, it was possible to preserve the formal apparatus of his theory by adopting a theory of probability with beliefs derived from actions, but not as definite and consistent as Ramsey had assumed (see also Sahlin 1990, 46-8, Runde 1994, and Kyburg 2003).

6. Conclusion

The canonization of Ramsey attributes to him a notion of rationality that simply means consistency. His role as innovator would consist in his ability to identify how ideal economic agents should be represented, in conformity with a normative view of decision-making. In the economics of the second half of the XX century, acting rationally under uncertainty has implied following what Marschak indicated as the "Ramsey Norm."

However, a more nuanced perspective emerges from a thorough examination of his oeuvre, wherein rationality may be replaced by the more elusive notion of reasonableness, meaning conformity to reality. In this view, human agents should aim not at the ideal decision indicated by axiomatically justified norms of behavior, but at the actual decision that a reasonable degree of belief justifies. And a reasonable degree of belief can be ascertained in terms of the success of habits. Habits such as "inference … observation and memory" (Ramsey 1931a [1990], 92) are as relevant as the attitude to action justified by formal logic. The reasonableness of habits is thus demonstrated by the results ensured by the actions they imply, the more successful the action the better the mental habit is.

All this, would make it Ramsey's theory more in tune with a descriptive view of decision-making. But the kind of rationality as reasonableness that it seems to imply still remains to be investigated.

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