

Whither Keynesian probability? Impolite techniques for decision-making

This is the peer reviewed version of the following article:

Original:

Zappia, C. (2016). Whither Keynesian probability? Impolite techniques for decision-making. EUROPEAN JOURNAL OF THE HISTORY OF ECONOMIC THOUGHT, 23(5), 835-862 [10.1080/09672567.2015.1068349].

Availability:

This version is available <http://hdl.handle.net/11365/984575> since 2021-04-05T20:09:27Z

Published:

DOI: <http://doi.org/10.1080/09672567.2015.1068349>

Terms of use:

Open Access

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. Works made available under a Creative Commons license can be used according to the terms and conditions of said license.

For all terms of use and more information see the publisher's website.

(Article begins on next page)

Whither Keynesian Probability? Impolite Techniques for Decision-Making

European Journal of the History of Economic Thought, Forthcoming

18 Pages Posted: 6 Aug 2015

[Carlo Zappia](#)

University of Siena - Department of Economics and Statistics

Abstract

This paper focuses on Keynes's understanding of individual decision-making under uncertainty and tries to address a question left mostly unexplained in the critical literature. On re-reading Keynes after the recent surge of interest induced by the financial crisis, a number of scholars have placed emphasis on well-known excerpts from the General Theory dealing with the "state of long-term expectations." Keynes evidenced that future events conditioning economic activity in general, and investment in particular, "can only be forecasted with more or less confidence," and that "the state of confidence, as they term it, is a matter to which practical men always pay the closest and most anxious attention," while economic theory does not. In these instances, Keynes argued, people may fall back on "conventions," which give them assurance that they are doing the right thing. As a result, "to behave in a manner which saves our faces as rational, economic men ... we have devised for the purpose a variety of techniques." But Keynes's main message is that under uncertainty "[a]ll these pretty, polite techniques, made for a well-panelled Board Room and a nicely regulated market, are liable to collapse." For instance, conventional behaviour easily turns into herd behaviour, and financial markets are pervaded by alternating currents of euphoria and panic. So a question remains: how should Keynesian agents behave in markets continuously on the brink to fail? Are there, rephrasing Keynes, "impolite techniques" to be used under uncertainty or must we end up admitting that "we simply do not know," and behave in a conventionally understood fashion? The paper offers a reading of Keynes suggesting that the thread going from Keynes's Treatise on Probability to the General Theory and its defence provides a positive analysis of decision-making under uncertainty, and argues that placing emphasis on this positive analysis simply means adhering to Keynes's long-standing commitment to interpret reasonable judgement in a (surely unconventional) probabilistic set-up.

Keywords: Keynes, probability, uncertainty, decision-making

JEL Classification: B21, D21

Suggested Citation:

Zappia, Carlo, Whither Keynesian Probability? Impolite Techniques for Decision-Making (March 2015). European Journal of the History of Economic Thought, Forthcoming, Available at SSRN: <https://ssrn.com/abstract=2639764>

Whither Keynesian probability? Impolite techniques for decision-making¹

CARLO ZAPPIA

Department of Economics and Statistics
University of Siena

March 2015

Abstract

This paper focuses on Keynes's understanding of individual decision-making under uncertainty and tries to address a question left mostly unexplained in the critical literature. On re-reading Keynes after the recent surge of interest induced by the financial crisis, a number of scholars have placed emphasis on well-known excerpts from the *General Theory* dealing with the "state of long-term expectations." Keynes evidenced that future events conditioning economic activity in general, and investment in particular, "can only be forecasted with more or less confidence," and that "the state of confidence, as they term it, is a matter to which practical men always pay the closest and most anxious attention," while economic theory does not. In these instances, Keynes argued, people may fall back on "conventions," which give them assurance that they are doing the right thing. As a result, "to behave in a manner which saves our faces as rational, economic men ... we have devised for the purpose a variety of techniques." But Keynes's main message is that under uncertainty "[a]ll these pretty, polite techniques, made for a well-panelled Board Room and a nicely regulated market, are liable to collapse." For instance, conventional behaviour easily turns into herd behaviour, and financial markets are pervaded by alternating currents of euphoria and panic. So a question remains: how should Keynesian agents behave in markets continuously on the brink to fail? Are there, rephrasing Keynes, "impolite techniques" to be used under uncertainty or must we end up admitting that "we simply do not know," and behave in a conventionally understood fashion? The paper offers a reading of Keynes suggesting that the thread going from Keynes's *Treatise on Probability* to the *General Theory* and its defence provides a positive analysis of decision-making under uncertainty, and argues that placing emphasis on this positive analysis simply means adhering to Keynes's long-standing commitment to interpret reasonable judgement in a (surely unconventional) probabilistic set-up.

¹ A revised version of this paper is forthcoming in the *European Journal of the History of Economic Thought*. A previous draft was presented at the Conference "75 Years of the General Theory", Université Paris 1, Panthéon Sorbonne, December 2011, revised while the author was the Shackle Scholar at St. Edmund's College, University of Cambridge, in the Easter Term of 2012. Comments by Roger Backhouse, Marcello Basili, Mario Cedrini, Annie Cot, Alberto Feduzi, Mark Hayes and Jochen Runde are gratefully acknowledged.

1. Introduction

When reacting to critics of his *General Theory*, Keynes (1937) emphasized that the revolutionary nature of his theory stayed in its property of being relevant for a world in which there is true uncertainty. In a rather unnoticed passage from his 1937 *Quarterly Journal of Economics* article, Keynes (1937, p. 215) summarizes his dealing of behaviour under uncertainty in the first part of that paper as such:

“Perhaps the reader feels that this general, philosophical disquisition on the behavior of mankind is somewhat remote from the economic theory under discussion. But I think not. Tho[gh] this is how we behave in the market place, the theory we devise in the study of how we behave in the market place should not itself submit to market-place idols. I accuse the classical economic theory of being itself one of these pretty polite techniques which tries to deal with the present by abstracting from the fact that we know very little about the future.”

Secondary literature on Keynes has provided extensive analysis of what Keynes meant for “polite techniques,” relating them to Keynes’s claim that “Benthamite calculus” was ineffective to deal with uncertainty. The Post-Keynesian literature, in particular, has pointed out that it is the *Treatise on Probability* that provides the rationale for Keynes’s critique of the mainstream on this issue (Carabelli 1988, O’Donnell 1989).

This paper concentrates instead on what Keynes may have meant for techniques of thought apt to deal with an uncertain future, which in the rest of the paper I will term “impolite techniques.” My claim is that a fresh reading of the *TP* makes it possible to conjecture about the kind of techniques Keynes may have had in mind when claiming that, although decision theory has to address “the fact that we know very little about the future,” he was nonetheless interested in a theory that “should not itself submit to market-place idols.” Thus the paper offers a reading of Keynes that suggests that the thread going from the *TP* to the *GT* and its defence provides a positive analysis of decision-making under uncertainty, one that is suitable for formal developments.

The investigation I provide in this paper can help shed light on two related interpretative issues. First, I think it is important to make clear whether Keynes really suggested that, when considering matters on which “we simply do not know” (Keynes 1937, p. 214), economic theory should simply retreat from trying to provide a formal treatment of decision-making under uncertainty. This point, which has been made by a majority of Keynesian interpreters, would place the very notion of Keynesian uncertainty on the brink of being considered a verbal notion, devoid of any formal content, at least with respect to the conventional stance that subjective expected utility offers a distinct criterion for decision-making under both risk and uncertainty (Hirshleifer and Riley, 1992). My argument is that in placing emphasis on the positive analysis of decision-making emerging from his works, and trying to put forward a criterion for decision making under uncertainty alternative to the maximization of subjective expected utility, one is simply adhering to Keynes’s long-standing commitment to interpret reasonable judgement in a (surely unconventional) probabilistic set-up.

Second, the spreading interest in behavioural decision-making and the contribution it can provide to the understanding of the current financial crisis has provoked a surge of interest in Keynes’s notion of animal spirits. Since behaviour in markets is typically conventional, it can easily turn into herd behaviour, and financial markets may end up pervaded by alternating currents of euphoria and panic. As a result, Keynes’s animal spirits have been interpreted as depicting the kind of “irrational” behaviour that may explain why standard economic theory could not address the

malfunction of financial markets, as in behavioural finance. This viewpoint, which emphasises the “non-economic motives” of economic decisions (Akerlof and Shiller 2009, p. ix), downplays the ability of Keynes’s analysis to translate into a usable theory for decision-making. My argument in what follows suggests that the statement that for Keynes individuals who want to avoid falling back on “conventions” can only react “irrationally” is misplaced.

The question this paper tries to address can then be put as follows. Suppose Keynesian agents reject the attitude to fall back on conventions, and to rely on non-rational behaviour in order to “save [their] faces as rational, economic men.” How should they behave in markets continuously on the brink to fail? What kind of criterion for decision-making would they adopt? In the rest of the paper I will take into account a number of classic references of Keynes’s dealing of uncertainty and try to show how they can help provide a positive analysis of “impolite techniques”.

2. The correspondence with Hugh Townshend

In what follows I will make extensive use of the link between the *TP* and the *GT*, placing emphasis on the analogies that can be envisaged among the two books as regards decision-making. In dealing with a number of issues I will assume a continuity of thought which has been questioned by many (among others, see Bateman 1996). The textual pieces used to deny that the *GT* can be seen as endorsing the same perspective of the *TP* are well known: first, the obituary of Frank Ramsey written by Keynes in 1931, and, second, the retrospective essay “My Early Beliefs” published in 1938. I am not going to comment on these pieces, since both have been made subject of controversy and did not appear conclusive to many commentators (see for instance Runde’s (1994) rebuttal of Bateman’s point about Keynes’s endorsement of Ramsey’s theory of subjective probability).

In order to offer an alternative, arguably sounder textual evidence of my assumption of continuity in Keynes’s thought I rely on another well-known piece, namely, the correspondence Keynes had with Hugh Townshend that is reproduced in the 29th volume of Keynes’s *Collected Writings*. Although the correspondence with Townshend refers to the years of the defence of the *GT*, it should be noted that Keynes’s acquaintance with Townshend originated in the years Townshend spent in Cambridge in the early 1910s while he was taking a first in mathematics, and that Townshend had been a pupil of Keynes while preparing for Civil Service examinations he took in 1914 (Chick 1987). Townshend did not enter academics and had a career as civil servant, but the correspondence reveals that Keynes (1936-1938, pp. 235-239) valued Townshend’s private comments on the *GT* highly.² When a letter from Townshend appeared in *The Economist*, in which Townshend suggested that the journal’s reviewer of the *GT* had misunderstood Keynes’s definition of investment, Keynes showed his appreciation for Townshend’s comments: “Once more you have shown a complete comprehension of what I am driving at, and I am very grateful” (letter to Townshend 24, March 1936).

In March 1937 Keynes, then editor of the journal, published an essay by Townshend in the *Economic Journal*. Among Keynesian interpreters Townshend’s 1937 essay is attributed a prominent role in presenting the subversive character of Chapter 17 of the *GT* and, on uncertainty issues, it is usually paired with Keynes’s own 1937 *QJE* essay (Shackle 1967, pp. 246-248). On the

2 On receiving from Townshend a note suggesting corrections to a number of points in the *GT*, Keynes replied: “I am very grateful for your corrections, all of which seem to me to be sound.” Keynes added that, as a reprint of his book was in order, “I sent off yesterday to the printer some corrections based on your notes” (letter to Townshend, 11 March 1936). For more on the significance of the Keynes-Townshend correspondence, see Zappia (2015).

occasion of the publication of Townshend's essay the correspondence with Keynes hinged on the dealing with uncertainty: "you have put your finger on the spot in saying that they [the classical economists] are trying to describe a world in which risks exists without uncertainty" (letter to Townshend, 11 April 1937). And when Keynes asked Townshend to review Shackle's volume *Expectations, Investment and Income*, the issue of how to deal with probability in the valuation of decision made in an uncertain environment is dealt with by Keynes (letter to Townshend, 27 July 1938) as follows:

"I fancy one has to tackle it on the basis of 'equivalent certainties' ... But a main point to which I would call your attention is that, on my theory of probability, the probabilities themselves, quite apart from their weight or value, are not numerical. So that, even apart from this particular point of weight, the substitution of a numerical measure needs discussion."³

I think it is fair to assume then that the exchange with Townshend reveals Keynes's actual thoughts about uncertainty issues in the years of the defence of the *GT*. This is especially true since in their exchange Keynes and Townshend are discussing the "alternative lines" examined by "those [scholars] who, following on the appearance of the *GT*, are trying to develop further an expectational economic analysis" (H. Townshend, letter to Keynes, 25 November 1938). This was suggested to Townshend by Keynes himself, who ended the letter to Townshend with a sentence that would serve well as a summary of Chapter 26 of the *TP* on the application of probability to conduct (letter to Townshend, 27 July 1938):

"... the economic problem is of course only a particular department of the general principles of conduct, although particularly striking in this connection because it seems to bring in numerical estimations. One arrives presumably at the numerical estimations by some system of arranging alternative decisions in order of preference, some of which will provide a norm by being numerical. But that still leaves millions of cases over where one cannot even arrange an order of preference. When all is said and done there is an arbitrary element in the situation."

In a letter of November 1938, Townshend offers to Keynes a long analysis of "methods hitherto used for expectational economic analysis," concluding that "the question whether, as you suggested in your letter, it may not be possible to develop a logical doctrine of equivalent certainties free from the assumption of numerical probabilities and perhaps of wider than economic application" is left open. Indeed "the element of arbitrariness in judgements of probability, to which you refer", Townshend argues "really implies a criticism, or at least calls for further analysis, of the basic concept of economic man, defined as *determinately* motivated by (his) judgements of maximum (in some sense) anticipated profitability" (letter to Keynes, 25 November 1938).

It should be stressed then that it is with reference to this notion of "economic man" and his reliance on a orthodox criterion for decision-making that Keynes (letter to Townshend, 7 December 1938) replied in conclusion that these are to be considered "elusive problems." And it is to the traditional notion of economic calculus that Keynes refers when he claims that "all this is not

3 The letter from Townshend to which Keynes is replying has not survived. But the use of the term equivalent certainties in quotation marks suggests that the discussion among the two related to Shackle's first attempt to deal with investment decisions under uncertainty (Shackle 1938). Indeed, in his review of Shackle's book appeared in the *Economic Journal*, it is Townshend (1938, p. 523) who emphasises that "Mr. Shackle's concept of 'equivalent certainties', ingenious and interesting as it is" does not survive the test of the numerical character of probability posed by Keynes in the *TP*.

particularly an economic problem, but affects every rational choice concerning conduct where consequences enter into the rational calculation,” to such an extent that “in making a decision we have therefore before us a large number of alternatives, none of which is demonstrably more ‘rational’ than the others”. Therefore Keynes’s conclusion that “to avoid being in the position of the Buridan’s ass, we fall back ... on motives of another kind,” concerns rational calculation and the picture of both the “non-economic” and “economic man.” Keynes is still talking of the established view of “economic man” when concluding that “it may well be, as you suggest, that when we remember all this we have to abate somewhat from the traditional picture of the latter.”

The issue of whether, after Ramsey, Keynes abandoned the logical epistemology on which his theory rested is controversial, although Keynes seems perplexed about defending it (Raffaelli 2006). Yet the correspondence with Townshend shows that Keynes never endorsed the view that rational beliefs imply the kind of consistency on which subjective probabilities are based and that non-numerical probabilities cannot guarantee. Had he yielded to Ramsey’s on the possibility to derive point probabilities from action in every instances, Keynes would not refer to non-numerical probabilities as such strong an objection to received analysis of decision-making under uncertainty.⁴ Moreover, the correspondence with Townshend indicates that his interest in methods for evaluating decision-making under uncertainty did not fade away even when considering the cases in which Benthamite calculus is inapplicable that he discussed in the *GT*. This is a point rarely made in the critical literature on Keynes, which insists that Keynes’s rejection of mathematical expectation and probability calculus necessarily entails endorsing a theory of decision-making which, in the *GT*, is mostly based on conventional valuations.

3. Instability due to speculation and instability due to human nature

There surely is a conventional element which is implied by Keynes in his correspondence with Townshend when talking of non-economic motives as motives “that are not ‘rational’ in the sense of being concerned with the evaluation of consequences, but are decided by habit, instinct, preference, desire, will etc.” This is reminiscent of a crucial chapter of the *GT* so this section turns to its examination.

Keynes’s analysis of uncertainty which Chapter 12 of the *GT* is central for the study of Keynesian uncertainty. The emphasis placed on it by a number of Keynesian followers has suggested to other commentators the tag “Chapter 12 Keynesians” (Coddington 1976).⁵ The impact of uncertainty on investment decision motivates Keynes’s concern with the rational grounds underlying decision-making, and conventional behaviour comes to the fore in Keynes’s analysis of financial markets, when discussing investor’s beliefs about prospective yields on assets.

The scale of investment depends on the prospective yield of an asset. When discussing

4 This is confirmed by another remark made by Keynes in his correspondence with Townshend, re-stating the connection between liquidity and the weight of argument he had made in Chapter 17 of the *GT*, which suggests the continuity of Keynes’s thought (Runde 1994 and Gerrard 2003). As it is well-known in his last letter to Townshend reproduced in the *Collected Writings*, Keynes (letter to Townshend, 7 December 1938) argues: “I am rather inclined to associate risk premium with probability strictly speaking and liquidity premium with what in my *Treatise on Probability* I called weight,” with the liquidity premium representing “a payment ... for an increased sense of comfort and confidence.” In Ramsey’s subjectivist perspective there is no room for a measure representing the degree of reliance on a probability assessment.

5 For instance, Dow (2003) argues that Keynes’s explicit treatment of convention in the *GT* substitutes for his explicit treatment of probability in the *TP* (see also Davis 1994)

changes in the type and quantity of the stock of capital-assets, expectations about future yields can be termed long-term, and in forming long-term expectations “it may be foolish ... to attach great weight to matters which are very uncertain” (Keynes 1936, p. 148). The confidence with which forecasts are made is as relevant as the ability to formulate a probable forecast and it is only the “actual observation of markets and business psychology” that can indicate what the actual state of confidence is. The stock market is therefore seen as reflecting valuations that depend on the decision taken by investors that are typically unable attach numerically defined probabilities to the possible outcomes of their investment decisions. “In practice” investors tacitly agree “to fall back on what is, in truth, a convention”, that is, they assume that “the existing state of affairs will continue indefinitely,” except in so far as they have “no specific reasons to expect a change” (ibidem, p. 152). It is the precariousness on which conventions are based that makes the economic system unstable.

Keynes examined two kinds of instability. First, there is instability due to speculation. This is exemplified through the famous parallel between stock markets and “those newspaper competitions in which the competitors have to pick out” a beauty among a group. Investment based on “genuine” long-term expectations can be “scarcely practicable” and considered “eccentric ... in the eyes of the average opinion,” since what will turn out profitable is the ability to forecast the psychology of the market, in much the same way as in a beauty contest (ibidem, pp. 153, 157). Second, there is instability due to the characteristic of human nature. It is only as a result of a “spontaneous urge to action” that decisions to invest are taken in an uncertain environment, and “individual initiative will only be adequate when reasonable calculation is supplemented and supported by animal spirits.” (ibidem 161, 162). This means that investment activity will depend on spontaneous optimism (or pessimism), thus adding to instability.

Both these instances of instability are reformulated by Keynes in his 1937 *QJE* essay and the emphasis placed on them in an essay explicitly presented as a summary of the *GT* makes it possible to conclude that Keynes’s aim was to characterise his revolution by means of his understanding of uncertainty (Backhouse and Bateman 2011). In the 1937 article, moreover, the emphasis on uncertainty is even stronger than in Chapter 12. Uncertainty is characterized as a situation where there are matters about which “there is no scientific basis on which to form any calculable probability whatever.” Under this kind of uncertainty the “necessity for action” may entail to devise a “variety of techniques” which consists, partly in relying on Benthamite calculus, even though its application is nonsensical, and partly on concentrating on the “state of opinion.” This renders “the psychology of a society of individuals each of whom is endeavoring to copy the others” something that “leads to what we may strictly term a conventional judgement.” The obvious consequence of a state of affairs “based on such a flimsy foundation,” may well be that “the forces of disillusion may suddenly impose a new conventional basis of valuation” and the techniques made for “a nicely regulated market” may collapse (Keynes 1937, pp. 214-215).

Both in Chapter 12 of the *GT* and in the *QJE* article, though, Keynes contrasts his apparently sceptical attitude with claims to the contrary. On instability due to speculation he suggests: “The social object of skilled investment should be to defeat the dark forces of time and ignorance ... The object of skilled investment to-day is ... to outwit the crowd.” (ibidem 155). On instability due to human nature he argues: “We should not conclude from this that everything depends on waves of irrational psychology ... We are merely reminding ourselves that human decisions affecting the future, whether personal or political or economic, cannot depend on strict mathematical expectation.”⁶ Finally, as already noted, on “polite techniques” he concludes that “the theory we

6 Moreover, it is well-known that Chapter 12 ends with a claim for policy intervention as a factor able to

devise in the study of how we behave in the market place should not itself submit to market-place idols.”

To sum up: on the one hand, the limits to the practical relevance of probability calculus is at the foundation of Keynes’s insistence on long-term expectations; on the other hand, Keynes refers to a theory of economic behaviour under uncertainty which is still based on his generalized notion of probability. Rational belief must be replaced by a kind of reasonable judgement influenced by non-economic motives. But this does not mean that no formal modelling of the behaviour of individual agents was suggested by Keynes. I would consider then unsound the point made, for instance, by Skidelsky when claiming that no formal model can be provided since, for Keynes, uncertainty is irreducible on ontological grounds, and some probabilities are not just unknown, but “non-existent” (Skidelsky 2011, p. 3). As it is typical of most post-Keynesian literature, Skidelsky’s insistence on the importance of individual behaviour under uncertainty is not substantiated by a specific indication of how Keynesian agents should behave in the market, apart from the reference to unqualified conventional habits. For this, a closer inspection of the *TP* is worth making.⁷

4. Keynes’s theory of probability and its application to conduct

In his *TP* Keynes questioned what contemporary frequency probability theory could encompass, and put forward an alternative, epistemic notion of probability. But retrospectively assessed his analysis has a much wider significance, since Keynes’s critical remarks in the *TP* constitute a challenge to any theory of probability which, like frequency probability, is based on a unique additive distribution. Similarly to some current criticism of the Bayesian mainstream, Keynes came to reject the idea that probability functions are always well-defined. This section aims to show in which sense Keynes’s critique of frequency probability has a positive content that is usually disregarded among commentators. In particular, as argued in Basili and Zappia (2009), Keynes’s view of the potential incomparability of alternative probability assessments hints at the crucial notion of decision weights that are not usual probabilities, but distortions of probability measures. Also, Keynes’s application of the theory of probability to “human conduct” goes further than a critique of maximisation, and discusses the need for every sensible decision rule under uncertainty to incorporate a measure of the degree of confidence in the probability assessment.

Keynes’s theory provides an interpretation of probability different from chance or frequency. Probability is seen as a property of the way individuals think about the world. In Keynes’s view (1921, p. 109), probability should not be identified with statistical frequency, as the theory of probability concerns the broader issue of inferring degrees of belief from the available evidence, whereas relative frequencies are a special kind of evidence. Probability, Keynes argued, “is concerned with arguments, that is to say, with the ‘bearing’ of one set of propositions upon another set.” The subject matter of the theory of probability, therefore, is the logical relation of implication between a certain evidence and a conclusion, “a relation, in virtue of which, if we know the first, we can attach to the latter some degree of rational belief” (Keynes 1921, pp. 6-7).⁸

mitigate the effects of uncertainty, the State being “in a position to calculate the marginal efficiency of capital-goods on long views.”

7 In what follows I will try to present a view that partly overlaps with Gerrard (1994 and 2003) and Runde (2001). A similar view can also be found in Carabelli (2002) and Carabelli and Cedrini (2011) when they concentrate on the need to identify what “reasonable judgement,” as opposed to “rational belief,” may mean in Keynes.

8 Depending on the knowledge on which it is based probability may appear subjective, Keynes (1921, p. 4) maintained, but “the theory of probability is logical ... because is concerned with the degree of belief which is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular

In the *TP* Keynes did not limit himself to a philosophical inquiry, but tried to develop his own formal logic of probability. These formal developments did not get attention among decision theorists after the subjectivist approach shifted emphasis towards consistency as an evaluation criterion for probabilistic axiomatic systems, and remained part of a minority viewpoint (Kyburg 1995). But the rationale of Keynes's attempt is of great interest nonetheless for at least three reasons. Firstly, Keynes rejected the idea, implicit in the definition of frequency probability, that probabilities can always be represented through real numbers, and pointed out the limited degree to which probability can be measured. Secondly, Keynes introduced the notion of weight of argument by arguing that even when a probability measure can be identified the degree of completeness of information is a crucial factor in the assessment of an uncertain environment. Thirdly, he discussed the issue of what kind of decision rule proves to be consistent with his critiques of frequency probability. In what follows, these aspects are briefly examined.

Keynes's rejection of the numerical character of probability was put forward in the first part of the *TP*, introducing his "fundamental ideas" on probability. Keynes (1921, p. 21) argued against the generally accepted opinion that "a numerical comparison between the degrees of any pair of probabilities is not only conceivable but it is actually within our power." Being critical of the frequentist viewpoint that the numerical character of probability is necessarily involved in the definition of probability as the ratio between "favourable cases" and the "total number of cases," he analysed various instances of ordinary life in which "no rational bases have been discovered for numerical comparison" (Keynes 1921, p. 23). Only in "very special case" where the principle of indifference can be applied, Keynes (1921, p. 32) argued, "a meaning can be given to a *numerical* comparison of magnitude." Keynes conceded that, probability being an intermediate stage between certainty and impossibility, when one argues that one probability is "greater" than another, "this precisely means that the degree of our rational belief in the first case lies *between* certainty and the degree of the rational belief in the second case" (Keynes 1921, p. 37). But he also stressed that the probabilities of two quite different arguments can be impossible to compare. Probabilities can be compared if they belong to the same "ordered series," that is, if they "belong to a single set of magnitude measurable in term of a common unit." But there may be more than one "path" from certainty to impossibility and probabilities cannot be compared if they belong to two different paths. Also, when there is more than one path probabilities can still be placed into order if they follow the same path, but cannot be numerically measurable.⁹

It is apparent that the issue of "non-numerical" probabilities is reminiscent of the Knightian distinction between risk and uncertainty, with the latter meant to describe situations in which information cannot be summarized by probabilities.¹⁰ However the Knightian case of unmeasurable

individuals." The *TP*, therefore, shows how to derive knowledge from probability arguments, and the goal of the "logical" approach is to identify the principles of inductive rationality leading different individuals sharing the same evidence to agree on definite probability judgements. This logical perspective met with strong resistance after the emergence of the subjectivist-personalist approach and became a minority viewpoint among philosophical theories of probability. As already noted, while advocating an epistemic approach to probability, Ramsey (1926) rejected Keynes's view of probability as a purely logical relation.

⁹ Keynes's discussion was summarized in a diagram, featuring different probabilistic paths. A linear path accounts for the usual probabilistic representation, ranging from impossibility to certainty, but other different non-linear paths between the extremes, that do not lie on the straight line, are drawn. These paths represent what Keynes (1921, p. 42) called a 'non-numerical probability' or a 'numerically undetermined probability'. Only probabilities lying on the same path, or on paths that have points in common, can be compared among themselves, but 'the legitimacy of such comparison must be a matter for special inquiry in each case' (Keynes 1921, p. 40).

¹⁰ That Keynesian uncertainty and Knightian uncertainty overlap on various points has been argued by several authors (in particular see O'Donnell 1989, and Runde 2001). This is an aspect emphasized by

uncertainty can be seen as a limit case in Keynes's taxonomy of probabilistic cases. Keynes identified two extremes delimiting the array of situations in decision-making, constituted by the possibility to derive numerical probabilities (Knightian risk) and the situation in which there is no reason available on which deriving probability of any kind (Knightian uncertainty). But he concentrated mainly on intermediate situations. Keynes's notion of probability covers cases ranging from non-numerical probabilities making possible qualitative comparisons, to non-numerical probabilities that do not allow for comparisons and may turn out to imply "rational dilemmas" (Carabelli 1998). Keynes's main aim in the *TP* was to provide a formal structure for comparisons between probability relations concerning these intermediate situations, that is, when the standard approach to probability fails, but there are grounds on which human action can be taken, something Knight did not aim at. This was presented in the second part of the book, intended to state the "fundamental theorems" of his theory. Keynes stressed the inductive process of deriving new probability comparisons on the basis of other comparisons constituting direct knowledge, and tried to establish the conditions for the emergence of an ordering of probabilities, well aware that the incompleteness of the probability relation induces only a partial order. Further, he attempted to account for a numerical measure of a relation of probability through the method of "numerical approximation," that he described as "the relating of probabilities, which are not themselves numerical, to probabilities, which are numerical."¹¹

The interpretative point here is that Keynes clearly stated that his concern with probabilities that are not numerical could be given theoretical content. Probabilistic weights like the ones represented by non-linear paths from impossibility to certainty can be approximated through the assigning of intervals of probabilities. Indeed, Keynes pointed to inexact numerical comparison rather than simply to the impossibility of attributing cardinal numbers and deriving probability comparisons: this approach has a precise formal meaning that survived the decline of interest in logical probability thanks to the works of a number of authors adhering to the subjectivist tradition, but critical of its strictly Bayesian version.¹²

A reading of the *TP* emphasising the philosophical foundations of Keynes's characterization of uncertainty as entailing an epistemic state of individuals in which, using Keynes's (1937, p. 214) words, "we simply do not know," misses the importance of "non-numerical" probabilities for an understanding of the kind of numerical comparisons Keynes might have in mind. But the traditional reading of the *TP* also misses out a related technical aspect. The Keynesian paths describing non-numerical probabilities closely resemble what nowadays would be identified with decision weights, that is, distortions of an usual probability measure. In modern decision theory this issue has come to the fore after Kahneman and Tversky's (1979) introduction of prospect theory. As it is well-known, in order to justify individuals' unwillingness to use objective probabilities as a basis for decisions not under risk, as in the experimental evidence concerning the Allais Paradox, Kahneman and Tversky represented the perception of probabilities through probability weighting functions. It has thus become common to use weighting functions to represent decision-makers who over-weight low

those Keynesian scholars who, following Shackle (1967), claim that "for Keynes uncertainty is an absence of probabilistic reasoning" (Hillard 1992, p. 69).

11 Keynes (1921, p. 176) maintained that "many probabilities, which are incapable of numerical measurement, can be placed nevertheless *between* numerical limits. And by taking particular non-numerical probabilities as standards a great number of numerical comparisons or approximate measurements become possible."

12 Keynes's attempt was taken up by Koopman (1940), Good (1950) and Smith (1961), who built an axiomatic system based on the intuition that probabilities usually provides only a partial ordering, and introduced the notion of upper and lower probabilities (Walley 1991). Brady (1993) and Brady and Arthmar (2012) examine the Keynesian rationale of this thread.

probabilities and under-weight high probabilities, a pattern of behaviour regularly observed in actual decision-making under both risk and uncertainty (Wu and Gonzales 1999).¹³ Behavioural theories of financial markets, like those referred to by Akerlof and Shiller (2009), have argued in recent years that most of what is left unexplained by the theory of efficient markets can be understood by assuming that individual agents deviate from the rationality of subjective expected utility in the specific way proposed by prospect theory and the ensuing developments. On these grounds, it can be maintained that the Keynesian origins of most recent developments have been substantially overlooked.

The second fundamental aspect of Keynes's rationale for criticizing frequency probability was the weight of argument. Keynes emphasized that the uncertainty surrounding the individual cannot be represented only through probability: the confidence in the probability assessment itself is another relevant dimension in the epistemic state of the individual. In Keynes's view the measurement of probabilities should encompass both the magnitude of the probability of an argument and the degree of confidence in it. Keynes (1921, p. 82) exemplified the way in which the standing of a probability assessment depends on the information on which the assessment is based through the example of coloured balls drawn from urns. Keynes claimed that, by virtue of the principle of indifference, the probability of drawing a white ball from an urn known to contain black and white balls in equal proportion is equal to the probability of drawing a white ball from an urn containing an unknown proportion of white and black balls. But he argued nonetheless that, in the first case, a greater weight supports the argument that the probability is $\frac{1}{2}$.¹⁴

In order to make clear that probability and weight are "independent properties" Keynes (1921, p. 345) added that the weight could be thought as "the degree of completeness of the information upon which a probability is based."¹⁵ The intent was to specify that the weight of argument is not a second order probability. But the distinction between a probability assessment and the degree of confidence in it has no place in a strictly Bayesian set-up, where the decision-maker chooses as if she was guided by precise numerical probabilities of the consequences of her action, and confidence is considered at most a probability distribution over the probability distribution on the set of events, and an axiom of compound probabilities is used. As a result this distinction has been long ignored in mainstream decision theory.¹⁶ However some recent developments endorsing a

13 An axiomatic representation of probability weight conforming with Kahneman and Tversky's evidence has been proposed by means of non-additive probability measures. A consistent theory of decision capable of accommodating the behaviour of Kahneman and Tversky's experimental subjects has ensued in the forms of Rank Dependent Expected Utility (Quiggin 1982) and Choquet Expected Utility (Schmeidler 1989), in which a non-additive probability measure is used to represent the individual's assessment of uncertainty. Gilboa and Schmeidler proved that there is an homomorphism between non-additive probability measures and interval-valued probabilities (Gilboa 2009).

14 This is the example later made known in the economics literature by Ellsberg (1961). Ellsberg suggested that decisions contemplating the "unknown" urn can be rationalized as if the probability of drawing a white ball is less than $\frac{1}{2}$. Since the same holds for the drawing of a black ball, either probabilities are non-additive or the decision-maker is using an interval of probability values (Ellsberg 1961, pp. 650)

15 As $p(a/h)$ is the probability of some proposition a , on the basis of the available evidence h , the weight of a certain proposition a given the available evidence h is $w(a/h)$. Following Runde (1990), by using K to denote knowledge and I to denote ignorance, the weight as degree of completeness of information is: $w(a/h)=K/(K+I)$. If $K+I$ is normalized to the unity, w ranges from 0 to 1, as suggested by Keynes in Chapter 26 of the *TP*.

16 It is a fundamental contribute of Keynesian scholars reading the *TP* in the late 1980s the stress on Keynes's claim that probability and weight are independent properties, and that the weight is intended to provide a deeper explanation of the way individuals assess uncertain phenomena (Lawson 1985; Carabelli 1988; O'Donnell 1989). As recalled above, Keynes himself provided an example of the distinction in his

less strict Bayesian viewpoint attribute a fundamental role to what Keynes termed weight of argument. This is apparent in the increase of literature on the so-called Ellsberg Paradox. In his study of agents deliberately violating the axioms of Bayesian rationality, Ellsberg (1961, p. 657) remarked that the nature of the individual's information concerning the likelihood of events is a relevant dimension of the decision problem, and proposed to call it the ambiguity of information, "a quality depending on the amount, type, reliability and 'unanimity' of information" expressing the individual's "degree of confidence in an estimate of relative likelihoods." Ambiguity, he argued, can be considered a special case of uncertainty.¹⁷ This is highly significant since all current developments of decision theory under ambiguity and uncertainty originated as attempts to account for Ellsberg's ambiguity (Wakker 2008).¹⁸

The third aspect of Keynes's rationale for criticizing frequency probability relates to his rejection of the use of mathematical expectation as a criterion for making decisions. To this aspect Keynes devoted his investigation of "the application of probability to conduct" in Chapter 26 of the *TP*. Here Keynes dealt with the interpretation of "goodness" of choice when "it is not rational for us to believe that the probable is true." Keynes (1921, p. 343) recalled that "normal ethical theory at the present day makes two assumptions: first, that degrees of goodness are numerically measurable and arithmetically additive, and second, that degrees of probability also are numerically measurable." As a result, ethical theory suggested deciding among alternative acts on the basis of their mathematical expectations, which Keynes presented as "a technical expression originally derived from the scientific study of gambling and games of chance, ... [that] stands for the product of the possible gain with the probability of attaining it."¹⁹ Of course, Keynes (1921, p. 344) disagreed with a generalized application of mathematical expectation, since assuming that "degrees of probability are wholly subject to the law of arithmetic, runs directly counter to the view which has been advocated in part I [of the *TP*]." In Keynes's view, "mathematical expectations, of goods or advantage, are not always numerically measurable, and hence even if a meaning can be given to the sum of a series of non-numerical mathematical expectations, not every pair of such sums are numerically comparable in respect of more and less." Also, Keynes contended that mathematical expectation cannot be used in actual conduct since it "ignores what I have termed the weights of arguments, namely the amount of evidence upon which each probability is founded." To sum up, "it is not always possible by a mere process of arithmetic to determine which of the alternative ought be chosen" (Keynes 1921, p. 344-345).

Keynes (1921, p. 349) admitted that an alternative to the notion of mathematical

analysis of liquidity preference in the *GT* (1936, p. 148 n., and p. 240). In his investigation of a Keynesian explanation of the current crisis, Skidelsky (2009, p. 88) signals its significance for the interpretation of individual behaviour in the markets when claiming: "the greater the amount of evidence supporting an expectation, the more confident we will be in having it."

- 17 Ellsberg did not quote Keynes's *TP* in the 1961 article introducing the paradox of choice, seemingly because he did not read it until working on the philosophical background of his own viewpoint in his Ph. D. thesis, submitted to the Harvard Department of Economics in 1962 (Ellsberg 2011). Ellsberg (2001 [1962], p. 9-13) recognized that the notion of weight of argument is "closely related" to his notion of ambiguity and noted that in situations where information is perceived to be vague the traditional approach to probability is inadequate. Keynes's discussion of probabilities become pivotal since Keynes introduced "formally the notion of *non-comparability* of beliefs."
- 18 Since Einhorn and Hogart's (1986) assessment of Ellsberg, ambiguity and uncertainty have become almost synonyms in decision theory literature.
- 19 Keynes discussion is based on George E. Moore's analysis of which are the appropriate behavioural rules to be used in ethics. He objected to Moore's contention that mathematical expectation was the appropriate behavioural rule in ethics. On the relationship between Moore and Keynes see Bateman (1989) and Raffaelli (2006).

expectations does not lie, in principle, “in the discovery of some more complicated function of the probability wherewith to compound the proposed good.” However, even in this case, he made an effort to analyse constructively. In order to move forward in the search for a decision rule, Keynes argued, probability and weight should be compounded into a coefficient to be used in the shaping of a normative theory of decision making, and he attempted to show how a weight of argument lower than a given maximum would possibly influence decision-making.²⁰ As noted by Brady (1993), Keynes’s coefficient incorporates the properties of a probability weighting function, like Kahneman and Tversky’s decision weights, and can thus be used to provide a decision criterion that solves the Ellsberg Paradox. It can be concluded then that even with respect to this third aspect of Keynes’s probability theory the rationale of Keynes’s criticism presents strict similarities with a much favoured theme in current decision theory, that is, Ellsberg’s rejection of expected utility maximization.

5. Which Keynes for understanding decision-making under uncertainty?

The previous section has pointed out that Keynes’s analysis of individual behaviour in the *GT* finds a consistent philosophical background in the treatment of probability in the *TP*. This background is instrumental to enlighten the issue of how people behave under uncertainty, as Keynes himself stated most clearly in his correspondence with Townshend. Under uncertainty, Keynesian decision-makers formulate subjective assessments of the external world that can take the form of “non-numerical” probabilities, that is, probabilities that are not standard additive probability functions. Keynes underlined the need to conceive these non-numerical probabilities in a consistent qualitative, if not quantitative, ordering, and suggested the method of approximation through intervals of probabilities, when possible. With the benefit of insight, non-numerical probabilities can be interpreted as probabilistic weights, that is, as “subjective” distortions of an “objective” probability function that either cannot be known or is not known due to the ambiguity of the environment perceived by individual agents.²¹

This framework entails the rejection of the ethical criterion for choice suggested by Moore, later endorsed in economics through the application of “Benthamite calculus” to decision under uncertainty. However, this is a prelude to the analysis of alternative criteria Keynes hinted at. Keynesian decision-makers evaluate prospective investments using the maximisation of expected values only when the weight of argument is at its maximum, something Keynes considered the exception rather than the rule. A low weight is associated with situations in which the state of confidence worries the decision-maker to the point of feeling unsure whether one probability distribution is as reliable as to be used for calculus. In this instance the decision-maker typically

20 Keynes introduced the following coefficient: $c=2pw/(1+q)(1+w)$, where p is the probability of an event, $q=1-p$ the probability of its complement, and w is the weight, ranging from 0 to 1. Keynes then argued that, in making a decision concerning a possible “amount of good” A which can be expected with probability p , the standard mathematical expectation $E=pA$ should be disposed of in favour of an alternative criterion for choice such as $E'=cA$. Keynes provided also a short discussion of how, for different values of w and p , E' would help order different goods A implying the same E (Keynes 1921, p. 348fn). As Keynes did not use utility values but monetary amounts, in his discussion the coefficient $c=p/1+q$ is intended to take into account the “risk” implicit in taking a decision based on E . Except for the taking of risk into account, when the weight of argument is at its maximum there is no other need to modify E as a decision rule. But when lower than 1, the weight and the distortion of p it implies must be taken into account.

21 In certain contexts this ambiguity can be solved by the passing of time, or the replication of an experiment, like in the urn example. However, Keynes seems to suggest that since decision-makers may find difficult to establish how complete their information is, different propensities to act will emerge in general (Feduzi 2010).

shows both risk aversion and uncertainty aversion: while the former can be represented as usual by the shape of the utility function,²² the latter is caught by a probability measure that is “distorted” in relation to the weight, the lower its value the more significant the distortion. Or else, when the weight is low, a set of probability functions are deemed to represent the individual’s understanding of the uncertain environment and probabilities associated to outcomes are interval valued. In both cases, the ordering of acts does not conform to a mathematical expectation and a more conservative decision rule would be selected. To sum up: it is “Benthamite calculus” that cannot be applied to uncertainty, not probability calculus as such.

Notably, Keynes suggested a philosophy of decision-making that, after Ellsberg and the related experimental evidence (Camerer 1995), has emerged extensively in an increasingly relevant part of modern decision theory criticizing the mainstream Bayesian set-up. Decision criteria devised to account for unknown probabilities (such as Wald’s maximin) or for probabilities with a low degree of reliability (such as Ellsberg’s and Hurwicz’s α -maximin) or for multiple probabilities (such as Gilboa and Schmeidler’s maximin expected utility), all can be considered different but related ways to account for the behaviour of agents in Keynesian settings. Most of these criteria can be viewed as instances of a generalized procedure consisting in maximizing an expected value where probabilities are non-additive like in Choquet expected utility models (Wakker 2008). The study of this class of what Keynes would term “impolite techniques” – and are defined “non-Bayesian” ones in an increasingly influential current literature (Gilboa, Postlewaite and Schmeidler 2008) – has been mostly motivated on axiomatic ground, since the aim was to account for what is perceived as a normative violation of Savage’s axioms.²³

The discussion of the *TP* provided in the previous section suggests an assessment of Keynes’s (1937, p. 214) famous statement that, on matters for which “there is no scientific basis on which to form any calculable probability whatever,” “we simply do not know” that is alternative to the conventional post-Keynesian one. What individuals do not know can be interpreted as an array of differently characterized situations entailing different degrees of confidence, all of which can be discussed without denying the fruitfulness of the Keynesian viewpoint. In uncertain settings, Keynesian agents are not forced to rely on “conventions” or “useful mental habits” as if complete ignorance would represent the entire spectrum of economic activity. In trying to focus on a positive analysis of decision-making under uncertainty, hence, one is simply adhering to Keynes’s long-standing commitment to interpret reasonable judgement in a, possibly unconventional, probabilistic set-up.

But putting aside the interpretative issue one main aspect remains to be examined. The main message of the decision rules devised to deal with individual’s aversion to ambiguity is that a conservative attitude, one that rejects making decision on the basis of subjective expected utility maximization, cannot be deemed “irrational.” When beliefs are ambiguous, and are represented by a set of probability functions, the expected payoff is measured with respect to more than one probability distribution, and the ambiguity averse decision-maker evaluates an act by the minimum expected value that can be associated with it: the non-Bayesian decision rule requires to compute all possible expected values for each action and then choose the act which has the best minimum expected outcome. When applied to financial markets, this theoretical set up can be used to show

22 As already noted in footnote 18, this was represented by Keynes via the “risk” associated with trying to get a profit taking part in a project that cannot yield it for sure, thus possibly originating a loss.

23 These decision rules have also been used to rationalize the behavioural theories proposed to account for experimental evidence. The literature insisting on behavioural explanations of market choices, though mostly aiming to account for descriptive “anomalies,” has been re-interpreted as an application of these decision rules, as in cumulative prospect theory (Tversky and Kahneman 1992).

that there exists a price interval at which an individual may prefer to take a zero position in risky assets (Dow and Werlang 1992) and the equilibrium allocation is not Pareto optimal, just like in models with incomplete markets (Mukerji and Tallon 2001). Ambiguity averse investors are shown to be inclined to choose actions whose consequences are more robust to the perceived ambiguity, that is, they hold a portfolio whose value is relatively less affected by the uncertainty about probability distribution governing the future payoffs. Also, if ambiguity emerges as a result of a feeling by the individual investor that her competence in assessing the relevant probabilities is low, and in comparative situations in general (Fox and Tversky 1995), it can be shown that the structure of the optimal portfolio depends on the different degrees of ambiguity across assets, so that the optimal portfolio may contain only the assets in which the investor feels more confident, i.e. the less ambiguous ones (Boyle et al. 2011).²⁴

This interpretation of how “irrational” individuals act in financial markets relates well to Keynes’s own record as an investor. As it is well-known even in the informed press after the endorsement by an investment giant like Warren Buffet, Keynes’s own investment strategy as Bursar of King’s College, Cambridge, and a director of the Provincial Insurance Company, did not conform to the practice of diversification of risk. After persuading his college fellows to increase the proportion of funds available for a discretionary strategy, he started investing heavily in equities, and within a few years of activity he came to follow a strategy summarised in a famous letter to F. C. Scott, managing director of the Provincial: “As time goes on I get more and more convinced that the right method in investment is to put fairly large sums into enterprises which one thinks one knows something about and in the management of which one thoroughly believes. It is a mistake to think that one limits one’s risk by spreading too much between enterprises about which one knows little and has no reason for special confidence” (Keynes 1934, p. 57). He constructed an highly idiosyncratic portfolio with pronounced size and value tilts that, from the early 1930s on, makes it possible for him to outperform the market on a consistent basis (Chambers and Dimson 2012).

Keynes’s quest for confidence in the probability assessment also emerges in his understanding of how government should act in the face of uncertainty. Indeed, the analysis of decision under uncertainty at the individual level can be productively applied to policy decisions. For instance, the policy-maker’s decision to intervene in order to regulate the market when a certain information is available can be examined in the following way. Let us assume that a governmental authority, concerned with the potential losses of a systemic crisis, calculates the expected utility of the decision to intervene and check exuberant financial markets, possibly because actual values do not seem to relate to fundamentals. It may well happen that, estimating a “very low,” maybe even potentially “unknown,” probability of the realisation of the admittedly rare event of the systemic crisis, the expected loss associated with this unfavourable event would be low, even in the face of potentially high losses. The option of inaction, i.e. not to check the bubble, may be considered a preferred choice in view of the expected utility of leaving business to continue as usual, since the expected value of the gains experienced by financial markets associated with it outweighs the expected value of the potential losses calculated on the basis of this “very low” probability. But this deliberate choice, made on the basis of a criterion apt to face familiar events, may well turn out to be difficult to justify when compared to the “irrational” precautionary option of avoiding the potentially enormously high losses associated with the catastrophic event. If one looks at the current crisis with the benefit of insight, a choice favouring a prudential attitude, by means of an intervention intended to check the bubble before it eventually burst, would have been advisable.

24 On the relationship between Markowitz and the ensuing capital assets pricing model and subjective expected utility, see Zappia (2012).

This is a point that conforms well to Keynes's philosophy of decision to such an extent that he (1921, p. 344) made it explicitly in the *Treatise* when contending that "the doctrine that the 'mathematical expectations' of alternative courses of action are the proper measures of our degrees of preference is open to doubt ... because it ignores the element of 'risk' and assumes that an even chance of heaven or hell is precisely as much to be desired as the certain attainment of a state of mediocrity." Keynes's argument cannot be interpreted simply as an allusion to risk aversion, or to the variance of the expected outcomes, as it preludes to the analysis of the coefficient compounding probability and weight. A more persuading interpretation is that a conservative attitude in making decisions is fundamental when the numbers at stake are huge and the probabilities are "unknown". Here Skidelsky's intellectual assessment of Keynes helps when pointing out that the main underlying idea behind Keynes's political philosophy was "prudence." In his early thoughts on Edmund Burke's political philosophy Keynes argued that the expected probable consequences of achieving one's goal must be pondered and "we should be very chary of sacrificing large numbers of people for the sake of a contingent end, however advantageous that may appear" (quoted in Skidelsky 2009, p. 156). Even with reference to other thoughts about government's action Keynes made during his political life, Skidelsky (2009, p. 158) summarizes this attitude as follows: "Prudence in the face of unknown is the key to Keynes's philosophy of statesmanship." Following on the interpretation of the *Treatise* just provided, Keynes's quest for prudence is reflected in his application of probability to human conduct.

6. Concluding remarks

The re-reading of Keynes originated by the current financial crisis includes a critical analysis of Keynes's decision theory under uncertainty. For instance, Akerlof and Shiller (2009) claim that the "spontaneous urge to action" of Keynes's animal spirits must find room in economic and financial modelling. However, Akerlof and Shiller cannot find any constructive element in Keynes's thought. Keynes was right in pointing out that most decisions depend on confidence and that when confidence is low a straightforward rational calculation of alternative expected outcomes is not available, but it is only through behavioural economics that his message can get substance in economic analysis.

The existing critical literature on Keynes and his dealing with uncertainty may justify this view. As a matter of fact even those scholars who correctly refer to the *TP* for the more persuasive motivation of Keynes's advocacy of uncertainty, usually rely on an assessment that rejects, sometimes even on ontological grounds, the possibility of discussing uncertainty in a formal context and place emphasis instead on the conventional aspects of decision-making presented in Chapter 12 of the *GT*.

This paper has pointed out that Keynes's discussion of probability in the *TP* can help enlighten this issue with specific regard to the kind of decision rule Keynes would have argued in favour of. A peculiar reading of this part of Keynes's work, placing emphasis on some technical aspects of his probability theory, can indeed contribute to an understanding of how actual individual agents behave under uncertainty.

REFERENCES

- Akerlof, G., and Shiller, R. J. (2009). *Animal Spirits*. Princeton University Press, Princeton, NJ.
- Backhouse, R. E. and Bateman, B. W. (2011). Methodological issues in Keynesian macroeconomics. In Davis J. B. and Hands D. W. (eds.), *The Elgar Companion to Recent Economic Methodology*. Cheltenham, Edward Elgar.
- Basili, M. and Zappia, C. (2009). Keynes 'non-numerical' probabilities and non-additive measures. *Journal of Economic Psychology*, 30: 419-430.
- Basili, M. and Zappia, C. (2010). Ambiguity and uncertainty in Ellsberg and Shackle. *Cambridge Journal of Economics*, 34: 449-474.
- Bateman, B. (1989). G. E. Moore and J. M. Keynes. A missing chapter in the history of the expected utility model. *American Economic Review*, 78: 1098-1106.
- Bateman, B. (1996). *Keynes's Uncertain Revolution*. Ann Arbor, MI: University of Michigan Press.
- Boyle, P., Garlappi, L., Uppal, R. and Wang, T. (2011), Keynes meets Markowitz: The trade-off between familiarity and diversification. *Management Science*, forthcoming.
- Brady, M. E. (1993). J. M. Keynes's theoretical approach to decision making under condition of risk and uncertainty. *The British Journal for the Philosophy of Science*, 44: 357-376.
- Brady, M. E. and Arthmar, R. (2012). Keynes, Boole and the interval approach to probability. *History of Economic Ideas*, 20: 65-84
- Camerer, C. (1995), Individual decision making. In J. Kagel and A. E. Roth (eds.), *Handbook of Experimental Economics*. Princeton: Princeton University Press.
- Carabelli, A. M. (1988). *On Keynes's Method*. London: Macmillan.
- Carabelli, A. M. (2002). Speculation and reasonableness: A Non-Bayesian theory of rationality". In S. Dow and J. Hillard (eds), *Keynes, Uncertainty and the Global Economy. Beyond Keynes* (Vol. II). Aldershot: Edward Elgar.
- Carabelli, A. M. and Cedrini, M. (2011). On the new appeal of Chapter 12 of the *General Theory*. Working Paper. Università del Piemonte Orientale.
- Chambers, D. and Dimson, E. (2012). Keynes the stock market investor. Working Paper. Available at SSRN: <http://ssrn.com/abstract=202301>.
- Davidson, P. (2009). *The Keynes Solution*. New York: Palgrave Macmillan.
- Davis, J. B. (1994). *Keynes's Philosophical Developments*. Cambridge: Cambridge University Press.
- de Finetti, B. (1937). Foresight: its logical laws, its subjective sources. In H. E. Kyburg, & H. E. Smokler (Eds.), *Studies in Subjective Probability*. New York: Wiley, 1964.
- Dow, S. C. (2003). Probability, uncertainty and convention. In Runde J. and Mizuhara, S. (eds.), *The Philosophy of Keynes's Economics*. London: Routledge.
- Dow, J. and Werlang, S. R. da Costa (1992). Uncertainty aversion, risk aversion, and the optimal choice of portfolio. *Econometrica*, 60: 197-204.
- Ellsberg, D. (1961), Risk, ambiguity, and the Savage axioms, *Quarterly Journal of Economics*, 75: 643-669.
- Ellsberg, D. (2001 [1962]), *Risk, Ambiguity and Decision*. New York: Routledge.
- Feduzi, A. (2010). On Keynes's conception of the weight of argument. *Journal of Economic Behaviour and Organization*, 76: 338-351.
- Gärdenfors, P. and Sahlin, N. E. (1982). Unreliable probabilities, risk taking and decision making. *Synthese*, 53, 361-86.
- Gerrard (1994). Beyond rational expectations. A constructive interpretation of Keynes's analysis of behaviour under uncertainty. *Economic Journal*, 104: 327-337.
- Gerrard (2003). Keynesian uncertainty: What do we know? In Runde J. and Mizuhara, S. (eds.), *The Philosophy of Keynes's Economics*. London: Routledge.
- Gilboa, I. (2009). *Theory of Decision under Uncertainty*. Cambridge: Cambridge University Press.

- Gilboa, I., Postlewaite, A. and Schmeidler, D. (2008). Probability and uncertainty in economic modelling. *Journal of Economic Perspectives*, 22: 173-188.
- Good, I. J. (1950). *Probability and the Weighing of Evidence*. London: Charles Griffin.
- Hillard, J. (1992). Keynes, orthodoxy and uncertainty. In B. Gerrard, and J. Hillard (eds.), *The Philosophy and Economics of J.M. Keynes*. Aldershot: Edward Elgar.
- Kahnemann, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47: 263-291.
- Kelsey, D. (1994). Maxmin expected utility and the weight of evidence. *Oxford Economic Papers*, 46: 425-444.
- Keynes, J. M. (1921). *A Treatise on Probability*. *The Collected Writings of John Maynard Keynes*, Vol. VIII. London: Macmillan, 1973.
- Keynes, J. M. (1934). Letter to F. C. Scott, 15 August 1934. *The Collected Writings of John Maynard Keynes*, Vol. XII. London: Macmillan, 1983.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. *The Collected Writings of John Maynard Keynes*, Vol. VII. London: Macmillan, 1973.
- Keynes, J. M. (1937). The general theory of employment. *Quarterly Journal of Economics*, 51: 209-223.
- Keynes, J. M. (1936-1938). Letters to H. Townshend, various dates. *The Collected Writings of John Maynard Keynes*, Vol. XXIX. London: Macmillan, 1979.
- Knight, F.H. (1921). *Risk, Uncertainty and Profit*. Chicago: University of Chicago Press, 1985.
- Koopman, B. O. (1940). The axioms and algebra of intuitive probability. *Annals of Mathematics*, 41: 269-92.
- Kyburg, H. E. Jr. (1995). Keynes as a philosopher. In A. F. Cottrell, and Lawlor, M. S. (Eds.), *New Perspectives on Keynes*. Durham: Duke University Press.
- Lawson, T. (1985). Uncertainty and economic analysis. *Economic Journal*, 95: 909-927.
- Levi, I. (1974). On indeterminate probabilities. *Journal of Philosophy*, 71: 391-418.
- Mankiw, G. (2008). Economic View: What would Keynes have done? *New York Times*, November 30.
- Minsky, H. P. (1992). The Financial Instability Hypothesis. *The Levy Economics Institute Working Paper Collection*, 74. Annandale-on-Hudson, NY.
- Mukerji, S. and Tallon, J.-M. (2001). Ambiguity aversion and incompleteness of financial markets, *Review of Economic Studies*, 68: 883-904.
- O'Donnell, R. M. (1989). *Keynes: Philosophy, Economics and Politics*. London: Macmillan.
- Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behaviour and Organization*, 3: 323-343.
- Raffaelli, T. (2006). Keynes and philosophers. In Backhouse, R. E. and Bateman, B. W. (eds.), *The Cambridge Companion to Keynes*. Cambridge: Cambridge University Press.
- Ramsey, F. P. (1926). Truth and probability. In *Foundations: Essays in Philosophy, Logic, Mathematics and Economics*, edited by D. H. Mellor. London: Routledge & Kegan Paul, 1978.
- Runde, J. H. (1990). Keynesian uncertainty and the weight of argument. *Economics and Philosophy*, 6: 275-92.
- Runde, J. H. (1994). Keynesian uncertainty and liquidity preference. *Cambridge Journal of Economics*, 18: 129-44.
- Runde, J. H. (2001). Chances and choices: notes on probability and belief in economic theory. In Maki, U. (ed.) *The Economic World View: Studies in the Ontology of Economics*. Cambridge: Cambridge University Press.
- Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica*, 57: 571-587.
- Shackle, G. L. S. (1938). *Expectations, Investment and Income*. Oxford: Oxford University Press.

- Shackle, G. L. S. (1967). *The Years of High Theory*. Cambridge: Cambridge University Press.
- Skidelsky, R. (2009). *Keynes: The Return of the Master*. London: Penguin Books.
- Skidelsky, R. (2011). The relevance of Keynes. *Cambridge Journal of Economics*, 35, 1-13.
- Smith, C. A. B. (1961). Consistency in statistical inference. *Journal of the Royal Statistical Society*, 23, 1-25.
- Townshend, H. (1936-1938). Letters to J. M. Keynes, various dates. *The Collected Writings of John Maynard Keynes*, Vol. XXIX. London: Macmillan, 1979.
- Townshend, H. (1938). Review of *Expectations, Investment and Income* by G. L. S. Shackle. *Economic Journal*, 48: 520-523.
- Tversky, A. and Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5: 297-323.
- Wakker, P. (2008). Uncertainty. In Blume, L. and Durlauf, S. N. (eds.), *The New Palgrave: A Dictionary of Economics*, 2nd Edition. London: Palgrave Macmillan.
- Wald, A. (1945). Statistical decision functions which minimize the maximum risk. *Annals of Mathematics*, 46: 265-280.
- Walley, P. (1991). *Statistical Reasoning with Imprecise Probabilities*. London: Chapman and Hall.
- Wu, G. and Gonzales, R. (1999). Non linear decision weights in choice under uncertainty. *Management Science*, 45: 74-85.
- Zappia, C. (2012). After the crisis: financial economics and decision theory. *History of Economic Thought and Policy*, 1: 125-137.
- Zappia, C. (2015). Keynes on probability and decision: evidence from the correspondence with Hugh Townshend, available at SSRN: <http://ssrn.com/abstract=2612951>