Letter

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Optimal Imprisonment with General Enforcement of Law

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Abstract: We study the optimal use of imprisonment when enforcement efforts are general (i.e. when the probability of detection is common for a range of acts). In contrast to the conventional wisdom that optimal imprisonment rises with the act's harmfulness and is equal to the maximum level only for the most harmful acts, we show that – when the distribution of criminal benefits exhibits a standard monotone hazard rate – optimal imprisonment can only be zero or maximal. Thus, having general as opposed to specific enforcement effort does not alter the fact that only extreme sanction levels should optimally be employed.

Keywords: imprisonment, general enforcement, maximal sanctions

JEL Classification: K14, K42

1 Introduction

Becker (1968) famously showed that it is optimal to deter crime by combining the lowest possible probability (to economize on enforcement costs) with the severest possible penalty (to maintain adequate deterrence). This proposition remains the basic tenet in the optimal law enforcement literature, although several papers have identified specific circumstances in which this combination of instrument levels is not optimal (Garoupa 1997).

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In an important contribution, Shavell (1991) argues that acknowledging the relevance of *general* instead of *specific* law enforcement can align prescriptions from theory with the real-world observation of non-maximal sanctions. According to Shavell (1991), specific enforcement concerns apprehending and penalizing individuals who have committed a single kind of criminal act as identified by its level of social harm (e.g. enforcers who write tickets for overtime parking). In contrast, general enforcement enables apprehending and penalizing individuals who have committed any of a range of acts (e.g. a police officer on patrol may assist in the conviction of perpetrators of minor or major crimes).[1] Shavell (1991) reports that the maximum imprisonment term is socially optimal when enforcement is specific. For each act in isolation, the Becker proposition applies, that is, any expected sanction should comprise the severest sanction possible in order to economize on the costs of enforcement. In contrast, he finds that, if enforcement efforts are general, optimal imprisonment rises with the act's harmfulness and is equal to the maximum term only for the most harmful acts. The intuition is that, under general enforcement, it is impossible to tailor the detection effort to the different acts, creating a need to adjust the sanction to the acts' severity. These results have become a corner stone in the analysis of optimal law enforcement.[2]

This paper challenges this conventional wisdom. We show that optimal imprisonment is either zero or maximal when the distribution of criminal benefits exhibits an increasing hazard rate, that is, fulfills the monotone hazard rate (MHR) assumption. Not only is this property of the distribution of a random variable usually assumed to hold in many fields of applied microeconomics (including law and economics), it is also fulfilled for a wide range of actual distribution functions.[3] The intuition for the extreme-imprisonment result is that welfare evolves in a U shaped manner with the imprisonment term when MHR applies, signifying that the interior solution *minimizes* welfare.

With the MHR assumption, the optimal pattern of enforcement and imprisonment is as follows: (i) no enforcement and any imprisonment term at all harms level, or (ii) positive enforcement and no (maximum) imprisonment for acts that

1 Mookherjee and Png (1992) similarly distinguish between monitoring (efforts not tailored to the severity of the act) and investigation (efforts undertaken when the severity of the act is known) in their study focused on monetary sanctions.

2 See, for example, Garoupa (1997, pp. 272–73) and Polinsky and Shavell (2007, pp. 431–32).

3 The most commonly used parametric distributions – such as the normal and the beta and gamma distributions with bell-shaped density – exhibit MHR on their support. The same holds for a distribution on a finite support such as the uniform distribution or triangular distribution with any mode.

cause harm below (above) a critical harm level. As a result, both specific and general enforcement of law are associated with the use of only extreme sanctions.

Whereas the result by Shavell (1991) that imprisonment increases in the harmfulness of the act remains conventional wisdom to this day, Kaplow (1990) already mentioned that interior solutions may require possibly strong assumptions. Our paper adds to Kaplow (1990), who analyzes only a single act (i.e. specific enforcement), by identifying the precise characteristics of the distribution functions that induce boundary solutions, by highlighting that these characteristics are taken for granted in many strands of the literature, and by describing the optimal regime when enforcement is general. Whereas Kaplow (1990, p. 247) states that the "most salient factor relevant to whether an intermediate sanction is optimal is the level of the social cost" of punishment, we suggest that the shape of the distribution function is the decisive factor.[4]

The present contribution is complementary to D'Antoni, Friehe, and Tabbach (2023), where we describe how the unobservability of wealth changes the optimal combination of fines and imprisonment when enforcement is specific. In the present paper, wealth levels are irrelevant as we focus on imprisonment as the only kind of sanction in order to understand optimal imprisonment when enforcement is general (i.e. when there is a range of acts to consider).

2 Model

We draw on the model suggested by Shavell (1991) in which risk-neutral individuals choose whether to commit harmful acts by comparing the expected sanction with the criminal benefit. Individuals differ regarding this benefit b and the act's harmfulness denoted h. The benefit is distributed on the support $B \subseteq [0, \infty)$ according to the cumulative distribution function $F(b)$ and density $F'(b) = f(b)$. We also refer to the hazard rate function

$$H(b) = \frac{f(b)}{1 - F(b)},$$

and assume that it is non-decreasing. This is the widely used monotone hazard rate assumption (see, for example, Tirole 1994, p. 156). The harm is distributed on the support $[0, \infty)$ according to $G(h)$ as the cumulative distribution function and $g(h)$

4 It is unclear why Kaplow (1990) had little influence on subsequent literature. Sometimes, his work is even cited in the context of arguments favoring an interior solution (see Garoupa 1997, pp. 271–72, Result 4). Possibly, that paper's small impact is due to the fact that Kaplow (1990) did not identify a clear-cut condition for the circumstances in which the extreme imprisonment level proves optimal, leaving the impression that it is a mere possibility. Our contribution makes some progress in this regard.

as density. We assume that the distribution of benefits is the same for different harm levels.

If an individual commits an act, she will be detected with probability p and receive an imprisonment term z. We focus on general enforcement, so p cannot depend on h. Let y denote the costs of enforcement and assume that $p'(y) > 0$ and $p''(y) \leq 0$, with $p(0) = 0$. The imprisonment term can depend on the level of harm (i.e. the kind of act) so we write $z(h)$. Imprisonment cannot exceed an upper limit $\bar{z}$, which may reflect physical limits or moral constraints and is the same for all individuals.

Imprisonment is costly for the convicted offender and for society. The cost per-unit amount to $\sigma > 1$. They comprise the individual's per-unit cost (equal to 1) and the additional social costs per unit ($\sigma - 1 > 0$).

3 Analysis

An individual will commit an act if and only if her benefit exceeds the expected sanction, that is, if $b > p(y)z(h)$, where $z(h)$ is the imprisonment term as a function of harm. Understanding this decision criterion, the social planner chooses enforcement efforts y and imprisonment $z(h)$ to maximize social welfare W defined to be the benefits individuals obtain from committing acts, less the harm done, less the social costs of imposing imprisonment, less the cost of enforcement. Formally,

$$W = \int_0^\infty \varphi(h, z(h), y) g(h) \mathrm{d}h - y. \tag{1}$$

where

$$\varphi(h, z(h), y) = \int_{p(y)z(h)}^\infty (b - h - \sigma\, p(y)z(h)) f(b) \mathrm{d}b \tag{2}$$

is social welfare at each level of harm.

When enforcement efforts are positive, the marginal welfare effect from a longer imprisonment term results as

$$\frac{\partial \varphi}{\partial z} = p\big(h + (\sigma - 1)pz\big) f(pz) - p\sigma\big(1 - F(pz)\big), \tag{3}$$

where the first term reflects the marginal benefit from deterring the marginal offender, and the second term reflects the marginal cost from increasing imprisonment on those who will continue to offend even at the higher sanction level. A

higher imprisonment term increases the term $h + pz(\sigma - 1)$ in the marginal benefits and lowers the marginal costs of imprisonment as it reduces the population of undeterred individuals.

Shavell (1991) considers an interior solution for the level of imprisonment (see equation (A17) in Shavell 1991, p. 1105). Assuming that the marginal welfare effect in (3) is equal to zero, we can also state the necessary requirement as

$$H(pz) - \frac{\sigma}{h + pz(\sigma - 1)} = 0, \tag{4}$$

using the hazard rate H. When we derive this term with respect to z for the second-order condition, we obtain (after dividing by p)

$$H'(pz) + \frac{(\sigma - 1)\sigma}{\big(h + p(\sigma - 1)z\big)^2}, \tag{5}$$

which is *positive* as long as the hazard rate function is non-decreasing (i.e. $H'(b) \geq 0$). A non-decreasing monotone hazard rate indicates that the ratio between the marginal probability of deterrence (captured by the criminal benefit density function $f(b)$) to the crime rate (captured by the survival function $1 - F(b)$) is non-decreasing. A very intuitive understanding results from noticing that the hazard rate is equal to the absolute value of the elasticity of the crime rate with respect to the expected sanction $\mathcal{S}$ divided by the expected sanction $\mathcal{S}$, that is,

$$H(\mathcal{S}) = \left| \frac{d(1 - F(\mathcal{S}))}{d\mathcal{S}} \frac{1}{1 - F(\mathcal{S})} \right|. \tag{6}$$

Assuming $H'(b) \geq 0$, our result rules out that an interior solution represents a global maximum, and points to boundary solutions. Namely, $z(h)$ is either zero or $\bar{z}$, with $z(h) = 0$ if and only if

$$\varphi(h, 0, y) - \varphi(h, \bar{z}, y) = \int_0^\infty (b - h) f(b) \mathrm{d}b - \int_{p(y)\bar{z}}^\infty (b - h - p(y)\bar{z}\sigma) f(b) \mathrm{d}b \geq 0. \tag{7}$$

After establishing that optimal imprisonment results in a corner solution under the MHR assumption, we are now ready to characterize the general pattern of imprisonment [5]

Proposition 1. *Suppose that general enforcement is employed, that imprisonment is used as the only kind of sanction, and that the distribution of benefits fulfills the*

5 Please note that an increasing hazard rate is sufficient to ensure that the second-order condition is positive, meaning that extreme imprisonment can be optimal even when H' is negative but small. In other cases, the interior solution assumed in Shavell (1991) applies.

monotone hazard rate condition. Then, optimal law enforcement features either (i) no enforcement ($y = 0$ and $p = 0$) or (ii) positive enforcement and maximum (no) imprisonment for acts imposing $h > h^*$ ($h \leq h^*$).

Proof. The fact that only extreme sanctions will be employed has been shown above.

Define h_c as the level of harm solving (7) with equality, or

$$\int_0^{p(y)\bar{z}} (b - h_c) f(b)\mathrm{d}b + p(y)\bar{z}[1 - F(p(y)\bar{z})] = 0. \tag{8}$$

Such value always exists for continuity of the integral. Because the left hand side is decreasing in h_c, the optimal level of imprisonment will be $z(h) = 0$ for all $h \leq h_c$ and $z(h) = \bar{z}$ for all $h > h_c$. Since (8) can never be satisfied for $h_c = 0$ when $p(y) > 0$, a scenario in which all acts are punished by the maximum sanction is not possible.

An optimal enforcement policy is characterized by $\left(h_c^*, y^*\right)$ maximizing

$$W = \int_0^{h_c} \int_0^{\infty} (b - h) f(b)dbg(h)\mathrm{d}h + \int_{h_c}^{\infty} \int_{p(y)\bar{z}}^{\infty} \left(b - h - \sigma\, p(y)\bar{z}\right) f(b)dbg(h)\mathrm{d}h - y, \tag{9}$$

where $y^* \geq 0$. This requires that either $y^* = 0$ (so that $p = 0$) or (8) is jointly satisfied with

$$\frac{\partial W}{\partial y} = \int_{h_c}^{\infty} \frac{\partial \varphi}{\partial y} g(h)\mathrm{d}h - 1 = 0 \tag{10}$$

where

$$\frac{\partial \varphi}{\partial y} = p'(y)\bar{z}\left[\left(h + (\sigma - 1)p(y)\bar{z}\right) f(p(y)\bar{z}) - \left(1 - F(p(y)\bar{z})\right)\sigma\right]. \tag{11}$$

The latter expression is similar to $\partial\varphi/\partial z$ in (3). In this case, however, with $p'(y)$ decreasing sufficiently fast, an interior solution can be ensured (for a discussion, see Kaplow 1989).

With $\partial\varphi/\partial y$ positive at all levels of $h \geq h_c$, the optimal y is decreasing in h_c and is zero for h_c sufficiently large. The optimal level h_c^* solves (8) at $y = y^* > 0$.

Scenario (i) obtains when (8) cannot be satisfied (i.e. the left hand side of (8) is strictly positive) at any h_c for which $y^* > 0$. Scenario (ii) results when both (11) and (8) are satisfied at $y^* > 0$; this requires that acts with $0 < b \leq p(y^*)\bar{z}$ create sufficiently high net harm. □

4 Discussion and Conclusion

According to the conventional wisdom, optimal imprisonment increases with the harmfulness of the act when enforcement effort is general, whereas optimal imprisonment is necessarily maximal when enforcement effort is specific. For the case of general enforcement, this paper shows that optimal imprisonment is either zero or maximal when the distribution of benefits fulfills a widely used and intuitive characteristic that is also fulfilled by many regularly used distribution functions. Our result implies that general as opposed to specific enforcement cannot unconditionally improve the alignment of theoretically prescribed and practically observed sanction regimes.[6]

Although our conclusions do not invalidate widely accepted results, they put them into perspective by pointing out that their validity relies on hypotheses that are implicit, unconventional, and could be easily violated. As clarified by our analysis, reliance on internal solutions in our context requires a discussion of the implied shape of the hazard rate of the criminal benefits.

This raises the issue of identifying circumstances under which the hazard rate decreases, such that an interior solution may result. We suggest some possibilities that could be further explored.

A first possibility is assuming a distribution of benefits that is "fat-tailed", for example, because it is a Pareto distribution.[7] The Pareto distribution is used in finance, to deal with catastrophic events, and in the analysis of income and wealth distributions, as it is consistent with empirical data for that context.[8] It can be debated in which contexts a fat-tailed distribution can reflect criminal benefits.

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6 It is important to emphasize the difference between the specific and general enforcement cases. In the specific enforcement case, an interior solution is not optimal, irrespective of the shape of the benefits distribution function, as long as enforcement is endogenous (e.g., Polinsky and Shavell 2007). This is clear from the observation that the two first-order conditions (the one concerning enforcement and the one concerning the sanction level) cannot be simultaneously satisfied at an interior solution for both variables. In the general enforcement case, this is no longer true, that is, an interior solution for the sanction level is possible in principle. However, such a solution is impossible if the benefits distribution function satisfies MHR.

7 This is the assumption made by Mungan (2017) in a model of optimal sanctioning in which social welfare does not include criminals' benefits.

8 Notably, the assumption of a Pareto distribution of income plays a role in reconciling the optimal taxation analysis of the income tax with the circumstance that the marginal tax rate is not declining at high levels of income (Diamond 1998).

A second possibility is that individuals may be unable to completely control their behavior and may commit a crime inadvertently.[9] If there is a probability that individuals commit a harmful act even when they do not want to (and, ex post, it is not possible or socially desirable to discriminate intentional and unintentional acts), then the frequency of violation will stop declining as sanctions increase even before full deterrence is reached. This amounts to a declining hazard rate.

Finally, some individuals may not engage in a rational cost-benefit comparison and thus be unresponsive to incentives provided by sanctions. This implies that some individuals remain undeterred at all levels of punishment. This kind of "irrational" behavior by some individuals may be realistic in some cases of criminal behavior (e.g. drug addicts), although it deviates from the standard economic framework. In these instances, an interior solution to imprisonment in the general enforcement model is possible.

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9 Think, for example, about speeding. A driver may occasionally speed even if he aims at not driving above the speed limit. This usually requires dealing with strict liability offenses.

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