Nonlinear Circuits and Systems with Memristors

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Nonlinear Dynamics and Analogue Computing via the Flux-Charge Analysis Method

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To Novella, Giorgia, and Sara
and in memory of Antonio—F.C.
To Emiliana and in memory of Silvano—M.F.
To Diana—L.O.C.

Foreword by Sung Mo (Steve) Kang

Nonlinear Circuits and Systems with Memristors is a timely contribution to the field of nanoelectronic circuits and systems. Personally, the contents of this book are dear to me, especially since I was one of the early graduate students of Professor Leon Chua at the UC Berkeley who at that time introduced memristors and memristive systems. The seminal paper on memristor by Leon Chua was published in 1971, followed by the Chua and Kang paper on memristive devices and systems in 1976. In microelectronics industry, CMOS Very Large-Scale Integrated (VLSI) circuits have been the dominant workhorse, the development of which has followed the Moore's law. However, as the downscaling faced its limitations and because of the volatility of charge storage in ultrasmall capacitors in VLSI chips, nonvolatile resistance has become critically important as a new state variable. The demands for Resistive RAMs (RRAMs) and other memory devices such as MRAM, PCRAM, STT-RAM, which do not depend on charge storage, have increased. Analog computing has also become of increasing importance for ultralow energy computing as in neuromorphic computing. Almost four decades later, when Stan Williams and his associates in HP announced nanoscale memristors in the May 2008 Nature paper, a new epoch for integrated memristor circuits was established. HP's memristor was the first solid-state realization of the "two-terminal memristor." It was my honor and pleasure to organize the first symposium on memristors and memristive systems in November 2008 under sponsorship of NSF and HP at the Berkeley campus, the fountain of the memristor research. Since then, memristor electronics has been pursued globally at a phenomenal growth rate. In particular, the application of crossbar arrays of memristors for analog neuromorphic computing has been pursued actively as one of the most promising approaches to the mimicry of brain functions. Two-terminal memristors are used in crossbar arrays with programmable memductances as learning weights. Coincidentally in their Proceedings of the IEEE paper (1976), Chua and Kang identified the conductive channels of the Hodgkin–Huxley model for neuromorphic signal (action potential) generation to be memristive. A few years ago, after giving a seminar on memristors at the Rowan University in New Jersey, its faculty members asked me when a textbook would be published on the subject matter for education of electronic circuits including memristors. Although this book is better suited for graduate students, I would expect that undergraduate textbooks on electronic circuits would include basic memristor circuits in the not too distant future.

In the beginning of this book, the authors take the axiomatic approach to describe the four basic circuit elements, namely R, L, C, and M (memristor), with focus on interesting cases involving nonlinear characteristics of elements. Through systematic formulation of circuit equations and rigorous analysis, the authors illustrate peculiar circuit behaviors of nonlinear RLC circuits, including oscillations and bifurcations. Theoretical discussions become even more interesting after including memristors for most general nonlinear RLCM circuits. The introduction of the flux-charge analysis method (FCAM) is not only novel but also uniquely revealing. The introduction of Kirchhoff's charge Law (KqL) and Kirchhoff's flux Law (KφL) in lieu of KCL and KVL is enlightening and prepares the readers for systematic formulation and analysis of RLCM circuits. The application of FCAM to neuromorphic systems uncovers numerous peculiar dynamic behaviors. Undoubtedly, this book is unique and provides a rich set of interesting and intriguing topics for both interested graduate students and researchers. I congratulate the authors for the publication of this book and hope that the readers will benefit from studying the contents to enrich the field of memristive nanoelectronics.

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Foreword by Ronald Tetzlaff

An increasing number of different two-terminal devices manufactured in distinct technologies can be classified as memristors with most popular developments so far, in technologies implementing dense, low-power nonvolatile memories and neuromorphic systems operating according to biological principles. Especially, recent investigations show a strong interest in new computing architectures required for future digital systems that are often based on communicating intelligent sensor–processor architectures. The development of new memristor computing concepts in order to overcome the limits of classical technology caused by the so-called von-Neumann bottleneck is based on principles in which processing and data storage are carried out in the same physical location. These technologies are ranging from crossbar arrays to artificial and bio-inspired neural networks, including models of biological computation in a human brain. The availability and application of these so-called in-memory computing systems would obviate the need for energy-expensive data transfer operations on the memory-CPU in future IOT networks. While different authors have shown in an increasing number of publications that certain mathematical operations can be performed rather efficiently on crossbar arrays, attractive concepts of universal computation are based on the emergence of complex behavior in strongly nonlinear dynamical arrays. By investigating the so-called reaction–diffusion systems, Leon Chua has derived the fundamental results that prove that the emergence of complexity in these structures is based on local activity and in particular on a parameter subset called the "Edge of Chaos." Typically, the treatment of such highly nonlinear, memristive spatiotemporal systems, which exhibit oscillations, pattern formation, and wave propagation phenomena, is based on the availability of compact device models and on new mathematical strategies to gain a deep understanding of the mechanisms of such systems in order to enable the derivation of highly efficient in-memory computing methods. Although mem-elements and memristive circuits have been addressed in a bulk of publications, an in-depth mathematical treatment and understanding based on circuit theory has been provided in only a few investigations. Mostly, present model-based simulators operating in the voltage-current domain make qualitatively incorrect predictions, especially when applied to the problems of nonlinear dynamics. There is a lack of accurate but stable compact device models that allow the simulation and design of future memristive dynamic circuits, which are needed for the digitalization of today's technology.

Starting from an axiomatic approach to introduce basic circuit elements and fundamental concepts of circuit theory, including the treatment of oscillatory RLC circuits, the authors continue with their original development, the flux-charge analysis method (FCAM), which is a unique approach to gain a deep understanding of nonlinear dynamics in memristive circuits. The circuit theoretic concept which is based on Kirchhoff's charge law and Kirchhoff's flux law is clearly outlined and prepares the reader for the following deep insight into the theory of nonlinear dynamics, including the bifurcation without parameters in memristive RLCM circuits. The application of the FCAM method to different circuits considering cellular neural networks completes this unique book, which offers a comprehensive insight into the concepts that allow an in-depth treatment of circuits with mem-elements.

It is my pleasure to congratulate the authors on this important contribution to the theory of memristors, dedicated to graduate students and a broad class of researchers.

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Ronald Tetzlaff

Foreword by R. Stanley Williams

I first became aware of the seminal papers on memristors by Leon Chua and the extension of memristive systems by Chua and Kang in 2004. I was struck by the similarity of the “pinched hysteresis loop” characteristic of the axiomatically defined memristor and the experimental data we had been collecting in my group for nearly 6 years, and I spent a couple of frustrating years trying to figure out if and how they were connected. After suffering an auto accident in 2006 and being incapacitated for the month of August, I spent several weeks downloading and reading about 300 papers. Somehow, my subconscious brain made a connection among them and I had a genuine eureka! moment, where I understood how and why our resistance switching devices were memristors. I spent the next year primarily in writing invention disclosures and patent applications, and in 2008 with my HP colleagues we published our “Missing Memristor Found” paper in Nature. The next couple of years were spent on expanding our understanding of the physical mechanisms behind memristance in transition metal oxides, developing improved compact models that we could use for circuit simulations, improving our devices, and exploring a wide range of potential applications for the “fourth fundamental circuit element.” During this period, I read many more papers by Chua and collaborators, and I realized that the memristor was the literal “tip of the iceberg” for an enormous and comprehensive body of work on nonlinear dynamical circuits and networks. I have spent much of the past decade studying and learning how to utilize nonlinear dynamics and chaos for computation in anticipation of the saturation of Moore’s scaling for CMOS circuits. I could see that for some functions, one or two memristors could emulate the properties of a neuron with better fidelity than a circuit with several hundred to thousands of transistors, thus my goal was not a one-off replacement for a transistor, but rather dramatic simplification and efficiency improvements in complete circuits for neuromorphic computing. However, the existing papers on nonlinear dynamical circuits were necessarily very formal mathematical developments of unfamiliar (to me) concepts that required extreme effort on my part to understand. In order to provide guidance for me and others, I invited Leon Chua in 2015 to present a series of lectures sponsored by HP that gave him the opportunity to provide an overview of his entire body of work. He gracefully accepted and devoted a huge amount of work into organizing and presenting the twelve-part series "The Chua Lectures: From Memristors and Cellular Nonlinear Networks to the Edge of Chaos," which is available on YouTube. During the lectures, the interesting topic of a Hamiltonian approach to electronic circuits that was based on flux and charge came up, which I found to be fascinating but had no bandwidth to pursue myself.

I was thus delighted when Fernando Corinto presented a copy of this book and asked me to provide some remarks for a preface. I started to read it and felt a warm glow through the first four introductory chapters. I had just finished teaching a graduate-level class for electrical engineers on nonlinear dynamics and chaos, and those chapters with their excellent figures cemented home the concepts that I had been trying to help my students learn. Just one example is the discussion of impasse points and how (and why) to break them. I wish I had the book before I taught the class, because the choice of topics and the presentation were ideal—much better than what I have found elsewhere in texts or papers and certainly better than my lecture notes. Then I began with chapter five, which is the inception of the core of the book. I was both challenged and intrigued as I continued to read. It took me quite a long time to finish the book, not because of lack of interest but rather I kept coming up with completely new ideas for experiments that I wanted to develop, which included designing special-purpose circuits and apparatus for performing measurements. Building a charge and flux meter, musing about the implications of Noether's theorems for electronic circuits, and analyzing a coupled oscillator circuit for computing were just a few of the projects I investigated. I spent a lot of time on these pursuits and had great fun in the process—at this stage, it is hard to know if any of my concepts will work, but I am excited by the prospects. As I write this, I am in a mode where my lab is closed because of the COVID-19 epidemic, but I hope to start work on the ideas inspired by this book as soon as possible. *Nonlinear Circuits and Systems with Memristors* has reinforced much that I have previously learned and taught, and it has also challenged me to consider the new ways of thinking about how to understand and even measure the properties of nonlinear dynamical systems. It is not the final elucidation of the field, but rather a portal and invitation to explore new opportunities.

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Preface

Conventional Von Neumann computing architectures based on CMOS technology are currently facing challenges termed “the heat and memory wall” in addition to the advent of Moore’s Law slowdown [1–3]. Von Neumann bottleneck originates in particular from the speed limitations due to the constant data movements between the memory (e.g., the Random Access Memory—RAM) and the Central Processing Unit (CPU), since RAM and CPU have physically distinct locations. Going beyond CMOS and overcoming Von Neumann restrictions are long-term visions aimed at developing completely new nanoscale components with unconventional functions and dynamics that are capable of outperforming similar CMOS implementations to sustain the growth of the electronics industry at the end of Moore’s Law.

The memristor (a shorthand for memory-resistor) is one of the most promising candidate information processing devices for beyond CMOS and more than Moore semiconductor technology. The memristor has been theoretically envisioned by L. O. Chua in 1971 [4] as the fourth basic passive circuit element, in addition to the resistor, inductor, and capacitor, using an axiomatic approach on device modeling and symmetry arguments on the basic electric quantities. A memristor is a state-dependent resistor, where the resistance (also named memristance) is not fixed but rather depends on the history of the voltage or current. A memristor is endowed with a number of new peculiar features that are not shared by the other basic circuit elements.

For a long time, the memristor remained basically an object of academic interest since no passive physical device was known behaving as a memristor. Almost four decades after the publication of the seminal paper [4], the researchers at Hewlett and Packard headed by R. Stanley Williams first announced and presented to the world a nanoscale device displaying memristive features [5]. This work has sprung a huge worldwide cross-disciplinary interest on memristor and its applications ranging from tunable electronics, neuromorphic/in-memory computing, biosensors, data storage, and complex nonlinear systems [6–8].

On-chip memory, biologically inspired computing and in-memory computing, i.e., the integration of storage and computation in the same physical location [9], are categories that are expected to significantly benefit from memristor developments.

This is in turn particularly relevant to future computing needs such as cognitive processing, big-data analysis, and low-power intelligent systems based on the Internet of Things.

Neuromorphic/synaptic electronics is an emerging field of research aiming to overcome Von Neumann platforms by building artificial neuronal systems that mimic the extremely energy-efficient biological synapses. Neuromorphic memristive architectures integrated into edge computing devices are expected to increase the data processing capability at lower power requirements and reduce several overheads for cloud computing solutions [10]. The introduction of photovoltaic/photonic aspects into neuromorphic architectures could produce self-powered adaptive electronics and open new possibilities in artificial neuroscience, neural communications, sensing, and machine learning. This would enable, in turn, a new era for computational systems owing to the possibility of attaining high bandwidths with much reduced power consumption [11].

Memristor devices operated as a nonvolatile memory (NVM) are emerging for data storage and unconventional computing systems. In this case, a memristor should display two (or more) largely different values of memristance and be a nonvolatile device. The memristor is driven from one memristance to others via a suitable current (or voltage) pulse, an operation that is often referred to as set/reset. This mechanism permits to exploit nonvolatile resistive states of memristor to encode information bits. A continuous range of resistive states enables the use of memristor devices as *analogue programmable resistors*. Such devices, whose resistance can be precisely modulated electronically, and which can support important synaptic functions (e.g., Spike-Timing-Dependent Plasticity—STDP), pave the way to denser low-power analogue circuits, multi-state memory, and large-scale synapse implementation in neuromorphic systems and cellular neural networks with adaptation capabilities [10, 12].

Memristor devices embedded in nonlinear circuits can generate a complex nonlinear dynamical evolution of their memristance that can be exploited to build nanoscale oscillators potentially described by low-order mathematical/circuit models. Networks of interconnected and interacting oscillators can develop cooperative and collective dynamics, e.g., phase synchronization and other self-organizing spatiotemporal phenomena for alternative computing schemes overpassing the limits of conventional digital and Boolean computation. A memristor-based nonlinear oscillator is proposed in [13] as a source of tunable chaotic behavior that can be incorporated into a Hopfield computing network to improve the efficiency and accuracy of converging to a solution for computationally hard problems. The breakthrough is the development of networks of interacting nanoscale memristor oscillators and their computational schemes based on cooperative and collective dynamics. It is then crucial to develop a technological platform comprehensive of functional materials and hardware memristors including physical/circuit models of their operation as a key in hand tool for the large-scale industrial exploitation.

Aims and Scope

The main goal of this book is to develop systematic theoretical methodologies to analyze nonlinear circuits including memristors as nonlinear dynamic elements. Such dynamical nonlinear circuits are referred to as *memristor circuits*. These methods are essential for understanding the peculiar dynamic properties and computational capabilities of memristors/mem-elements and exploit their dynamics to implement future unconventional computing systems.

It is known from circuit theory that it is natural and effective to analyze a nonlinear RLC circuit, i.e., a circuit with linear/nonlinear resistors (R), inductors (L), and capacitors (C) (but without memristors), in the traditional voltage-current (v, i)-domain, i.e., using constitutive relations of circuit elements and Kirchhoff laws expressed in terms of voltages v and currents i. However, since the definition of a memristor involves a link between flux φ (the integral of voltage or voltage momentum) and charge q (the integral of current or current momentum), it is natural to ask the following:

1. Is there a more effective domain, with respect to the traditional (v, i)-domain, to analyze the nonlinear dynamics of memristor circuits?
2. Do we expect to observe new and peculiar nonlinear dynamic phenomena when adding one (or more) memristors to a classical RLC circuit?

An extended literature is available on the analysis of dynamic memristor circuits. Several papers point out via experimental or numerical means that, generally speaking, including a memristor in an RLC circuit greatly enriches the dynamics. In particular, it appears that due to the memristor there can coexist different dynamics and regimes (e.g., convergent, oscillatory, complex dynamics) for the same set of circuit parameters. Several basic aspects of the observed behaviors are however elusive and remain unclear and in large part unexplained. It is not even clear from a mathematical viewpoint whether the order of the dynamics of an RLC circuit increases or not when a memristor is added to it.

The chief aim of the book is to answer these fundamental questions and provide an analytic treatment and clear explanation of dynamic phenomena reported in the literature via numerical or experimental means. The treatment is rigorous and based on tools and techniques with foundation in nonlinear circuit theory. Whenever possible, we try however to keep mathematics at the minimum indispensable level for an accurate description.

In this book, we identify and select progressively relevant classes of memristor circuits, widely investigated in the literature and used in the applications, and describe a new method for their dynamic analysis. The new analysis method introduced in this book is based on suitable forms of Kirchhoff laws and constitutive relations of circuit elements expressed in the *flux-charge* (φ, q)*-domain* rather than in the traditional (v, i)-domain. Thus the name *Flux-Charge Analysis Method*, or FCAM, in short. FCAM has been mainly developed in a series of recent articles [14–20].

The authors' aim is to make clear to the readers that there are several main advantages when using FCAM with respect to the traditional analysis in the voltage-current domain. Two key advantages are related to the principle of *reduction of order for the dynamics* and the possibility to deal with a *smoother dynamics*, in the (φ, q)-domain, with respect to the (v, i)-domain. In addition, any fundamental property of a memristor circuit proved via FCAM in the (φ, q)-domain has a corresponding one for an RLC circuit in the (v, i)-domain. Via this analogy, systematic methods for writing the dynamic equations of memristor circuits in the form of a Differential Algebraic Equation (DAE), or a State Equation (SE), are developed.

Moreover, FCAM permits to highlight and rigorously show the existence of new peculiar dynamic behaviors displayed by memristor circuits. These include the presence of *invariant manifolds* and the *coexistence of different nonlinear dynamics, attractors, and regimes*, a complex dynamic scenario that is sometimes referred to in the literature as *extreme multistability*. The coexistence of different attractors is shown to be related to a new type of bifurcations, due to changing the initial conditions for a fixed set of circuit parameters, which are named *bifurcations without parameters*.

Given the extremely rich dynamic scenario in memristor circuits, it is natural to wonder if there is the possibility to control and programme different attractors and regimes in an effective way. This book shows that there is a positive answer to this question by developing a simple programming procedure using impulsive voltage or current sources (a natural way for transmitting signals from a neuromorphic viewpoint).

Applications of the results are considered to oscillatory and chaotic memristor circuits and also to arrays of memristor oscillators and neuromorphic architectures. Finally, FCAM is generalized to higher-order elements with memory (also named *mem-elements*) as memcapacitors and meminductors and also to some classes of extended memristors.

Next, we discuss in some more detail the organization of the book, the content of each single chapter, and the prerequisites and the audience for which the book is intended.

We then conclude the preface by reporting a slightly abridged version of an article by Chua [21] devoted to a reminiscence of the genesis and the thought process he followed to theoretically introduce the fourth circuit element.

Organization of the Book

The book is mainly organized in three parts:

- Foundation of Nonlinear Circuit Theory
- Flux-Charge Analysis Method (FCAM)
- Applications and Extension of FCAM

The first part is a summary of nonlinear circuit theory pillars, so that readers are gently introduced to concepts underlying the FCAM via a self-contained book. Although expert researchers might pass over this part, the holistic approach used reveals the generality of the FCAM and poses the basis for its extension to nonlinear circuits with mem-elements. The FCAM is the core of the second part, whereas the third part is devoted to FCAM applications. A brief summary of the chapters included in each part is reported in the following.

Foundation of Nonlinear Circuit Theory

Chapter 1 provides the reader with an axiomatic definition of the basic nonlinear circuit elements that could be used to model a wide variety of nonlinear devices. A black-box approach is used, independent of the internal composition, material, geometry, and architecture of each device. The axiomatic approach leads naturally to the definition of the fourth basic circuit element, i.e., the memristor, and also higher-order circuit elements, such as the memcapacitor and meminductor.

Chapter 2 deals in more depth with the behavior of a memristor by examining some of its main properties and signatures. A brief account of the HP memristor is provided, as well as a discussion on extended memristors and a classification of memristive devices. The technological realization of real memristive devices as well as materials and physics phenomena underlying the memristor behavior are briefly discussed.

Chapters 3 and 4 are devoted to synthetically give the needed theoretic background on nonlinear RLC circuits (without memristors). Especially, Chap. 3 deals with a synthetic description of the main methods to analyze nonlinear RLC circuits, i.e., tableau, nodal, and mesh analysis. Special emphasis is then paid to give conditions for the existence of the state equation (SE) representation and the techniques to write the SEs of nonlinear RLC circuits.

Chapter 4 briefly discusses some main dynamical phenomena that can be observed in autonomous nonlinear RLC circuits of first, second, and third order, including convergence of solutions, oscillations, and complex dynamic behavior. The chapter ends with some basic considerations on local bifurcations of equilibrium points and global bifurcations of limit cycles in autonomous RLC nonlinear circuits depending on parameters.

Flux-Charge Analysis Method (FCAM)

Chapters 5–7 are at the core of the book, and they are devoted to develop FCAM for the dynamic analysis of nonlinear circuits containing memristors.

In particular, Chap. 5 starts with a critical review of Kirchhoff Laws in the (φ, q)-domain, and, on this basis, it then proceeds to develop the new method of analysis in the (φ, q)-domain (FCAM) for wide classes of memristor circuits. The chapter also establishes an analogy between RLC circuits in the (v, i)-domain and memristor circuits in the (φ, q)-domain and, via this analogy, gives a general formulation of memristor circuit equations.

Chapter 6 discusses the applications of FCAM to some basic low-order autonomous memristor circuits highlighting the main advantages of FCAM related to the reduction of order, and smoother dynamics, in the (φ, q)-domain.

New peculiar and intriguing dynamical features of memristor circuits, such as the presence of invariant manifolds, coexisting dynamics and attractors (extreme multistability), and bifurcations without parameters, are addressed and characterized analytically.

Finally, Chap. 7 develops a systematic analytic procedure for programming different dynamics and regimes in non-autonomous memristor circuits by means of impulsive voltage or current sources.

Applications and Extension of the FCAM
Chapter 8 describes the application of FCAM to study synchronization phenomena in arrays of locally connected oscillators, while Chap. 9 exploits FCAM to design a class of memristor neural networks that are able to process signals in the (φ, q)-domain according to the principle of in-memory computing.

Chapter 10 develops a circuit model of a class of extended memristors by interconnecting basic circuit elements as an ideal memristor and a nonlinear resistor. FCAM is then used to analyze the dynamics in the case the nonlinear resistor has a piecewise-linear voltage-current characteristic. Finally, Chap. 11 discusses the extension of FCAM to nonlinear circuits containing also higher-order elements such as memcapacitors and meminductors, showing that such circuits display even richer dynamic features with respect to memristor circuits.

Prerequisites and Audience

From a didactic viewpoint, the present book is intended for a graduate-level course in engineering on memristors including relevant aspects of nonlinear circuit theory connected with the emerging nanoscale devices. It may also be used for self-study or as a reference book by engineers, physicists, and applied mathematicians. From a research viewpoint, the book is an account of the state of the art on some main research directions in the analysis of memristor circuits. As such it may be used as a solid basis to start and pursue researches on the challenging and rapidly evolving field of memristors and nanoscale devices.

The prerequisite of this book is a graduate-level course in circuit theory introducing basic aspects of the analysis of nonlinear circuits at the level of classical textbooks (e.g., [22]). The needed mathematical background corresponds to the essential level of calculus, ordinary differential equations, and algebra that are part of the graduate student curricula in engineering, physics, and mathematics. Whenever possible, mathematics is kept to a minimum without losing rigor and accuracy in the description. Ad hoc mathematical textbooks are adequately referred to in order to give the reader the possibility to further elaborate on some of the presented topics.

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Memristor: Remembrance of Things Past

This part is mainly based on the work [21] written by L. O. Chua to present his personal scientific and historical account of memristor concept.

> Postulated in 1971, the memristor did not see the light of day until a serendipitous discovery at HP nearly four decades later. Here, I reminisce on the crisis that inspired me to develop an axiomatic nonlinear circuit theory where the memristor emerges naturally as the fourth basic circuit element.

The dawn of nonlinear electronics was ushered during the 1960s by a Cambrian-esque explosion of newly minted two-terminal electronic devices, bearing such intimidating monikers as Esaki diode, Josephson junction, varactor diode, thyristor, impact ionization avalanche transit time (IMPATT) diode, Gunn diode, and ovonic threshold switch. Trained in the old school of linear circuit analysis, hordes of electronics engineers were awed and shocked upon witnessing a parade of such strongly nonlinear and dynamical electronic devices unfolding at such a breathless rate. To many, the surreal proliferation of exotic devices would conjure the opening scene of Dickens's tale of yore: "It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before. . . ."

The surreal proliferation of these exotic devices was an exciting time full of opportunity and challenge. At this time, I was working on my PhD research to make sense of the cornucopia of exotic nonlinear devices. Rather than charting a taxonomy to pigeonhole them, I opted for an axiomatic definition of a few basic nonlinear circuit elements that could be used to model a broad variety of nonlinear devices. I joined the Purdue University upon graduation in 1964 and was assigned to revamp its outdated circuit analysis curriculum, thereby providing me an ideal launching pad for teaching nonlinear circuit theory through my device-independent black-box approach. The axioms that would predict the memristor had made their debut in the world's first textbook on nonlinear circuit theory [23] in 1969. It took a year for me to derive and prove mathematically the unique circuit-theoretic properties and memory attributes of this yet unnamed device, earning its accolade as the fourth circuit element [24, 25] and its justification for submission to the *IEEE Transactions on Circuit Theory* on November 25, 1970 [4]. Its publication in the following year coincided with my move to the University of California, Berkeley, to spearhead research in a new frontier dubbed nonlinear circuits and systems. The memristor was soon relegated to the back burner due to lack of research funding, where, like Rip Van Winkle, it would slumber until awakened. This was despite recognition by the IEEE of the potential of the memristor back in 1973, when they awarded me the prestigious IEEE W.R.G. Baker Prize Paper Award for the most outstanding paper reporting the original work in all IEEE publications.

To analyze circuits made of strongly nonlinear and dynamical electronic devices, it is necessary to have realistic device models made of well-defined nonlinear circuit elements as building blocks [23, 26], which did not exist then, because the electrical engineers from that bygone epoch were taught to overcome nonlinearities by expanding them in a Taylor series and then retaining only the linear term that neatly maps into a linear circuit model. But just like the ancient parable about the blind men and the elephant, such models invariably gave rise to grossly inaccurate and misleading results. Unfortunately, the "linearize then analyze" culture endowed upon the electronic engineers of the day had made it impossible to devise such generalizations due in part to the lack of a circuit-theoretic foundation for basic nonlinear circuit elements. Even Richard Feynman would sloppily use the oldfashioned name "condenser"—instead of capacitor—in his 1963 classic, "The Feynman Lectures on Physics," when he wrote on pages 6–12, vol. II, "The coefficient of proportionality is called the Capacity, and such a system of two conductors is called a Condenser" [27]. Electronic circuit theory is concerned only with the prediction of the voltage $v(t)$ and current $i(t)$ associated with the external terminals sticking out of an enclosing cocoon (henceforth called a black box) whose interior may contain some newly minted nonlinear electronic device or an interconnection of various electronic devices, batteries, and so on. And thus, basic nonlinear circuit elements must be defined from a black-box perspective, independent of their internal composition, material, geometry, and architecture.

From the circuit-theoretic point of view, the three basic two-terminal circuit elements are defined in terms of a relationship between two of the four fundamental circuit variables, namely, the current i, the voltage v, the charge q, and the flux φ. Out of the six possible combinations of these four variables, five have led to well-known relationships [23]. Two of these relationships are already given by $q(t) = \int_{-\infty}^{t} i(\tau)d\tau$ and $\varphi(t) = \int_{-\infty}^{t} v(\tau)d\tau$. Three other relationships are given, respectively, by the axiomatic definition of the three classical circuit elements, namely, the resistor (defined by a relationship between v and i), the inductor (defined by a relationship between φ and i), and the capacitor (defined by a relationship between q and v). Only one relationship remains undefined: the relationship between φ and q. From the logical as well as axiomatic points of view, it is necessary for the sake of completeness to postulate the existence of a fourth basic two-terminal circuit element, which is characterized by a $\varphi - q$ relationship (see [21, Fig. 1(a)]). This element is christened the memristor because it behaves like a nonlinear resistor with memory, remembering things past.

As a proof of principle, three memristors were built in 1969 using operational amplifiers and off-the-shelf electronic components. They demonstrated three distinct $\varphi - q$ curves on a bespoke memristor curve tracer that I had designed for this purpose [4]. However, the challenge of fabricating a passive monolithic memristor remained unfulfilled for 37 years until a team of scientists from the HP lab, under the leadership of Dr. R. Stanley Williams, reported in May 1, 2008, issue of *Nature* the world's first operational memristor made by sandwiching a thin film of titanium dioxide between platinum electrodes [5, 28]. The pinched hysteresis loop fingerprint of this seminal HP memristor is reproduced below the four-element quartet in [21, Fig. 1(a)]. It is truly remarkable that the pinched hysteresis loop is in fact an endearing signature that nature endowed upon all basic nonlinear circuit elements [29, 30] capable of remembering their past, including the memcapacitor and the meminductor that I identified to be lossless at the opening lecture of the first UC Berkeley Memristor and Memristive Systems Symposium in November 2008 [31].

Prior to Dr. Williams's *Nature* paper, no one understood how certain experimental two-terminal solid-state devices could remember and switch between two or more values of resistance without a power supply. The seminal *Nature* publication has provided a unifying foundation for all nonvolatile memory devices, which go by such names as ReRAM, PCRAM, STT-RAM, RRAM, MRAM, FRAM, PCM, and the Atomic Switch [32], where the device's high and low resistance states are used to code the 0 and 1 binary bits, instead of the conventional voltage or current, which collapses to zero when power is interrupted.

News of the nanoscale HP memristor has breathed new life into a device of antiquity that had been relegated to the dustbin of history. Overnight, it has triggered a torrent of research and development activities on memristors worldwide in both industry and academia (see Figure 1b, c in [21]), in anticipation of its disrupting potential in AI and neuromorphic computing. Companies such as Panasonic and Fujitsu have sold several hundred million chips with embedded memristor memory based on tungsten oxide since 2013, and Taiwan Semiconductor Manufacturing Company (TSMC) has announced that it has developed a 22-nm memristor crossbar process for ASIC embedded memories that will be available for mass production in 2019. Aside from its predicted eventual replacement of the over-extended flash memories, DRAMs, and even hard drives [2] with disrupting nonvolatile memristor technology, memristors can emulate synapses and ion channels in neurons and muscle fibers [33], sweat ducts in human skin, and even the primitive amoeba's amazing counting ability [29]. Moreover, the memristor's scalable diminutive physical size makes it the right stuff for building brain-like intelligent machines. Furthermore, because even plants can remember events and communicate through memristors [34], could memristors in fact be the *sine qua non* for emulating life itself?

I close my above reminiscence (which is an embellishment on a previous work [35]) by reproducing the last paragraph of my 1971 article, "Memristor-the missing circuit element" [4]. In hindsight, this passage foreshadowed the pinched hysteresis loop fingerprints not only of the ideal memristor predicted in the article but also of all memristors cited in my later works [29–31, 33, 36, 37]. *Although no physical memristor has yet been discovered in the form of a physical device without internal power supply, the circuit-theoretic and quasi-static electromagnetic analyses presented in Sections III and IV make plausible the notion that a memristor device with a monotonically increasing $\varphi - q$ curve could be invented, if not discovered accidentally. It is perhaps not unreasonable to suppose that such a device might already have been fabricated as a laboratory curiosity but was improperly identified! After all, a memristor with a simple $\varphi - q$ curve will give rise to a rather peculiar—if not complicated hysteretic—$v - i$ curve when erroneously traced in the current-versus-voltage plane. (Moreover, such a curve will change with frequency as well as with the tracing waveform.) Perhaps, our perennial habit of tracing the $v - i$ curve of any new two-terminal device has already misled some of our device-oriented colleagues and prevented them from discovering the true essence of some new device, which could very well be the missing memristor.*

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Contents

List of Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	Set of n-dimensional vectors
$\mathbf{x}$	$= (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$, vector in $\mathbb{R}^n$
$\mathbf{x}^T$	Transpose of vector $\mathbf{x}$
$\mathbf{M}$	$= (m_{ij}) \in \mathbb{R}^{n \times m}$, $n \times m$ matrix
$\mathbf{M}^T$	Transpose of matrix
$\mathbf{M}^{-1}$	Inverse of matrix
$\det \mathbf{M}$	Determinant of matrix
$\mathrm{tr}\mathbf{M}$	Trace of matrix
$f(x)$	$: \mathbb{R} \to \mathbb{R}$, function of scalar variable x
$f'(x)$	Derivative of f with respect to its argument x
$f^{-1}(x)$	Inverse function
$\mathbf{f}(\mathbf{x})$	$: \mathbb{R}^n \to \mathbb{R}^m$, vector function
$\mathbf{J}_\mathbf{f}$	$= (\partial f_i / \partial x_j)$, Jacobian of $\mathbf{f}$
∇f	$= (\partial f/\partial x_1, \partial f/\partial x_2 \ldots, \partial f/\partial x_n)^T$, gradient of $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$
j	$= \sqrt{-1}$
Re	Real part
Im	Imaginary part
$\forall$	For all
$\in$	Is a member of
$\cap$	Intersection
$\doteq$	Definition
$\dot{x}(t)$	Derivative with respect to time t. The explicit notation $dx(t)/dt$ is also used
$\ddot{x}(t)$	$= d^2x(t)/dt^2$
ODE	Ordinary differential equation
DAE	Differential algebraic equation
SE	State equation
IVP	Initial value problem
EP	Equilibrium point

IC	Initial condition
BWP	Bifurcation without parameters
1D	One dimensional
i	Current
A	Ampere
v	Voltage
V	Volts
$p(t)$	Instantaneous power at t
W	Watt
q	Charge, or current momentum
C	Coulomb
φ	Flux, or voltage momentum
Wb	Weber
Ω	Ohm
S	Siemens
F	Farad
H	Henry
J	Joule
ac	Alternate current
dc	Direct current
σ	Charge momentum
ρ	Flux momentum
$v^{(\alpha)}$	where α is an integer, α-derivative (or integral) of voltage v
$i^{(\beta)}$	where β is an integer, β-derivative (or integral) of current i
KCL	Kirchhoff current law
KVL	Kirchhoff voltage law
KqL	Kirchhoff charge law
KφL	Kirchhoff flux law
$\mathcal{G}$	Graph
$\mathcal{C}$	Cut-set
$\mathcal{T}$	Tree
Q	Fundamental cut-set matrix
A	Reduced incidence matrix
B	Fundamental loop matrix
(v, i)-domain	Voltage-current domain
(φ, q)-domain	Flux-charge domain
t_0	Initial instant for transient analysis
$q(t; t_0)$	$= q(t) - q(t_0)$, incremental charge
$\varphi(t; t_0)$	$= \varphi(t) - \varphi(t_0)$, incremental flux
CR	Constitutive relation
FCAM	Flux-charge analysis method
CMOS	Complementary metal-oxide-semiconductor
ASIC	Application-specific integrated circuit
CPU	Central processing unit
RAM	Random-access memory

DRAM	Dynamic random-access memory
PCM	Phase-change memory
NVM	Non-volatile memory
STDP	Spike-Timing-Dependent Plasticity
CNN	Cellular neural network
MIM	Metal-Insulator-Metal structure
IoT	Internet of Things
HP	Hewlett Packard