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Three Essays on Behavioral Economics

Candidate:
Francesco Strati

Supervisor:
Professor Nicola Gennaioli
Bocconi University

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To Laura

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Abstract

This thesis consists of three essays about the application of the heuristic of representativeness in finance and economics.

Nowadays, behavioral economics has lured the attention of an increasing number of economists. This is due to the recent financial crisis of 2007-09 that forced economists to realize that psychological biases could modifiy rational equilibria since, on avarage, arbitrageurs are not always able to bring back the economy to the rational path. Taking into account limits of arbitrage and limits in cognitive capacities, an economy can be vulnerable to the *sensitiveness* of agents to new information. If this sensitiveness is of a certain size, such that arbitrageurs behavior is negligible, then sensitive agents (also called local thinkers in the following essays) can enter into the formation of an equilibrium modifying its dynamics.

The first essay concerns a real market model in which agents are of two types: lenders and borrowers. They maximize the intertemporal consumption path and the level of debt of each period by exchanging debt contracts collateralized by the borrowers' houses with numeraire the only consumption good needed. Borrowers are impatient so that debt persists in equilibrium. The problem is that both types are sensitive to news distorting the rational density distribution. In this essay I shall set up a condition for which a bubble can arise in the collateral market even if the initial state is Pareto-efficient. It will be clear how this bubbly dynamics is a sufficient condition for agents to deleverage. In particular applying the heuristic of representativeness a bubble arises due to extrapolative expectations (over-optimism) even in case in which the transversality condition does hold. Moreover, agents deleverage because of the neglection of the down-

side risk (a slump in this case). Representativeness thus triggers extrapolation in expectations and neglection of some states maintaining a positive bubble component until a bad shock unveils the unfeasible path. I called this kind of bubble: *Representative Bubble*.

The second essay tackles the issue of representative bubbles in a more general fashion. In this essay I shall compare rational bubbles to the representative one. The last model is not driven by the classic *explosive path* and, moreover, a representative bubble may rise and burst at any time. It will be clear that the excess of volatility which sets off a predictable error (extrapolations) causes a mispricing that maintains an increase in prices that is not justified by changes in dividends, and thus driven by a behavioral component. This essay extends, in a theoretical fashion, a recent work which claims to new (behavioral) models of bubbels since it demonstrates that there are cases in which even when the transversality condition is fulfilled for the classic rational bubble model, nonetheless advanced econometric tests detect bubbles dynamics leaving the movement in prices unexplained by rational models.

The last essay applies the heuristic of representativeness to auction theory. In particualr it sets up a crucial theorem for answering to the following question: what if in a first-price (high bid) auction there are local thinkers, rational bidders and aggressive types? In few words, this essay demonstrates the existence of the pivotal point: if players are nonrational, then their valuations will follow their sensitivity to new information. There will be asymmetric sensitivity to symmetric information. This extends the previous works on auction theory in which asymmetric and private information modify the equilibrium bid. Here the equilibrium is driven only by the extent of sensitiveness to new pieces in-

formation. If an agent is highly sensitive to a signal (he is a local thinker) then he will revise upward his valuation if he supposes that there exists at least an aggressive type among bidders. This will be a fundamental step in defining a pairwise single crossing condition in case of nonrational agents. The main theorem of this essay is a requirement such that local thinkers strictly prefer to increase their equilibrium bids for which the payoff is still the same but with a higher probability to win. In few words, local thinkers shade until the absolute amount of shading is equal to that before the new information shows up. It is thus defined a rationale by which an increase in the maximal valuation triggers a symmetric increase in the equilibrium bid until the shading is relatively the same as before, but notice that this payoff is higher in terms of probability to win.

Contents

Chapter 1

Representative Bubbles and Deleveraging

I explore the causes for the formation of a bubble in the collateral market (in which house values work as collateral) when agents are provided with homogeneous expectations. Furthermore, I shall define a sufficient condition for deleveraging: a bubble driven by a *representative* bubbly collateral. The role of the heuristic of representativeness in shaping expectations turns out to be crucial: the bubble's formation is driven by a gap between the fundamental value defined in terms of rational expectations and the error in processing information driven by representativeness. This behavioral bubble can explain bubbles even when the transversality condition on the bubbly component is fulfilled.

1.1   Introduction

The present paper aims for defining the condition of existence of non-rational[1] bubbles and their role for the subsequent existence of deleveraging. In particular the main contribution of this work is twofold: *i*) it is defined the condition for which bubbles arise even in case of Pareto-efficient initial conditions, *ii*) it is defined a sufficient condition from which a deleveraging may occur: the existence of a (behavioral) bubble (proposition 5). This work is divided into two parts: one in which it is defined the bubbles' formation in the collateral market and the second in which this bubble sets off a deleveraging.

The classic model of rational bubbles defines them as a positive (sequential) difference between market prices and fundamental values fueled by agents expectations of being able to resell that inflated claim at a higher price to another agent. The looming capital gains depict the root of the bubbles in a speculative motive, from the Kaldor-Keynes definition: "investors exhibit speculative behavior if the right to resell (an) asset makes them willing to pay more for it than they would pay if obliged to hold it forever" (that is their market fundamental). From Tirole 1982 it is known that this kind of speculative behavior cannot be observed in a fully dynamic rational expectations equilibrium (REE). The present work does not divert from Tirole's finding, but it treats the issue differently, by exploring the consequences of a non-rational dynamics of the economy. It thus defined a *dynamically inefficient* path which follows non-rational expectations (defined as diagnostic (Bordalo, Gennaioli, and Shleifer 2016)) and a *dynamically efficient* equilibrium which follows rational expectations.[2][3]

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[1]I define non-rationality as an (endogenous) overstatement of a true probability distribution. In this sense a non-rational agent forms his expectations following the path of a rational (true) distribution by only increasing its magnitude without changing its sign (see Bordalo et al. 2016; Gennaioli and Shleifer 2010).

[2]I shall use the terms: rational, dynamic efficient and true as referring to the same equilibrium path (vice versa for: non-rational, dynamic inefficient and not true).

[3]I shall consider dynamic inefficiency as Pareto-inefficiency since, in the particular model

In general, the latter can be broadly defined as an equilibrium consistent (in the sense of definition 4) with the needs of the economic activities. The former is defined upon a neglected risk (in this case the decline in houses value) and extrapolation from which optimism (pessimism) is overstated. In Giglio, Maggiori, and Stroebel 2016 the presence of classic rational bubbles in housing markets is ruled out. Following their results, this work proposes a new way of considering bubbles on real estate prices.

Similar to Giglio, Maggiori, and Stroebel 2016, here I shall refer to classic rational bubble models as those models which require a failure of the transversality condition so as to account for a positive bubbly component. In the following model the market value will be greater than the fundamental one even if the transversality condition is fulfilled: the bubble is driven by a positive error in expectations for which agents overstate positive fundamental news, what Kindleberger 1978 defined as "displacement" (see also Pansera and Strati 2016). In this spirit, the bubble formation hinges on a sequence of large positive cash-flow shocks (later on called "trees dividends" shocks). I shall treat expectations as homogeneous (both for the rational and for the non-rational case) processed by an infinitely-lived measure one of agents. The agents's rationality is slightly relaxed at the beginning of the economy in their ability to choose and understand which path follows the dynamics of the true efficient economic model (the fully rational one). This reasoning follows from Shiller 1990 in which the econometric model and the popular model may not coincide. Agents are thus allowed to rely on a "dynamic inefficient" model fueled by positive news which drives their expectations to different magnitudes with respect to a rational (true) one. The dynamically inefficient path is followed temporary because an exogenous shock which unveils the overstated model triggers an abrupt adjustment towards the true (rational) path. The inefficient path overstates the efficient one because of a gap between the fundamental

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which follows, in a dynamic inefficient economy a marginal increase in the tradings (and value) of collateral contracts implies a marginal decrease in the lifetime utility of some agents. However this decrease in utility is lagged of one period.

value defined in terms of rational expectations and the agents's error in processing the expectations of future houses value. In this overheated economy there is an excess of volatility which triggers predictable forecast errors since they are not orthogonal with the information currently available (see (1.18)): besides the neglection of downside risks, diagnostic expectations are also extrapolative in the sense that past values (of the collateral) are crucial in the formation of the expectations for the future (collateral) values.

If bubbles are driven by speculative motives, and thus by transferring risk between a finite numbers of agents (as an insurance), then an initial Pareto-efficient condition implies that no one agent trades because he would be worse off. But if the bubble is triggered by the fact that all of the agents believe to an over optimistic model, π^d say, while it is not covered by fundamentals, then the economy follows a path which diverts from the actual true model π implying a spread. This spread is set off by dynamic inefficient paths. The dynamic inefficiency considered in this work can be seen as follows: at an initial position agents' strategies are Pareto efficient and define a Nash equilibrium at $t = 0$. However since expectations become diagnostic (non-rational), the Nash equilibrium can be improved given that expectations are homogeneous, that is: by current representative information, the equilibrium at $t = 0$ is strictly dominated by the equilibrium strategy that can be reached with homogeneous diagnostic expectations.[4] This possible enhancement towards a strictly representative dominant strategy triggers a non-rational Nash equilibrium that is not feasible, thus the new position is Pareto-inefficient at $t = 1$. Moreover at $t = 2$ the dynamic inefficient equilibrium will come back on a feasible Nash equilibrium. Notice that from $t = 0$ to $t = 1$ the economic activities (consumption and tradings) rely on a Nash Pareto-inefficient equilibrium, this lag in understanding the mistake allows the econ-

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[4]This effect of representative pieces of information strongly departs from the findings of Milgrom and Stokey 1982 who state that: "...the information conveyed by the change in relative prices *swamps* each agent's private information taken individually...".

omy to overheats. Since the new equilibrium is not feasible, then the economy cannot roll with the punches and thus knocks out. Notice that this reasoning is similar to the model of Cooper and John 1988, but here I shall account of a limited ability of processing information for which over-optimistic (biased) strategic complementarities and a positive representativeness give rise to a multiplicity of equilibria which are not feasible. The mistake in processing the rational efficient path is possible since in the sequential equilibrium (implicit in this work) the beliefs defined over agents' strategies can be non-rational (diagnostic). In this sense, at the initial position, agents can choose a node not encompassed in a rational belief assessment, hence overestimating the Bayesian beliefs of the rational sequential equilibrium.

After a bubble arises and bursts, deleveraging follows since the fundamental value of the financial contracts does not cover the inflated one. In theory the deleveraging should be neutral on the economic activity (Maffezzoli and Monacelli 2015), in the sense that a decrease in the potential of one side of an economy is perfectly offset by a symmetric increase of that potential from the other side. This work studies bubbles formations and deleveraging for agents in a very simplified economy, and thus a decrease in the borrowers's consumption must be offset by an increase in the lenders's consumption so as to maintain the aggreagate demand unchanged (assumption 3). For this neutrality it is implicitly assumed that both kinds of agent have an equal marginal propensity to consume (see Kumhof, Ranciére, and Winant 2015 for the effects of a more realistic assumption). In this case the problem is not on the level of the aggregate demand, but on its composition. This issue of redistribution keeps unaltered the real effect on the economic activity and thus in the case of a perfect-offsetting, the deleveraging is neutral on the economy. The most famous frictions on the "perfect-offsetting" state (that do not follow assumption 3) are two

- the fall in consumption of the constrained agents, the borrowers, is larger than the increase in consumption up to the unconstrained ones, the lenders, dragging

the aggregate demand down;

- the interest rate cannot decrease enough for luring lenders into consuming more and lending less since the presence of the zero lower bound constraint.

In this paper I shall not deal with the problem of the presence of frictions, rather the existence of bubbles and thus of deleveraging (see proposition 5) do not imply that the latter should be non-neutral.

After having defined the model and the notion of diagnostic expectations (section 1.2), the work will be divided into a bubble rise-and-burst model (section 1.3) and the subsequent borrowers deleveraging (section 1.4).

1.1.1   Related literature

This work has been built from the definition of rational sequential equilibrium of Radner 1972. I shall follow the infinitely-lived model of Tirole 1982 and I allow for the possibility that agents follow a dynamic inefficient path: it is the root of a bubble formation in the model.

I shall take into account credit fluctuations driven by fluctuations in borrowing constraints as in Martin and Ventura 2016. The crowding-in effect in Martin and Ventura 2016 of an "optimistic" bubbly collateral allows agents to take on an excess of credit over their fundamental collateral. I shall follow this reasoning with a bubbly collateral steered by diagnostic expectations.

The present work is a theoretical one, but it is closely linked to an increasing empirical literature about the links between households credit expansion and the macroeconomy. In particular it has been studied the effects that a sudden increase in households debt triggers on the output growth. In the presence of fiscal and monetary constraints it has been proved that those effects are negative on growth. Empirically I shall pertain to the work of Mian and Sufi 2014 in general and to Mian and Sufi 2010; Mian, Sufi, and Verner 2016 as for the effects of increasing households debt. In so far as this

excess in credit growth is linked to a future decrease in consumption, the referring empirical papers are Kumhof, Ranciére, and Winant 2015; Mian, Rao, and Sufi 2013 and for a further explanation of the causes of this households debt glut Mian and Sufi 2009. In a recent paper Di Maggio and Kermani 2015 study the relationship between a credit expansion and the boom and bust cycle in house prices and real economic activities. During $2004 - 2006$, an increase in lendings has been followed by a 3.3% rise in annual house price growth rate and a 2.2% expansion of employment in the non-tradable sectors (Di Maggio and Kermani 2015), but during the bust the mortgage delinquency rates increases and economic activities decrease.

This paper is also linked to an increasing theoretical literature about the effects of a highly indebted economy which, mostly after observing a negative shock, faces a deleveraging on the economic activity. The deleveraging equilibria defined in section 1.4 are based upon Krugman and Eggertsson 2012; Korinek and Simsek 2016, provided with expected values of the constraint as in Maffezzoli and Monacelli 2015. It is plain that I shall touch the problem of inefficiency for an ex-ante decision of borrowing, and in this flavor this work is very similar to Lorenzoni 2008 in which it is studied the possibility of an inefficiency in those decisions concerning ex-ante borrowings that do not lead to Pareto improvements.

In so far as the collateralized debt is concerned, this paper is similar to Simsek 2013 from the point of view of considering optimism and (or) pessimism in the model. However this work is different from Simsek 2013 since here I shall take into account homogeneous (non-) rational expectations, while in Simsek 2013 the heterogeneity (optimists and pessimists) in beliefs is assumed as a natural root of the financial constraint (which is closer to Harrison and Kreps 1978).

The expectations on the future value of the collateral are assumed to be "diagnostic" in the sense of Bordalo, Gennaioli, and Shleifer 2016 (see section 1.2.1 of the present work). The heuristic behind these kind of expectations has been proposed by

Kahneman and Tversky 1972; Kahneman and Tversky 1983 and has been formalized in Gennaioli and Shleifer 2010. In Gennaioli, Shleifer, and Vishny 2012 has been modeled the possibility that a neglected risk in the tail of a distribution may trigger a sort of Wile E. Coyote fall (in the sense of Krugman 2007) in the asset prices. I shall follow a similar reasoning, but here, as in Bordalo, Gennaioli, and Shleifer 2016, I shall take into account a different formalization of the representativeness of Gennaioli and Shleifer 2010 called: representativeness based discounting (see Bordalo et al. 2016)[5]. In Gennaioli, Shleifer, and Vishny 2012 has been used a rank-based truncation, while in Gennaioli, Shleifer, and Vishny 2015 a rank-based discounting. In order to understand the different kinds of representativeness see Bordalo et al. 2016.

This work will take into account empirical papers of extrapolative expectations which are not rational as provided in Gennaioli, Ma, and Shleifer 2015; Greenwood and Shleifer 2014 and it is related to those works which study empirically the problem that an increasing optimism can be a catalyst for piling up debt, leading to a fall in the economic activities when negative news arrive. This latter effect have been empirically provided in Baron and Xiong 2014; Schularick and Taylor 2012; Jorda, Schularick, and Taylor 2013.

1.2 The model

There is a real economy populated by two infinitely-lived kind of finite number of agents of equal measure normalized to one: borrowers (b) and lenders (l), with $a \in [b, l]$. It is assumed that there is one efficient path which is rational with respect to the

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[5]The notion of representativeness will be defined in section 1.2.1. For now it suffices to know that it is a distortion of the true (rational) distribution which affects its magnitude but not its sign. The representativeness based discounting defines a weighing function which describes the extent of the rational distortion. I have chosen this form of weighing since it is convenient when dealing with continuous distributions.

environment. At the beginning of the economy there are more than one identical models whose paths' dynamics can either be or not efficient. Agents are assumed to be in a Pareto-efficient initial condition but they cannot know it, thus agents are assumed to not understand a priori the dynamics of the initial condition so they can choose a dynamic inefficient model which overstates (understates) the rational efficient one. In this work I shall assume that agents choose, at the beginning of the economy, the dynamic inefficient model so as to be focused only on this issue (differently from Milgrom and Stokey 1982).

There exists just one good which is in the form of a not storable endowment, every intertemporal exchange is made in terms of that good only. It will be clear that this good has not an *infra*-temporal process which defines a price, that is: it is not traded by buying and selling at a point of time t say. However, the only price is the real interest rate, the *inter*-temporal price, and since there is no money, the model allows to exchange the amount of that good through time, from t to $t + 1$ say.

This economy is divided into two different spans of time: the long run and the short run. The rationale for doing so is the following: an economy begins at $t = 0$, here $t = 0, 1, 2, \ldots$ is assumed to be discrete. At $t = 0$ agents settle on their choices with respect to an infinite discrete time ahead as if the dynamics of this economy was assumed to closely fluctuate around a long run trend. However, at $t = 1$ something happens, a shock (either positive or negative) hits the economy and the path estimated for the long run changes abruptly. From $t = 0$ to $t = 1$ the economy is in its short run. Since this sudden and unexpected change in the long run trend, agents must adjust their choices accordingly. Of course these choices will depend upon the kind of the shock has just occurred and its extent. The long run is defined when a bad signal forces a drift in the decision made at the beginning of $t = 1$, thus expectations for $t \geq 1$ are set for the long run again. Notice that if the shock would never occur, short and long run should not diverge too, therefore the splitted economy would have been

intended as an only long run one. In few words:

- at $t = 0$ agents trade loans that are due at the beginning of $t = 1$;

- at $t = 1$ credit market clears and new contracts are traded, they are due at $t = 2$. Two cases are allowed:

  a) If no bad signals arrive, for intertemporal consistency, at each date the new contracts worth at least as much as the latest traded at the market value.

  b) If a bad signal arrives, then it is assumed to be observed after the new contracts have been treaded, hence borrowes are forced to deleverage.

With a slight abuse of terminology, I shall refer to short and long period with the notation $\mathcal{S}$ and $\mathcal{L}$ respectively (as in Krugman and Eggertsson 2012).

There is only one source of heterogeneity among agents, and it lies in their rates of time preferences: borrowers are assumed to be impatient, while lenders to be patient. Formally $\beta_b < \beta_l \leq 1$, in which β_a is the intertemporal discount factor. The patient-impatient structure is useful in order to maintain the necessity to borrow also in the case of equilibrium without assuming heterogeneity in beliefs and speculative behavior.

The two kinds of agents are provided with a constant endowment $e/2$, and the general problem that agents $a \in [b, l]$ have to solve is the following

$$\max_{C_t^a, B_t^a} \sum_{t=0}^{\infty} \beta_a^t U(C_t(a))$$

$$\text{s.t.} \quad B_t(a) = (1 + r_{t-1})B_{t-1}(a) - \frac{1}{2}e + C_t(a) \tag{1.1}$$

with a debt constraint which will be considered as binding naturally[6]

$$(1 + r_t)B_t \leq \mathbb{E}_{t-1}(\phi_t). \tag{1.2}$$

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[6]I consider a constraint as "binding naturally" when it is fully exploited for consumption motives (that is there are no residuals), but its upper bound is not exogenously given since it follows an endogenous equilibrium of the economy considered (in this case the collateral market).

The notations are simple: B is a debt, while $-B$ is an asset holding. Moreover, $U(\cdot)$ is a linear utility function and C is consumption. In (1.2), the parameter ϕ is crucial for the following analysis: this is the maximum level of debt a borrower can borrow from and a lender can lend to. Differently from Krugman and Eggertsson 2012, this work analyses the value of this constraint from a behavioral point of view: defining rational and non-rational paths in the collateral market, their dynamic relation can fuel a bubble whose inflated value, assumed as temporary, triggers a deleveraging. As precisely defined in section 1.3, the market of collateral contracts ϕ is a place in which lenders and borrowers exchange those contracts at their market value (always in terms of the only good in the economy). Borrowers sell a contract ϕ to lenders by promising to buy back it at value $\alpha\phi$, with $\alpha > 1$, the next period. Borrowers may not fully commit themselves to pay. With a slight abuse of terminology I shall call debts and lendings the tradings in this kind of market.

It is assumed, for simplicity, that the operator $\mathbb{E}$ is homogeneous, in the sense that both agents will agree upon the value of the expected constraint. This is a simplification that forces me to treat the constraint imposed on borrowers as a unique value, that is: the borrowing capacity is equal for every borrower. Notice that $\mathbb{E}_{t-1}[\phi_t + d_t] \approx \phi_{t-1}$, in the sense defined in (1.8), that is to say: the current collateral value is equal to the expected value of the contract at t when traded at $t-1$, and a $d_t \in \mathbb{R}_+$ as cash flows (later on trees dividend). More formally: there is a forward commitment at $t-1$ in a state space ω_{t-1} in which it is specified the amount of consumption good that borrowers promise to deliver to the market at a date $t > t-1$ contingent on the occurrence of some future state space ω_t.

1.2.1 The representativeness

The constraint is assumed to be the value of the houses owned by borrowers: the house's value is the collateral for the new debts. A borrower (b) may take on a new

debt (a mortgage) for buying a house which, in turn, will be the collateral of that debt, or he may use the already owned home as collateral for consumption motives. I shall take into account the latter motive. It is important to notice that neither investment nor savings are allowed in the model. Houses cannot be rent, but they are provided with a tree which produces a stock of consumption good contingent on the current state: a movement in the value of the houses is a movement in the perception of wealth. In particular, if a house value increases and so their "trees dividend", this value can be turned into consumption or into a collateral contract. As for the latter application, the house value is the actual collateral for borrowing, then this value comes into the disposable income by means of debt or credit. Suppose that ϕ_1 (which encompasses the trees dividend, see Equation (1.8)) is the house value at time $t = 1$ and $\mathbb{E}_0$ the current expectation operator for the house value at $t = 1$. There are two states of the world which can occur at $t = 1$: growth (g) and recession (r). By denoting $\pi_{g,1}$ and $\pi_{r,1}$ the probability of growth or recession at $t = 1$ respectively,

Assumption 1. at $t = 0$, $\pi_{g,1} > \pi_{r,1}$.

Suppose now that at $t = 1$ there is an exogenous signal $s \in [\bar{s}, \underline{s}]$ with $\underline{s} < \bar{s}$. This signal gives information on which of the two states will occur: $\bar{s}$ denotes an incoming growth, while $\underline{s}$ denotes an incoming recession. The signal is observed by all of the agents at the same time. If $\underline{s}$ is observed, then assumption 1 is reversed.

Assumption 2. At $t = 1$

   $\underline{s}$: $\pi_{g,2} < \pi_{r,2}$;

   $\bar{s}$: $\pi_{g,2} > \pi_{r,2}$.

Denote with Ω the state space in which ω_0 is a realization. In particular $(\Omega_1 = \omega_1)$ is the realization of the state ω_1 at $t = 1$. If $(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0)$ then it is intended that the realization ω_1 at $t = 1$ hinges on the occurrence of the state ω_0 at $t = 0$. Now suppose that there exists a smooth density function $f(\cdot)$ for which in a Bayesian

framework

$$\pi_{g,1} \equiv F(\bar{s} \mid \omega_1) = \int_{\underline{s}}^{+\infty} f(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0)d\omega \qquad (1.3)$$

is the rational cumulative distribution which gives the probability that the state ω_1 is a growth state. The survival distribution $1 - \pi_{g,1} \equiv \pi_{r,1}$ denotes the probability of a recessionary state at $t = 1$. The support information of (1.3) seems to be a rational one, in the sense that a σ-algebra $\mathcal{F} \subset \Omega$ defines a unique rational filtration for the probability function. The perception upon this probability is however distorted by representativeness. The house value is a function of the evaluation of the collateral given the expected state space (see equation (1.8)): if $\omega_0 \geq \bar{s}$ then the house value tends to rise, while if $\omega_0 \leq \underline{s}$ the housing value is going to decrease. In order to grasp the behavioral aspect of either optimism or pessimism in the determinance of ϕ, the difference between the actual ω_0 and its expected value plays a crucial role.

Diagnostic Expectations

Define $\Gamma \equiv \{\Omega_0 = \omega_0\}$ as the group which describes all the possible future states ω_1 whose values hinge on the current state $\Omega_0 = \omega_0$.

Definition 1. The density function $f(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0)$ is the true distribution function.

The function in definition 1 is assumed to be known by agents, the representativeness defines the overweighted probabilily attached to the true distribution that is more representative for Γ than for its comparison group $\Gamma^c \equiv \Omega \setminus \Gamma$. As in Bordalo, Gennaioli, and Shleifer 2016, the comparison group may be defined as the rational expectation formed at $t = -1$ for ω_1 in which no new pieces of information occur, that is $\Gamma^c \equiv \{\Omega_0 = \mathbb{E}_{-1}(\omega_0)\}$.

Definition 2. The representativeness of a state ω_0 at $t = 0$ for a group Γ is defined as

$$\mathcal{R}(\omega_t, \Gamma, \Gamma^c) = \frac{f(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0)}{f(\Omega_1 = \omega_1 \mid \Omega_0 = \mathbb{E}_{-1}(\omega_0))}. \qquad (1.4)$$

It is plain that if $\omega_0 > \mathbb{E}_{-1}(\omega_0)$, then (1.4) increases. It is thus assumed that the monotone likelihood ratio property (MLRP) holds in (1.4). Thus if Γ is the most representative group for future ω_1, then the right tail of the distribution function turns out to gain more attention, neglecting the left tail (vice versa for the case of a $\mathcal{R}(\omega_t, \Gamma, \Gamma^c)$ monotonically decreasing). The representativeness is characterized by attaching to $\mathcal{R}(\omega_t, \Gamma, \Gamma^c)$ a weighing function $p_t : \mathcal{R}_+^t \to \mathbb{R}_+$. In this sense

$$p_0 = \left[\frac{f(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0)}{f(\Omega_1 = \omega_1 \mid \Omega_0 = \mathbb{E}_{-1}(\omega_0))} \right]^\theta$$

for which $\theta \geq 0$ describes the magnitude of the distortions of beliefs. In particular, following Bordalo, Gennaioli, and Shleifer 2016, the stereotype of time $t + 1$, denoted by $\pi_{1,\Gamma}^d$ (the diagnostic expectation (d) for a state ω_1 at $t = 1$ with respect to the representative group Γ), is thus defined as

$$d(\omega_1) \equiv f(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0) \times \left[\frac{f(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0)}{f(\Omega_1 = \omega_1 \mid \Omega_0 = \mathbb{E}_{-1}(\omega_0))} \right]^\theta \frac{1}{Z}, \qquad (1.5)$$

notice that, as in Bordalo, Gennaioli, and Shleifer 2016, $1/Z$ is simply a normalizing constant which assures the integrability to one of (1.5). The stereotype described by (1.5) is very useful for the issue arisen by the present work.

Agents (borrowers and lenders) are provided with the same expectation at t towards a probable state at $t + 1$. Since they share the same symmetric information and the MLRP holds, a news by which $\omega_0 > \mathbb{E}_{-1}(\omega_0)$ triggers an increase in the expected value of the collateral since growth is the incoming state so that $d(\omega_1)$ increases too (vice versa for a recessionary state). In few words, following Bordalo et al. 2016, in case of diagnostic expectation (d), for a monotonic increasing likelihood ratio in (1.4)

$$\mathbb{E}_t^d(\omega \mid \Gamma) > \mathbb{E}_t(\omega \mid \Gamma) > \mathbb{E}_t(\omega \mid \Gamma^c) > \mathbb{E}_t^d(\omega \mid \Gamma^c), \qquad (1.6)$$

vice versa for a monotonic decrease (1.4). For a $\theta = 0$, there is place for the only rational expectation part (the true distribution), while for a $\theta > 0$, the representativeness gooses the representative states and deflates the likelihood of those states that

are not representative (belonged to Γ^c). It is thus plain that $\mathbb{E}_0^d(\omega_1 \mid \Gamma) \geq \mathbb{E}_0(\omega_1 \mid \Gamma)$ depends on the value of θ. Since the density function is normal ($f(\cdot) \sim \mathcal{N}(0, \sigma^2)$), its mean is defined as

$$\mathbb{E}_0^d(\omega_1) \equiv \mathbb{E}_0(\omega_1) + \theta[\mathbb{E}_0(\omega_1) - \mathbb{E}_{-1}(\omega_1)]. \tag{1.7}$$

In (1.7) the case $\mathbb{E}_0^d(\omega_1 \mid \Gamma) = \mathbb{E}_0(\omega_1 \mid \Gamma)$ can occur if and only if either $\theta = 0$ or no news has arrived. It is straightforward now that diagnostic expectations overweight a true distribution either positively or negatively, in the sense that there is an exaggeration in the level of the path followed by rational expectations: diagnostic expectations determines an overheating or a severe cooling down of economic activities.

1.3 Equilibria in the collateral market $t = 0$ to $t = 1$

I consider a market of contracts called "collaterals", traded at $t = 0, 1, 2 \ldots$, in which a patient agent (lenders) buys a collateral contract ϕ_0 at $t = 0$ and an impatient agent (borrowers) sells that contract ϕ_0 at $t = 0$ as well. Borrowers promise to buy back ϕ_0 from lenders by an amount $\alpha\phi_0$ at $t = 1$ with an $\alpha > 1$ contingent on the realization of an event $\omega_1 \geq \omega_0 \equiv \bar{s}$ otherwise $\alpha < 1$. The possibility for $\alpha \in \mathbb{R}_{\geq 0}$ to be either above or under the threshold 1 reflects the fact that agents may not fully commit themselves to buy back the contract at the price they have promised. Agents meet and trade at the beginning of each date $t = 0, 1, 2 \ldots$ Eventually, collateral ϕ reflects the housing value, which I assumed homogeneous among agents $a \in [l, b]$.

Define the value of the collateral as

$$\phi_0 = \gamma\mathbb{E}_0[\phi_1 + d_1] \tag{1.8}$$

where ϕ_0 is the real value of the collateral at $t = 0$, γ is a positive discount factor and $\mathbb{E}_0$ are the expectations conditional to the available information at time $t = 0$. Each

house is provided with an infinitely-lived tree (and zero depreciation) which produces the only consumption good (a trees dividend d) by an additional amount $d_1 \geq 0$ whose values $\{d_0, d_1, \ldots, d_t, \ldots\}$ hinge on a stochastic process with signals $s \in [\underline{s}, \overline{s}]$. In particular, for positive news $\omega_t \geq \overline{s}_t$, then $d_t > d_{t-1}$ (vice versa otherwise). Each stock of tree's consumption good is given at the end of the period.

The fundamental value of the collateral is given by solving (1.8)

$$\Phi_0 = \sum_{j=1}^{\infty} \gamma^j \mathbb{E}_0 d_{0+j} \tag{1.9}$$

and the general solution is given by

$$\phi_0 = \Phi_0 + \Theta_0 \tag{1.10}$$

in which Θ_0 is a random variable defined as

$$\Theta_0 = \gamma \mathbb{E}_0 \Theta_1. \tag{1.11}$$

Equation (1.11) can be thought of as a "rational bubble" if $\mathbb{E}_0$ defines a rational expectation model with respect to information at time $t = 0$. This bubble may either depend on d or on any other exogenous variables. Since I shall define an equilibrium of the collateral market with an infinite horizon, the bubble is defined as the difference between the market value of the collateral and its market fundamental. Rather in a finite-horizon case, the market value should be always equal to its market fundamental. Moreover, since I have assumed that the Bayesian agents have the same prior and follow the same heuristic, the market fundamentals are identical among agents. This reasoning means that the value of ϕ does not impart any extra information than that provided by the observation of a signal $s_t \in [\overline{s}, \underline{s}]$. It follows that agents base their expectations on s_t. Define a forecast function ψ_0 which associates with any signal s_0 a collateral value $\phi_0 = \psi_0(s_0)$. The forecast function is the statistical relationship between market prices and their signals, and it is justified by an assumed common knowledge about the structure of the market, hence an equilibrium arises when each

agent believes that the other agents use their equilibrium strategies. Moreover, by observing ϕ_0 the information indicates that s belongs to $S_0(\phi_0) = \psi^{-1}(\phi_0)$ (in which S_0 is the set of possible signals at $t = 0$). It will be clear, as already stated, that when there are homogeneous beliefs, then the two observations coincide.

1.3.1 REE of the collateral market

The fundamental value of (1.8) can be rewritten in terms of the two pieces of information s^a and $S(\phi)$ (it is possible to drop the superscript a since both kinds of agent observe the same signal, and re-write $S(\phi)$ as S for notational convenience)

$$\Phi(s_0, S_0) = \mathbb{E}_0 \left(\sum_{j=1}^{\infty} \gamma^j d_{0+j} \mid s_0, S_0 \right). \tag{1.12}$$

For a consistent ϕ_1 with S_0, from (1.10) the collateral value bubble is

$$\Theta(s_0, S_0) = \phi_0 - \Phi(s_0, S_0) \tag{1.13}$$

from which

$$\Theta(s_0, S_0) \geq 0.$$

In order to define the equilibrium, denote by x_0^a the stock of transactions between lenders and borrowers at time $t = 0$. In a dynamic environment, markets clear when

$$\sum_a x_0^a = \bar{x},$$

in which $\bar{x} = \sum_a x_{-1}^a$.[7]

Definition 3. A dynamic REE is a sequence of self-fulfilling forecast functions (ψ_t)[8] for which

$$\phi_t = \psi_t(s_t) \Leftrightarrow s_t \in S_t(\phi_t) \equiv \psi^{-1}(\phi_t)$$

such that there exists a sequence of strategies $x_t^a(s_t^a, \phi_t)$ satisfying

    i) market clearing condition: $x_t^a(s_t^a, \phi_t)dx = \bar{x}$ for each $a \in [l, b]$;

---

[7]In a static case $\sum_a x_0^a = 0$ since no transaction is on the market.

[8]It is a similar reasoning as Radner 1972 and Jordan and Radner 1982.

ii) maximizing behavior: for any information (s_t^a, S_t) (and common prior), an a's strategy maximizes a's expected present discount gain from t on.

From Tirole 1982 follows the following crucial proposition:

Proposition 1. *Collateral value bubbles do not exist in a dynamic REE*

$$\Phi(s_t, S_t) = \phi_t \ \text{ i.e. } \ \Theta(s_t, S_t) = 0$$

Proof. Assume a set of optimal strategies $\{x_t(s_t, \phi_t)\}$, it follows that the aggregate gains[9] G equal the fundamental value for every $a \in [l, b]$

$$G^a \equiv \Phi_t \bar{x} \tag{1.14}$$

in general. Define the transversality condition under REE

$$\mathbb{E}(G_t \mid s_t, S_t) \geq x_t \phi_t \tag{1.15}$$

from which an agent cannot gain by selling x_i contracts and leaving the market at t, then

$$\mathbb{E}(\Phi \bar{x} \mid S_t) \geq \phi_t \bar{x} \ \text{ which means } \ \Phi(S_t) \geq \phi_t.$$

The last result assures that the fundamental value which stems from the market information exceeds (or is equal to) the collateral value. Moreover, if an agent is provided with a stock $x_t^0 < \bar{x}$ and from taking into account and assuming $\Phi(s_t^0, S_t) > \phi_t$, then by contradicting the implication of (1.14), that agent can make a profit by buying an underpriced collateral contract with higher expected value (in terms of trees dividend), triggering an arbitrage benefit. It follows that $\Phi(s_t, S_t) = \phi_t$ from which it is clear that an agent cannot produce a benefit by only holding a stock of contracts of the precedent period, that is (from (1.14))

$$\mathbb{E}(G_t \mid s_t, S_t) = \mathbb{E}(G_t(x_{t-1}) \mid s_t, S_t).$$

---

[9] A lender buys at price ϕ, x claims from which he will receive $\tilde{\phi} = \alpha \phi$ as soon as the state ω occurs. The ex-post realized gain is thus defined as $G = (\tilde{\phi} - \phi)x$. For the transversality condition (market clearing) then $\sum_a G^a$ equals the fundamental value.

Now, $\Phi(s_t, S_t) = \phi_t$ is exactly the condition for which an REE cannot set off a bubble, that is $\Theta = 0$ (Tirole 1982). $\qquad\square$

1.3.2 Dynamics with Diagnostic Expectations

Suppose that at $t = 0$ agents do not know that they are in a Pareto-efficient equilibrium. However they receive news by which diagnostic expectations divert from rational expectations following (1.7), that is by $\theta[\mathbb{E}_0(\phi_1) - \mathbb{E}_{-1}(\phi_1)]$. Suppose that at $t = 0$ the new pieces of information trigger optimism among agents as for the future value of the collateral $\phi_1 \geq \phi_0$. In order to show this optimism effect, define the cumulative distribution of (1.5) as

$$F_0^d(s) = \int_{s \geq \bar{s}}^{+\infty} d(\omega_1)d\omega_1,$$

it is the probability that positive news continue to hit the economy. According to assumption 1 better news boost agents' perception of safer state, that is

$$\frac{\partial \ln F_0^d(s)}{\partial \omega_0} = [\mathbb{E}_0^d(\omega_1 \mid \omega_1 \geq \bar{s}) - \mathbb{E}_0^d(\omega_1)]\frac{(1+\theta)b}{\sigma^2} \ \ \text{with} \ \ \frac{\partial \ln F_0^d(s)}{\partial \omega_0} > 0. \qquad (1.16)$$

Notice that $\partial \ln F_0^d(s)$ is strictly related (increasing) with $\mathbb{E}_0^d(\omega_1)$, that is better news trigger more probable safer states which means higher collateral values and housing prices (vice versa in case of negative news). Moreover lenders feel safer since they know that if a borrower does not pay, they can seize his collateral contract albeit at its current market value (the value at the date the credit is seized).

Given very positive news, the overall optimism increases (since the assumption of homogeneous expectations) for which agents assume collateral contracts as safe, pushing the housing and collateral value up. For homogeneous expectations: optimism in lenders makes them willing to grant a loan and optimism in borrowers makes them confident in fulfilling their commitment. In this way positive pieces of informations trigger an upward path of the collateral's value in an economy provided with homogeneous expectations.

Rewriting (1.7) in terms of the model

$$\mathbb{E}_0^d(\phi_1) \equiv \mathbb{E}_0(\phi_1) + \theta[\mathbb{E}_0(\phi_1) - \mathbb{E}_{-1}(\phi_1)]. \tag{1.17}$$

It follows an important property of diagnostic expectations, namely they encompass excess volatility since they distort the magnitude of the direction of the observed news.

Proposition 2. The variance of future diagnostic expectations $\mathbb{E}_0^d$ at $t = -1$ is defined as

$$\text{Var}_{-1}[\mathbb{E}_0^d(\phi_1)] \approx (1 + \theta)^2 \text{Var}_{-1}[\mathbb{E}_0(\phi_1)]$$

which increases with representativeness θ.

From proposition 2 follows that excess volatility drives to predictable forecasting error $\mathbb{E}_0[\phi_1 - \mathbb{E}_0^d(\phi_1)]$ that, by substituting 1.17 and applying the law of iterated expectations, gives

$$\mathbb{E}_0[\phi_1 - \mathbb{E}_0^d(\phi_1)] = -\theta[\mathbb{E}_0(\phi_1) - \mathbb{E}_{-1}(\phi_1)], \tag{1.18}$$

for very good news at $t = 0$, the forecasting error is negative. From proposition 2 it is straightforward that diagnostic expectations are endowed with higher volatility than that exhibited by rational fundamentals, this excess in volatility is driven by the latest observed news. Proposition 2 implies predictable forecast errors (1.18) which means that it produces predictable anomalous contracts' values. It is a crucial property since by the presence of a systematic pattern of errors, expectations do not just fluctuate randomly like a noise. Rather, beside the possibility of a noisy exogenous shock, a bad "shock" here can be caused by the extrapolative nature of diagnostic expectations by simply stopping the flow of good news (es. see Giglio and Shue 2014 for the importance of the information encompassed in the absence of news). In few words, differently from rational expectations, here forecast errors are not orthogonal to the information available at the time of the forecast. The error predictability is thus an important diversion to the rational benchmark since future errors are predictable by relying on the information available ex ante. This approach is also consistent to

Manski 2004 in stressing the importance of agents expectations (what they think of) beside the "revealed preference analysis" (what they actually do) in shaping economic outcomes.

It is now clear that under diagnostic expectations the market value of the collateral inflates (deflates) the equilibrium of proposition 1, hence a solution for (1.8) is re-defined as

$$\phi_0^d = \Phi(s_0, S_0) + \epsilon_0^d \ \text{ with } \ \Theta(s_0, S_0) = 0. \tag{1.19}$$

where ϵ^d is the error in the agents' expectations caused by diagnostic expectations. In particular

$$\epsilon_0^d = [\mathbb{E}_0^d(\phi_1) - \mathbb{E}_0(\phi_1)]$$

that, from (1.17), can be rewritten as

$$\epsilon_0^d \equiv \theta[\mathbb{E}_0(\phi_1) - \mathbb{E}_{-1}(\phi_1)]. \tag{1.20}$$

The gap between the fundamental value defined in terms of rational expectations and the error of (1.20) is driven by representativeness θ, that is the extent of the *surprise effect* relative to news. Notice that $\Theta(s_t, S_t) = 0$ suggests that there is not a rational bubble, in the sense that the solution of (1.8) rules out a rational bubbles as before, there is however a bubble driven by a temporary error in expectations ϵ^d.

Proposition 3. *Collateral value bubbles exist in a dynamic REE distorted by Diagnostic Expectations*

$$\phi_t^d = \Phi(s_t, S_t) + \epsilon_t^d \ \text{ in which } \ \epsilon_t^d \equiv \theta[\mathbb{E}_{t-1}(\phi_t) - \mathbb{E}_{t-2}(\phi_t)] > 0.$$

Proof. Suppose that expectations follow an AR(1) process of the form $\mathbb{E}_0(\phi_1) = a + b\phi_0$, moreover $\Phi(s_0)$ is defined upon rational expectations and equates $\mathbb{E}_0(\phi_1)$, that is $\Phi(s_0) \equiv a + b\phi_0$. Put the AR(1) processes into (1.19), then follows that

$$\frac{\partial \mathbb{E}_0^d(\phi_1)}{\partial \phi_0} = (1 + \theta)b$$

which means that the non-rational value of ϕ_t depends upon the extent of the representativeness, and on the serial correlation of the data $b > 0$, that is: a positive

overestimation produces its effects over more periods. Notice that for rational expectations $\epsilon_0 \equiv [\mathbb{E}_0(\phi_1) - \mathbb{E}_{-1}(\phi_1)] = 0$, hence

$$\frac{\partial \mathbb{E}_0(\phi_1)}{\partial \phi_0} = b$$

that if defined as a random walk ($b = 1$) implies that ϕ_1 and ϕ_0 do not change values. For $\theta > 0$ and $b = 1$

$$(1 + \theta) > 1,$$

which means that representativeness of positive news affects positively the expected value of the contracts, that is

$$\phi_0^d > \phi_0 \equiv \Phi(s_0, S_0)$$

$\square$

It is plain that proposition 3 implies that the trees dividend in terms of consumption good d is overvalued and exceeds the fundamental value. The error term ϵ^d gives rise to a bubble. The problem arises if at $t = 1$, before a bad signal $\underline{s}$ is observed, market clears and new loans are contracted. Since no bad signals have shown up yet, agents trade at the latest *bubbly* value of collaterals

$$\phi_1^d = \gamma \mathbb{E}_1^d[\phi_2 + d_2]. \tag{1.21}$$

Suppose that after the trading, a bad signal $\underline{s}$ shows up and unveils the inflated nature of ϕ_1^d for which there is an excess of leverage with respect to the rational fundamental value. The safe collateral market turns out to be risky and highly volatile (proposition 2) implying a predictable abnormal decrease in trees dividends (Equation (1.18)).

It is crucial to notice that bubbles arise because of the following causality for each *bubbly* period

$$\underbrace{\text{Positive News}}_{\mathbb{E}^d} \Rightarrow \underbrace{\text{Solid Trees Dividends} \Rightarrow \text{Safe Collateral Values}}_{d_t > d_{t-1} \therefore \phi_t^d > \phi_{t-1}}$$

Since the time preferences are assumed as $\beta^b < \beta^l \leq 1$, then safe financial contracts attract both lenders who prefer to postpone consumption and borrowers who prefer to anticipate consumption. Diagnostic expectations inflate the magnitude of the positive news causing an increasing demand for collateral contracts whose value is pushed up. It is expected that trees dividends will rise too, but eventually they will not because of a negative exogenous shock, causing a bubble to burst (as assumed in assumption 2.25). In this latter effect borrowers took on an excessive leverage which cannot be fulfilled and thus they are forced to deleverage.

The next section studies what happen from $t = 1$ ahead, that is from the time in which the economy has been hit by a negative shock.

1.4 On the existence of deleveraging $t \geq 1$

I left the last section at time $t = 1$, when a negative shock $\underline{s}$ has been observed. This unexpected shock has the effect to burst the bubble. There is thus a natural fall in the debt limit from ϕ_1^d to $\underline{\phi}_2$ since borrowers cannot pay off that inflated amount given that the trees dividends worth less than expected. I shall solve the problem of deleveraging in an economy in which from $t = 0$ ahead, the model is driven by rational expectations. Then I shall define the case in which diagnostic expectations, and Equation (1.19), occur.

1.4.1 Deleveraging with "rational expectations"

Assume that agents are fully rational and assume that, after having observed $\underline{s}$, the debt limit fall from $\bar{\phi}_1$ (in case of growth) to $\underline{\phi}_2$ (in case of a slump). Agents a follow rational expectations so that they define the REE value of the collateral contracts on the market.

A very useful implication of proposition 1 is that the condition of equality between the fundamental value of a collateral contract and its market value, implies that the

transactions and the value of the contracts are *consistent* with the necessity of the model.

Definition 4. The strategies (plans) of the agents are "consistent" if for each date, each contract and each event, the excess of the planned supply of contracts is zero at that date for that event.

It is plain that definition 4 defines consistency for a dynamic REE with the necessity of having $\Theta = 0$. From the proof of proposition 1 this condition can be written as $\mathbb{E}(\Phi \bar{x} \mid S_t) = \phi_t \bar{x}$ from which it is clear that the minimum number of aggregate a's transactions $\bar{x}$ for the collateral contract ϕ_t at t, is perfectly covered by its fundamental value. Rather, in the following model of deleveraging, in case of $\mathbb{E}(\Phi \bar{x} \mid S_t) < \phi_t \bar{x}$ the excess of traded contracts will turn out to be a deadweight loss for agents from which the economic activity decreases. However any strict or weak inequalities cannot occur in a model in which an REE is fulfilled, and it is what this section is concerned. In what follows I shall define a model of deleveraging with rational expectations and I define condition (1.23) that must be fulfilled by a neutral deleveraging. The main result of the section is that that condition cannot be fulfilled since a deleveraging in a model provided with a REE cannot occur.

The natural real interest rate is defined with respect to the lenders' preferences

$$1 + r_2 = \frac{1}{\beta}. \tag{1.22}$$

If price r is below β^{-1}, then no one will be willing to lend any amount of consumption good to borrowers. While if r is above β^{-1} lenders will not consume at t at all, lending the whole stock of goods.

Starting backwards, in the long run $t \geq 2$, borrowers face a consumption

$$C_{\mathcal{L}}^b = \frac{1}{2}e - \frac{r}{1+r}\underline{\phi}_2 = \frac{1}{2}e - (1-\beta)\underline{\phi}_2$$

here $\underline{\phi}_2$ has already been set and stays constant until a new shock triggers a deviation from this new path. However, borrowers must deleverage to satisfy the new upper

borrowing limit

$$\underline{\phi}_2 = \overline{\phi}_1 - \frac{1}{2}e + C_{\mathcal{S}}^b$$

Since borrowers deleverage within one period $t = 1$ to $t = 2$, they have to pay off $\underline{\phi}_2/1 + r_2$ for which

$$C_{\mathcal{S}}^b = \frac{1}{2}e - \left[\overline{\phi}_1 - \frac{\underline{\phi}_2}{1 + r_2}\right]$$

Similar reasoning for the lenders. In the long run

$$C_{\mathcal{L}}^l = \frac{1}{2}e + (1 - \beta)\underline{\phi}_2$$

The short run equation can be easly derived from the fact that all production in the short run is consumed $C_1^l + C_1^b = e$ by which

$$C_{\mathcal{S}}^l = \frac{1}{2}e + \left[\overline{\phi}_1 - \frac{\underline{\phi}_2}{1 + r_2}\right]$$

From the Euler equation for lenders,

$$U'(C_{\mathcal{S}}^l) = (1 + r_2)\beta U'(C_{\mathcal{L}}^l).$$

Assume that $C_{\mathcal{L}}^l$ and $C_{\mathcal{S}}^l$ are different each other and thus a borrowers deleveraging in the short run needs to be offset by an increase in the short run consumption of the unconstrained agents (lenders) which can only be triggered by a decrease in the real interest rate r_2. Since the only price is the *inter-temporal* one (in terms of the only good in the economy), the real interest rate is considered as a flexible price and it should not, in general, face a lower bound until the increase in lenders consumption and the fall in borrowers consumption clear. This perfect elasticity implies an unconstrained agent and no frictions on consumption. It is however reasonable to assume an upper bound on consumption $U'(C_{\mathcal{L}^l}) \leq \overline{C}$ and then assume that it never binds

Assumption 3. $\overline{C} \geq \frac{e}{2} + \overline{\phi}_1 - \underline{\phi}_2$,

so as to ensure that the unconstrained agent will consume as much as needed for the economic activity to stay in equilibrium. For assumption 3 the deleveraging is neutral. It is interesting to notice that the non-neutrality of the deleveraging does

not simply depend upon the solely implication of dynamically inefficient paths. A deleveraging can be defined as follows

$$\bar{\phi}_1 - \underline{\phi}_2 \lessgtr \overline{C} - \frac{e}{2}. \tag{1.23}$$

The left hand side of (1.23) is the borrowers's deleveraging, while the right hand side is the lenders's maximum spending. From assumption 3 the left hand side cannot be greater than the right one, therefore interest rates always induce lenders' spending to be sufficient for offsetting borrowers deleveraging. However, following definition 3, definition 4 and proposition 1, in a model provided with REE, condition (1.23) cannot be fulfilled.

Proposition 4. *In a deleveraging model provided with REE, the only intertemporal equilibrium condition which can occur is the following*

$$\bar{\phi}_t - \underline{\phi}_{t+1} \equiv 0.$$

Deleveraging does not exist in a fully REE.

From condition (1.22) and Equation (1.8), then

$$\beta\bar{\phi}_1 \equiv \frac{1}{1+r_2}\mathbb{E}_1[\phi_2 + d_2] = \Phi(s_1, S_1) + \Theta(s_1, S_1)$$

therefore proposition 4 can be plainly re-defined as

$$\Phi(s_1, S_1) - \underline{\phi}_2 = 0,$$

which, for $\Theta(s_1, S_1) = 0$, fulfills definition 4.

Proof. From definition 3, the maximizing behavior (point (ii)) is defined by discounting the information (s_t, S_t) whose values, following definition 4, take into account any probable state. By this framework, given that proposition 1 implies $\Phi(s_t, S_t) = \phi_t$, and since in a rational expectations environment agents take into account the possibility of a bad state, then $\Phi(s_t, S_t) = \bar{\phi}_t \equiv \underline{\phi}_{t+1}$ for which a deleveraging cannot exist

satisfying proposition 4. The result is plain if it is assumed that rational expectations follow a random walk AR(1) process ($b = 1$), for which

$$\frac{\partial \mathbb{E}_t(\phi_{t+1})}{\partial \phi_t} = 1$$

that is to say $\bar{\phi}_t \equiv \underline{\phi}_{t+1}$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

1.4.2 A sufficient condition for deleveraging

Assume now that expectations do not follow a rational heuristic, but either inflate or deflate their rational counterpart following optimism or pessimism respectively. Albeit diagnostic expectations distort the rational path by overstating the sign without changing the direction, this new kind of expectation is crucial for explaining why deleveraging exists.

The steady state interest rate still be (1.22) in terms of lenders since they are the unconstrained agents. Moreover, at $t = 0$ the collateral market is driven by diagnostic expectations, Equation (1.19), causing an inflated collateral value $\bar{\phi}_1^d$. But at $t = 1$ a significant bad news $\underline{s}$ hits the economy making clear that $\bar{\phi}_1^d$ is not covered by actual fundamentals. It will be shown that it triggers a deleveraging towards the rational collateral value $\underline{\phi}_2$.

The consumption of both borrowers and lenders in the long and in the short run is defined by the following equations

$$C_{\mathcal{L}}^{b,d} = \frac{1}{2}e - (1-\beta)\underline{\phi}_2 \quad \text{and} \quad C_{\mathcal{S}}^{b,d} = \frac{1}{2}e - \left[\bar{\phi}_1^d - \frac{\phi_2}{1+r_2}\right]$$

$$C_{\mathcal{L}}^{l,d} = \frac{1}{2}e + (1-\beta)\underline{\phi}_2 \quad \text{and} \quad C_{\mathcal{S}}^{l,d} = \frac{1}{2}e + \left[\bar{\phi}_1^d - \frac{\phi_2}{1+r_2}\right]$$

Now the lenders Euler equation will be $U(C_{\mathcal{S}}^{l,d}) = \beta(1+r_2)U(C_{\mathcal{L}}^{l,d})$ from which

$$\frac{C_{\mathcal{S}}^{l,d}}{\beta \cdot C_{\mathcal{L}}^{l,d}} = 1 + r_2.$$

The problem is that agents who apply the maximizing behavior of definition 3 consider now a biased extrapolation of information out of (s_1, S_1), from which, before

$\underline{s}$ shows up, a possibility of a recession is ruled out. It is plain that $\phi_1^d > \phi_1$ and thus from Equation (1.19)

$$\epsilon_1^d = \phi_1^d - \Phi(s_1, S_1) \text{ from which } \epsilon_1^d > 0,$$

then

Proposition 5. *In a model with no frictions, a sufficient condition for a deleveraging is the existence of a bubble on the credit market.*

Proof. Given the inflated value of ϕ_t^d over $\overline{\phi}_t$, Equation (1.23) becomes valid. It follows directly by considering the following strict inequality

$$\underline{\phi}_{t+1} < \Phi(s_t, S_t) + \epsilon_t^d \text{ for } \epsilon_t^d > 0.$$

since from proposition 4 $\Phi(s_t, S_t) \equiv \underline{\phi}_{t+1}$, then it follows that $\phi_t^d - \underline{\phi}_{t+1} > 0$ contradicting proposition 4. $\qquad \square$

Proposition 5 defines a sufficient condition for triggering a deleveraging in a supersimple economy: the existence of a bubble. In order to become non-neutral, a collateral bubble does not suffice since no frictions on consumption and interest rates have been set up.

1.5 Conclusions

Following the result of Giglio, Maggiori, and Stroebel 2016, I have set up a model which can be consistent for describing a bubble even when rational models are not able to detect it. In particular I have defined sufficient conditions for a behavioral rational bubble's formation in the collateral market and the subsequent deleveraging. The crowd-in effect of the representative bubble is caused by errors in extrapolating information and thus by representativeness, while the crowd-out effect of deleveraging is set off by reverting to a rational heuristic. Moreover, this work takes into account

a bubble formation in an environment of homogeneous expectations, that in case of overoptimistic agents can cause a general euphoria that in turn overheats the market. The cooling off of this homogeneous euphoria leads to deleveraging towards a feasible level of debt. In this work I have considered a neutral deleveraging, but a future research can explore how (and under which conditions) the crowd-out effect of the behavioral bubble affects the aggregate demand and thus letting the deleveraging to be non-neutral.

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1.6 Appendix

Equation (1.7)**:** Let ω_0 follow an AR(1) process $\omega_1 = a + b\omega_0 + \epsilon_1$ i.i.d. distributed and endowed with normal shocks $\epsilon_t \sim (0, \sigma^2)$. The rational distribution at $t = 1$ is

$$f_1(\omega_1) = f(\Omega_1 = \omega_1 \mid \Omega_0 = \omega_0) \equiv \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\omega_1 - \mathbb{E}_0(\omega_1))^2}{2\sigma^2} \right)$$

in which $\mathbb{E}_0(\omega_1) = a + b\omega_0$. By using the law of iterated expectations define the comparison distribution

$$f_{-1}(\omega_1) = f(\Omega_1 = \omega_1 \mid \Omega_0 = \mathbb{E}_{-1}(\omega_1)) \equiv \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\omega_1 - \mathbb{E}_{-1}(\omega_1))^2}{2\sigma^2} \right).$$

Putting the distributions into the general formula (1.5)

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\omega_1 - \mathbb{E}_0(\omega_1))^2}{2\sigma^2} \right) \times \frac{\left[\exp\left(-\frac{(\omega_1 - \mathbb{E}_0(\omega_1))^2}{2\sigma^2} \right) \right]^\theta}{\left[\exp\left(-\frac{(\omega_1 - \mathbb{E}_{-1}(\omega_1))^2}{2\sigma^2} \right) \right]^\theta} \frac{1}{Z}$$

and by exploiting the exponential property for which

$$e^{-x} \left(\frac{e^{-x}}{e^{-y}} \right)^\theta \Rightarrow e^{\theta y - (\theta+1)x}$$

then

$$d_1(\omega_1) = \frac{1}{Z} e^{-\frac{1}{2\sigma^2}[(1+\theta)(\omega_1 - \mathbb{E}_0(\omega_1))^2 - \theta(\omega_1 - \mathbb{E}_{-1}(\omega_1))]^2}.$$

Notice that

$$[(1+\theta)(\omega_1 - \mathbb{E}_0(\omega_1))^2 - \theta(\omega_1 - \mathbb{E}_{-1}(\omega_1))]^2 = (\omega_1 - \mathbb{E}_0(\omega_1) + \theta[\mathbb{E}_0(\omega_1) - \mathbb{E}_{-1}(\omega_1)])^2$$

$$= (\omega_1 - \mathbb{E}_0^d(\omega_1))^2$$

from which

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}\left[(\omega_1-\mathbb{E}_0^d(\omega_1))^2\right]}.$$

Eventually notice that the mean of the diagnostic expectation is exactly (1.7), that is

$$\mathbb{E}_0^d(\omega_1) \equiv \mathbb{E}_0(\omega_1) + \theta[\mathbb{E}_0(\omega_1) - \mathbb{E}_{-1}(\omega_1)].$$

Equation (1.16): Consider the cumulative distribution

$$F_0^d(s) = \int_{s\geq\bar{s}}^{+\infty} d(\omega_1)d\omega.$$

It is important to define the reasoning behind (1.16). Notice that

$$\frac{\partial \ln F_0^d(s)}{\partial \omega_0} = \frac{1}{\partial F_0^d(s)}\frac{\partial F_0^d(s)}{\partial \mathbb{E}_0^d(\omega_1)} \cdot \frac{\partial \mathbb{E}_0^d(\omega_1)}{\partial \omega_0}$$

from which:

- the first term on the right hand side is

$$\frac{1}{\partial F_0^d(s)}\frac{\partial F_0^d(s)}{\partial \mathbb{E}_0^d(\omega_1)\omega_0} = \int_{s\geq\bar{s}}^{\infty}\frac{\omega_1 - \mathbb{E}_0^d(\omega_1)}{\sigma^2} \cdot \exp\left[-\frac{(\omega_1 - \mathbb{E}_0^d(\omega_1))^2}{2\sigma^2}\right]\frac{d\omega_1}{F_0^d(s)}$$

$$= \frac{1}{\sigma^2}\int_{s\geq\bar{s}}^{\infty}\omega_1 \exp\left(-\frac{1}{2\sigma^2}\left[(\omega_1 - \mathbb{E}_0^d(\omega_1))^2\right]\right)\frac{d\omega_1}{\partial F_0^d(s)} - \frac{\mathbb{E}_0^d(\omega_1)}{\sigma^2}$$

that is to say

$$\frac{\partial \ln F_0^d(s)}{\partial \omega_0} = [\mathbb{E}_0^d(\omega_1 \mid \omega_1 \geq \bar{s}) - \mathbb{E}_0^d(\omega_1)]\frac{1}{\sigma^2};$$

- the second term, still assuming normal densities and AR(1) process, stems from

  Equation (1.7) and is defined as

$$\frac{\partial \mathbb{E}_0^d(\omega_1)}{\partial \omega_0} = b(1 + \theta) > 0.$$

It follows that

$$\frac{\partial \ln F_0^d(s)}{\partial \omega_0} > 0$$

Proposition 2 The variance is of the form $\text{Var}[aX + b]$ from which

$$\text{Var}[aX + b] = \mathbb{E}[(aX + b)^2] - [\mathbb{E}(aX + b)]^2$$

$$= a^2 \text{Var}(X).$$

Applying the above reasoning

$$\text{Var}_{-1}[\mathbb{E}_0^d(\omega_1)] = \text{Var}_{-1}[(1 + \theta)\mathbb{E}_0(\omega_1) - \mathbb{E}_{-1}(\omega_1)]$$

$$= (1 + \theta)^2 \text{Var}_{-1}[\mathbb{E}_0(\omega_1)].$$

Equation (1.18) Taking into account that

$$\mathbb{E}_0^d(\omega_1) = \mathbb{E}_0(\omega_1) + \theta[\mathbb{E}_0(\omega_1) - \mathbb{E}_{-1}(\omega_1)].$$

and using the law of iterated expectations

$$\mathbb{E}_0[\omega_1 - \mathbb{E}_0^d(\omega_1)] = \mathbb{E}_0\left(\omega_1 - (\mathbb{E}_0(\omega_1) + \theta[\mathbb{E}_0(\omega_1) - \mathbb{E}_{-1}(\omega_1)])\right)$$

$$= \mathbb{E}_0(\omega_1) - \mathbb{E}_0(\mathbb{E}_0(\omega_1)) + \mathbb{E}_0(\theta\mathbb{E}_0(\omega_1)) - \mathbb{E}_0(\theta(\mathbb{E}_{-1}(\omega_1))$$

$$= -\theta[\mathbb{E}_0(\omega_1) - \mathbb{E}_{-1}(\omega_1)],$$

Chapter 2

Representative Versus Rational Bubbles

The present paper aims for comparing two models of bubble: the rational and the representative one. The latter model is capable of detecting a broader set of states which bring about the blossom of a bubble in both equity and debt markets. It will be clear that this model extends that of classical rational bubbles by a behavioral component which reveals a possible bubble formation even in those cases in which the transversality condition is fulfilled.

JEL: E03, D84

Keywords: Bubbles, Diagnostic Expectations

2.1 Introduction

Since the work of Giglio, Maggiori, and Stroebel 2016 it has been clear that rational bubbles are not the solely bubbles which can occur in a market. Classic rational bubble models are those which rely on failures of the transversality condition so as to justify the increase in the market value with respect to the fundamental one. What Giglio, Maggiori, and Stroebel 2016 fund is that, for the housing market in the UK and Singapore between 1996 and 2013, there have not been failures in the transversality condition, that is to say: the bubble in those markets cannot be explained by classic rational bubble models. This plain result stresses the importance of finding models capable to explain those distortions in the market values which classic models are not able to detect. The paper meets this need of the macroeconomic literature by proposing a kind of bubble that detects both possible failures in the transversality condition, but also over-optimism (resp. pessimism) by means of a behavioral component, called representativeness. Even in case of fulfillment of the transversality condition, there can be a positive non-rational component which maintains the market value at a higher level than the fundamental value. This behavioral component can be thought of as a positive predictable error caused by an excess volatility that in turn stems from over-optimistic expectations which exaggerate the direction of the rational path neglecting bad future states: in the model good news cause neglect of downside risk. Since the presence of excess volatility, the model is also capable of detecting the possibly extrapolative nature of the expectations: past values of the assets are crucial in the formation of the expectations for their future values. As for this characteristic of extrapolation, Case, Shiller, and Thompson 2012 investigated the role of homebuyers' expectations in the US housing market in which "...there is a strong correlation between the respondents stated perceptions of price trends and actual movements in prices ..." (Case, Shiller, and Thompson 2012).

A model of bubbles which detects extrapolations is that of Barberis et al. 2016. In

this model, as in this work, bubbles give rise because of "displacements"[1], that is a sequence of positive fundamental news. However, Barberis et al. 2016 can be classified as an *endogenous* disagreement-based model, in the sense that it extends models à la Harrison and Kreps 1978; Scheinkman and Xiong 2003 to an endogenous (rather than exogenous) formation of disagreement during the rising of a bubble. This model, in contrast, can be classified as an extension of the classic rational bubble models based on the failures of the transversality condition in which beliefs can be homogeneous. A crucial difference between the model of Barberis et al. 2016 and the general form of the present one, is that this model does not explain the elevated trading volume which characterize *loud* bubbly episodes (in the sense of Hong and Sraer 2013). However, it will be clear that this model can be simply extended to one based on disagreement. Rather, a crucial characteristic of representative bubbles is that they can explain several bubbly dynamics within a unified model triggering a blurred taxonomy of bubble models. Following some points pinned down in Barberis et al. 2016, let us make clear the position of my definition of bubble with respect to the aforementioned taxonomy:

- differently from rational bubbles, the representative ones explain how a bubble starts. As in Barberis et al. 2016, bubbles arise because of displacement (see Pansera and Strati 2016 for a thorough reasoning on this point);

- rational models are silent on the empirical findings of extrapolative expectations (es. Greenwood and Shleifer 2014; Gennaioli, Ma, and Shleifer 2015; Case, Shiller, and Thompson 2012). This model explains this possibility;

- Rational bubbles cannot explain high trading volumes, while Barberis et al. 2016 and in general disagreement models do. Since this model can be easly extended

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[1] "Displacement consists of events that change the situation, extend the horizon, and alter expectations ... *it* is an outside event or shock. . . A surge in the oil price is a displacement. An unanticipated devaluation is another displacement. . ." (Kindleberger 1978)

to disagreement, then representative bubbles may explain elevated volume.

This flexibility of the model extends to a second-order taxonomy of bubbles: loud-high volume (mostly in equities) and quiet-low volume (mostly in debt). Following Hong and Sraer 2013, debt upside payoffs are bounded and are concave in the investor beliefs about fundamentals. Since this cap tenders rational speculation, then by a rational disagreement model the debt bubble has to be smaller than an equity one. However by taking into account optimism, then a bubble in debt can arise even if the resale option is smaller[2]: representative bubbles can explain both quiet and loud bubbles. This model distorts the rational path by taking into account the heuristic of representativeness of Kahneman and Tversky 1972; Kahneman and Tversky 1983, as formalized in Gennaioli and Shleifer 2010; Bordalo et al. 2016.

2.2 A Classic Bubble Model

In this Section I will define what I mean with classic models of rational bubbles. Consider an asset which pays dividends d_t for every $t = 1, 2 \ldots$, its price at time t is denoted by p_t. Moreover $\alpha_{t,t+i} = \prod_{j=0}^{i-1} \alpha_{t+j,t+j+1}$ is the stochastic discount factor for the asset between the discrete period t and $t + i$. The market value of an asset equals the present discounted value of dividends and prices

$$p_t = \mathbb{E}_t[\alpha_{t,t+1}(p_{t+1} + d_{t+1})]. \tag{2.24}$$

From solving the stochastic difference Equation (2.24) and applying the law of iterated expectations,

$$p_t = \sum_{i=1}^{\infty} \mathbb{E}_t[\alpha_{t,t+i}d_{t+i}] + B_t, \tag{2.25}$$

in which

$$B_t = \lim_{T \to \infty} \mathbb{E}_t[\alpha_{t,t+T}p_{t+T}]. \tag{2.26}$$

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[2]In Strati 2016, representative bubbles are quiet since homogeneous diagnostic expectations on safer debt claims.

In the right hand side of Equation (2.25), the first element is called the *fundamental value F_t* and the second is called the *bubble component B_t*. As usual, Equation (2.25) can be rewritten as

$$p_t = F_t + B_t.$$

As already known (see for example Giglio, Maggiori, and Stroebel 2016; Diba and Grossman 1987), a bubble evolves following an explosive path $\mathbb{E}_t[\alpha_{t,t+1}B_{t+1}] = B_t$ with $B_0 > 0$. If $B_t = 0$ there are not bubbles and the transversality condition is fulfilled, that is

$$\lim_{T\to\infty} \mathbb{E}_t[\alpha_{t,t+T}p_{t+T}] = 0, \tag{2.27}$$

while if $B_t > 0$, then there is a bubble and the transversaility condition does not hold. In the latter case, an agent can gain by only departing from the optimal path, by exploiting the overvaluation of the asset. In this case, the solution $p_t = F_t$ is only one of the possible ones since the positivity of B_t. From Diba and Grossman 1987 (see also Pansera and Strati 2016), a bubble cannot be negative because of the Walrasian free disposal of assets for which a negative price is ruled out, that is $p_t \geq 0$.

In Giglio, Maggiori, and Stroebel 2016, the price of an asset with maturity T at time t is denoted by $P_t^T = \sum_{i=1}^{T} \mathbb{E}_t[\alpha_{t,t+i}d_{t+i}]$, while P_t is the price with infinite maturity at t. The transversality condition must be

$$\lim_{T\to\infty} \mathbb{E}_t[\alpha_{t,t+T}p_{t+T}] = \lim_{T\to\infty} (P_t - P_t^T) = B_t, \tag{2.28}$$

see the online Appendix A.4 of Giglio, Maggiori, and Stroebel 2016 for the proof. What has been demonstrated by Giglio, Maggiori, and Stroebel 2016 is that $P_t - P_t^T \approx 0$ from which follows that there is no failure in the transversality condition: $B_t = 0$.[3]

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[3] *"... Our test exploits a unique feature of housing markets in the United Kingdom and Singapore, where property ownership takes the form of either very long-term leaseholds or freeholds. Leaseholds are finite-maturity, pre-paid, and tradeable ownership contracts, often with initial maturities of 999 years, while freeholds are infinite-maturity ownership contracts... We estimate the price difference between freeholds and extremely-long leaseholds to obtain a direct*

Consistently with the transversality condition and the free disposal, it is now important to notice that in this classic rational bubble environment, a bubble has to be already present at the first trading of the assets, at the inception, otherwise it cannot give rise in any subsequent periods (as demonstrated in Diba and Grossman 1987). This framework does not detect bubbles that arise from motives that do not fall within the failure of the transversality condition. It is thus easy to be tangled up in a surprise effect triggered by a positive difference between market and fundamental values. It will be clear that even if Equation (2.28) is zero (the transversality condition is fulfilled), then an overvaluation of the market value is still possible by a positive *error component*. I follow Strati 2016 in defining the existence of a bubble called *representative bubble*, that is an overvaluation of an asset for which its market value will be higher than its fundamental value even for the absence of failures in the transversality condition: the bubble is driven by a positive error in expectations for which agents overstate positive fundamental news, what Kindleberger 1978 defined as "displacement".

2.3 Representative Bubbles

2.3.1 A brief digression: The Representativeness

In Kahneman and Tversky 1983 the representativeness is defined as "an assessment of the degree of the correspondence between a sample and a population..., more generally, between an outcome and a model". In few words: "A person who follows this heuristic evaluates the probability of an uncertain event, or a sample, by the degree to which it is: (i) similar in essential properties to its parent population; and (ii) reflects the salient features of the process by which it is generated" (Kahneman and Tversky 1972). For a distribution of a trait T in a group Γ, a decision maker

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estimate of the price of the bubble claim, and test whether it is indeed positive..."(Giglio, Maggiori, and Stroebel 2016 p.1049)

(DM) will assess the *relative* frequency of a particular trait $T = t$ in Γ with respect to the frequency of the same trait $T = t$ in a relevant comparison group Γ^c. The true distribution is $\Pr(T = t \mid \Gamma)$. The heuristic of representativeness $\mathcal{R}(\cdot)$ has been formalized in Gennaioli and Shleifer 2010 and Bordalo et al. 2016, in particular, from Gennaioli and Shleifer 2010 the representativess can be formalized as

$$\frac{\Pr(T = t \mid \Gamma)}{\Pr(T = t \mid \Gamma^c)}.$$

It is plain that the representativeness defines the inflation of the likelihood of traits whose *objective* probability rises the most in Γ relative to the reference *context* Γ^c (Bordalo, Gennaioli, and Shleifer 2016). It is worth noting that an event A is perceived as more probable than an event Z if A is more representative than Z, but this perception is biased since agents (in the model) are prone to confusing likely with representative. A very simple example can be that of judging the probability that the order of tails and heads in the next eight coin tosses will be $THTHTHTH$. It is clear, as demonstrated in Kahneman and Tversky 1972, that this regularity is not representative since it does not reflect the feature of randomness of the process by which it is generated. A sequence of coin tosses $TTHTHHHT$ is more representative of that randmoness and thus it is wrongly percieved as more likely.

Diagnostic Expectations   The heuristic of representativeness can thus be applied in order to define a formal model of expectations, called diagnostic from Bordalo, Gennaioli, and Shleifer 2016.

There is a discrete time $t = 1, 2, \ldots$, and two states of the world: growth g and recession r. At any time there is a signal $s \in [\bar{s}, \underline{s}]$ with $\underline{s} < \bar{s}$ about the next state space, with growth or recession accordingly. The signals are characterized in the following way: $\Pr(\underline{s} \mid r) = \gamma$, $\Pr(\underline{s} \mid g) = 1 - \beta$ for which $\gamma > \beta \geq 1/2$: a bad signal $\underline{s}$ reduces the probability of expecting a growth rate and it is a very strong signal for a looming recession.

Assumption 4. There is a prior probability π_k with $k = \{g, r\}$ for which $\pi_g > \pi_r$.

As in Strati 2016, denote a generic state space Ω in which ω_t is a realization. The random variable ω_t is assumed to follow an AR(1) process $w_t = a + bw_{t-1} + \zeta_t$ with ζ_t as i.i.d. normal $(0, \sigma^2)$ shocks. Moreover, $(\Omega_{t+1} = \omega_{t+1})$ is the realization of the state ω_{t+1} at $t+1$. If $(\Omega_{t+1} = \omega_{t+1} \mid \Omega_t = \omega_t)$ then it is intended that the realization ω_{t+1} at $t+1$ hinges on the occurrence of the state ω_t at t. Now suppose that there exists a smooth density function $f(\cdot)$ for which in a Bayesian framework

$$\pi_{g,t+1} \equiv F(\overline{s} \mid \omega_{t+1}) = \int_{\underline{s}}^{+\infty} f(\Omega_{t+1} = \omega_{t+1} \mid \Omega_t = \omega_t)d\omega \qquad (2.29)$$

is the rational cumulative distribution which gives the probability that the state ω_{t+1} is a growth state.

Define $\Gamma \equiv \{\Omega_t = \omega_t\}$ as the group which describes all of the possible future states ω_{t+1} whose values hinge on the current state $\Omega_t = \omega_t$. Moreover, define its relevant comparison group $\Gamma^c \equiv \Omega \setminus \Gamma$. As in Bordalo, Gennaioli, and Shleifer 2016, the comparison group may be defined as the rational expectation formed at $t-1$ for ω_{t+1} in which no new pieces of information occur, that is $\Gamma^c \equiv \{\Omega_t = \mathbb{E}_{t-1}(\omega_t)\}$.

Definition 5. The representativeness of a state ω_t at t for a group Γ is defined as

$$\mathcal{R}(\omega_t, \Gamma, \Gamma^c) = \frac{f(\Omega_{t+1} = \omega_{t+1} \mid \Omega_t = \omega_t)}{f(\Omega_{t+1} = \omega_{t+1} \mid \Omega_t = \mathbb{E}_{t-1}(\omega_t))}. \qquad (2.30)$$

In particular, following Bordalo, Gennaioli, and Shleifer 2016, an agent overstates the representative state and understate the less representative ones by distorting the rational density as follows

$$d(\omega_{t+1}) \equiv f(\Omega_{t+1} = \omega_{t+1} \mid \Omega_t = \omega_t) \times \left[\frac{f(\Omega_{t+1} = \omega_{t+1} \mid \Omega_t = \omega_t)}{f(\Omega_{t+1} = \omega_{t+1} \mid \Omega_t = \mathbb{E}_{t-1}(\omega_t))} \right]^\theta \frac{1}{Z}, \; (2.31)$$

notice that, as in Bordalo, Gennaioli, and Shleifer 2016, $1/Z$ is a normalizing constant which assures the integrability to one of (2.31). Moreover, $\theta \in [0, +\infty)$, that is: when $\theta = 0$, then agents are fully rational, while if $\theta > 0$ agents overweight in (2.31) the

representative states and underwheight the less representative ones. In this work, good fundamental news cause that the former are high future states, while the latter are low future ones, neglecting the downside risk.

2.3.2 The Representative Case

From the bubble defined in Strati 2016, Equation (2.24) has to be rewritten as

$$p_t = \mathbb{E}_t^d[\alpha_{t,t+1}(p_{t+1} + d_t)]. \tag{2.32}$$

in which $\mathbb{E}_t^d[\cdot]$ is the diagnostic expectation from which the representative density function (2.31) is explicitly defined as a normal distribution

$$d(p_{t+1}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(p_{t+1} - \mathbb{E}_t(p_{t+1}))^2}{2\sigma^2}\right) \times \frac{\left[\exp\left(-\frac{(p_{t+1} - \mathbb{E}_t(p_{t+1}))^2}{2\sigma^2}\right)\right]^\theta}{\left[\exp\left(-\frac{(p_{t+1} - \mathbb{E}_{t-1}(p_{t+1}))^2}{2\sigma^2}\right)\right]^\theta} \frac{1}{Z}$$

that can be rewritten as

$$d_1(p_{t+1}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}[(p_{t+1} - \mathbb{E}_t^d(p_{t+1}))^2]\right). \tag{2.33}$$

In this linear environment it is plain how the heuristic of representativeness causes an exaggeration in the judgment based on rational expectations, that is

$$\mathbb{E}_t^d(p_{t+1}) \equiv \mathbb{E}_t(p_{t+1}) + \theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})], \tag{2.34}$$

in which θ shifts the (gaussian) rational distribution. Since it is the feasible one for the market, then this shift causes a neglection of the tail risk encompassed in the (gaussian) rational distribution: good news cause neglect of downside risks. In Strati 2016 has been shown that for positive shocks to fundamentals, the so called *displacements* (see Kindleberger 1978), there is an excess of volatility which brings about a positive forecast error that, since it is not orthogonal to the information available at the time of the forecast, triggers the presence of an additional positive component directly into the fundamental solution. This result tells us that there is no need of a positive bubble component: a representative bubble (driven by representativeness) can maintain the values overvalued anyway. Let us see this reasoning formally.

Extrapolative Expectations   The excess volatility is justified by the fact that rational expectations are distorted in the direction of realized news. Positive news can cause over-optimistic expectations (vice versa for negative news). Consider the cumulative distribution

$$F_t^d(s) = \int_{s \geq \overline{s}}^{+\infty} d(p_{t+1}) dy, \tag{2.35}$$

with $s \in [\underline{s}, \overline{s}]$ as usual. Equation (2.35) describes the probability that positive news continue to hit the model. Notice that for a contingent current value p_t

$$\frac{\partial \ln F_t^d(s)}{\partial p_t} > 0 \tag{2.36}$$

from which there is an over-optimistic value driven by θ (the extent of the representativeness) which triggers an excess of volatility

$$\mathrm{Var}_{t-1}[\mathbb{E}_t^d(p_{t+1})] = (1 + \theta)^2 \mathrm{Var}_{t-1}[\mathbb{E}_t(p_{t+1})]. \tag{2.37}$$

that in turn, taking into account (2.34), sets off a positive predictable error

$$\mathbb{E}_t[p_{t+1} - \mathbb{E}_t^d(p_{t+1})] = -\theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})]. \tag{2.38}$$

Since past values of p affect the formation of the current expectations, then the orthogonality condition is violated and the expectations are called extrapolative. Moreover, the negative term of the error is justified by the fact that an ex-ante overvaluation is symptomatic of a future fall of the value (see for example Baron and Xiong 2014 or Gennaioli, Ma, and Shleifer 2015). It is a crucial property since by the presence of a systematic pattern of errors, expectations do not just fluctuate randomly like a noise. Rather, beside the possibility of a noisy exogenous shock, a bad "shock" here can be caused by the extrapolative nature of diagnostic expectations by simply stopping the flow of good news (es. see Giglio and Shue 2014 for the importance of the information encompassed in the absence of news). In few words, differently from rational expectations, here forecast errors are not orthogonal to the information available at the time of the forecast. The error predictability is thus an important diversion from the rational

benchmark since future errors are predictable by relying on the information available ex ante. This approach is also consistent to Manski 2004 in stressing the importance of agents expectations (what they think of) beside the "revealed preference analysis" (what they actually do) in shaping economic outcomes.

2.3.3 The Representative Bubble

As for the rational case, let us find a forward solution for Equation (2.32), that is

$$p_t = \mathbb{E}_t^d[\alpha_{t,t+T} p_{t+T}] + \sum_{i=1}^{\infty} \mathbb{E}_t^d[\alpha_{t,t+i} d_{t+i}], \qquad (2.39)$$

Since I am interested in the case in which a bubble exists also for a zero bubble component, consider that the transversality condition is fulfilled on it, and thus even for a $\theta > 0$, Equation (2.34) goes to zero since the free disposal at $t-1$.

Lemma 1. If

$$\lim_{T \to \infty} \mathbb{E}_t[\alpha_{t,t+T} p_{t+T}] = 0$$

then

$$\mathbb{E}_t(p_{t+1}) + \theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})] = 0$$

also in case of $\theta > 0$.

Proof. Since $\mathbb{E}_t(p_{t+1}) = 0$, and given by the free disposal $p_{t+i} \geq 0$ and the law of iterated expectations, then $\mathbb{E}_{t-1}(p_{t+1}) = 0$ because

$$\mathbb{E}_{t-1}[\mathbb{E}_t((p_{t+1}))] = \mathbb{E}_{t-1}(p_{t+1}) \equiv 0.$$

$\square$

From Lemma 1, it follows that

$$p_t = \sum_{i=1}^{\infty} \mathbb{E}_t^d[\alpha_{t,t+i} d_{t+i}]. \qquad (2.40)$$

Differently from Equation (2.26), in (2.40) the expectations are assumed to be diagnostic. In particular, agents are sensitive to positive news that hit fundamentals

(displacements) causing an overconfidence in the market which in turn triggers both: a neglection of downside risks and a positive predictable errors since the extrapolative nature of these expectations. By implictly assuming that the feasible path is that which follows rational expectations, then the difference between the path driven by representativeness and the rational one suggests that for Equation (2.40) the reasoning behind the forward solution has to be different. We know that the fundamental value of (2.40) is biased by a positive error. This error, as I stated, derives from the excess of volatility of (2.37); that is

$$\epsilon_t^d = [\mathbb{E}_t^d(p_{t+1}) - \mathbb{E}_t(p_{t+1})]$$

that, from (2.34), can be rewritten as

$$\epsilon_t^d \equiv \theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})]. \tag{2.41}$$

where ϵ^d is the error in expectations caused by diagnostic expectations which affects directly the expectations about market values. Because of this direct distortions in the fundamental value, the solution of Equation (2.40) becomes

$$p_t = \sum_{i=1}^{\infty} \mathbb{E}_t[\alpha_{t,t+i} d_{t+i}] + \epsilon_t^d \tag{2.42}$$

Proposition 6. *Representative Bubbles (RB) are positive if in a classic rational bubble model there exist: extrapolation and neglection of less representative states. The RB exist also in case of fulfillments of the transversality condition.*

Proof. The expectations follow an AR(1) process for which $\mathbb{E}_{t-1}(p_t) = a + b p_{t-1}$. Since the fundamental component is defined upon rational expectations, then it equates $\mathbb{E}_{t-1}(p_t)$, that is $F_t \equiv a + b p_{t-1}$. Putting this formalization into Equation (2.42) and differentiating with respect to p_{t-1}, then

$$\frac{\partial \mathbb{E}_{t-1}^d(p_t)}{\partial p_{t-1}} = (1 + \theta)b$$

from which $p_t > F_t$ even if $B_t = 0$. For a homogeneous euphoria of the market, which can be not detected by rational bubbles (since the transversality condition on B_t), then there can exist a bubble driven by representativeness which grows at a pace $(1 + \theta)b$ with the serial correlation of the data $b > 0$. $\qquad\square$

In case of rational expectations $\epsilon_t \equiv [\mathbb{E}_{t-1}(p_t) - \mathbb{E}_{t-2}(p_t)] = 0$, from which

$$\frac{\partial \mathbb{E}_{t-1}(p_t)}{\partial p_{t-1}} = b,$$

that if defined as a random walk ($b = 1$) implies that p_t and p_{t-1} do not change values. For $\theta > 0$ and $b = 1$

$$(1 + \theta) > 1,$$

which means that representativeness of positive news affects positively the expected prices.

2.4 Conclusions

Classic rational models should not be set aside, rather the sound economic meaning underpinning those models has to be exploited in order to spot different kind of bubbles. The characteristic of the bubble defined in Equation (2.42) is that representaive ones take into account both rational and behavioral components within the same model without overlooking the rational ones rendering it a possible candidate for a parsimonious behavioral model of bubbles. The model may be suitable for econometric tests in order to grasp the possibly extrapolative nature of the expectations.

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2.5 Appendix

2.5.1 A Hint on Destabilizing Speculation

In this appendix I shall sketch the case in which there are two kind of agents capable of affecting the market: local thinkers with homogeneous expectations and rational agents called arbitrageurs. In the fully homogeneous expectations model, arbitrageurs can be thought of as a small group of people who do not affect the market. Now, rational agents can affect it: by exploiting the expected distorted future price, they can destabilize the market value already overvalued by nonrational agents. Local thinkers have costly limits to acquire and process information, then their fundamental value is

$$p_t^d = F_t + \epsilon_t^d,$$

while for arbitrageurs who have complete information and no limits in processing it

$$\bar{p}_t = F_t.$$

Rationally, arbitrageurs can ride the bubble if there are not limits of arbitrage. In particular, if arbitrageurs know that $s_{t+1} \geq s_t$ at $t+1$, then they expect that $\mathbb{E}[p_{t+1}^d] \geq p_t^d$. If they are risk neutral, then they can exploit the error in expectations of the local thinkers obtaining higher gains. In this sense, representative bubbles may open an exploitable market for bubbles if Assumption 5 is fulfilled, that is

Assumption 5. The rate of return on bubbles $r^b > r$, in which r is the rate of return for fundamentalists.

Denote by $D(p, \theta)$ the assets demand from local thinkers which depends on: prices recent history and representativeness. The demand for arbitrageurs $\bar{D}(\Delta, G_{t+1})$ will depend on current difference between diagnostic and rational prices (Δ), and on the distribution (G_{t+i}) on the next future diagnostic prices which supports $\mathbb{E}[p_{t+1}^d]$.

Assumption 6. $\lim_{p_t \to \infty} \bar{D}(\Delta, G_{t+1}) = 0$ uniformly in G_{t+1}.

Here, Assumption 6 is crucial since rational agents will not stay in the market of bubbles if the price grows too much indipendently from the probability distribution. Define a cap in exploitable prices as $\tilde{p} > p_{t+i} \in G_{t+i}$ for which

Assumption 7. If arbitrageurs expect a price $p_{t+i} \geq \tilde{p}$ then their demand will be $\bar{D}(\Delta, G_{t+1}) = 0$.

If X_t is the overall stock of assets held by agents at t, then the market clearing condition is

$$X_t = D(p, \theta) + \bar{D}(\Delta, G_{t+1}). \tag{2.43}$$

Define by $\mathbb{D}$ the per-capita demand of assets of the local thinkers, and denote with $\bar{\mathbb{D}}$ the per-capita demand of rational agents. The latters maximize a CARA utility function with α the coefficient of absolute risk aversion

$$\max_{\mathbb{D}} e^{-\alpha[\mathbb{D}(\mathbb{E}(p_t^d) - p_{t-1})]}. \tag{2.44}$$

Rational agents have an incentive to exploit a highly probable subsequent increase of assets' prices. Since they are better informed, assume that they understand a signal $s \in [\bar{s}, \underline{s}]$ just before the local thinkers. Assume that they observe a $s_t > s_{t-1}$. Rational agents know that the market is populated by local thinkers and thus anticipate them and enter in the market. Consider a paramenter $\delta \in \mathbb{R}_{\geq 0}$ such that if $s_{t+i} > s_t$ then $\delta > 0$ while if $s_{t+i} \leq s_t$ then $\delta = 0$. Eventually suppose that there are $\bar{n} = 1 - n$ rational traders and n local thinkers. Following Barberis et al. 2016, the first order condition of Equation (2.44) is

$$\mathbb{D} = \frac{\mathbb{E}_{t-1}(p_t^d) - p_{t-1}}{\alpha \sigma^2}, \tag{2.45}$$

in which $\sigma^2 = \text{Var}(\mathbb{E}_{t-1}(p_t^d) - p_{t-1})$ and, crucially, $\mathbb{E}_{t-1}(p_t^d) = F_{t-1} + \delta$. Note that here ϵ^d is not perfectly foreseeable: rational agents know with certainty that it will be positive and can guess a range of values it can assume (see Assumption 8). By this imperfect knowledge, rational agents will buy at a price F_{t-1} with a δ high enough so as to assume arbitrageurs with strictly positive preferences for entering the market of bubbles *a la* greater fool speculation, that is

Assumption 8. $F_{t-1} + \delta < \tilde{p}$ with $\delta \in [\epsilon^d - \lambda, \epsilon^d + \lambda]$ with $\lambda \in \mathbb{R}_+$ small as you want.

Here λ depends inversely on α. The market clearing condition can be rewritten as

$$\bar{n}\left(\frac{\delta}{\alpha\sigma^2}\right) + n\mathbb{D} = X_t. \tag{2.46}$$

Notice that if $\delta = 0$ then $F_{t-1} \equiv p_{t-1}$ and $\sigma^2 = 0$ and thus the bubble is driven only by local thinkers. For $\delta > 0$ then rational agents exploit the market of bubbles since $F_{t-1} - p_{t-1} = \delta$. Define δ as an incentive prameter for speculation. Since $X_t \leq \tilde{X}$ with $\tilde{X}$ as the maximum stock supply, then if rational agents buy assets that are limited, by the law of demand, the prices of those assets increase triggering a further increase just after local thinkers observe $\bar{s}$ and an inflated p_t.

2.5.2 Mathematical Appendix

Equation (2.34) Let ω_0 follow an AR(1) process $p_{t+1} = a + bp_t + \gamma_{t+1}$ i.i.d. distributed and endowed with normal shocks $\gamma_t \sim (0, \sigma^2)$. The rational distribution at $t = 1$ is

$$f_1(p_{t+1}) \equiv \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(p_{t+1} - \mathbb{E}_t(p_{t+1}))^2}{2\sigma^2}\right)$$

in which $\mathbb{E}_t(p_{t+1}) = a + bp_t$. By using the law of iterated expectations define the comparison distribution

$$f_{t-1}(p_{t+1}) \equiv \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(p_{t+1} - \mathbb{E}_{t-1}(p_{t+1}))^2}{2\sigma^2}\right),$$

that from the general formula (2.31),

$$d(p_{t+1}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(p_{t+1} - \mathbb{E}_t(p_{t+1}))^2}{2\sigma^2}\right) \times \frac{\left[\exp\left(-\frac{(p_{t+1} - \mathbb{E}_t(p_{t+1}))^2}{2\sigma^2}\right)\right]^\theta}{\left[\exp\left(-\frac{(p_{t+1} - \mathbb{E}_{t-1}(p_{t+1}))^2}{2\sigma^2}\right)\right]^\theta} \frac{1}{Z}$$

that can be rewritten as

$$d_1(p_{t+1}) = \frac{1}{Z} \exp\left(-\frac{1}{2\sigma^2}[(1+\theta)(p_{t+1} - \mathbb{E}_t(p_{t+1}))^2 - \theta(p_{t+1} - \mathbb{E}_{t-1}(p_{t+1})]^2\right),$$

and since

$$[(1+\theta)(p_{t+1} - \mathbb{E}_t(p_{t+1}))^2 - \theta(p_{t+1} - \mathbb{E}_{t-1}(p_{t+1})]^2 =$$

$$= (p_{t+1} - \mathbb{E}_t(p_{t+1}) + \theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})])^2$$

$$= (p_{t+1} - \mathbb{E}_t^d(p_{t+1}))^2,$$

then

$$d_1(p_{t+1}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}[(p_{t+1} - \mathbb{E}_t^d(p_{t+1}))^2]\right).$$

from which follows

$$\mathbb{E}_t^d(p_{t+1}) \equiv \mathbb{E}_t(p_{t+1}) + \theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})].$$

Equation (2.36)   Consider the cumulative distribution

$$F_t^d(s) = \int_{s \geq \bar{s}}^{+\infty} d(p_{t+1})dp, \qquad (2.47)$$

with $s \in [\underline{s}, \bar{s}]$ as usual. Equation (2.47) describes the probability that positive news continue to hit the model. Notice that for a contingent current value p_t

$$\frac{\partial \ln F_t^d(s)}{\partial p_t} = \frac{1}{\partial F_t^d(s)} \frac{\partial F_t^d(s)}{\partial \mathbb{E}_t^d(p_{t+1})} \cdot \frac{\partial \mathbb{E}_t^d(p_{t+1})}{\partial p_t}$$

from which:

i) the first term on the right hand side is

$$\frac{1}{\partial F_t^d(s)} \frac{\partial F_t^d(s)}{\partial \mathbb{E}_t^d(p_{t+1})p_t} = \int_{s \geq \bar{s}}^{\infty} \frac{p_{t+1} - \mathbb{E}_t^d(p_{t+1})}{\sigma^2} \cdot \exp\left[-\frac{(p_{t+1} - \mathbb{E}_t^d(p_{t+1}))^2}{2\sigma^2}\right] \frac{dp_{t+1}}{F_t^d(s)}$$

$$= \frac{1}{\sigma^2} \int_{s \geq \bar{s}}^{\infty} p_{t+1} \exp\left(-\frac{1}{2\sigma^2}\left[(p_{t+1} - \mathbb{E}_t^d(p_{t+1}))^2\right]\right) \frac{dp_{t+1}}{\partial F_t^d(s)} - \frac{\mathbb{E}_t^d(p_{t+1})}{\sigma^2}$$

that is to say

$$\frac{\partial \ln F_t^d(s)}{\partial p_t} = [\mathbb{E}_t^d(p_{t+1} \mid p_{t+1} \geq \bar{s}) - \mathbb{E}_t^d(p_{t+1})]\frac{1}{\sigma^2};$$

ii) as for the second term, assume normal densities and AR(1) process for $\mathbb{E}_t(p_{t+1}) = a + bp_t$, rewriting (2.34) in these terms, then

$$\frac{\partial \mathbb{E}_t^d(p_{t+1})}{\partial p_t} = b(1 + \theta) > 0. \tag{2.48}$$

It follows that

$$\frac{\partial \ln F_t^d(s)}{\partial p_t} > 0.$$

Equation (2.37) The variance is of the form $\text{Var}[aX + b]$ from which

$$\text{Var}[aX + b] = \mathbb{E}[(aX + b)^2] - [\mathbb{E}(aX + b)]^2$$

$$= a^2 \text{Var}(X).$$

Applying the above reasoning

$$\text{Var}_{t-1}[\mathbb{E}_t^d(p_{t+1})] = \text{Var}_{t-1}[(1 + \theta)\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})]$$

$$= (1 + \theta)^2 \text{Var}_{t-1}[\mathbb{E}_t(p_{t+1})].$$

Equation (2.40) Taking into account that

$$\mathbb{E}_t^d(p_{t+1}) \equiv \mathbb{E}_t(p_{t+1}) + \theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})]$$

and using the law of iterated expectations

$$\mathbb{E}_t[p_{t+1} - \mathbb{E}_t^d(p_{t+1})] = \mathbb{E}_t\left(p_{t+1} - (\mathbb{E}_t(p_{t+1}) + \theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})])\right)$$

$$= -\theta[\mathbb{E}_t(p_{t+1}) - \mathbb{E}_{t-1}(p_{t+1})].$$

Chapter 3

Sensitive Bidders

This work aims at defining a pairwise single crossing condition in a first-price auction in which some players' expectations of the distribution of other bidders valuations are driven by the heuristic of representativeness: some bidders are not rational. An implication of this psychological bias is that even if the information is symmetric, the asymmetric bidders' capacity of processing one-dimensional signals becomes relevant for the definition of the equilibrium. It will be demonstrated that an asymmetric sensitivity of agents to news affects the private valuation of the object.

3.1   Introduction

Following Maskin 1992 and Dasgupta and Maskin 2000, in an efficient equilibrium for a first-price auction, a pairwise single crossing condition implies that players's equilibrium bid must be increasing in their signal and that their information is symmetric.[1] It is known that if the symmetric condition drops, the equilibrium may not be efficient. In this paper I shall not tackle directly the issue of efficiency, rather I shall investigate if a pairwise single crossing condition is satisfied in case of nonrationality of some bidders who will follow the heuristic of representativeness. In this spirit, Theorem 1 shows that a pairwise *single crossing condition* is satisfied even in case of nonrationality, and thus it is crucial in defining how valuations can change. In particular this work sets up the condition for which the *sensitivity* to new information is relevant for the definition of the equilibrium. I shall apply this reasoning to a first-price auction, in which informative signals about types are symmetric among bidders, while their capacity of processing this information is asymmetric, jeopardizing the efficiency even for one-dimensional signals. Rather, this kind of nonrationality implies that the valuation is increasing with respect to representativeness other than signals.

What do I mean with *sensitivity*? I shall refer to it as the ability of processing the information, in the sense that a rational agent is perfectly able to do it but a small unpredictable error. Each rational bidder has not cognitive limits and can rely on a symmetric sensitivity: if information is symmetric and players are equally and rationally sensitive to news, then the latter has no or negligible effects on the formation of the equilibrium. It follows that one has to focus only on the quality and distribution of news. However if some agents are less rational, then there can be cognitive hindrances in processing new information correctly (as a rational man). In this case the sensitivity may be asymmetric and thus it also obtains a key role in shaping the equilibrium.

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[1]An *efficient auction* "...maximizes surplus conditional on all available information..." (Dasgupta and Maskin 2000)

In order to cope with the thorny definition of nonrationality, I shall apply the heuristic of representativeness which has been studied in Kahneman and Tversky 1972; Kahneman and Tversky 1983 and formalized in Bordalo et al. 2016; Gennaioli and Shleifer 2010. In general, representativeness distorts rational expectations (es. over-optimism (resp. pessimism)) since a representative agent (or local thinker) neglects some part of information overweighing the relatively most frequent, or the *representative* part to our mind. A representative event is thus confused with a probable one. For example, think about a sequence of regular (fair) coin tosses (heads H and tails T) as $THTHTHTH$ or $TTHHTTHHTT$. It is not so representative to our mind. People view chance as unpredictable but essentially fair and those sequences are too regular for being representative. Beware: regular (fair) coin tosses $THTHTHTH$ and $TTHTHHHT$ are endowed with the same probability of happening. This reasoning can be applied to many models and situations (es. Gennaioli, Shleifer, and Vishny 2012; Gennaioli, Shleifer, and Vishny 2015; Gennaioli, Ma, and Shleifer 2015; Bordalo, Gennaioli, and Shleifer 2016 and Strati 2016; Ferrara and Strati 2016; Ferrara, Pansera, and Strati 2016).

In Engelbrecht-Wiggans, Milgrom, and Weber 1983 there exists a bidder B say, who is privately and better informed than another bidder A say. This private information is an accurate estimate of the object's value and allows B to know the information possessed by A. The problem to study how information either gathered *covertly* or *overtly* affects payoffs has been further investigated in Milgrom and Weber 1982. In this work I shall not treat information as inefficiently distributed among agents, information is symmetrically accessible to everyone, the issue is that not all of the agents are able to understand it correctly, like a perfect rational man. Following the case of the valuation of an object auctioned, in this work some bidders fail to process rationally the information observed causing an overreaction to news. Since the value is not known with certainty, the sensitivity distorts the expectations of the valuation of

the object. In this paper I shall give the rationale (Theorem 1) which explains this distortion.

3.2   On Representativeness

Assume $n \geq 2$ bidders and a set of ordered types $V = \{v_1, \ldots, v_n\}$ with $v_1 < \cdots < v_n$ and a set of states $S \subset V$ for which, in turn, $\Gamma \subset S$. Define a probability distribution such that $\pi : V \times S \to [0, 1]$. Define a conditional distribution $\Pr(V = v \mid \Gamma)$ when it is restricted to Γ. For notational convenience denote $\pi_{v,\Gamma} = \Pr(V = v \mid \Gamma)$ and $\pi_\Gamma = (\pi_{v,\Gamma})_{v \in V}$ the vector of conditional distributions.

A bidder considers a distribution of the types of the other bidders taking into account only a particular restriction Γ. By the representativeness heuristic, bidders overweight the probability of the types most representative in Γ with respect to a comparison group $\Gamma^c \subseteq S \setminus \Gamma$.

Definition 6. The representativeness of type v for a set Γ is the likelihood ratio

$$\mathcal{R}(v, \Gamma, \Gamma^c) \equiv \frac{\pi_{v,\Gamma}}{\pi_{v,\Gamma^c}}. \tag{3.49}$$

In definition 6 it is plain that a type v is representative in Γ if it is more likely to occur in Γ than in Γ^c (see Kahneman and Tversky 1983). The term "diagnostic" expectations stems from the fact that a representative type is *diagnostic* of the target group: if $\mathcal{R}(v, \Gamma, \Gamma^c)$ increases, then the bidder (decision maker in general) is more confident that a type v belongs to Γ rather than to Γ^c (see Bordalo et al. 2016). Moreover denote by $\mathbf{R}(v, \Gamma, \Gamma^c) \equiv (\pi_{v,\Gamma}/\pi_{v,\Gamma^c})_{v \in V}$ the vector of representativeness of all types of V, then from Bordalo et al. 2016 it follows that

Definition 7. For each type $v \in V$ in Γ, bidders attach a distorted probability

$$\pi_{v,\Gamma}^d = \pi_{v,\Gamma} \cdot \frac{e_v(\mathbf{R}(v, \Gamma, \Gamma^c))}{\sum_{s \in V} \pi_{s,\Gamma} e_s(\mathbf{R}(v, \Gamma, \Gamma^c))} \tag{3.50}$$

in which $e_v : \mathcal{R}_+^V \to \mathbb{R}_+$ is a weighting function for which[2]

i) e_v is a symmetric function of the representativeness of the types $s \neq v$, that is

$$e_v \equiv e\left(\frac{\pi_{v,\Gamma}}{\pi_{v,\Gamma^c}}; \left(\frac{\pi_{s,\Gamma}}{\pi_{s,\Gamma^c}}\right)_{s \in V \setminus \{v\}}\right)$$

in which $e_v : \mathcal{R}_+ \times \mathcal{R}_+^{V-1} \to \mathbb{R}_+$ is invariant to a permutation of the last $V - 1$ arguments;

ii) $e_v(\cdot)$ is weakly increasing in its first argument, and weakly decreasing in its $V - 1$ arguments.

3.3 The Model

Suppose that the bidders observe a signal $s \in [\underline{s}, \overline{s}]$ for which $\underline{s} < \overline{s}$. A signal $\overline{s}$ means that there is a positive probability such that among the $N = \{1, 2, \dots n\}$ bidders there is one that is aggressive a. While if $\underline{s}$ is the signal, then there is a positive probability such that among the n bidders there is one that is relaxed b.

Definition 8. If the js's maximal bid is $\max\{b_j\}$, then

i) player d is aggressive if $b_d > \max\{b_j\}$ and it is denoted by a;

ii) player d is relaxed if $b_d < \max\{b_j\}$ and it is denoted by b.

The signal brings to the model the same symmetric information to each bidder, that is: $\mathbf{s} \equiv (s_i, \dots s_n)$ with $n \in N$ and $\{s_1 = s_2 = \dots = s_n\}$. The difference lies in the sensitivity of processing this information of a *less* rational bidder called local thinker. Assume that among the active bidders (those whose bids are above the reserve price) there is at least one who is a local thinker d: there exists a nonempty subset of bidders $D \subset N$ who are local thinkers.

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[2]It is the function which assigns a weight to the representativeness (its extent). It will be used in the form

$$e_v = \left(\frac{:}{:}\right)^\theta.$$

Notice that θ measures the level of bidders' sensitivity.

Definition 9. The value of the object is unkown, while S is the information from which arises a signal s, then $v \equiv \mathbb{E}[V \mid S = s]$.

After the observation of $S = s$, each active bidder will solve the problem of finding a bid b which maximizes his expected profit

$$\Pi^i = \mathbb{P}(b_i > b_j) \cdot (\mathbb{E}_i[V \mid S = s] - b_i).$$

If there is an aggressive type, then, by definition, his valuation must outbid rational players who value the object less. I shall show that if there is a local thinker $d \in D$, then his valuation can in turn outbid the aggressive one. This finding is crucial for defining an equilibrium bid in an auction in which there are some bidders whose bids are driven by rapresentativeness. As shown in Ferrara and Strati 2016, representativeness will trigger an increase in the hammer price and can bring about jump bids at the beginning of the auction. In particular, it is a necessary condition for modeling a possibly bubbly component by which a hammer price is higher than its real market fundamental.

Rational Bidders   Define an ex-ante rational valuation of the object auctioned as $v = \mathbb{E}[V \mid S = s]$, for which

$$\frac{\partial v}{\partial s}(\mathbf{s}) > 0. \tag{3.51}$$

Since the signal is observed by everyone at the same moment, then

$$\sum_{k=1}^{n} \frac{\partial v_k}{\partial s}(\mathbf{s}) > 0, \tag{3.52}$$

that is each bidder is affected by the new observation. Define by

$$Q^{i,j}(\mathbf{s}) = \left[\frac{\partial v_i}{\partial s_j}(\mathbf{s}) \right] \tag{3.53}$$

the *influence matrix* (see Krishna 2003) whose (i, j) elements is the measure of the j's signal influence to bidder i's value. In what follows, signals $\mathbf{s}$ can be seen as a vector in which all of the elements are the same. This assumption helps to stress the role of the bidders's ability of processing the information rather than focusing on

the information itself. Since $D = \emptyset$, then bidders are all rational and $Q^{i,j}(\mathbf{s})$ is a matrix whose elements are all the same, I denote it by $Q^{i,s}(\mathbf{s})$. It means that in a model with rational expectations, a homogeneous new piece of information influences each value by the same magnitude affecting only the level rather than the relative valuations between bidders. Of course, prior beliefs of the bidders need not be the same, and valuations can be different from the beginning. In this sense, a $Q^{i,s}(\mathbf{s}) > 0$ does not mean that every v is the same, but that every v is homogeneously affected by s maintaining the relative differences as defined by their subjective priors. Each bidder has a subjective prior valuation $\pi_k(v)$ which he updates by the Bayesian rule.

Assumption 9. Assume that $\pi_i(v) > \pi_j(v)$

then

$$Q^{i,s}(\mathbf{s}) > Q^{j,s}(\mathbf{s}).$$

However, following Krishna 2003, define

$$W^{i,j}(\mathbf{v}) = \{\mathbf{s} : \quad v_j(\mathbf{s}) = v_i(\mathbf{s}) = \max_{k \in N} v_k(\mathbf{s})\} \tag{3.54}$$

so that the values of j and i equal the maximal value. For the pairwise single crossing condition it follows that if bidders i and j have equal maximal values, and if the signal of bidder i is increased, then the effect on the value of that bidder is the highest.[3] However, I have assumed that the information (and a signal s) is the same for everyone, in this case, if the priors are undefined, nothing can assure that the single crossing condition will be satisfied.

Proposition 7. If prior valuations are the same $\pi_i(v) = \pi_j(v)$, and information is homogeneously and symmetrically distributed, then for the final valuation $\mathbf{v}$, for any $i \neq j$

$$Q^{i,s}(\mathbf{s}) = Q^{j,s}(\mathbf{s})$$

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[3] "...the buyer i's signal has a greater marginal effect on his own valuation than on that of any other buyer j (at least at points where buyer i's and buyer i's valuations are equal) ..." Maskin 2003.

at every $\mathbf{s} \in W^{i,j}(\mathbf{v})$.

Proof. Since $v_j(s) = \mathbb{E}_j[V_j \mid S = s]$ and $v_i(s) = \mathbb{E}_i[V_i \mid S = s]$, then if the signal is homogeneous and $\mathbf{s} \in W^{i,j}(\mathbf{v})$, it means that $\pi_i(v) \equiv \pi_j(v)$, and thus, by the homogneity of $\mathbf{s}$, $Q^{i,s}(\mathbf{s}) = Q^{j,s}(\mathbf{s})$. $\qquad\square$

Local Thinkers

Assumption 10. From now on, I shall assume that the new signal brings information about the probable aggressive valuation, that is $s \equiv \overline{s}$.

Applying Equation (3.50), define

$$h_i^\theta(s) = h(V \mid S = s) \times \left[\frac{h(V \mid S = s)}{h(V \mid S = v(s_\pi))} \right]^\theta \frac{1}{Z}, \qquad (3.55)$$

the distorted density for which local thinkers overweight the rational expectation of the valuation of the object auctioned, with $1/Z$ a normalizing constant. Notice that in Equation (3.55), if $\theta = 0$ then agents are rational because only $h(V_i \mid S = s)$ appears, while if $\theta > 0$, then agents are representative (local thinkers). The part in square brakets of the right hand side of Equation (3.55) is the distortion. In particular, at the denominator appears $v(s_\pi)$ which is the valuation based on the prior information of the opponent's type, before news arrive. The numerator is instead sensitive to new information, thus if the denominator is smaller than the numerator, then there is a sensitivity to new information at a level θ. Moreover define the probability of attaching higher valuations if signals are of aggressive types $\overline{s}$, that is

$$f_i^\theta(v) = \int_v^{+\infty} h_i^\theta(s) ds \qquad (3.56)$$

It is crucial to notice that ($t \in \mathbb{R}$ is the current time)

$$\frac{\partial \ln f_i^\theta(v)}{\partial s_t} > 0 \qquad (3.57)$$

Proof. see Appendix. $\qquad\square$

Assuming s as a signal of a valuation $\hat{v}$ which affects v, define

$$\mathbb{E}^\theta(v) = \int_S v \cdot h_i^\theta(\hat{v}) dv \tag{3.58}$$

the expected value v of a local thinker. I shall define the expected value which follows from Equation (3.58) as v^θ. Defining

$$\mathbb{Q}^{i,s}(\mathbf{s}) = \left[\frac{\partial v^\theta}{\partial s_t}\right] \tag{3.59}$$

as the influence matrix when a bidder i is representative, then it is possible to set up the single crossing condition when bidders' beliefs are driven by representativeness.[4] Define

$$\mathbb{W}^{i,j}(\mathbf{v}) = \{\mathbf{s}: \quad v_i^\theta(s) = v_j^\theta(s) = \max_{k \in D} v_k^\theta(\mathbf{s})\}. \tag{3.60}$$

Lemma 2. If $\theta_i > \theta_j$, then for a generic homogeneous signal $\mathbf{s}$, $\mathbb{Q}^{i,s}(\mathbf{s}) > \mathbb{Q}^{j,s}(\mathbf{s})$.

Proof. If it is assumed that $\theta_i > \theta_j$, and the signal is equal and homogeneously distributed among agents, then

$$\left[\frac{h(V_i \mid S = s)}{h(V_i \mid S = v(s_\pi))}\right]^{\theta_i} > \left[\frac{h(V_j \mid S = s)}{h(V_j \mid S = v(s_\pi))}\right]^{\theta_j}$$

reflecting the higher sensitivity to news of player i relatively to player j. It is easly shown if one looks at the proof of the positivity of the probability of facing up with an aggressive type (a) for a new signal $\bar{s}$ (Equation (3.57)). The second term of this derivative, Equation (3.63), is

$$\frac{\partial \mathbb{E}_i^\theta(v_a)}{\partial v_a} = b(1 + \theta) > 0,$$

from which it is clear that the sensitivity is driven by representativeness. The higher θ, the higher the sensitivity. Thus for $\theta_i > \theta_j$, it turns out that $\mathbb{Q}^{i,s}(\mathbf{s}) > \mathbb{Q}^{j,s}(\mathbf{s})$ for any $\bar{s}$. $\qquad\square$

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[4]Notice that, since $s = \hat{v} \in V$, then

$$v^\theta = \frac{1}{Z} \int_S v \cdot \left(h(V \mid S = \hat{v}) \times \left[\frac{h(V \mid S = \hat{v})}{h(V \mid S = v(s_\pi))}\right]^\theta\right) dv$$

from which, following Equation (3.57), $\partial v^\theta / \delta s_t > 0$ too.

Theorem 1. *The valuation $\boldsymbol{v}^\theta$ satisfies the single crossing condition, since Lemma 2, for any $i \neq j$*

  i) $\mathbb{Q}^{i,s}(\boldsymbol{s}) > \mathbb{Q}^{j,s}(\boldsymbol{s})$ *for every* $\boldsymbol{s} \in \mathbb{W}^{i,j}(\boldsymbol{v})$.

  ii) $\mathbb{Q}^{i,s}(\boldsymbol{s}) > Q^{j,s}(\boldsymbol{s})$ *for every* $\boldsymbol{s} \in \mathbb{W}^{i,j}(\boldsymbol{v})$ *and* $\boldsymbol{s} \in W^{i,j}(\boldsymbol{v})$.

Proof. A rational valuation, since the actual value of the object is unknown, is $v = \mathbb{E}[V \mid S = s]$. If bidders are local thinkers, then $v^\theta = \mathbb{E}^\theta[V \mid S = s]$ for which the operator $\mathbb{E}^\theta(\cdot)$ follows Equation (3.55). Now, player i is a local thinker, and a signal $\bar{s}$ comes about. His valuation is thus positive for any aggressive signal following Equation (3.57).

  i) If also player j is a local thinker, then

$$\frac{\partial v_i^\theta}{\partial s_t} > 0 \quad \text{and} \quad \frac{\partial v_j^\theta}{\partial s_t} > 0.$$

  Following Lemma 2, assume that these two local thinkers are endowed with different sensitivity and thus their extent of representativeness is different $\theta_i > \theta_j$. From Lemma 2, it implies that in general $\mathbb{Q}^{i,s}(\mathbf{s}) > \mathbb{Q}^{j,s}(\mathbf{s})$. Now suppose that $\mathbf{s} \in \mathbb{W}^{i,j}(\mathbf{v})$ and thus $v_i^\theta(s)$ and $v_j^\theta(s)$ are at their maximal value for the information gathered until present say. Suppose that the information is updated by $\bar{s}$, then the probability of facing up with an aggressive type increases. By the very definition of representativeness, the second part of the right hand side of Equation (3.55) increases too. In particular

$$\left[\frac{h(V_i \mid S = s)}{h(V_i \mid S = v(s_\pi))} \right]^{\theta_i} > \left[\frac{h(V_j \mid S = s)}{h(V_j \mid S = v(s_\pi))} \right]^{\theta_j}$$

  because even if s is the same, the extent of representativeness θ is not.

  ii) Assume now that player i is representative, while player j is rational and, as for *i*), assume a signal $\bar{s}$. From the definition of representativeness

$$\left[\frac{h(V_i \mid S = s)}{h(V_i \mid S = v(s_\pi))} \right]^{\theta_i} > 1$$

and thus $h_i^\theta(s) > h_j(s) \equiv h(V_i \mid S = s)$, that is to say

$$v_i^\theta(s) > v_j(s),$$

rather $v_j(s)$ is the rational valuation for which the sensitivity in $h_j(s)$ is $\theta_j \equiv 0$ and thus the ratio in the square brakets of the right hand side of Equation (3.55) is always one. If $\mathbf{s} \in \mathbb{W}^{i,j}(\mathbf{v})$, it follows the same reasoning of i) for which $\mathbb{Q}^{i,s}(\mathbf{s})$ is increasing following θ_i, while for a symmetric $\mathbf{s} \in W^{i,j}(\mathbf{v})$, $\mathbb{Q}^{i,s}(\mathbf{s}) > Q^{j,s}(\mathbf{s})$ because $h_i^\theta(s) > h_j(s)$.

$\square$

It is important to notice that Theorem 1 sets a condition for defining an equilibrium, but the "efficiency" is not generally guaranteed yet. The efficiency can be defined as the mechanism for which resources go in the hands of who values them most. It is thus needed a study on how local thinkers submit their equilibrium bids. In particular, since their valuations are higher (in case of aggressive types) is their equilibrium bid increasing with respect to these higher values? If it is so, then local thinkers will win the auction if their value $v^\theta > \max v_k$. Albeit I shall not focus on the formation of the equilibrium bid, the following example will show heuristically why equilibrium bids should follow the bidders' sensitiveness.

Example   Consider two bidders i and j with private values, from which $s \in [\underline{s}, \overline{s}]$. The equilibrium bid functions $(b_i(s), b_j(s))$ in the high-bid auction satisfies $b_i(s) = b_j(s)$. Suppose now that i is a local thinker with $\theta_i > 1$ for a signal $\overline{s}$ from which j is aggressive (a) with posterior probability $h_i^a(\overline{s}) > 1/2$. Now, $b_i(s) < b_a(s)$ since $b_a(s)$ is the equilibrium bid of the aggressive player. By Theorem 1, the local thinker is willing to value the object $v^\theta > v^a$, from which follows that:

$$v_i^\theta(\overline{s}) - b_i(s) > v_i(s) - b_i(s).$$

The higher payoff triggers the (risk neutral) local thinker to increase his equilibrium bid to b_i^θ until

$$v_i^\theta(\bar{s}) + b_i(s) = v_i(s) + b_i^\theta(\bar{s}) \tag{3.61}$$

that is: the higher sensitivity of Theorem 1 sets off an increase in the equilibrium bids until it is offset by a higher bid maintaining the absolute value of the payoff relatively equal to the case of the absence of new signals. Notice that this does not mean that local thinkers are indifferent between the pre-signal payoff and that of the post-signal. Since their sensitiveness $v_i^\theta(\bar{s})$, then the probability to win is higher for higher bids than by bidding $b_i(s)$. For this reason $b_i^\theta(\bar{s}) \geq b_a(\bar{s})$ as long as

$$v_i^\theta(\bar{s}) - b_i^\theta(\bar{s})) \geq v_a(\bar{s}) - b_a(\bar{s}).$$

3.4 Conclusions

In this work I have demonstrated that taking into account different agents' sensitivities to news may affect the formation of an equilibrium. In particular I have applied the model to a first-price auction, but it can be tested to several models. For example can be interesting to see what are the implications of this nonrationality in a second-price auction. Rational players are not sensitive in this model $\theta = 0$, this means that a rational man learns the model, processes objectively the information needed, and bids rationally expecting that the other bidders are rational too and so on. Local thinkers in turn, will process the information under cognitive limits for which they cannot extract a complete and objective meaning from the information gathered. Even if this information is complete, then local thinkers are not able to take it into account as a whole and proccess it correctly. This work shows that this asymmetry of sensitiveness among agents should be taken into account since it may affect the decision making. More research is needed, for example it will be interesting to understand how much the outcome of a model changes if one considers an environment in which coexist

both asymmetric sensitivity and asymmetric information. Moreover it can be useful
to understand how this sort of *sensitivity bias* can be mitigated. I hope that this work
will foster further research on the subject.

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3.5 Appendix

Define $v(s_\pi)$ as the valuation based on a prior probability π given an information s. Assume that arrives a new information encapsulated in a signal $\bar{s}$, then it is important to understand how this news affects the posterior valuation. Define the rational probability distribution, after $\bar{s}$ has been observed, as

$$h_i(s) = h(V_i \mid S = \bar{s}) \equiv \frac{1}{\sigma\sqrt{2\pi}} e^{\left[-\frac{(v - \mathbb{E}_i(v))^2}{2\sigma^2}\right]},$$

and the rational prior probability distribution

$$h_i(s_\pi) = h(V_i \mid S = v(s_\pi)) \equiv \frac{1}{\sigma\sqrt{2\pi}} e^{\left[-\frac{(v - v(s_\pi))^2}{2\sigma^2}\right]}.$$

It is possible to put the distributions into the formula of representativeness, that is to say

$$h_i^\theta(s) = h(V_i \mid S = \bar{s}) \times \left[\frac{h(V_i \mid S = \bar{s})}{h(V_i \mid S = v(s_\pi))}\right]^\theta \frac{1}{Z}.$$

After some simple computations

$$h_i^\theta(s) = \frac{1}{Z} e^{\frac{1}{\sigma\sqrt{2\pi}}[(1+\theta)(v - \mathbb{E}_i(v))^2 - \theta(v - v(s_\pi))]^2},$$

notice that the term inside the square brakets can be simplified

$$(1 + \theta)(v - \mathbb{E}_i(v))^2 - \theta(v - v(s_\pi)]^2 = [v - \mathbb{E}_i(v) + \theta(\mathbb{E}_i(v) - v(s_\pi))]^2$$

$$= (v - \mathbb{E}^\theta(v))^2$$

from which

$$h_i^\theta(s) = \frac{1}{Z} e^{\frac{1}{\sigma\sqrt{2\pi}}[v - \mathbb{E}_i^\theta(v)]^2}.$$

It follows that the mean of the diagnostic expectation is exactly

$$\mathbb{E}_i^{\theta}(v) = \mathbb{E}_i(v) + \theta[\mathbb{E}_i(v) - v(s_{\pi})]. \tag{3.62}$$

Equation (3.57)

For a general local thinker, let me set the proof in terms of time (where t is the current time), in which v_i follow an AR(1) process $v_i = a + bv_t + \epsilon$, i.i.d. distributed and is endowed with normal shocks $\epsilon \sim (0, \sigma^2)$. In order to easly compute the proof, I shall consider that a signal s is a description on the valuation (or type in general) v for which $s \equiv v$. Consider the cumulative distribution

$$f_i^{\theta}(v) = \int_{v_1}^{\infty} h_i^{\theta}(s)ds.$$

Notice that

$$\frac{\partial \ln f_i^{\theta}(v)}{\partial v_t} = \frac{1}{\partial f_i^{\theta}(v)} \frac{\partial f_i^{\theta}(v)}{\partial \mathbb{E}_i^{\theta}(v)} \cdot \frac{\partial \mathbb{E}_i^{\theta}(v)}{\partial v_t}$$

from which:

- the first term on the right hand side is

$$\frac{1}{\partial f_i^{\theta}(v)} \frac{\partial f_i^{\theta}(v)}{\partial \mathbb{E}_i^{\theta}(v)v_t} = \int_{v_i \geq \overline{v}}^{\infty} \frac{v_i - \mathbb{E}_i^{\theta}(v)}{\sigma^2} \cdot \exp\left[-\frac{(v_i - \mathbb{E}_i^{\theta}(v))^2}{2\sigma^2}\right] \frac{dv_i}{f_i^{\theta}(v)}$$

$$= \frac{1}{\sigma^2} \int_{v_i \geq \overline{v}}^{\infty} v_i \exp\left(-\frac{1}{2\sigma^2}\left[(v_i - \mathbb{E}_i^{\theta}(v))^2\right]\right) \frac{dv_i}{f_t^{\theta}(v)} - \frac{\mathbb{E}_i^{\theta}(v)}{\sigma^2}$$

  that is to say

$$\frac{\partial \ln f_i^{\theta}(v)}{\partial v_i} = [\mathbb{E}_i^{\theta}(v \mid v \geq \overline{v}) - \mathbb{E}_i^{\theta}(v)]\frac{1}{\sigma^2};$$

- still assuming normal densities and AR(1) process with $\mathbb{E}_i(v) = a + bv_t$, the second term stems from equation (3.62) and is defined as

$$\frac{\partial \mathbb{E}_i^{\theta}(v)}{\partial v_t} = b(1 + \theta) > 0. \tag{3.63}$$

It follows that

$$\frac{\partial \ln f_i^{\theta}(v)}{\partial v_t} > 0$$