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Sub-nanometer length metrology for ultra-stable ring laser gyroscopes

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Declaration

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November 2015

Rosa Santagata

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*Dedicated to my family,
without whom nothing is so beautiful!*

*Three people were at work on a construction site.
All were doing the same job, but when each was asked
what the job was, the answers varied.
"Breaking rocks", the first replied.
"Earning my living", the second said.
"Helping to build a cathedral", said the third.*

Peter Schultz (co-inventor of the fiber optics)

Sub-nanometer length metrology for ultra-stable ring laser gyroscopes

by
Rosa Santagata

ABSTRACT

Large-frame ring laser gyroscopes are extremely sensitive inertial detectors of rotational motion. When properly coupled to the ground, they provide precise measurements of the Earth rotation rate and give important informations to geodesy and geophysics. Recent advances in this technology led to consider the application of ring laser gyroscopes to fundamental physics. In this context is GINGER (Gyroscopes IN GENeral Relativity), a scientific proposal for testing General Relativity (local observation of the Lense-Thirring effect) with a ground-based array of ring laser gyroscopes.

The experimental target is to locally measure the Earth rotation rate with a relative precision better than one part in 10^9 , corresponding to an absolute rotational resolution of 10^{-14} rad/s. The main factor limiting the performances of the presently most stable ring laser gyroscopes is the uncontrolled deformation of their optical cavity, since instabilities in the cavity geometry introduce systematic errors in the rotational signal. Cavity geometry is typically kept stable by using monolithic frames made of ultra-low thermal expansion materials, and operating in very well isolated environments. An alternative approach is based on the active control of the shape in heterolithic cavities, by measuring and stabilizing the mirrors positions by means of laser-based length metrology.

The goal of this thesis is the development of a stabilization system based on interferometric length metrology, with a view to improve the sensitivity of the new generation square ring laser gyroscopes, going beyond the level achievable with passive methods. The main idea proposed in this work is to exploit the diagonal resonators formed by opposite cavity mirrors, and to use their lengths as observables to constraint, against an optical reference standard, residual deformation degrees of freedom of the square cavity.

As a first step, a detailed model of the light propagation along the square cavity has been developed. This allowed us to quantify the effectiveness of the

fixed length constraint of the diagonal resonators, and gave precise indications for the optimization strategy of the residual degrees of freedom.

The optical frequency reference is a helium neon laser stabilized to the iodine molecular absorption. Since the power emitted by this laser is of only 300 μW , an optical amplifier, based on the injection locking of a 15 mW diode laser, has been developed to guarantee a proper signal to noise ratio in the interferometric absolute length measurements.

To stabilize the absolute lengths of the two diagonals, we proposed an original experimental method for the determination of both the optical resonance frequency and the free spectral range of each cavity. It is based on a triple-frequency modulation of the interrogating laser beam by electro-optic modulators. In a first tabletop experiment, the method has been verified on two Fabry-Perot resonators that, composed by couples of spherical mirrors typically used in the He-Ne ring cavities, simulate the diagonals of a ring laser gyroscope on an optical bench. Here, the capability of setting equal the two lengths at the level of 500 nm, with residual fluctuations only limited by the laser frequency noise, has been experimentally proved.

As a final result, we have applied the developed method to lock the diagonal cavities lengths of GP2 ring laser gyroscope, a square cavity 1.6 m in side length dedicated to the interferometric control of the cavity geometry deformations, and fully set up at the INFN laboratories in Pisa in June 2015.

Keywords: ring laser gyroscopes, general relativity, length metrology, laser beam steering, interferometry, laser stabilization, active control

SOMMARIO (*Italian abstract*)

I giroscopi laser di grandi dimensioni sono sensori inerziali di moto rotazionale estremamente sensibili. Se correttamente accoppiati al suolo, essi forniscono misure precise della velocità rotazionale della Terra e danno importanti informazioni per la geodesia e la geofisica. Recenti progressi in questo campo tecnologico hanno condotto a considerare l'applicazione dei giroscopi laser alla fisica fondamentale. In tale contesto nasce GINGER (Gyroscopes IN GEneral Relativity), una proposta scientifica per testare la Relatività Generale (l'osservazione locale dell'effetto Lense-Thirring) con un insieme di giroscopi laser disposti sulla superficie terrestre.

L'obiettivo sperimentale è misurare localmente la velocità rotazionale della Terra con una precisione migliore di una parte in 10^9 , che corrisponde ad una risoluzione rotazionale assoluta di 10^{-14} rad/s. Il fattore principale che limita le prestazioni dei più stabili giroscopi laser ad oggi esistenti è la deformazione incontrollata della cavità ottica, dal momento che instabilità nella geometria della cavità introducono errori sistematici nel segnale rotazionale. La geometria della cavità è solitamente tenuta stabile utilizzando strutture monolitiche costituite da materiali aventi espansione termica ultra-bassa, e lavorando in ambienti molto ben isolati. Un approccio alternativo si basa su un controllo attivo della forma di cavità eterolitiche, misurando e stabilizzando le posizioni degli specchi mediante l'utilizzo della metrologia di lunghezza facente uso della tecnologia laser.

L'obiettivo di questa tesi è lo sviluppo di un sistema di stabilizzazione basato sulla metrologia interferometrica di lunghezza, al fine di migliorare la sensibilità dei giroscopi laser di nuova generazione aventi cavità quadrata, andando oltre il livello ottenibile con metodi passivi. L'idea principale proposta in questo lavoro è sfruttare i risonatori lineari formati lungo le diagonali da specchi opposti della cavità, e utilizzare la loro lunghezza come osservabili per bloccare, rispetto ad uno standard di frequenza ottica, i gradi di libertà residui di deformazione della cavità quadrata.

Come primo passo, è stato messo a punto un modello dettagliato della propagazione della luce lungo la cavità quadrata. Questo ci ha permesso di quantificare l'efficacia del vincolo di lunghezza fissa delle cavità diagonali, e ha

fornito precise indicazioni ai fini di una strategia di ottimizzazione dei gradi di libertà residui.

Il riferimento di frequenza ottica è un laser elio neon stabilizzato sull'assorbimento molecolare dello iodio. Dal momento che la potenza emessa da questo laser è di soli $300\ \mu\text{W}$, è stato sviluppato un amplificatore ottico, basato sull'iniezione ottica di un diodo laser avente potenza di emissione di $15\ \text{mW}$, al fine di garantire un rapporto segnale-rumore adeguato alle misure interferometriche di lunghezza assoluta.

Per stabilizzare le lunghezze assolute delle due diagonali, abbiamo proposto un metodo sperimentale originale che permette di determinare sia la frequenza ottica di risonanza che il free spectral range di ciascuna cavità. Esso è basato su una tripla modulazione di frequenza del fascio laser di misura, utilizzando modulatori elettro-ottici. In un primo esperimento da banco, il metodo è stato verificato su due risonatori Fabry-Perot che, composti da coppie di specchi sferici tipicamente usati nelle cavità He-Ne ad anello, simulano su un banco ottico le diagonali di un giroscopio laser. In questo caso, è stata sperimentalmente dimostrata l'abilità della tecnica di porre uguali le due lunghezze ad un livello di $500\ \text{nm}$, con fluttuazioni residue limitate solo dal rumore di frequenza del laser di riferimento.

Come risultato finale, abbiamo applicato il metodo sviluppato per bloccare le lunghezze delle cavità diagonali in GP2, un giroscopio laser con cavità quadrata, di lato $1.6\ \text{m}$, dedicato al controllo interferometrico delle deformazioni della geometria della cavità, e la cui messa in funzione è stata completata nei laboratori INFN di Pisa a Giugno 2015.

Parole chiave: giroscopi laser, relatività generale, metrologia di lunghezza, analisi dei cammini ottici, interferometria, stabilizzazione laser, controllo attivo

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Introduction

The General Theory of Relativity (GTR), developed by Albert Einstein between 1905 and 1915 [1, 2], has determined a revolution in the fundamental physics. After one hundred years, it continues to be one of the most fascinating scientific field. The theory has been verified from the beginnings, for example through the explanation of the anomalous perihelion precession of Mercury's orbit by Einstein itself in 1915, the confirmation of the doubling of the deflection of light by the Sun made by Sir Arthur Eddington in 1919, and the demonstration of gravitational redshift in 1960 thanks to the Pound-Rebka experiment. In 1970, new tests have been driven by the advances in spacecraft tracking, high precision astrometric observation and space-based experiments. Today, what makes the GTR an active research area is the possibility to experimentally test the theoretical models with very high accuracy, since recent advances in several applied scientific disciplines make available new instruments and technologies. One of the most powerful tool in this sense is metrology.

Recent years have seen increasing efforts to directly measure the so-called *Lense-Thirring effect* [3]. First predicted by the austrian physicists Joseph Lense and Hans Thirring in 1918, it is one of the most famous and empirically investigated feature of the general relativistic gravitomagnetic interaction induced by the Earth rotation. It consists in a dragging of the local inertial frame of reference caused by the perturbation of the local metric in the proximity of a spinning massive body. As consequence, the geometrical fabric of space-time twists like a vortex, making the orbits of small body precess and the axis of a gyroscope wobble. It affects also in the orbital precession of X and gamma-rays emitting gas in the vicinity of rotating collapsed objects, like neutron stars and black holes, resulting in kHz quasi-periodic-oscillations (QPO) of their brightness. However, these effects take place at very fast rate, making very difficult the interpretation of phenomena.

To date, the Lense-Thirring effect has been observed as global effect on LA-

GEOS and LAGEOS II satellites' orbits [4, 5] within an accuracy of 20 – 30%, and as a precession of an orbiting gyroscope by the Gravity Probe B experiment [6, 7] within an accuracy of 15%. Further improvements are expected with the LARES satellite mission [8], based on the laser ranging technology. A local measurement of this effect is in principle possible in an Earth-based laboratory but, since on the Earth surface it is a corrective term of the order of 1 part in 10^9 of the rotation of our planet, it has not been implemented yet because of the stringent requirements in terms of sensitivity and accuracy. Earth-based experiments aiming at measure the frame dragging on the plane of swing of a Foucault pendulum [9], or the effect on a Gyromagnetic Electron Gyroscope (GEG) using the SQUID technology [10] have been proposed in the past.

Promising detectors for a local high-accuracy measurement of the Lense-Thirring effect are the ring laser gyroscopes (RL) [11]. At present, they are the most sensitive devices for measuring inertial rotation rates, taking the frequency shift between two laser beams counterpropagating in an active ring laser cavity. Therefore, they can provide an accurate measurement of relativistic effects on rotation.

These considerations inspired the GINGER proposal [12, 13]. Acronym of Gyroscopes IN GEneral Relativity, it aims at measuring, for the first time in a ground-based laboratory, the Lense-Thirring effect locally, by using an array of large ring laser gyroscopes. Even if the ring laser technology is mature and its capability in detecting very tiny effects has been experimentally demonstrated, several improvements are necessary in order to reach the stability and the resolution needed to achieve the GINGER goal. The present limit of the most stable devices is given by the deformations of the ring cavity geometry that, induced mainly by temperature, pressure and aging of materials, affect the path of the laser beams. To measure the Lense-Thirring effect requires a very high precision control of the cavity mirrors positions. This goal can be achieved only by means of interferometric methods, taking advantage of modern optical metrology.

The accuracy of the Lense-Thirring effect measurement using an Earth-based RL depends strongly on the geometrical stability of the optical cavity. In fact, in a RL the rotation rate is converted in frequency Sagnac beat through a scale

factor $k_S \propto A/p$, where A is the area enclosed by the optical path p of the laser beams along the cavity. Since the magnitude of the frame dragging term is of 1 part in 10^9 of the Earth rotation, a long-term accuracy on ring cavity mirrors positions better than 1 nm is required.

The fluctuation associated with the deformation of the optical cavity is a limiting factor also for the today's best performing ring laser, the Gross Ring 'G' (Wettzell, Bavaria), made of a semi-monolithic structure with high mechanical stability and operating in a well passively stabilized laboratory. To solve this problem, several approaches have been followed in the last years to actively control the cavity shape. The used observable is typically the ring laser optical frequency, that is connected to the perimeter length. In G-Pisa (Pisa, Italy), the long term perimeter control has been obtained by comparing the RL optical frequency with a frequency-stabilized He-Ne reference laser. The correction is applied to the ring cavity by acting on two piezo-electric devices, moving two opposite mirrors boxes along the diagonal of the ring [14]. In G, the RL optical frequency has been stabilized against an optical frequency reference, by tuning the pressure inside a vacuum tank enclosing the whole monolithic cavity [15], and later by locking the ring laser frequency to an optical frequency comb referenced to a hydrogen maser [16]. However, to lock the RL optical frequency, and so the cavity perimeter, is not the ultimate solution to the problem of the cavity deformation. In fact, variations of the area vector remain uncontrolled and the residual relative variation of the cavity side-lengths, even at constant perimeter, are source of noise in the rotational measurement.

The experience made with G and the other large frame RLs developed up to now has shown that a sub-nanometer accuracy on cavity geometry can be achieved only with the use of precise external laser metrology techniques, combined with an innovative sensor design. The development of a sub-nanometer length metrology for high-sensitivity next-generation square ring laser gyroscopes has been the goal of my PhD studies.

The primary step towards this direction is to find suitable observables to constraint the cavity deformations. The originality of my studies relies mostly on using, as observables for the control, the distance between opposite mirrors

in a ring cavity. Square geometry, in fact, guarantees that opposite mirrors are parallel so that they form along the diagonals two extra Fabry-Perot cavities, that can be interrogated by using an external laser source (see figure 1).

First, through a detailed study of the effects of geometrical distortions on the optical path, we demonstrated that these two observables can be used to monitor the geometrical stability of the whole ring system. If the two RL diagonals are locked to the same absolute length, high rejection of the mirrors positions perturbations from the scale factor is obtained, and the fluctuations can be reduced at level of 1 part in 10^{10} . In addition, a control procedure can be implemented to optimize the residual geometrical deformations.

To lock the absolute length of the two diagonals, we worked out an original interferometric technique, taking advantage of optical metrology tools. The technique is based on the use of an external reference laser beam to interrogate the diagonal resonators. The laser beam is stabilized with respect to a frequency standard, and phase-modulated to recover the error signals for the cavities length control.

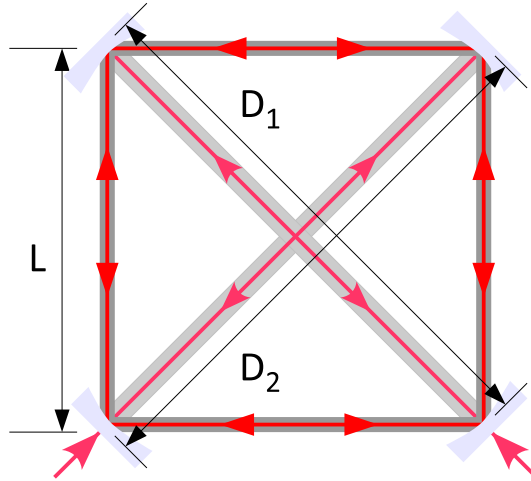


Fig. 1: Square ring laser gyroscope; the diagonal Fabry-Perot cavities, probed by external laser beams, are shown.

The technique has been worked out to be applied on GP2, a single-axis square cavity RL dedicated to the optimization and stabilization of the geometrical scale factor and, in particular, to the experimental implementation of

the length stabilization of the diagonal cavities. Its ring cavity is specifically designed for controlling the scale factor under the constraint of locked diagonal lengths.

Before to use the method as length metrology tool for ring laser, we made a tabletop experiment to verify the control technique on two resonators formed by curved mirrors used typically as corner reflectors in He-Ne RLs. In this first step, a diode laser frequency-stabilized to a reference frequency, via optical injection-locking, has been developed too.

The experimental results have shown the feasibility of the diagonal lengths control. In the last months of my PhD, I started to apply the technique to GP2.

The thesis is organized as follows.

In **Chapter 1**, I give an introduction to ring laser gyroscope technology, with particularly attention to the recent advances. After a brief dissertation about the role of Lense-Thirring effect on gyroscopes, I report about the state of the art of the large frame RLs and the proposed next-generation multi-axial devices.

The theoretical studies are reported in **Chapter 2** and **Chapter 3**. **Chapter 2** is dedicated to the model of the light propagation along a square optical cavity. It has been developed to pose the basis for the active control of an heterolithic RL, and to demonstrate the keyrole of the diagonal cavities. In **Chapter 3**, I discuss the absolute length measurement of a linear optical resonator, and I give an estimation of the systematic errors that limit the accuracy on the measurement of the RL diagonals and perimeter lengths.

In **Chapter 4**, I report about the experimental setups developed in this work and the achieved results: the design of *GP2* RL, and the characterization of its ring optical cavity e piezo nano-positioning system; the *light source* developed in our laboratory in Pisa; the *tabletop experiment*, used to test the measurement technique we worked out, exploiting the use of two Fabry-Perot resonators on an optical bench; the length stabilization of the *GP2 diagonal resonators*.

Conclusions and future perspectives are deduced in **Chapter 5**.

CHAPTER 1

Ring laser gyroscopes

The aim of this chapter is to provide an introduction about the ring laser gyroscope technology. In section 1.1, I discuss about the Sagnac effect, the fundamental sensitivity of a ring laser in measuring rotations, and the systematic errors that can limit its performance. A brief dissertation about the Lense-Thirring effect, whose accurate measurement is the goal of GINGER detector, is reported in section 1.2. In section 1.3, I present the state of the art of the large frame RLs. In particular, I report the most important features of: the Gross Ring ‘G’, that is to date the best performing RL; the GEOsensor, on which design the today’s heterolithic RLs are based; G-Pisa and its upgraded version GINGERino, developed in Italy. The design of GINGER, that fulfills all the requirements for a high sensitivity multi-axial sensor, is described in section 1.4.

1.1 Ring laser response and noise

1.1.1 The Sagnac effect

The Sagnac effect, observed for the first time in 1913 [17], consists in the time difference between the propagation times (measured by a clock at rest in the rotating frame) of two light beams counter-propagating along a closed path in a rotating interferometer (figure 1.1). The Sagnac time-delay δt is given by the following formula:

$$\delta t = \frac{4}{v^2} \mathbf{A} \cdot \boldsymbol{\Omega}, \quad (1.1)$$

where $\boldsymbol{\Omega}$ is the rotation relative to the local Lorentz inertial frame, $\mathbf{A} =$

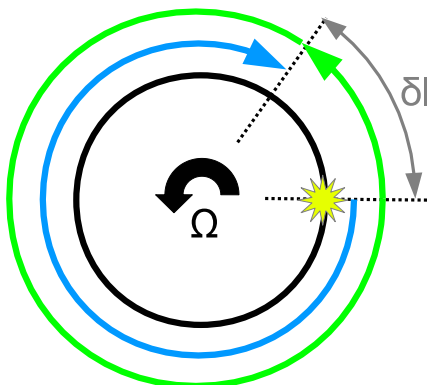


Fig. 1.1: Sagnac effect representation for two laser beams counterpropagating in a rotating cavity. δl : shift in path length.

$A\mathbf{u}_n$ is the area vector enclosed by the ring optical path and oriented according to its normal vector $\mathbf{u}_n$, and v is the speed of propagation around the loop.

As a consequence, a phase shift Φ_S arises

$$\Phi_S = \frac{4}{\lambda v} \mathbf{A} \cdot \boldsymbol{\Omega}, \quad (1.2)$$

where λ is the wavelength of the light in the medium filling the loop.

The Sagnac phase shift can be seen as flux of the angular velocity $\boldsymbol{\Omega}$ through the cross section of the apparatus. It is purely a classical phenomenon, since the higher-order corrections of special relativity apply equally on both directions, and hence cancel out of the phase difference; it is purely geometric [18], valid for any shape of the path encircling an enclosed area A [19, 20]. Its non-reciprocal nature implies that a Sagnac interferometer can be used to measure several effects which break time-reversal symmetry, such as magneto-optical and nonlinear effects. On the other side, any source of noise acting in a non-reciprocal way, such as mechanical and thermal fluctuations, is parasitic and, behaving as sources of noise in measurements, need to be removed or minimized.

Finally, the Sagnac effect is independent of specific material properties, and it has been observed not only for photons but also between two split paths of a coherent massive particle beam [21–26]. Interfering beams having a given velocity experience the same Sagnac time delay independently of the physical

nature [27]. For both photons and massive particles having energy E , the Sagnac phase shift can be written as [28]:

$$\Phi_S = \frac{4\pi E}{hc^2} \mathbf{A} \cdot \boldsymbol{\Omega}. \quad (1.3)$$

Being $E = \hbar\omega$ the energy of a photon of angular frequency ω , and $E = mc^2$ that of a particle of rest mass m , equation 1.3 shows that the rotational sensitivity per unit area of a matter-wave gyroscope exceeds that of an optical one by the factor $mc^2/\hbar\omega \sim 10^{11}$. Despite of this, at present they suffer from the smallness of the achievable loop areas and from fluctuations of the atomic sources that, resulting often in inhomogeneous dephasing washing out fringes contrast, limit the long term sensitivity. Promising results are coming out of the use of cold and ultracold atomic systems, such as Bose-Einstein condensates (BECs) of cesium or rubidium, confined in microchip waveguides [29–31].

1.1.2 Optical gyros

Optical gyros are traditionally divided in two main classes: passive devices, such as interferometric and passive resonant gyros, and active devices, such as ring laser gyroscopes.

Interferometric gyros typically exploit an optical fiber waveguide (FOG - fiber optic gyroscope) along which two beams, coming from an external source, are injected in opposite directions [32]. The sensitivity in the rotation induced phase shift (equation 1.2) is enhanced by using a multiple-turn fiber coil, so that the counter rotating beams travel around a larger area. However, they suffer from an high sensitivity to external perturbations and Brillouin scattering. These sensors are mostly used in applications requiring low performance such as robotics, space applications, inertial navigation of submarines and spacecrafts.

Passive resonant gyros [33, 34] and ring laser gyroscopes [35–37] measure both the rotation induced difference between the frequencies of two counter-propagating modes resonating in a ring cavity. The Sagnac beat frequency is

given by the following expression [17]:

$$f_S = \frac{4}{\lambda p} \mathbf{A} \cdot \boldsymbol{\Omega}, \quad (1.4)$$

where p is the ring optical path enclosing the area A , and λ is the wavelength of the laser. The quantity

$$k_S = \frac{4A}{\lambda p}, \quad (1.5)$$

is called *scale factor* of the gyro.

While in the passive resonant gyro the light source, as in the interferometric gyro, is external to the resonator, in a ring laser gyroscope the two counter-propagating beams are emitted by an active medium that fills the cavity. The presence of the medium could limit the performance of the gyro due to gain dispersion, dissipative losses and additional non-reciprocal phenomena. On the other hand, the intracavity generation of the beams assures a more reciprocity and equal optical paths of them. Passive resonant gyros and ring laser gyroscopes are both widely adopted, limited in their further applications in terms of size and stability. However, all the high sensitive gyros developed until now are large ring laser gyroscopes. The most usual choice for the active medium is a helium-neon gas mixture, because it is a well known laser system and efficiently emits visible light with good beam quality. The first He-Ne ring laser operated at a wavelength of $1.153 \mu\text{m}$ [35]. Today the 632.8 nm transition is mostly used, since the optical components designed to operate at this wavelength are largely available.

I conclude this section mentioning that a Sagnac interferometer is not a rotation sensor only, even if this is its natural application. In particular, a modified topology can be set up such that the interferometer has an effective zero area, and so it is insensitive to mechanical rotations. A zero-area Sagnac interferometer has been recently proposed for gravitational wave detection [39, 76], being sensitive to displacements of its mirrors, and for high resolution magneto-optic measurements [40], by measuring the nonreciprocal phase shift between the incident and the reflected beams travelling along the same fiber loop with orthogonal polarization.

1.1.3 Sensitivity of a ring laser gyroscope

The fundamental sensitivity of a RL is limited by its Schawlow-Townes linewidth Γ_{ST} :

$$\Gamma_{ST} = \frac{\hbar\omega\Gamma^2}{P}, \quad (1.6)$$

where Γ is the linewidth of the cavity and P the emitted power. The resulting sensitivity limit is

$$\Omega_{sn} = \frac{vp}{4AQ} \sqrt{\frac{hf}{P_{out}T}}, \quad (1.7)$$

where $v = c/\mu$ is the velocity of the laser beam along the cavity (being c the velocity of light in vacuum and μ the refractive index of the medium filling the cavity), Q is the quality factor of the resonator (that depends on cavity mirror reflectivity), h the Planck constant, f the operating laser frequency, P_{out} the detected optical power, and T the measuring time.

Hence, for a given sampling time, the minimum uncertainty in the rotation rate depends upon the size of the cavity, the losses of the mirrors and the optical power level. Laser cavity with side length of several meters, the use of supermirrors with reflectivity better than 99.999%, and a long term stability are required to improve the sensitivity. Figure 1.1.3 shows the possible application of the RLs depending on their sensitivity. The sensitivities achieved by the G-Pisa (Pisa, Italy) and G (Wettzell, Germany) ring lasers are indicated, as well as the target required to detect relativistic effects on rotation. A review about the large frame RLs developed up to now is reported in section 1.3.

The possibility of operation at a wavelength shorter than 632.8 nm to increase the scale factor has also been considered but not implemented yet, since this could make worse the ring laser stability through the backscatter process (see section 1.1.4). Recently, several methods have been proposed to enhance the sensitivity of RLs using superluminal effects [41–44]. The fast-light ($v > c$) is generated by inserting inside the cavity a medium with anomalous dispersion. In this case the group refractive index $\mu_g \simeq \mu$ is very small, and then an enhancement of the scale factor is obtained. However, further investigations reported in reference [45], show that this enhancement does not imply a significant improvement of the RL fundamental sensitivity. The major advantage of the use of a linear dispersion medium is the reduction of the angular random walk induced by the lock-in phenomenon that, as we will see in the next

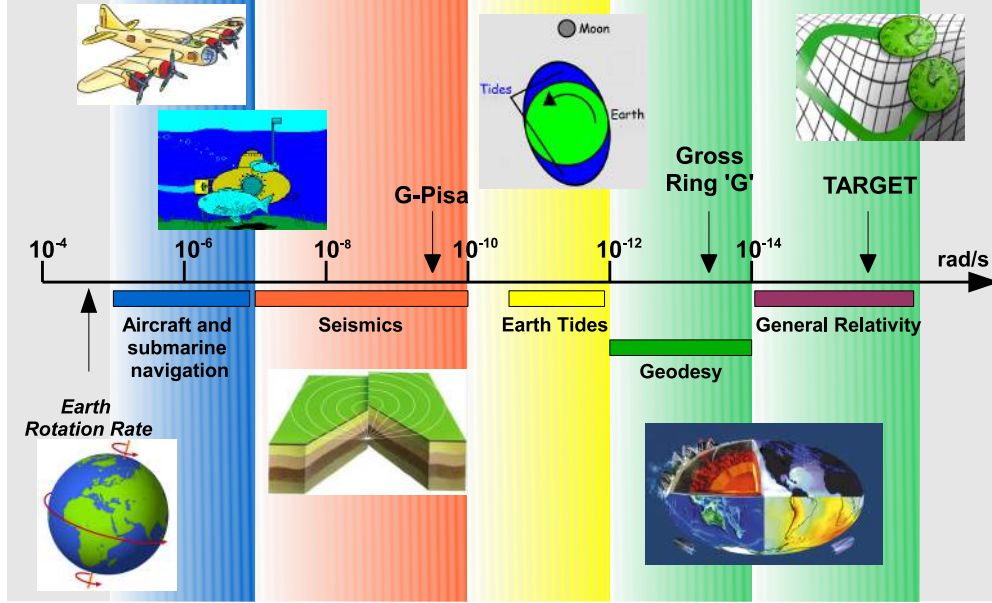


Fig. 1.2: Orders of magnitude of rotations & ring laser gyroscopes. The sensitivities in measuring rotations achieved by G-Pisa and G are indicated, as well as the target needed to measure the Lense-Thirring effect.

section, is one of the error sources that limit the RL performance.

1.1.4 Errors in ring laser response

The long-term perturbations that can corrupt the relation between the measured Sagnac frequency and the rotation rate, expressed by equation 1.4, have been analyzed in detail in several previous works [36, 37, 46]. These can be classified as scale factor error, non linear coupling effects between the frequency of the two circulating beams, and bias error. Following the *sensor error model* developed in reference [46], these errors can be treated as corrective terms, reformulating the RL response as:

$$f_S = k_S(1 + k_A)\mathbf{u}_n \cdot \boldsymbol{\Omega} + \delta f_{BS} + \delta f_0. \quad (1.8)$$

- *Scale factor error.* The systematic errors related to the scale factor are due to fluctuation of the cavity geometry and laser active medium parameters. This two contributions are represented by k_S and k_A in equation 1.8.

The fluctuation of k_S is caused by both reciprocal and non-reciprocal effects in the propagation of the two laser beams affecting the stability of the area over perimeter ratio. Slow deformations of the optical cavity are induced by temperature, pressure and aging of materials. These affect directly the measured Sagnac signal, and are the present limit to the performance of large devices. The stabilization of the geometrical scale factor k_S with high accuracy is the target of the work here presented.

The term k_A takes into account the change in the optical length of the cavity due to the dispersion properties of the plasma discharge. These are mainly induced by fluctuation of active medium parameters, such as gain, temperature, pressure and frequency detuning. To monitor the fluctuation of the atomic scale factor k_A a spectroscopic diagnostic of these parameters have been developed. The induced systematic effects can be subtracted from the Sagnac data using off-line denoising methods [47, 48].

- *Mode coupling.* The second main source of error in a RL is the frequency pulling δf_{BS} due to the coupling between the two beams. This is a consequence of the trading of energy between the two modes of oscillation that naturally occurs when the frequencies approach each other, so that they tend to lock. The mode coupling is induced by backscatter processes arising from mirror imperfections, that cause the scattering of one laser beams into the other.

The problem of backscatter has been extensively treated in the ring laser community [49]. The approaches used to minimize it range from the application of frequency dithering or pulse timing, to the development of schemes for off-line corrections [47, 48, 50]. Alternatively, a bidirectional optical diode combined with spatial resolution of eigenstates has been proposed to achieve a RL free of frequency locking [51]. More recently, the use of longer wavelength laser radiation has been investigated [52], being the backscatter amplitude proportional to the reciprocal of the fourth power of the laser wavelength [53]. The drawback of this last approach is that it causes also a reduction of scale factor k_S .

- *Bias error.* A non zero frequency beat δf_0 can be detected in the case of a null rotation. The null shift is mainly caused by the anisotropies

in the laser system that act in a non-reciprocal way. The main source of anisotropies is the non homogeneity of cavity mirrors in surface planarity, causing that the counter propagating beams experience two different quality factors Q . Other sources of anisotropy are the Faraday and Langmuir effects. A Faraday effect in the plasma can occurs in the presence of an external magnetic field, due to the elliptical polarization of the two beams. The Langmuir effect, instead, consist in a flow occurring in any laser active medium; as a consequence, the two counter propagating beams see opposite motions for the lasing medium, and hence different indices of refraction (Fresnel drag). A significant reduction of the Langmuir effect is obtained if a RF discharge, instead of a DC excitation, is used for laser excitation. The bias error exhibits as a difference between the intensities of the circulating beams.

1.2 Relativistic corrections to the Sagnac effect

The GTR represents a generalization of the Special Theory of Relativity for the description of accelerated reference frames and gravitation. It introduces a new concept of the fabric of space-time, that interacts with the dynamical distributions of matter and energy. Gravitation appears as curvature of the space-time fabric, which is shaped by the matter distribution and its motion. In particular, in the weak-field and slow motion approximation, the Einstein field equations of GTR look like the Maxwell equations of electromagnetism and, in the same way that a magnetic field is created when a charged object rotates, a gravitomagnetic field is created when a massive body rotates.

The frame dragging in the proximity of a spinning massive body, or Lense-Thirring effect, is the most famous gravitomagnetic effect. It is due to the fact that when a mass rotates, the space-time around is twisted a bit. As a consequence, the plane of a pendulum placed at the north pole is dragged along with the Earth's rotation, and the axis of a torque-free gyroscope placed near the Earth, as well as the orbit of a satellite, undergoes a small precession (see figure 1.3). This implies that the Earth rotation rate measured against the "fixed-stars" inertial frame differs from the rotation rate measured in the laboratory frame.

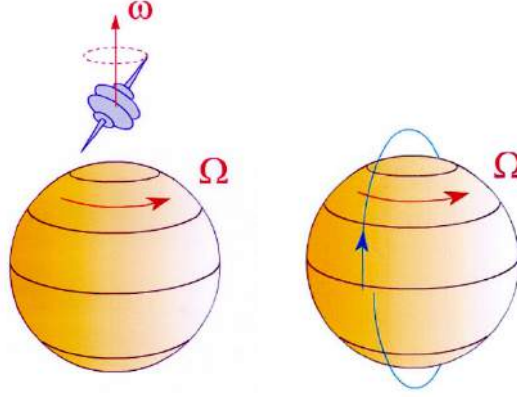


Fig. 1.3: Lense-Thirring effect in the axis of a gyroscope (on the left) and the motion of a satellite (on the right) [54].

Let's consider a RL in a ground-based laboratory. As in the case of a mechanical gyroscope, the difference in the two times of flight depends on the rotation of the axes of the local reference frame with respect to distant stars and on the gravitomagnetic drag. Hence, in the local frame the device measures an effective rotation rate

$$\Omega = \Omega_{\oplus} + \Omega', \quad (1.9)$$

where $\Omega_{\oplus}$ is the rotation rate of the Earth with respect the "fixed-stars" inertial frame, that gives rise to the purely kinematic Sagnac term, while $\Omega' = \Omega_G + \Omega_B + \Omega_W + \Omega_T$ are the relativistic precessions, as discussed in [12].

The Lense-Thirring drag is represented by the term Ω_B , while Ω_G is the geodetic term, Ω_W the preferred frame term and Ω_T the Thomas term. On the Earth surface, the Lense-Thirring and geodetic terms are both of order 1 part in 10^9 of $\Omega_{\oplus}$ but, while the geodetic term is radially directed, the dragging one has a dipolar shape. The Thomas and preferred term are respectively 3 and 2 orders of magnitude smaller of these. The relativistic terms Ω' can be recovered by subtracting from the RL data the Sagnac term provided by VLBI - Very Long Baseline Interferometry, that measures the Earth angular velocity with respect to the fixed stars with very high accuracy.

Since in a RL the rotation rate is recovered rescaling the measured fre-

quency beat by the scale factor k_S , to measure the Lense-Thirring k_S must be known and kept at 10^{-10} long term accuracy. This implies an accuracy on mirror positions better than 1 nm and on the ring cavities absolute orientation better than 1 nrad. In the case of a multi-axial device, as GINGER, the invariant modulus of the Earth rotation can be reconstructed and compared with VLBI data to recover the relativistic terms.

In terms of the angular momentum of the Earth $\mathbf{J}_\oplus$, the drag term reads

$$\boldsymbol{\Omega}_B = -\frac{1 + \gamma + \alpha_1/4}{2} \frac{G}{c^2 R^3} [\mathbf{J}_\oplus - 3(\mathbf{J}_\oplus \cdot \mathbf{u}_r)\mathbf{u}_r], \quad (1.10)$$

where $R = R\mathbf{u}_r$ is the position of the laboratory with respect to the center of the Earth, G is the gravitational constant, γ and α_1 are the PPN (Parameterized post-Newtonian) parameters, which account for the amount of space curvature produced by a unit rest mass and the effect of preferred reference frame, respectively.

In GTR, $\alpha_1 = 0$ and $\gamma = 1$. A high precision measurement of the drag term in the meridian plane, could provide new constraints on these parameters too.

1.3 Large frame RLs: state of the art

In an effort to attain ever higher sensitivity, several large frame RLs have been developed around the world over the last decade, in particular by a collaboration among Germany (U.K. Schreiber and coworkers, Technische Universität München and Fundamentalstation Wetzell), Italy (A. Di Virgilio and coworkers, INFN Pisa) and New Zealand (G.E. Stedman and coworkers, University of Christchurch). Detailed reviews of the large ring laser gyros constructed up to now are reported in references [46, 55] and summarized in table 1.1. All are horizontal devices, apart from G0 that works in the vertical configuration, and G-Pisa that can work in both horizontal and vertical ones. Here, I focus the attention on G, the best ring laser at present, and on the devices developed by the ring laser group in Italy. These represent the starting point for the design of the next-generation high-sensitivity RLs.

Table 1.1: Large frame He-Ne (633 nm) ring laser gyroscopes in the world. R : radius of curvature of mirror. f_S : Sagnac beat frequency.

	C-II (1997)	PR1 (2004)
Geometry	Square 1 m $\times$ 1 m	Square 1.60 m $\times$ 1.60 m
R	2 m	4 m
f_S	79 Hz	160 Hz
Location	Christchurch (NZ)	Christchurch (NZ)
	AR-1 (1998)	UG2 (2005)
Geometry	Triangular 15 m side	Rectangular 21.0 m $\times$ 39.7 m
R	9 m	20 m 70 m (alternated)
f_S	580 Hz	2177 Hz
Location	Conway (US)	Canterbury (NZ)
	G0 (1998)	GEOsensor (2005)
Geometry	Square 3.50 m $\times$ 3.50 m	Square 1.60 m $\times$ 1.60 m
R	6 m	4 m
f_S	287 Hz	103 Hz
Location	Christchurch (NZ)	Pinion Flat (US)
	G (2001)	G-Pisa (2008)
Geometry	Square 4 m $\times$ 4 m	Square 1.40 m $\times$ 1.40 m
R	4 m	2 m
f_S	340 Hz	111 Hz
Location	Wettzell (DE)	Pisa (IT)
	UG1 (2003)	GINGERino (2014)
Geometry	Rectangular 21.0 m $\times$ 17.5 m	Square 3.60 m $\times$ 3.60 m
R	20 m	4 m
f_S	1513 Hz	280 Hz
Location	Christchurch (NZ)	L'Aquila (IT)

1.3.1 The Gross Ring ‘G’

To this day, the best RL is the Grossring ‘G’, located at the Geodätisches Observatorium in Wettzell, Bavaria [56, 57]. It has achieved a resolution better than 5×10^{-13} rad/s with an integration time of few hours, very close to that

required to detect the relativistic effects on rotation, becoming of geodetic interest for measuring short-term fluctuation, with periods of hours to days, in Earth rotation.

This stability record is mainly due to its building material, that allows a strong passive stabilization of the optical cavity. G, in fact, is a semi-monolithic device made in Zerodur, a glass ceramic that exhibits a very small relative thermal expansion ($\sim 5 \times 10^{-8} \text{ K}^{-1}$), high mechanical stability and consistency of shape and length. Four bars are rigidly connected to a base plate forming the edges of a square 4 m length in side; spherical supermirrors are attached to the face sides of the bars by molecular adhesion, ensuring a stable vacuum seal. It is kept in an underground controlled room, and equipped with an active control of the cavity perimeter that stabilizes the circulating laser beams frequency against an optical frequency reference. Recent advances in G resolution have been obtained using a pressure stabilizing vessel that encloses the entire ring laser structure [15] and, more recently, by frequency locking the ring laser to a single tooth of an optical frequency comb referenced to a hydrogen maser, obtaining a frequency stability of 1 kHz over several days [16].

1.3.2 Research activities in Italy

In Italy, the research activities in the field of large RLs started in 2008. The challenge has been to show that an heterolithic design can be used for high-sensitivity applications. In this effort, several tools needed to achieve this goal have been developed.

All the RLs developed up to now in Italy have a square optical cavity and, except for GP2, are based on a design, called GEOsensor, conceived after the G experience [46, 58]. In fact, the good performance of G achieved, after few years of operation, in geodesy and geophysics showed the potentiality of the detector but, on the other side, the limit of its design. An upscaling of the size of the optical cavity was needed to improve the sensitivity, but a larger monolithic block in Zerodur is too expensive, not transportable and not usable for a multi-axial device.

The development of the GEOsensor has been stimulated by two requirements: to find a compromise between the sensitivity of G and the flexibility of



Fig. 1.4: The Gross Ring G. Above: overview of the semi-monolithic Zerodur structure whereon the optical cavity is installed (on the left); detail of the setup for Sagnac beat detection; a prism is used to combine the clockwise and counter clockwise beams. Below: pyrex plasma tube for radio frequency excitation of the gain medium (on the left); pressure stabilizing vessel enclosing the ring laser structure (on the right) [courtesy of U.K. Schreiber and A. Gebauer].

a smaller instrument, and to adjust a RL for specific seismological applications. The main feature of its design is that it is heterolithic, since it is constructed from several components. This makes the sensor independent from its concrete platform base, in the sense that its stability is mainly determined by that of the base. In addition, its actual area is not limited a priori, being the size length scalable. The constance of the length of the cavity, guaranteed in G by Zerodur, is actively obtained by a folded lever system, that allows the alignment of the turning cavity mirrors, located in solid corner boxes. To allows the observation of seismological events, it is equipped with additional sensors: a GPS, a seismometer and a tiltmeter. A photo of GEOsensor is reported in figure 1.5. The optical cavity has the shape of a square with a side length of 1.60 m. It is located at the seismological observatory in Pinion Flat (California, Unites States) and is currently in operation.

Based on GEOsensor, the optical cavity of the RLs developed in Italy is made of four supermirrors each one contained in a steel holder placed at the corner of a granite support, fixed on a concrete base. Steel pipes connected by

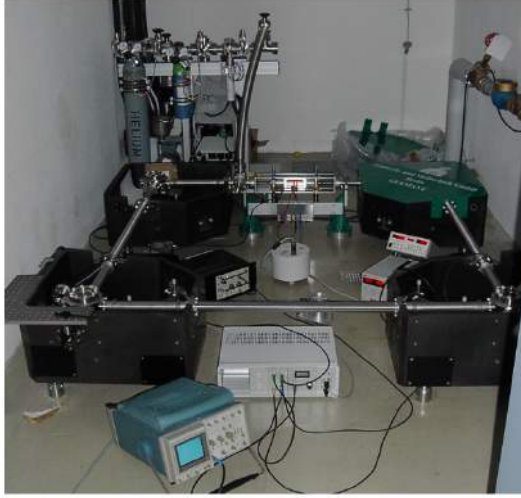


Fig. 1.5: GEOsensor ring laser [59]. The turning cavity mirrors, key feature of its design, are contained in the big gray corner box.

the mirror holders define the vacuum chamber, that encloses the optical path of the circulating beams along a square loop. The vacuum chamber is filled with a mixture of He and a 50% isotopic mixture of ^{20}Ne and ^{22}Ne ; the capacitive discharge for laser excitation, consisting in a pyrex capillary, is located in the middle of a side of the cavity. The laser frequency is stabilized with respect to an optical frequency reference, in order to actively control the ring cavity perimeter. The reference used in the first RL was a He-Ne laser self-stabilized on the Doppler gain profile, replaced in 2012 by a iodine stabilized He-Ne one (described in subsection 4.2.2).

The experimental activities started with the middle-size RL ‘G-Pisa’ [60–62], 1.40 m length in side. It was the first heterolithic actively controlled [14] and transportable large frame RL in the world, able to work in the horizontal as well as in the vertical orientation (figure 1.6). The flexibility of the design allowed to change the length of the optical cavity side from 1.40 m to 0.90 m. A rotational sensitivity at the level of 10^{-8} rad/ $\sqrt{\text{Hz}}$ below 1 Hz was demonstrated for the horizontal configuration [63]. After two years of operation at INFN laboratories in Pisa, in June 2010 it was moved at the site of the Virgo gravitational wave interferometer, located in Cascina (Pisa, Italy), to investigate the ability to improve the low-frequency sensitivity of the Grav-



Fig. 1.6: G-Pisa ring laser. Above: horizontal and vertical configuration of G-Pisa inside the Virgo area. Below: its upgraded setup at the INFN laboratories in San Piero a Grado (Pisa, Italy).

itational Wave Antenna [64]. The achieved high sensitivity and low noise level in the spectral range between 1 mHz and 1 Hz allowed G-Pisa to detect also rotational signals associated with the passage of seismic waves [65].

The upgraded setup of G-Pisa was later moved in 2011 in a cleanroom of the INFN laboratories in San Piero a Grado (Pisa, Italy). In this location it operated first in the horizontal configuration (as shown in figure 1.6); then, it was installed on a granite slab oriented with the normal to the ring plane almost parallel to the Earth rotation axis, to maximize the Sagnac effect and minimize the contribution of the orientation error. The studies and experimental activities made in that period regarded mainly the optimization of ring cavity perimeter stabilization against a primary frequency standard, and the control of the systematic errors related to the fluctuation of the laser



Fig. 1.7: GINGERino and GP2 italian ring lasers. Above: GINGERino, installed in the LNGS underground laboratory in Assergi (L'Aquila, Italy). Below: GP2, installed in the INFN laboratories in Pisa.

active medium parameters. As regards the last issue, a set of spectroscopic diagnostic of the active medium parameters and an offline denoising method have been developed [47, 48].

The results of the studies made on G-Pisa demonstrated that an heterolithic design equipped with precise diagnostics is a viable solution for the next-generation RLs aiming at detect tiny effects on Earth rotation. This mo-

tivated the design of the large frame multi-axial ring laser ‘GINGER’ [12]. Details about this proposal are given in subsection 1.4. To accomplish GINGER’s goal needs a strict control of the systematic errors related to the fluctuation of the cavity geometry and the laser active medium parameters, and a low-noise laboratory. The two RLs operating in Italy at present, GINGERino and GP2, both showed in figure 1.7, have been developed as intermediate prototypes of GINGER to supply to this two requests. GP2, being devoted to test the active control strategies, is the pivot of this work, and its innovative design, not based on GEOSensor one, will be fully characterized in the following chapter (see chapter 4). GINGERino, instead, has been developed to test the adequacy of the LNGS underground laboratory, where it is operating since the end of 2014. The task is to analyze the seismic noise at the site, but also to tryout the signal to noise ratio (SNR) improvement of a larger cavity. It is, in fact, an evolution of G-Pisa; holders and pipes are those of the first prototype, increased in size to get a cavity side length of 3.60 m, and placed on a suitable new granite support installed on a concrete monument well connected to the rock floor. Contrary to the others prototype, the slab consists in four arms that has been inserted in loco into a central square granite block. The operation of GINGERino in a low local noise laboratory gives the possibility to detect geodetic signals (Polar motion, Chandler wobble, tidal effects,...) and seismic signals (S-wave phase velocity,...) [66] for which a sensitivity of $10^{-9} - 10^{-13}$ rad/s is sufficient. A comparison with G measurements would be very interesting in the case of non-local effects, such as tele-seismic events.

1.4 Next-generation multi-axial RLs

Even if small size multi-axial RLs (with optical cavities few centimeters length) are used since a long time for Inertial Navigation Systems, the idea to develop an array of large frame RLs (with optical cavities some meters in length) for high accuracy applications has been promoted in the last years only. In this context, GINGER is a pioneering project. Its design study and scientific goals have been presented in reference [12]. A schematic drawing of GINGER is shown in figure 1.8. It consists in an array of three large square heterolithic RLs arranged in a octahedral structure; each couple of opposite

mirrors is shared between two rings. The length of the side of each square cavity will be > 6 m.

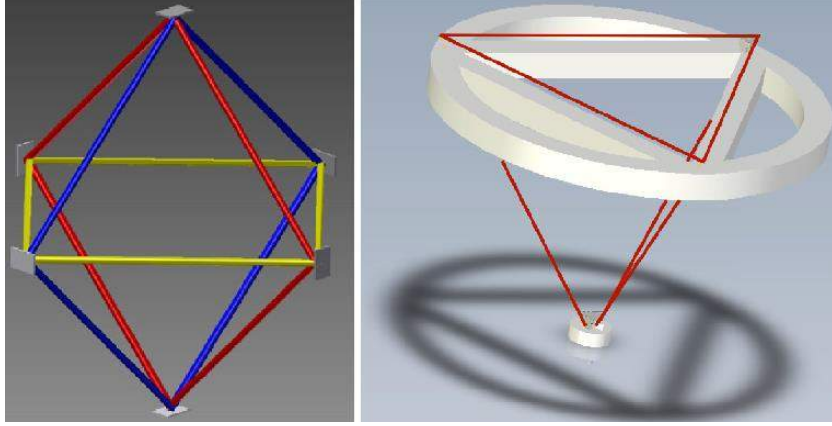


Fig. 1.8: Design of next-generation multi-axial RLs. Schematic drawing of GINGER (on the left) and ROMY (on the right). In GINGER, each mirror is shared between two square rings; ROMY, instead, consists in four independent triangular rings.

The design of GINGER fulfills all the requirements that a RL aiming to measure the Lense-Thirring drag have to satisfy.

The first requirement concerns the number and the size of ring optical cavities. As stated in section 1.2, the achievement of GINGER goal needs a stabilization of the scale factor k_S better than 10^{-10} . Therefore, the angles between vectors in equation 1.4 must be measured at the corresponding accuracy level, and this requests an absolute measurement of the area vector in the fixed stars reference system with the accuracy of nano-radians. This requirement, very hardly to achieve experimentally, can be relaxed using a multi-axial array of RLs, in order to reconstruct the modulus of the Earth rotational vector, and compare the Earth rotation rate measured locally with the one provided by IERS (International Earth Rotation and Reference Systems Service).

The array needs to have a large frame structure. As follows from equation 1.7 in fact, to increase the size of the ring cavity implies to increase the signal to noise ratio, being the signal proportional to the ratio A/p via the scale factor, and the shot noise of the sensor proportional to p via the quality factor Q . Low-loss ‘five-9s quality’ supermirrors must be used to guarantee a high- Q

resonator ($> 10^{11}$).

The second requirement is to reduce all the instrumental errors in the measurement of rotation rate to less than $\boldsymbol{\Omega}'/\boldsymbol{\Omega}_{\oplus}$. This needs a long-term strict control on the fluctuation of cavity geometry, laser active medium and of relative dihedral angles. The first issue emerges clearly from the dependence of the Sagnac frequency on the laser path geometry via the scale factor $k_S \propto A/p$. To provide a tool to guarantee the control of cavity geometry is the topic of this work. As regards the fluctuation of the laser active medium, a set of spectroscopic diagnostic of the active medium parameters and an offline denoising method that, based on the Kalman filter approach, subtracts from the raw Sagnac data the systematic effects induced by the non-linearity of the RL dynamics, have been developed in the last years and successfully applied on the single-axis RLs [47, 48]. To control the fluctuation of relative dihedral angles, a picometer accuracy laser metrology based on SIM (Space Interferometry System) technology is in development, and will be tested on an optical bench in the near future.

The last requirement concerns the location of the detector. To reduce all the sources of Earth-surface and environmental noise, the detector has to be installed in a very stable geological and low-noise environment. A suitable location could be the underground facility of LNGS (Laboratori Nazionali del Gran Sasso - Italy), where the GINGERino prototype is to date installed.

In addition to GINGER, a second multi-axial large frame RL, called ROMY-ROtational Motions in seismologY [67] has been conceived (LMU Munich and TU Munich, Fundamentalstation Wettzell; financed by European Research Council (ERC)). The main objective of this project is to make, for the first time, high resolution measurements of broadband rotational ground motions. The target sensitivity of ROMY is of 10^{-12} rad/s. A schematic representation of the device is shown on the right of figure 1.8. It is a tetrahedral set of four triangular ring lasers, each one with a side length of 12 m. To this day, its sub-surface installation in the Geophysical Observatory Furstenfeldbruck (Bavaria, Germany) is in progress. The cross-validation of the observations made by the two RL arrays would be of great interest too.

CHAPTER 2

Study of the geometry of a square cavity RL

This chapter is dedicated to the model of the light propagation along a square optical cavity, that we developed to pose the basis for the active control of an heterolithic RL, and to demonstrate the keyrole of the diagonal cavities. Most of the results here reported have been published in reference [68].

In section 2.1, I refer about what has been published on this topic up to now, and I introduce the points of originality of our approach. In section 2.2, I present our theoretical analysis about the lasing path along the cavity under mirror misalignment. A study of the degrees of freedom of the optical system is made, and the diagonals lengths stabilization constraint is analyzed. In section 2.3, finally, I show the use of the developed formalism to optimize the pre-alignment of a ring laser by centering the laser spots on the mirrors surfaces, starting from a generic lasing condition of the ring in the TEM_{00} mode.

2.1 Introduction and related works

A detailed study about the effects of geometrical distortions on the optical path in a RL has been made by Stedman's group. The authors considered in reference [69] a regular and then in reference [70] a non regular polygonal cavity, defined by an arbitrary number of mirrors having an arbitrary radius, and studied the stability of the lasing path both in a perfectly aligned ring and under mirror misalignment. However, their analysis, that makes use of the ABCD matrix formalism, is essentially based on a study of the sensitivity of the lasing path to the misalignment as a function of the cavity design, and hence aims at a stabilization of the geometry by an optimization of design

parameters.

Another important step in this context has been made by Rodloff in reference [71]. In that paper the author studied, in a triangular resonator, the lock-in threshold and the distribution of the backscatter sources as a function of the geometry. An active optimization of the resonator geometry is suggested, by moving the mirrors through piezotransducers, in order to cancel the effect of backscattering by destructive interference.

In recent literature, one can find many papers that, following an approach similar to Stedman's one, analyze the effects of mirror misalignment on the performance of RLs with different cavity features. Several papers have been published by Yuan et al. In [72], for example, they studied the characteristics of the Gaussian modes of a non-planar square RL as a function of design parameters, such as the cavity side length to the mirror curvature radius ratio. In a subsequent work [73], the authors proposed a generalization of the ray matrix formalism that takes into account the effects of perturbation sources of both the mirror's angular misalignments and the mirror's displacements, and they applied it to improve the alignment stability of single-axis and tri-axial square resonators through an optimized optical cavity design. Square RLs have been studied also by Wen et al. in [74] and Lu et al. in [75]. The first, identifies sensitivity factors considering the radial and axial displacement of spherical mirrors in non-planar cavities. The latter, takes into account planar cavities, and studies the rigid rotation effect induced in a RL when its mirrors are driven in opposite directions to optimize the geometry. The effects of misalignment, in particular the mirror-tilting induced power loss, are instead analyzed in a triangular RL by Sun et al. in [76].

The geometrical model of the optical cavity we developed here addresses to pose the basis for optimizing the stability of an heterolithic square RL with identical spherical mirrors actively controlled. It demonstrates that a minimization of the scale factor k_S variations, against fluctuation of the mirror positions, can be obtained by setting equal the absolute lengths of the diagonal cavities and controlling the perimeter, in order to get a regular square shape of the closed optical path. In this geometry, the residual cavity deformations contribute to k_S with quadratic terms, so that a control procedure can be implemented to optimize the regular cavity shape from a generic one. For the

first time, the suggested optimization of the geometry is based not only on the cavity design parameters, but on the use of the passive diagonal cavities lengths as external observables too. Our analysis is made in geometric optics approximation, that describes the light in term of rays whose propagation is governed by Fermat's principle.

In the ring cavity the light rays propagate along the cavity optical axis, that is determined uniquely by the position of the centers of curvature of the mirrors. Based on this observation, we choose the curvature centers configuration as set of dynamical variables that completely describes the optical system. We don't use the ray matrix formalism. The starting point of our theoretical analysis is the perimeter length, written as a function of this set of variables. By using the Fermat's principle, the effective optical length along the ring cavity is obtained, hence it is studied under the diagonal stabilization constraint.

2.2 Theoretical analysis

2.2.1 Optical path along a square cavity

We consider a square ring cavity made of four curved mirrors, that can be represented as spheres. In the case of concave mirrors, as I consider here, the curved reflecting surface is towards the center of the sphere of which the mirror is a part. Let $r_k \in \mathbb{R}^+$ and $\mathbf{c}_k \in \mathbb{R}^3$ denote the radius and the position of the center of curvature of the k_{th} mirror, with $k = \{1, 2, 3, 4\}$. The positions of the curvature centers determine the optical axis of the system, along which the laser beam propagates. Since each mirror displacement can be described as a variation of the position of its curvature center, the four vectors $\mathbf{c}_k$ represent a set of dynamical variables of the system. For a regular square cavity, the optical axis coincides with the perimeter contour line and intercepts each mirror in its pole. In this case, the intersection points coincide with the positions of the light spots. The four spots can be usefully represented as the elements X of $\mathbb{S}^{2 \times 4}$, i.e. the 2-dimensional coordinates of 4 points on the spheres representing the mirrors; let $\mathbf{s}_k = r_k \mathbf{x}_k + \mathbf{c}_k$ be the position vector of the k_{th} spot with respect to a reference frame, and $\mathbf{x}_k \in \mathbb{S}^2 = \{\mathbf{x} \in \mathbb{R}^3, \mathbf{x}^T \mathbf{x} = 1\}$ the directional cosines of the spot position with respect to its mirror center. We introduce the

compact notation for the directional cosine matrix $X = [\mathbf{x}_1, \dots, \mathbf{x}_4] \in \mathbb{S}^{2 \times 4}$, the mirror configuration matrix $C = [\mathbf{c}_1, \dots, \mathbf{c}_4] \in \mathbb{R}^{3 \times 4}$, and the curvature radii matrix $R = \text{diag}[r_1, \dots, r_4] \in \mathbb{R}^{3 \times 4}$. In this notation the spot position matrix can be written as $S = XR + C$. Since $\mathbf{x}_k$ are points of spherical surfaces, the 4 geometrical constraints $\|\mathbf{s}_k - \mathbf{c}_k\|^2 = r_k^2$ ($k = 1, \dots, 4$) exist. This reduces the dimensions of the set of X from $\dim(\mathbb{R}^{3 \times 4}) = 3 \times 4 = 12$ to $\dim(\mathbb{S}^{2 \times 4}) = 2 \times 4 = 8$.

The Euclidean distance among two consecutive light spots is given by

$$\begin{aligned} l_{kj}(X; C, R) &= \|\mathbf{s}_k - \mathbf{s}_j\| \\ &= \|r_k \mathbf{x}_k - r_j \mathbf{x}_j + \mathbf{c}_k - \mathbf{c}_j\|, \end{aligned}$$

being $\|\cdot\|$ the Euclidean norm in $\mathbb{R}^3$. The optical path length p and the modulus of the enclosed area A reads, respectively:

$$\begin{aligned} p(X; C, R) &= l_{12}(X; C, R) + l_{23}(X; C, R) \\ &+ l_{34}(X; C, R) + l_{41}(X; C, R), \end{aligned}$$

and

$$\begin{aligned} \|\mathbf{A}(X, C, R)\| &= \frac{1}{2} \|(\mathbf{s}_1 - \mathbf{s}_3) \wedge (\mathbf{s}_2 - \mathbf{s}_4)\| = \\ &= \frac{1}{2} l_{13}(X; C, R) l_{24}(X; C, R) \cos \vartheta, \end{aligned}$$

where ϑ is the angle between the vectors $\mathbf{s}_1 - \mathbf{s}_3$ and $\mathbf{s}_2 - \mathbf{s}_4$.

Our aim is to study the dependence between the geometrical path of the laser, represented by the spot position matrix S , and the geometry of the cavity, represented by the mirror configuration matrix C . As also remembered in [77], this idea is well expressed by the *Principle of Least Action* [78]. Formulated in 1662 by Pierre de Fermat, it is based on the assumptions of Heron from Alexandria, about the hypothesis that light takes the shortest path between two points, and of the arab Alhazen, about the fact that light moves with finite speed. This principle stated that the path of a light ray connecting two points is the one for which the time of transit is a minimum. Light travels along

paths of stationary optical path length (where the optical path length is a local maximum or minimum, depending on the refractive index of the medium along it propagates) with respect to any small variation. This means that any optical medium and any optical element, as a mirror in a ring laser, establishes a geometry. In the formalism we developed, the connection between geometry and optics is given by the directional cosine matrix $X(C, R)$. For a given mirrors configuration C and curvature radii R , the effective $X(C, R)^*$ is found by using the Fermat's principle, that is

$$\nabla p(X; C, R) = \mathbf{0} , \quad (2.1)$$

where $\nabla(\cdot)$ is the gradient with respect to a coordinate set of $X \in \mathbb{S}^{2 \times 4}$. The vector components are referred to a right-handed orthonormal coordinate system $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, with the origin in the center of the square, the axes $\mathbf{e}_1$ and $\mathbf{e}_2$ along the two diagonals, and $\mathbf{e}_3$ perpendicular to the square (see figure 2.1).

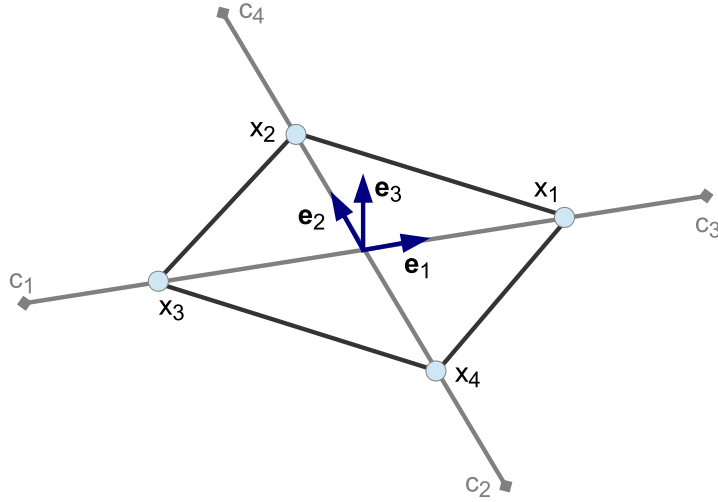


Fig. 2.1: Schematic of a square optical cavity. A right-handed orthonormal reference system has been chosen, with the origin in the center of the square, the axis $\mathbf{e}_1$ and $\mathbf{e}_2$ along the two diagonals, the axis $\mathbf{e}_3$ perpendicular to the square. $\mathbf{x}_k$: light spot position; $\mathbf{c}_k$: mirror center of curvature position.

In the case of a regular square cavity, with side length L and all the mirror curvature radii equal to r , the mirror configuration is represented by

the matrix:

$$C_0 = r \left(1 - \frac{L}{\sqrt{2}r} \right) \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (2.2)$$

and the corresponding solution of equation 2.1 for the spot positions is:

$$X_0 = \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (2.3)$$

We consider the case of a misalignment of the cavity mirrors by introducing a matrix $Z = X_0 + Y$, where the matrix Y is defined as follows:

$$Y = \begin{pmatrix} 0 & y_3 & 0 & y_7 \\ y_1 & 0 & y_5 & 0 \\ y_2 & y_4 & y_6 & y_8 \end{pmatrix}. \quad (2.4)$$

The matrix Z can be considered as a function of the vector $\mathbf{y} = (y_1, \dots, y_8)^T$, that represents the response of the optical cavity to a displacement δC of the centers of curvature from the ideal mirrors configuration. An analytic solution of equation 2.1 is not yet available, but for small mirror misalignments, as the case we are dealing with, the optical length can be expressed as second order Taylor expansion of $p(X; C, R)$ in a neighborhood of X_0 in the $\mathbf{y}$ variable:

$$p(X'(\mathbf{y}); C, R) = p(X_0; C, R) + \mathbf{s}(C, R)^T \mathbf{y} + \frac{1}{2} \mathbf{y}^T F(C, R) \mathbf{y} + o(\|\mathbf{y}\|^2). \quad (2.5)$$

Here, the vector $\mathbf{s}(C, R)$ is the gradient of the function p with respect to $\mathbf{y}$. The matrix $F(C, R)$ is the Hessian of p ; its inverse $F^{-1}(C, R)$ represents the ray matrix of the RL cavity. In this approximation, the stationary point of the system provided by equation 2.1 is

$$\mathbf{y}^* = -F(C, R)^{-1} \mathbf{s}(C, R).$$

Hence, the general expressions of optical length p^* and spot positions S^* as a

function of the mirrors configuration are:

$$p^*(C, R) = p(X_0; C, R) - \frac{\mathbf{s}(C, R)^T F^{-1}(C, R) \mathbf{s}(C, R)}{2}, \quad (2.6)$$

and

$$S(C, R)^* = X'(\mathbf{y}^*)R + C, \quad (2.7)$$

2.2.2 Effects of cavity misalignment on light path geometry

The system configuration is uniquely determined by twelve variables, that are the position coordinates of the centers of curvature of the four mirrors. In order to separate the rigid body movements from the deformation ones, we selected an orthogonal eigenvectors basis of the tangent space in C_0 parametrized by the columns of Y . We express the mirror configuration matrix for a general configuration of the cavity as

$$C(\tau_1, \dots, \tau_{12}) = C_0 + \sum_{\alpha=1}^{12} \tau_{\alpha} E_{\alpha}, \quad (2.8)$$

where E_{α} is the α_{th} element of the basis of the 3×4 matrices, and $\tau_{\alpha} \in \mathbb{R}$. In table 2.1 the matrices representing the eigenvector basis elements $E_{\alpha}(\alpha = 7, \dots, 12)$ of rigid body cavity movements are reported. By definition, to the first order in τ_{α} these does not affect the distances $l_{kj}(X; C, R)$ among the light spots, and their functions, as p and A . The deformation of the cavity is therefore expressed by $C(\tau_1, \dots, \tau_6)$. In table 2.2 the eigenvector basis elements $E_{\alpha}(\alpha = 1, \dots, 6)$ of non-rigidly body cavity deformations are listened; for each basis element, the matrix is reported and a graphical representation of its effect on the optical path geometry is shown.

The second order expansion of $p^*(C, R)$ in power series of the τ_{α} reads

$$p^*(C, R) = 4L \left[1 - \frac{\tau_1}{\sqrt{2}L} + \frac{\tau_2^2}{4L^2} - \frac{\tau_3^2 + \tau_4^2}{2L(\sqrt{2}r - 2L)} - \frac{\tau_6^2}{2L(2\sqrt{2}r - L)} \right] + o(\tau_{\alpha}^2). \quad (2.9)$$

Table 2.1: Eigenvector basis elements of rigid body ring cavity deformations. Each part of the table shows the eigenvector matrix related to the labelled degree of freedom. As said in the text, these does not affect the distances $l_{kj}(X; C, R)$ among the light spots.

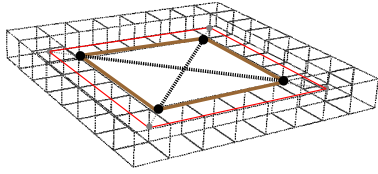
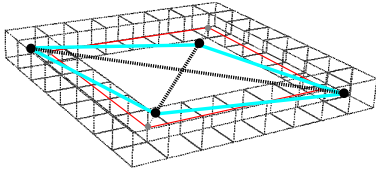
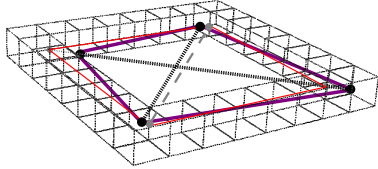
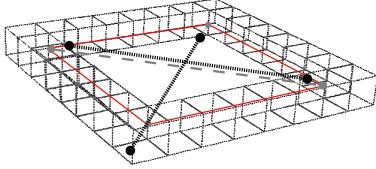
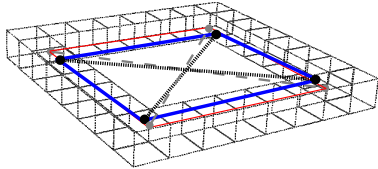
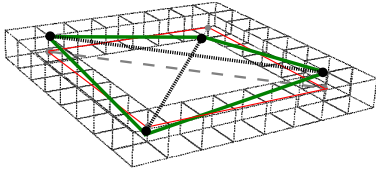
Rigid body ring cavity deformations	
<i>Rigid body translations</i>	<i>Infinitesimal rigid body rotations</i>
$E_7 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$E_{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$
$E_8 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$E_{11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$
$E_9 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$	$E_{12} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Combining equation 2.6 and equation 2.1, and assuming the cavity oriented along the direction of maximum rotational signal ($\mathbf{A} \parallel \mathbf{u}_\Omega$), the ring laser scale factor reads

$$\begin{aligned}
k_S^*(C, R) &\equiv \frac{4}{\lambda p^*(C, R)} \mathbf{A}^*(C, R) \cdot \mathbf{u}_\Omega \\
&= \frac{L}{\lambda} \left[1 - \frac{\tau_1}{\sqrt{2}L} + \frac{\tau_1^2}{2L^2} - \frac{\tau_2^2}{4L^2} - \frac{2L + \sqrt{2}r}{4L(r - \sqrt{2}L)^2} (\tau_3^2 + \tau_4^2) \right. \\
&\quad \left. - \frac{L + 2\sqrt{2}r}{L(4r - \sqrt{2}L)^2} \tau_6^2 \right] + o(\tau_\alpha^2) .
\end{aligned} \tag{2.10}$$

Equation 2.9 and equation 2.10 demonstrate that p^* and k_S^* depend linearly on the diagonal common stretching, while depend quadratically on differential stretching, shears and tilts.

Table 2.2: Eigenvector basis elements of non-rigid body ring cavity deformations. Each part of the table shows the eigenvector matrix (above) and the graphical representation of the optical paths (below) related to the labelled degree of freedom. The red line represents the optical path of the regular square cavity.

Non-rigid body ring cavity deformations	
<i>Common and differential stretching of the diagonals</i>	
$E_1 = \frac{1}{2} \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 	$E_2 = \frac{1}{2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 
<i>Shear planar deformations</i>	
$E_3 = \frac{1}{2} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 	$E_4 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 
<i>Diagonal tilts and out of plane motions</i>	
$E_5 = \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 	$E_6 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix}$ 

2.2.3 The diagonals stabilization constraint

Let's analyze what the constraint of equal diagonals involves. In this case, the distances $\|\mathbf{c}_3 - \mathbf{c}_1\|$ and $\|\mathbf{c}_4 - \mathbf{c}_2\|$ between the centers of curvature of the two pairs of opposite mirrors have to satisfy the condition

$$\|\mathbf{c}_3 - \mathbf{c}_1\| = \|\mathbf{c}_4 - \mathbf{c}_2\| = \sqrt{(2r - \sqrt{2}L + \tau_1 + \tau_2)^2 + \tau_5^2} = d_0, \quad (2.11)$$

being d_0 the diagonal length.

From equation 2.11 follows that $\tau_2 = 0$ and $\tau_1 = \tau_5^2/d_0$. Defining $\tilde{\tau}_\alpha = \tau_\alpha/d_0$, the expressions for the constrained perimeter and scale factor, normalized with respect to their nominal values of the square $4L$ and L/λ , can be rewritten as:

$$\begin{aligned} \hat{p}^*(C, R) &= 1 - \frac{h}{(\sqrt{2} - 2h)}(\tilde{\tau}_3^2 + \tilde{\tau}_4^2) - \tilde{\tau}_5^2 - \frac{\sqrt{2}h}{(4 - \sqrt{2}h)}\tilde{\tau}_6^2 \\ &= \sum_{\alpha=3,4,5,6} Q_\alpha \tilde{\tau}_\alpha^2, \end{aligned} \quad (2.12)$$

and

$$\begin{aligned} \hat{k}^*(C, R) &= 1 - \frac{h(2h + \sqrt{2})}{2(1 - \sqrt{2}h)^2}(\tilde{\tau}_3^2 + \tilde{\tau}_4^2) - \tilde{\tau}_5^2 - \frac{2h(h + 2\sqrt{2})}{(4 - 2\sqrt{2}h)^2}\tilde{\tau}_6^2 \\ &= \sum_{\alpha=3,4,5,6} S_\alpha \tilde{\tau}_\alpha^2, \end{aligned} \quad (2.13)$$

where $h = L/r$.

Let's observe that $\hat{p}^*(C, R)$ and $\hat{k}^*(C, R)$ are quadratic forms of $\tilde{\tau}_\alpha$ with no cross terms, and the regular square geometry corresponds to a saddle-point of $\hat{p}^*(C, R)$. The presence of a saddle-point demonstrates that, once the diagonal lengths are set to the same value, it is possible to approach the square configuration, by monitoring the perimeter length.

The dependence of perimeter and scale factor on h , expressed by equation 2.13 and equation 2.14, suggests that the design of a RL optical cavity plays an important role. In order to optimize the scale factor by monitoring the perimeter length, a good choice of h is that minimize the ratio S_α/Q_α . In fact, large values of Q_α guarantee a high sensitivity of the perimeter length

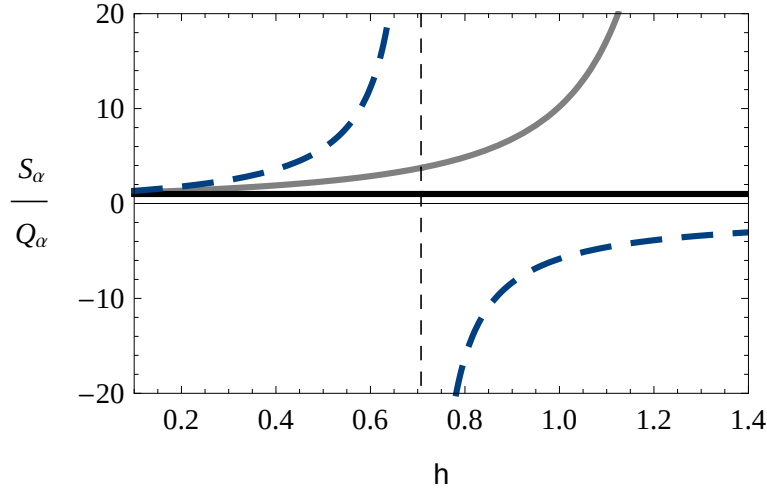


Fig. 2.2: Plot of the ratio of the second order expansion coefficients of the constrained scale factor S_α to that of the constrained perimeter Q_α as a function of the ratio h of the cavity side length to the mirror curvature radius. Blue dashed line: $\alpha = 3, 4$, shear planar deformations. Continuous black line: $\alpha = 5$, diagonal tilts. Continuous gray line: $\alpha = 6$, out of plane motions.

to deformations, while small values of S_α limit the sensitivity of the scale factor to residual deformations. A plot of S_α/Q_α as a function of h , in the range of cavity stability, reported in figure 2.2 shows that this condition is satisfied for $h < \sqrt{2}/2$, i.e. large radius of curvature are preferable. Values of h around $\sqrt{2}/2$ must be avoided; for $h = \sqrt{2}/2$ the cavity becomes instable, being confocal in the sagittal plane [69].

In figure 2.3 it is shown the simulated relative variation of the scale factor $k_S = 4A/(\lambda P)$ induced by a random displacement noise applied to the four mirrors centers of curvature. The simulation has been performed both for GP2 and the single-axis GINGER cavity. The position of each mirror has been changed along the three directions of space by three independent random processes with root-mean-square amplitude δ ; the case of a cavity with equal absolute diagonal lengths is compared to the uncontrolled case, and to the case where the two lengths are both stabilized better than 1 part on 10^{11} but differ one from the other by a quantity δD . The simulation shows the meaning of equation 2.10: since the scale factor depends linearly only on the cavity isotropic expansion E_1 , once the diagonal lengths are set to the same

value, the effects of the remaining deformations on k_S^* are quadratic in their amplitudes τ_α . Figure 2.4 shows the corresponding 3D plot. What is evidenced is that, even in the case of a systematic error in the difference between the two diagonal cavities lengths of 10^{-6} m, the scale factor stability is within the region of $\simeq 10^{-10}$. This is compliant with the requirements of GINGER.

2.3 Pre-alignment of a square optical cavity

The motivation of the theoretical analysis I reported in this chapter is to find a control strategy for the geometry of a square ring cavity which could meet the specifications of the GINGER project. Hence, the purpose has been to find a suitable set of observables in order to implement a precise active stabilization in a RL equipped with a piezo nano-positioning system, as GP2 (see chapter 4). However, the developed model of the cavity provides an useful tool for the pre-alignment of any square optical cavity. The scope of this operation is to place the positions of the four laser spots in the centers of the four mirror surfaces.

It is not uncommon in fact, above all at the starting operations of a newly installed RL, that the laser spots are not in the centers of the mirrors. In this case, acting on screws moving mirrors, the configuration of the spot positions is rearranged implementing an iterating procedure. This could be not an easy task, since acting on each mirror causes the repositioning of the spot on all four of them. All of the four mirrors of the ring have a finite transmission and, typically, all of the eight output beams are analyzed for different purposes (clock- and counterclock-wise beam combining, power monitors, laser emission frequency analysis). Operating the ring laser with centered mirrors is essential to guarantee a clean transmission of the laser. In this way, possible diaphragm effects introduced by the second surface mirror or by the holders frame are avoided. In addition, the surface specifications, as cleanliness and roughness, are typically guaranteed in a clear aperture of about central $\sim 25\%$ of mirror diameter.

Basing on the formalism reported in section 2.2, I have written a @Mathematica code that receives as input the position of the laser spots on the mirrors, and calculates as output the corrections to the curvature centers needed

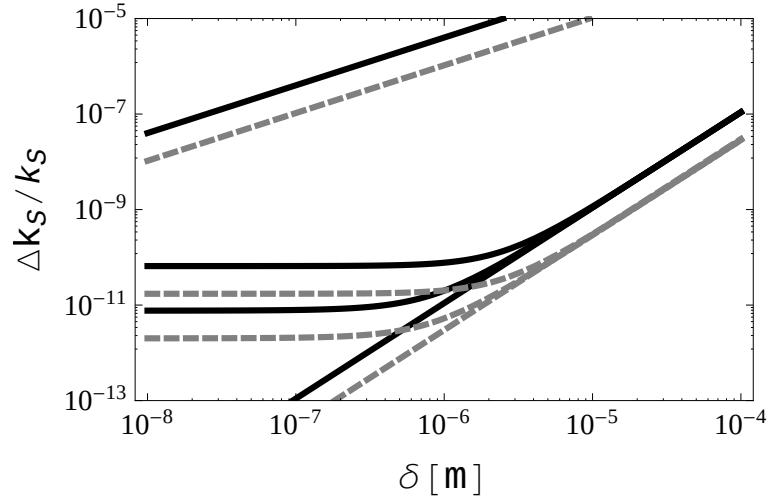


Fig. 2.3: Relative scale factor variations induced in GP2 and GINGER ring laser by a displacement noise with root mean square amplitude δ acting independently on the four mirrors positions. The cavities are simulated: GP2 cavity ($L = 1.60$ m, $r = 4$ m; continuous black line); GINGER single-axis cavity ($L = 6$ m, $r = 6$ m; dashed line). For each cavity, the different plotted curves correspond to the following cases (from top to bottom): free geometry; diagonal lengths stabilized but unbalanced of 10^{-6} m; diagonal lengths stabilized but unbalanced of 10^{-7} m; equal absolute diagonal lengths.

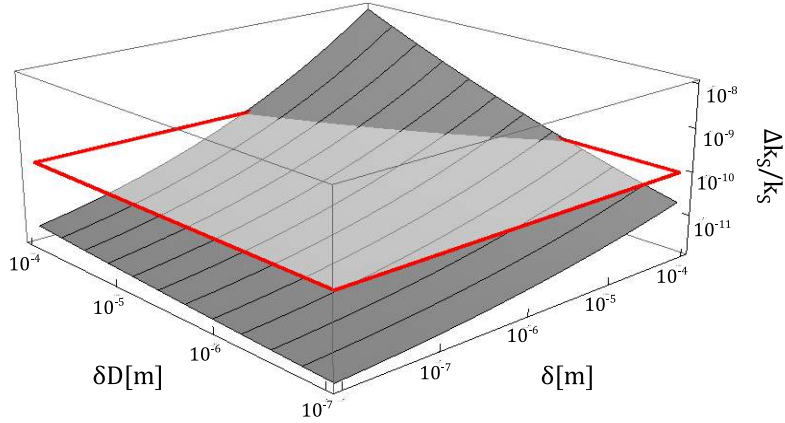


Fig. 2.4: Relative variation of the ring laser scale factor in function of the standard deviation δ of the random displacements applied independently to each mirror position and the unbalance δD between the two diagonal lengths. The plane plotted with a red contour delimits the region where the scale factor is $\Delta k_S/k_S = 10^{-10}$, that is the stability required for GINGER.

to align the cavity geometry, acting on screws tilting the mirrors. The procedure, described in the following, has been successfully implemented to align the optical cavity of GINGERino at LNGS. In figure 2.5 a picture of the laser spots observed on the cavity mirrors in GINGERino, at its starting condition, is shown. In a GEOsensor, the mirrors can be tilted to align the cavity by

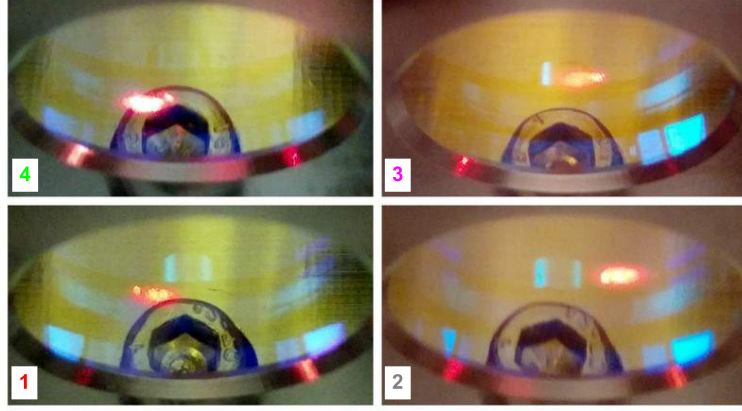


Fig. 2.5: Picture of laser spots observed on the cavity mirrors in GINGERino ring laser at its starting condition.

means of levers manually controlled by micrometric screws (figure 2.6). Each mirror has two tilting d.o.f.. It is convenient to introduce, for each vertex of the regular square, a left-handed orthonormal coordinate system $\tilde{\mathbf{K}}_k$ ($k = 1, \dots, 4$) with $\tilde{\mathbf{x}}_k$ and $\tilde{\mathbf{y}}_k$ horizontal axes, and $\tilde{\mathbf{z}}_k$ vertical axis (figure 2.7). We require $\tilde{\mathbf{y}}_k$ points along the diagonal itself toward the square centre. The vector $(\tilde{x}_k, \tilde{z}_k)$ identifies the position of the light spot on the surface of the k_{th} mirror. Let's denote by $\mathbf{s}_{k,b}$ the spot position in the reference system $\mathbf{K}_b$, introduced in 2.2.1, and by $\tilde{\mathbf{s}}_k = (\tilde{x}_k, \tilde{y}_k, \tilde{z}_k)$ the position in $\tilde{\mathbf{K}}_k$. The transformation law from one reference to the other is given by

$$\mathbf{s}_{k,b} = \mathbf{R}^{k-1} \cdot \left(-\frac{L}{\sqrt{2}} \tilde{\mathbf{y}}_k + \tilde{\mathbf{s}}_k \right), \quad (2.14)$$

with $\mathbf{R}$ the $\pi/2$ counter-clockwise rotation matrix:

$$\mathbf{R} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.15)$$

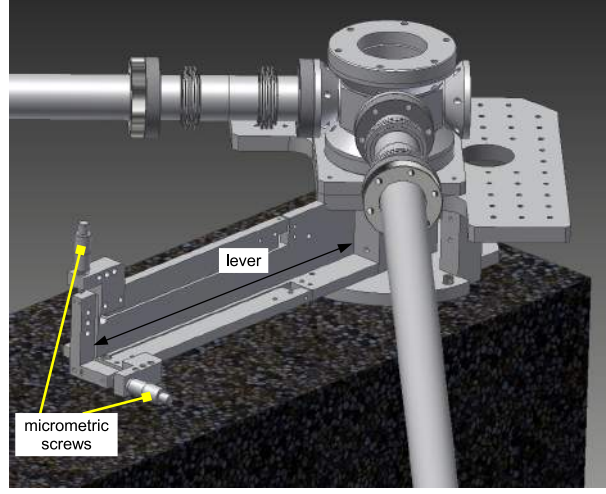


Fig. 2.6: Detail of the lever system, controlled by micrometric screws, used to align the ring cavity of a GEOSensor.

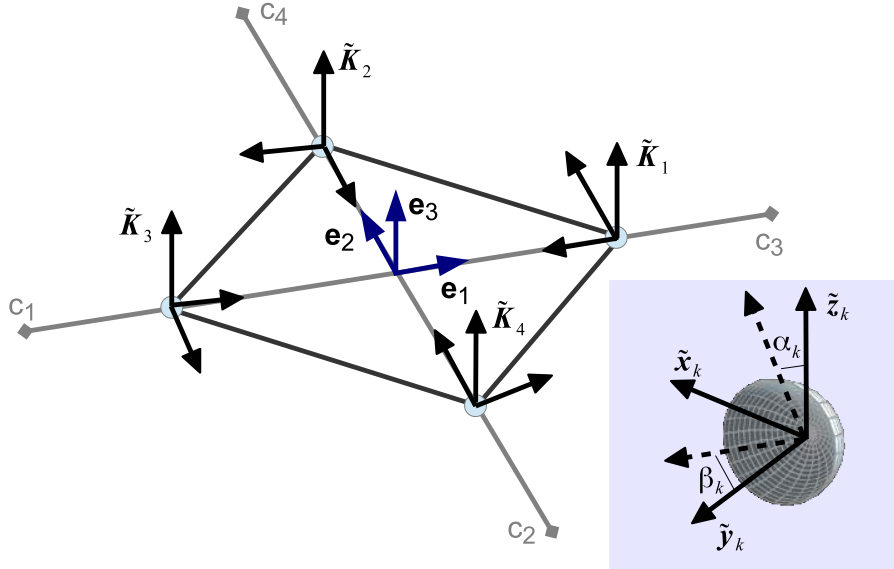


Fig. 2.7: Schematic representation of the four reference systems $\tilde{K}_k$ ($k = 1, \dots, 4$) located at the vertices of the regular square. The baricentric reference system $\mathbf{K}_b = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is also shown. A schematic representation of the two tilting d.o.f.s of each cavity mirror is shown on the right.

With reference to figure 2.7, I label α_k the rotation angle along $\tilde{\mathbf{y}}_k$ -axis and β_k the rotation angle along $\tilde{\mathbf{z}}_k$ -axis. The position $(\tilde{x}_k, \tilde{y}_k)$ of the light spot

on the surface of k_{th} mirror, with radius of curvature r , is:

$$\begin{pmatrix} \tilde{x}_k \\ \tilde{z}_k \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & -r \end{pmatrix} \times \begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} \quad (2.16)$$

where I have used the small angles approximation.

Let's introduce the matrix M_s and M_c ; their components are the light spot and center of curvature positions expressed in the local reference frames:

$$\begin{aligned} M_s &= [\tilde{x}_1, \tilde{z}_1, \tilde{x}_2, \tilde{z}_2, \tilde{x}_3, \tilde{z}_3, \tilde{x}_4, \tilde{z}_4], \\ M_c &= [x_{c1}, z_{c1}, x_{c2}, z_{c2}, x_{c3}, z_{c3}, x_{c4}, z_{c4}]. \end{aligned}$$

The following equation applies:

$$M_s = M \times M_c, \quad (2.17)$$

where M is a symmetric matrix, whose six independent coefficients ς_j ($j = 1, \dots, 6$), being function of the side length of the cavity and of the curvature radius of the mirror, are calculated for a defined geometry.

In figure 2.8 the user interface of the @Mathematica program is reported. The program receives as input the spot position on the four surface mirrors, and gives as output the correction to apply to the micrometric screws, in order to implement the needed displacement of the centers of curvature positions. The light spot and center of curvature configurations, corresponding to a given input, are visualized on a circle of diameter one inch, that is the mirror diameter. An animation option allows also to visualize the two configurations after that the correction is applied to one mirror at a time.

The procedure is the following:

- the mirrors are coarsely tilted in order to make the ring cavity resonant and to observe the laser spots on their surfaces; this is typically made with the help of a green laser injected from the window of one of the four corners, tilting the mirrors to close the green laser beam path on itself;
- from the observation of the laser spot positions M_s , the positions of

2.3. Pre-alignment of a square optical cavity

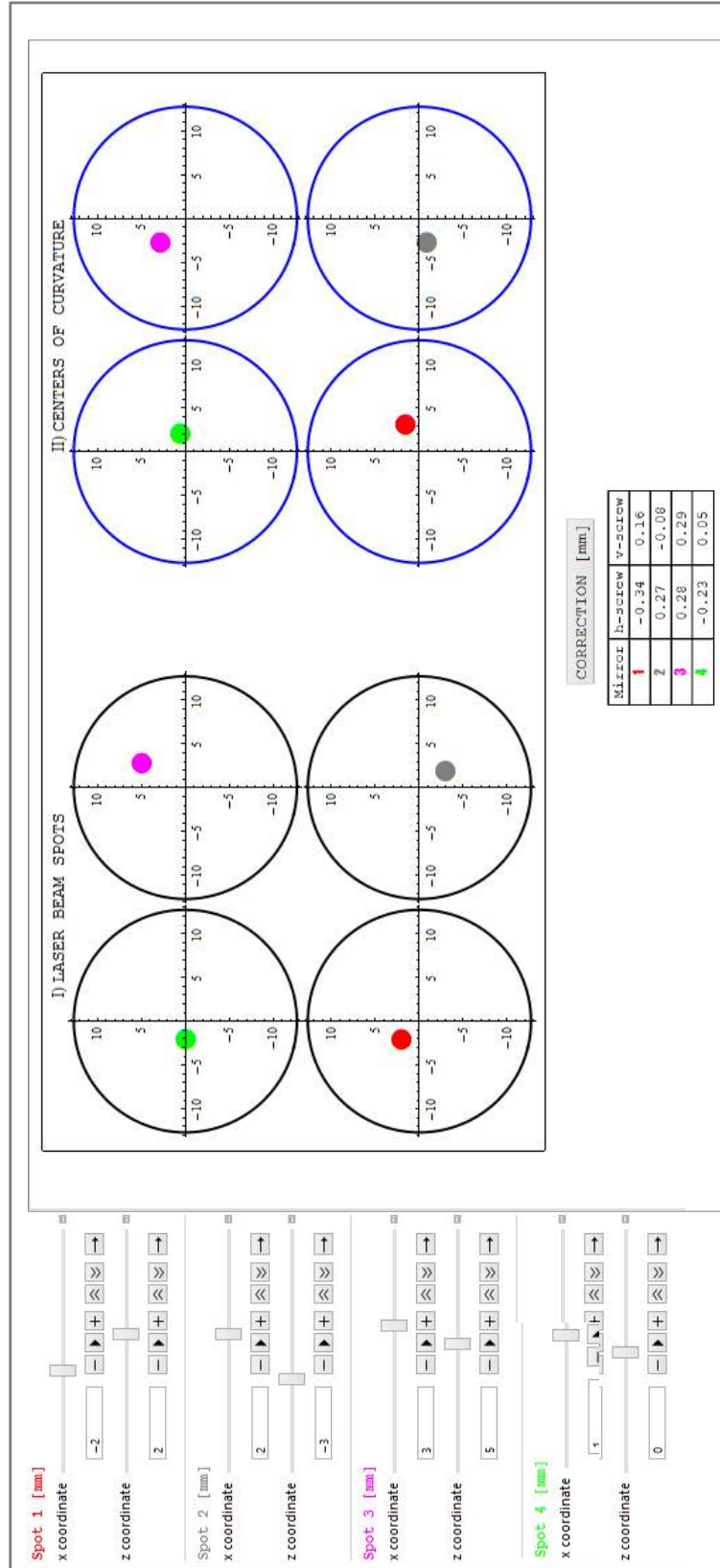


Fig. 2.8: Interface of @Mathematica program to align a square cavity RL.

centers of curvature M_c are calculated using the matrix equation

$$M_c = M^{-1} \times M_s; \quad (2.18)$$

- the correction to apply to mirrors is determined by using the driving matrix

$$M_{driv} = -\frac{l}{R}(M_d \times M_c), \quad (2.19)$$

where l is the length of the lever controlled by screws ;

- finally, the new spot M'_s and mirror positions M'_c can be calculated:

$$\begin{aligned} M'_s &= M_s - M \times M_c \\ M'_c &= M^{-1} \times M'_s. \end{aligned}$$

If the obtained configuration does not correspond to the geometry of the regular square cavity, the optimization procedure is iterated.

The performance of the method can be improved measuring the position of the laser spot, here rough estimated, by a position sensor, as a four-quadrant photodetector or a CCD. These provide an high-precision measurement of the deviation of the spot position with a measurement error of few μm . Experiments of this kind have been done in the past on UG-2 ring laser, by installing video cameras in the output beams at all the corners of the device, in order to monitor changes in positions of the emerging beams [79].

CHAPTER 3

Diagonals and perimeter length measurement - Theory

In this chapter I discuss about the high-accuracy measurement of the diagonals and perimeter lengths of a RL.

In section 3.1, I describe the theoretical principle for the absolute length measurement of a linear optical resonator. I report an estimation of the systematic errors that could limit the accuracy on this measurement and, above all, on the length difference between two linear resonators. In section 3.2, I briefly discuss about the measurement of the ring cavity perimeter, whose control is obtained by comparing the ring laser emission optical frequency with a frequency-stabilized reference laser. A review of the methods used for high-precision absolute length metrology, based on one-wavelength interferometry, are briefly reviewed in section 3.3. Finally, in section 3.4 I present the phase-modulation model we developed and implemented to measure and control the length of two passive linear resonators by using an external laser source.

3.1 Diagonals measurement

3.1.1 Length measurement of a linear resonator

Let's consider a laser beam resonating in a linear optical resonator (i.e. Fabry-Perot resonator) made of two identical spherical mirrors with radius of curvature R . The eigenfunctions of the stationary waves reflecting between the two mirrors are called *modes* of the resonator [80, 81]. They are solution of the paraxial Helmholtz equation:

$$\nabla^2 U(\mathbf{r}) + k^2 U(\mathbf{r}) = 0, \quad (3.1)$$

where $U(\mathbf{r}) = A(\mathbf{r})\exp(-jkz)$ is the complex amplitude of the wave with wavelength λ , and $k = 2\pi f/v$ is the wavevector, being $v = c/\mu$ the speed of light in the medium filling the cavity and μ its refractive index. In the paraxial approximation $A(\mathbf{r})$ is assumed to be a slowly function of position, so that the envelope is constant within a region of size λ .

The eigenfunctions of the linear resonator, as considered here, are described by the Hermite-Gaussian modes. These exhibit a non-Gaussian intensity distribution, but with the same wavefronts and angular divergence as the Gaussian beam [83]. In figure 3.1 a graph of the Gaussian beam profile resonating in a linear cavity is reported. The apex of the two spherical mirrors that forms the cavity are located at z_1 and z_2 along the z -axis, that is the propagation axis, so that their separation is $z_2 - z_1 = L$.

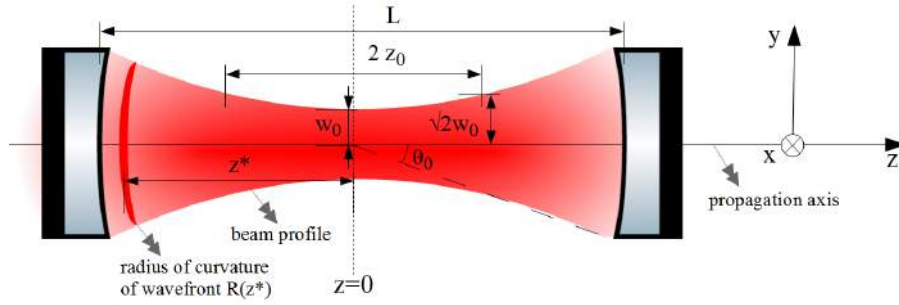


Fig. 3.1: Graph of the Gaussian beam profile resonating between two equal identical mirrors. At each point along the propagation axis z the optical intensity is a function of the radial distance $\rho = \sqrt{x^2 + y^2}$. As usual, the origin of the z -axis is at half length of the resonator, where the beam has its minimum width w_0 , called *waist radius*, and its best focus. L : length of the linear resonator. $2z_0$: *depth of focus* or *confocal parameter*, that is the axial distance within which the beam radius lies within a factor $\sqrt{2}$ of its minimum value. θ_0 : divergence angle. $R(z^*)$: wavefront radius of curvature at z^* .

The phase of the Hermite-Gaussian mode of order $(0,0)$ (TEM_{00}) propagating along the optical axis is [83]:

$$\varphi(\varrho, z) = kz - \xi(z) + \frac{k\varrho^2}{2R(z)}, \quad (3.2)$$

where: $\xi(z) = \tan^{-1}(z/z_0)$ is a phase retardation term corresponding to an

excess of delay of the wavefront in comparison with a plane or a spherical wave (*Gouy effect*) [84, 85], being z_0 the Rayleigh length; $k\rho^2/2R(z)$ represents the deviation of the phase at off-axis points in a given transverse plane from that at the axial point (*wavefront bending*), where $R(z)$ measures the wavefront radius of curvature and $\rho = \sqrt{x^2 + y^2}$ the radial distance.

Since the mirror surfaces coincide with the wavefronts, all points on each of the two mirrors share the same phase, $\varphi(0, z_1)$ and $\varphi(0, z_2)$ respectively. The resonance condition implies that the phase shift after a single round-trip of propagation must be a multiple of 2π :

$$2k_n L - \Psi_R - \Phi_n = 2\pi n, \quad (3.3)$$

where $k_n = 2\pi f_n \mu / c$ defines the resonance frequencies f_n , $\Psi_R = 2[\xi(z_2) - \xi(z_1)]$ is the *Gouy phase-shift* per round trip, while Φ_n takes into account the phase shift upon reflection from the dielectric mirrors per round trip (*dielectric phase-shift*) [86]. The positive integer n is called the *mode number*.

From equation 3.3 the length separation between the cavity mirrors can be written as

$$L = \frac{c}{2\mu f_n} \left[n + \frac{\Psi_R + \Phi_n}{2\pi} \right]. \quad (3.4)$$

The intracavity optical length L can be measured with an accuracy ultimately limited by the knowledge of the reference laser frequency if the cavity is locked to the laser and the mode number n is estimated together with Ψ_R and Φ_n .

3.1.2 Systematic measurement errors

The systematic errors that can change the cavity length are due to Gouy phase-shift, dielectric mirror phase shift and refractive index correction.

- *Gouy phase shift.* The Gouy phase shift Ψ_R is a function of the cavity geometry, and for a two-mirror Fabry-Perot cavity made of equal concave spherical mirrors, can be written as

$$\Psi_R = 2 \arccos(g), \quad (3.5)$$

where $g = 1 - (L/R)$ is the cavity g-factor, being R the mirror curvature radius.

For a stable resonator, the condition $g^2 \leq 1$ applies [83], and the Gouy term $\Psi_R/2\pi$ is always less than 1. The Gouy phase changes the length of the linear resonator by a factor $\Psi_R/2\pi(L/\lambda)$, being $2\pi(L/\lambda)$ the round trip length measured in phase. In the case of the diagonals cavities of GP2 ($L = 2.26$ m), the relative length change is of $\delta L/L = -1.52 \times 10^{-7}$. The uncertainty on the cavity length change ε is set by the accuracy on the knowledge of the curvature radius R . For typical ring laser supermirrors, R is given with 1% accuracy, that means, for GP2, an uncertainty $\varepsilon \sim 10^{-9}$. If needed, it is possible to measure R with a much higher accuracy. Clearly, this measurement error is reduced in the case of a large RL, as GINGER, since the relative length change and the uncertainty on it are proportional, respectively, to $1/\sqrt{L}$ and $1/L^2$.

- *Dielectric mirror phase shift.* The dielectric phase shift Φ_n is a function of the mirror coating fabrication properties. In principle, the dielectric phase shift term Φ_n can be evaluated from the reflectivity curve of the mirror [87] and then subtracted to the length measurement. The coating of the typical supermirrors considered here is made of a stack of alternating tantalum pentoxide (Ta_2O_5 , $\mu_H = 2.0411$) and silicon dioxide (SiO_2 , $\mu_L = 1.455$) dielectric layers. As discussed in reference [87], near the design wavelength, as in our case, the cavity length change due to coating is $\lambda/[2(\mu_H - \mu_L)]$ that, for a He-Ne RL operating at $\lambda = 633$ nm, implies a relative change of $5.4 \times 10^{-7}/L$. The uncertainty on this estimation is set by the time-dependent stability of the coating layer. Sources of fluctuations are the environmental instabilities (temperature, pressure, humidity), and the material aging, that causes a long-term length contraction, with a rate that is a function of the time, decreasing as the material gets old. The mirrors considered here are produced by the Ion-Beam Sputtering, that provides very stable amorphous coatings versus time and environment. The variations of these dielectric constants are within 0.5%, that implies an uncertainty $\varepsilon \sim 2 \times 10^{-9}/L$. As regards the material aging, in the literature [86], an aging rate of low-expansion glasses of $\sim \text{few ppbyr}^{-1}$ is reported.

- *Refractive index correction.* The refractive index μ of the medium filling the cavity introduces a correction on the measured length with respect to the case of an empty cavity with length L_0 . By using the Edlén equation [88, 89] or, as suggested in [90], the NIST calculation program [91] based on the Ciddor equation [92], the refractive index can be calculated as a function of various input parameters (as wavelength, temperature, pressure,...) with an accuracy of few parts in 10^{-8} . For a gas mixture of He-Ne at room temperature and at the pressure of 0.2 mbar Ne and 3.3 mbar He, in the perfect gas approximation, the refractive index correction respect to an empty vacuum cavity is $\mu_{HeNe} = 1 + 6 \times 10^{-8}$ [93, 94]. Hence, the correction to cavity length is $\delta L/L_0 = (\mu_{HeNe} - 1)/\mu_{HeNe} = 6 \times 10^{-8}$. The uncertainty on this estimation depends mainly on the environmental conditions, and is expected to be less than 1 ppb in a laboratory with a close temperature control.

3.1.3 Length difference between two linear resonators

Let's consider now the accuracy on the estimate of the length difference between two linear resonators. In order to measure L with high accuracy, one needs to measure the cavity resonance $f_n = c/\mu\lambda_n$ with the same accuracy and define univocally the integer n . This can be done by locking f_n to a reference laser having frequency $\omega_0/2\pi$, and measuring the free spectral range $FSR = c/(2\mu L)$ with the required accuracy. Once the two cavity resonance frequencies are locked to the laser frequency, the lengths of the two cavities can be rewritten as

$$\begin{aligned} L_1 &= \frac{\pi c}{\mu_1 \omega_0} \left[n_1 + \frac{\Psi_{R,1} + \Phi_{n,1}}{2\pi} \right] \\ L_2 &= \frac{\pi c}{\mu_2 \omega_0} \left[n_2 + \frac{\Psi_{R,2} + \Phi_{n,2}}{2\pi} \right], \end{aligned}$$

hence the mode numbers are:

$$n_i = \frac{\omega_0}{2\pi FSR_i} - \Upsilon_i, \quad (3.6)$$

where $\Upsilon_i = (\Psi_{R,i} + \Phi_i)/2\pi$.

Since the phase-shift correction Υ_i can be calculated and is typically less than 1, the integer mode number is univocally determined if the FSR_i is measured with an uncertainty better than $1/n_i$.

The length difference is

$$\begin{aligned} D &= L_2 - L_1 \\ &= \frac{\pi c}{\omega_0} \left[\left(\frac{n_2}{\mu_2} - \frac{n_1}{\mu_1} \right) + \left(\frac{\Upsilon_2}{\mu_2} - \frac{\Upsilon_1}{\mu_1} \right) \right]. \end{aligned}$$

In the case of two identical cavities filled with the same medium, one can state that the refractive indices μ_1, μ_2 are slightly different:

$$\begin{aligned} \mu_1 &= \mu \\ \mu_2 &= \mu + \delta\mu. \end{aligned}$$

By expanding around $\delta\mu = 0$, it gives

$$D \simeq \frac{\pi c}{\omega_0} \left[\left(\frac{n_2 - n_1}{\mu} \right) + \frac{n_2}{\mu^2} \delta\mu + \frac{\Upsilon_2 - \Upsilon_1}{\mu} \right]. \quad (3.7)$$

In this work, we estimate the length difference between two cavities having mirrors with the same design and produced in the same run of coating process. In this case, assuming that the dielectric shift and the Gouy phase-shift terms are equal for the two cavities, their difference does not contribute to the overall length uncertainty, so that $\Upsilon_2 - \Upsilon_1 = 0$. In addition, the mode numbers n_1 and n_2 , that are integers univocally determined by using equation 3.6, must be equal, so that $n_2 - n_1 = 0$. Hence, the length difference between the two cavities is

$$D \simeq \frac{c\pi n}{\omega_0} \frac{\delta\mu}{\mu^2}. \quad (3.8)$$

The limiting factor to length measurement accuracy could be given by the difference of the refractive indices of the two cavities. It depends on environmental parameters over the optical path, primarily the temperature. For a temperature difference of 0.5 °C between the two gas media filling the cavities we have a change of 2×10^{-10} in μ_{HeNe} . In the case of the diagonal cavities of GP2 ($n \simeq 7.15 \times 10^6$, $\omega_0 = 474$ THz), this sets the uncertainty on the length

difference to 10^{-10} , that is compliant with the needed scale factor stability. Clearly, a lower uncertainty is attainable in a laboratory equipped with a temperature stabilization system or, as expected for GINGER, in an underground environment with a natural stable temperature.

3.2 Perimeter measurement of a ring laser resonator

The control of the ring cavity perimeter length is obtained, as said in chapter 1, by comparing the ring laser emission optical frequency with a He-Ne frequency-stabilized reference laser. In the case of a ring cavity, to estimate the shift of the emission frequency we have to take into account, in addition to the effects discussed in the section 3.1.2 for the linear resonators, the *gain medium dispersion* and the *non-reciprocal effects*, due to the violation of the spatial invariance of the propagation of waves travelling in opposite directions [95]. The non-reciprocal effects, as for the reciprocal ones, can be calculated or estimated, and are expected to be not a limitation to the feasibility of the GINGER experiment.

The knowledge of the absolute perimeter of the ring cavity is not strictly required for the control of the RL geometry as here proposed. Actually, as discussed in chapter 2, our strategy consists into exploiting the fact that, once the diagonals stabilization is enabled, the regular square geometry corresponds to a saddle point of the perimeter. Hence, it's possible to implement an experimental procedure to lock the perimeter length to the saddle point.

3.3 Absolute length measurement experimental techniques

In the recent literature several techniques have been proposed for high-precision absolute length metrology. The large number of available methods, and of possible applications in different research area, makes difficult an exhaustive review and a comparison between the different approaches. Here, I

focus the attention on the one-wavelength interferometry for FSR measurement of long linear cavities involving frequency modulation. The technique we developed fits in this framework. A different approach consists in using multi-wavelength interferometry techniques. In this last case, absolute distance measurements, with demonstrated accuracy of 10^{-9} , are performed by use of a laser whose emission frequency can be tuned continuously. Diode lasers [96, 97] or CO_2 lasers are typically used [98, 99]. Multi-wavelength interferometry has become an active research area especially thanks to the development of *optical frequency comb* technology and its application to length metrology [100–103].

A new scientific impulse in one-wavelength interferometry, particularly in the field of gravitational wave detectors, led in recent years to the development of several strategies for the absolute length measurement of a linear optical cavity. These employ a single laser source, frequency stabilized to an optical reference standard and modulated by a combination of electro-optic modulators (EOM) and acousto-optic modulators (AOM).

Araya et al. [104] measured the absolute length of an approximately 300 m long FP cavity by FSR measurement, using two EOM for a double-frequency modulation of the laser beam, obtaining a measurement uncertainty of 10^{-8} . In a subsequent work [105], they applied this technique to the development of an highly accurate laser strainmeter for geophysical observations. It features the precise length measurement of a 100 m long optical cavity with reference to a standard rubidium atomic clock. Aketagawa et al. [106, 107] worked out a single-frequency modulation technique to measure with an uncertainty of 10^{-6} the FSR of an approximately 0.170 m long cavity. The technique makes use of a single EOM and the *null method*, that is based on the fact that the demodulation of the transmitted light signal by a lock-in amplifier become null when the modulation frequency equals the FSR, under the condition that the carrier frequency is slightly detuned from the resonance of the FP cavity.

Ozdur et al. [108] measured the FSR of a ~ 10 GHz FP etalon using a modified Pound-Drever-Hall (PDH) method with an uncertainty of 10^{-6} or less. The same group, in another work [109], obtained a measurement uncertainty of 10^{-8} or less by improving the technique making use of a narrow linewidth (1 kHz) laser swept in frequency via an AOM. Stochino et al. [110] developed

a new interferometric technique to measure both the FSR and the Transverse Mode Spacing of optical cavities of various lengths. The technique, tested on the optical cavities forming the LIGO 40 m gravitational wave-detector interferometer at California Institute of Technology, allowed the measurement of the cavity length with a precision of 1 ppm. A better result has been obtained in LIGO very recently by Staley et al. [111] that, using light doubly phase modulated via a single EOM, have measured the cavity length to a precision of 70 ppb.

An important application of the FSR measurement involving a double-frequency modulation scheme is the NICE-OHMS technique, acronym of noise-immune cavity enhanced optical heterodyne molecular spectroscopy. It was originally developed at JILA in Boulder, USA [112–114] for frequency standard applications, in particular for absolute frequency stabilization of near-infrared solid state lasers [115]. Today, it is widely used for detection of molecular compound in gas phase [116, 117], thanks to the fact that it combines frequency modulation (for reduction of noise) with cavity enhancement (for prolongation of the interaction length between a sample and the light).

3.4 Diagonals length measurement model

As seen in 3.1, the cavity length L can be measured with an accuracy ultimately limited by the knowledge of the reference laser frequency if the resonance frequency is measured with the same accuracy and the integer number n is univocally estimated. Our technique makes use of two *error signals* in order to implement a two frequency measurement in closed loop. The first error signal locks the cavity resonance to the laser carrier frequency, while the second one locks the phase modulation frequency to a harmonic of the FSR, in order to measure it with a precision better than $1/n$. This can be done phase-modulating the interrogating laser source, so that the light power is divided from one frequency component, the *carrier*, in a set of *sidebands*.

In the phase-modulation scheme we used, the laser electric field injected into the cavities has the form:

$$E_{in}(t) = E_0 e^{i\{\omega_0 t + \alpha \sin(\omega_A t) + \beta \sin[(\omega_B + \Delta \sin(\omega_C t))t]\}}, \quad (3.9)$$

where:

- ω_0 is the optical frequency;
- $\Delta\omega_{cav} < \omega_A < FSR$ is the Pound-Drever-Hall modulation frequency [118, 119] for the carrier lock, where $\Delta\omega_{cav}$ is the cavity linewidth;
- $\omega_B = mFSR$ is the modulation frequency for the cavity dynamic resonance excitation;
- α and β are the modulation indices for the modulations at ω_A and ω_B ;
- ω_c is the frequency of the dithering applied to ω_B for the low-frequency conversion and lock-in detection of the FSR resonance signal;
- Δ is the frequency amplitude of the dithering at ω_C .

The equation 3.9 can be expanded using the identity

$$e^{ix\sin\phi} = \sum_{k=-\infty}^{+\infty} (-1)^k J_k(x) e^{ik\phi}, \quad (3.10)$$

where $J_n(x)$ are the Bessel functions of the first kind.

Hence, we can write

$$E_{in}(t) = E_0 e^{i\omega_0 t} \left[(-1)^p \sum_p J_p(\alpha) e^{ip\omega_A t} \right] \left[(-1)^q \sum_q J_q(\beta) e^{iq(\omega_B + \Delta \sin \omega_C t)t} \right]. \quad (3.11)$$

As the equation 3.11 shows, the phase modulation creates an infinite number of sidebands. However, for small modulation indices ($\alpha, \beta < 1$) the Bessel functions rapidly decrease with increasing k . For small values of α and β , almost all the power is in the carrier and the first order sidebands, and hence the first sidebands only of the modulated spectrum ($p = -1, 0, 1$ and $q = -1, 0, 1$) can be considered. Thus, the incident field can be written in terms of 9 modes, i.e. the carrier optical frequency and two group of four sidebands, as shown in figure 3.2.

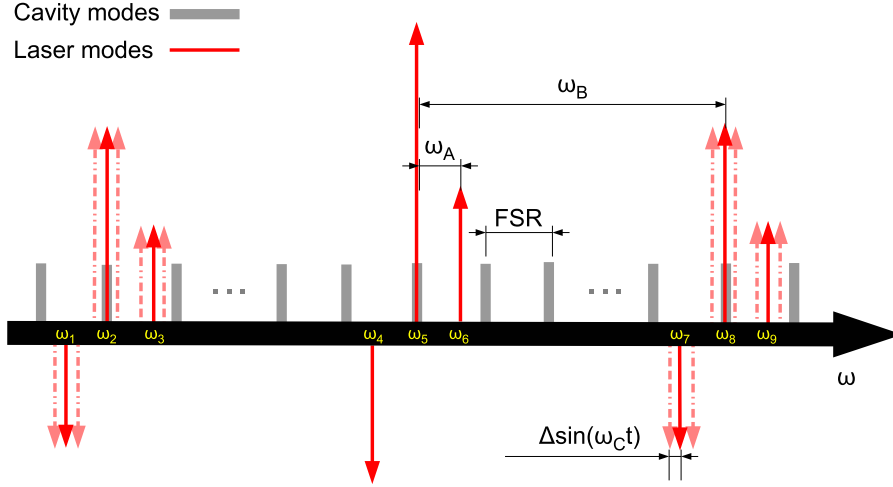


Fig. 3.2: Schematic diagram of the cavity modes and laser spectral structure in the developed phase-modulation scheme. Gray line: cavity modes, spaced by the free spectral range (FSR). Red line: laser modes resulting from the triple frequency modulation.

Within this representation we can write:

$$E_{in}(t) = E_0 \sum_{k=1}^9 A_k e^{i\omega_k t}, \quad (3.12)$$

where the 9 pairs of A_k , ω_k are defined in table 3.1.

The component amplitudes $J_0(\alpha)J_0(\beta)$, $J_1(\alpha)J_0(\beta)$ and $J_1(\alpha)J_1(\beta)$ are shown in figure 3.3.

The reflected field $E_r(t)$ is obtained by applying to each of the k_{th} field components the cavity reflection transfer function $R(\omega_k)$, defined as:

$$R(\omega) = \frac{\sqrt{r_{in}^2 r_{out}^2} (e^{i\omega/FSR} - 1)}{1 - r_{in}^2 r_{out}^2 e^{i\omega/FSR}}, \quad (3.13)$$

where r_{in} , r_{out} are the amplitude reflection coefficients for the input/output mirror and ω the laser frequency.

The reflected field is:

$$E_r(t) = E_0 \sum_{k=1}^9 A_k R(\omega_k) e^{i\omega_k t}. \quad (3.14)$$

Table 3.1: Amplitudes and frequencies of the electric field spectral components.

k	A_k	ω_k
1	$J_1(\alpha)J_1(\beta)$	$\omega_0 - (\omega_B + \Delta \sin \omega_C t) - \omega_A$
2	$-J_0(\alpha)J_1(\beta)$	$\omega_0 - (\omega_B + \Delta \sin \omega_C t)$
3	$-J_1(\alpha)J_1(\beta)$	$\omega_0 - (\omega_B + \Delta \sin \omega_C t) + \omega_A$
4	$-J_1(\alpha)J_0(\beta)$	$\omega_0 - \omega_A$
5	$J_0(\alpha)J_0(\beta)$	ω_0
6	$J_1(\alpha)J_0(\beta)$	$\omega_0 + \omega_A$
7	$-J_1(\alpha)J_1(\beta)$	$\omega_0 + (\omega_B + \Delta \sin \omega_C t) - \omega_A$
8	$J_0(\alpha)J_1(\beta)$	$\omega_0 + (\omega_B + \Delta \sin \omega_C t)$
9	$J_1(\alpha)J_1(\beta)$	$\omega_0 + (\omega_B + \Delta \sin \omega_C t) + \omega_A$

The signal detected by the photodetector is proportional to the reflected field intensity $I_r \propto |E_r|^2$, i.e.:

$$I_r(t) \propto E_0^2 \sum_{k,k'=1}^9 A_k A_{k'} R(\omega_k) R^*(\omega_{k'}) e^{i(\omega_k - \omega_{k'})t}, \quad (3.15)$$

where $R^*(\omega_k)$ is the conjugate of the cavity reflection transfer function.

Carrier error signals are obtained demodulating at frequency ω_A the light reflected from cavities. Sidebands error signals for ω_B are obtained by demodulating the same beam at ω_C .

The components of the reflected intensity oscillating at ω_A come from the interference of the three groups of modes centered respectively at ω_0 , $\omega_0 + \omega_B$, $\omega_0 - \omega_B$:

$$\begin{aligned} I_r^{\omega_A} \propto & 2E_0^2 \{ A_1 A_2 \{ \text{Re}[S_2] \cos \omega_A t - \text{Im}[S_2] \sin \omega_A t \} + \\ & + A_4 A_5 \{ \text{Re}[S_5] \cos \omega_A t - \text{Im}[S_5] \sin \omega_A t \} \\ & + A_7 A_8 \{ \text{Re}[S_8] \cos \omega_A t - \text{Im}[S_8] \sin \omega_A t \} \}, \end{aligned} \quad (3.16)$$

where

$$S_i = R(\omega_{i-1})R^*(\omega_i) - R(\omega_i)R^*(\omega_{i+1}). \quad (3.17)$$

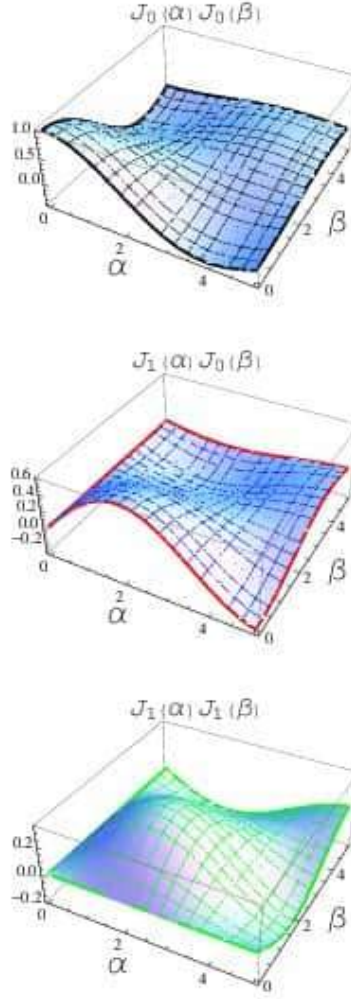


Fig. 3.3: Amplitudes of the electric field spectral components as a function of the modulation indices α and β (see 3.1). Black line: $(p, q) = (0, 0)$; red line: $(p, q) = (1, 0)$; light green line: $(p, q) = (1, 1)$. Modulation indices range: $\alpha, \beta \in [0 - 5]$.

For small values of Δ/ω_B , the component of detected intensity oscillating at ω_C is written as:

$$\begin{aligned}
 I_r^{\omega_C} \sim & E_0^2 \left[A_1^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_1, \Delta=0)} + A_2^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_2, \Delta=0)} \right. \\
 & + A_3^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_3, \Delta=0)} + A_7^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_7, \Delta=0)} \\
 & \left. + A_8^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_8, \Delta=0)} + A_9^2 \frac{d|R(\omega)|^2}{d\omega} \Big|_{(\omega=\omega_9, \Delta=0)} \right] \Delta \sin \omega_C t.
 \end{aligned} \tag{3.18}$$

If we select, by means of a phase sensitive detection technique, the components of the signal oscillating in phase with $\sin(\omega_A t)$ and $\sin(\omega_C t)$, we obtain the error signal for the carrier $\epsilon_0(\omega)$ and for the sidebands $\epsilon_s(\omega)$:

$$\begin{aligned}\epsilon_0(\omega) &\propto \sum_{i=1,4,7} 2A_i A_{i+1} \text{Im}[S_{i+1}] \\ \epsilon_s(\omega) &\propto \sum_{i=1,2,3,7,8,9} A_i^2 \left. \frac{d|R(\omega)|^2}{d\omega} \right|_{(\omega=\omega_i, \Delta=0)}\end{aligned}$$

In figure 3.4 the simulated error signals are reported. It is clearly shown that the frequencies ω_A and ω_B modulate independently the phase of the laser electric field. The modulation at ω_B does not affect the carrier error signal, when $\omega_B \simeq mFSR$.

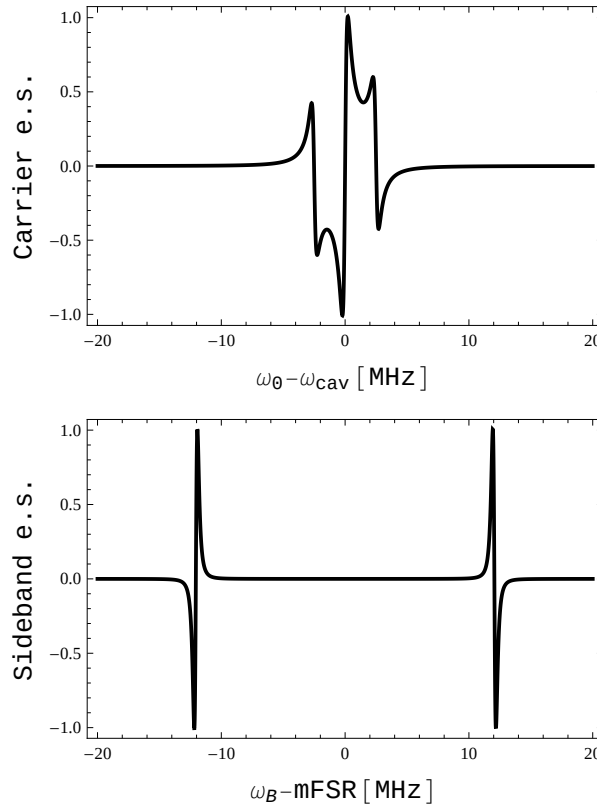


Fig. 3.4: Simulated error signals resulting from the triple frequency modulation scheme. Above: carrier error signal; below: sidebands error signal. Modulation frequencies: $\omega_A = 17$ MHz, $\omega_B = 15$ MHz, $\omega_C = 70$ kHz. Cavity features: $FSR = 1.56$ GHz, $r_{in} = r_{out} = 0.997$.

The typical error signals generated interrogating a scanning Fabry-Perot

interferometer by the triple frequency modulation scheme are shown in figure 3.5.

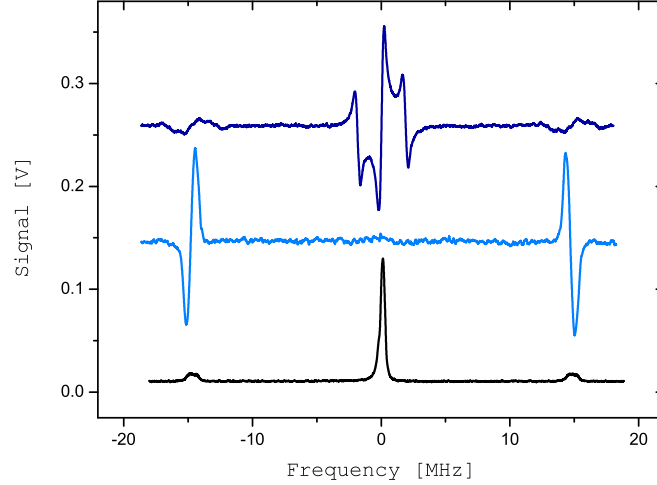


Fig. 3.5: Triple frequency modulation signals. The showed signals are generated interrogating a Fabry-Perot interferometer ($FSR = 1.56$ GHz) with the developed light source (see 4.2), and acquired using a digital oscilloscope. From top to bottom: carrier error signal (blue line), sideband error signal (light blue line), cavity transmission (black line). Modulation frequencies: $\omega_A = 17$ MHz, $\omega_B = 15$ MHz, $\omega_C = 70$ kHz.

3.4.1 Choice of modulation indices values

In the triple-frequency modulation scheme we worked out, two modulation indices α and β have been introduced. Both the modulation indices affect the amplitude of the carrier and sidebands error signals. Assuming that the noise at ω_A is the same of that at ω_C , the PDH sensitivity to the detection of the optical resonance is enhanced maximizing the slopes of the error signals.

The carrier slope can be calculated as the derivative of the carrier error signal with respect to the modulation frequency ω_A , evaluated in the resonant condition ($\omega_A = 0$). Similarly, the sideband slope can be calculated as the derivative of the sideband error signal with respect the modulation frequency ω_B , evaluated in the resonant conditions ($\omega_A = 0$, $\omega_B = mFSR$). In figure 3.6

I reported the surface of the error signal slopes as a function of the modulation indices α and β , and its contour plots showing the level curves of the bivariable functions. Both surfaces and level curves have been evaluated for the cavity parameters and frequency modulation values used in the experiment, as indicated in the caption.

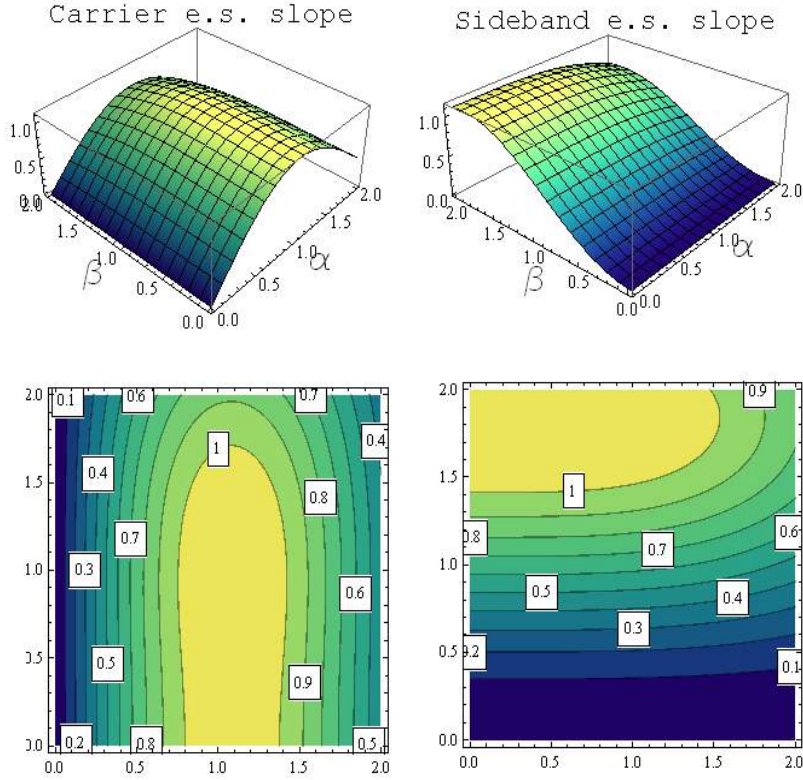


Fig. 3.6: Plot of the error signals slopes provided by the triple-frequency modulation scheme as a function of the modulation indices α and β . On the left: surface of carrier slope (above) and its contour plot (below). On the right: surface of sideband slope (above) and its contour plot (below). Both surfaces and level curves have been evaluated for the cavity parameters and frequency modulation values used in the experiment: $FSR = 113$ MHz, $r_{in} = r_{out} \simeq 0.99$.

It is worth noting that, for small modulation indices, the carrier slope is a near-flat function of β while the sideband one is a near-flat function of α . The carrier slope is maximized when $\alpha = 1.08$. This value is the same that maximizes the slope of the error signal in the standard PDH technique ($\beta = 0$). As pointed out in [119], in this case the power in each sidebands is half the power in the carrier. The sideband slope is maximized for high values of β .

Being the identified maxima fairly broad, a good choice for the modulation indices is provided by values in the following range: $\alpha, \beta \in [0.7 - 1.5]$. For $\beta = 0.7$ the sideband slope is reduced to half the maximum that occurs for $\beta = 1.44$, while the carrier slope is maximized for each value of β in the identified range.

However, values $\beta \leq 1$ are preferable since as modulation increases the power loss in the higher-order sidebands increases, reducing the error signals power. If I_0 is the light intensity in the carrier, $I_{A,B}$ are those in each of the first-order sidebands at ω_A and ω_B respectively, and I_{tot} is the total intensity when the light is not modulated ($\alpha, \beta = (0, 0)$), the efficiency of the modulation for the signals generation is given by:

$$\eta = \frac{I_0 + 2(I_A + I_B)}{I_{tot}}. \quad (3.19)$$

Figure 3.7 shows the simulated spectra of the signal detected by the photodetector when different values of index β are used in the triple-frequency modulation scheme. The case where the used modulation indices are $(\alpha, \beta) = (1.08, 0.7)$ is compared with that $(\alpha, \beta) = (1.08, 1.5)$. The efficiency is of 95% in the first case and of 85% in the second one. The optimum modulation depths are finally determined by finely adjusting the carrier and sidebands power during the experimental stage.

In figure 3.8 the simulated error signals are plotted for different values of modulation frequencies ω_A and ω_B . It is shown that a detuning of ω_B from an harmonic of the cavity FSR causes a deformation and amplitude reduction of the carrier e.s.. In figure 3.9 and 3.10 the simulated error signals are plotted for different values of modulation indices α and β . It is shown that the choice of α in the selected range maximizes the amplitude of both carrier and sidebands e.s.. While in the case of the carrier e.s. this is true also for the choice of β , in the case of the sidebands e.s. that choice provides a good signal but, in general, its amplitudes increases as β increases.

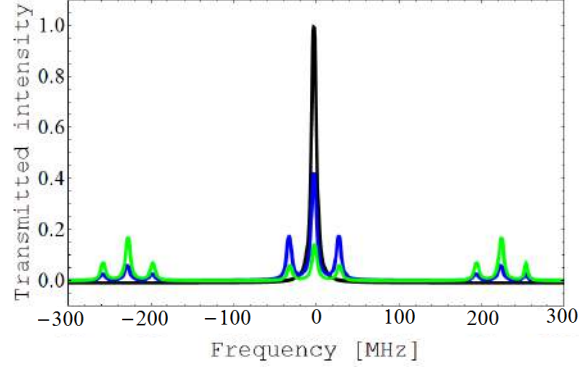


Fig. 3.7: Simulated spectra of the cavity transmitted signal detected by the photodetector when different values of index β are used in the triple-frequency modulation scheme. Different plotted curves correspond to the following choice of modulation indices: $(\alpha, \beta) = (0, 0)$ (black line); $(\alpha, \beta) = (1.07, 0.7)$ (blue line); $(\alpha, \beta) = (0, 1.5)$ (green line).

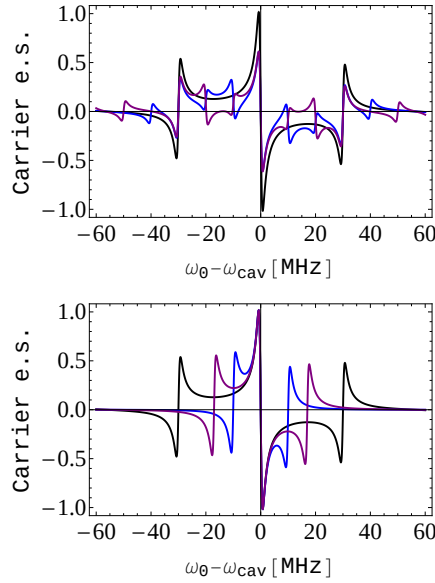


Fig. 3.8: Simulated carrier error signals plots for different values of modulation frequencies. Modulation indices: $(\alpha, \beta) = (0.7, 1)$. Cavity features: $r_{in} = r_{out} = 0.99$, $FSR = 113$ MHz. Above: $\omega_B = 2FSR = 226$ MHz; $\omega_A = 30$ MHz (black line), $\omega_A = 10$ MHz (blue line), $\omega_A = 17$ MHz (purple line). Below: $\omega_A = 30$ MHz; $\omega_B = 2 \times FSR = 226$ MHz (black line), $\omega_B = 236$ MHz (blue line), $\omega_A = 246$ MHz (purple line).

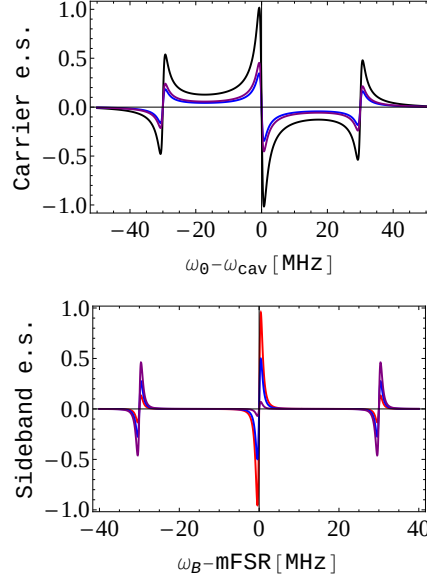


Fig. 3.9: Simulated error signals plots for different values of modulation indices α . Modulation parameters: $\omega_A = 30$ MHz, $\omega_B = 226$ MHz, $\beta = 1$. Cavity features: $r_{in} = r_{out} = 0.99$, FSR = 113 MHz. Above (carrier error signal): $\alpha = 0.7$ (black line), $\alpha = 0.2$ (blue line), $\alpha = 2$ (purple line). Below (sideband e.s.): $\alpha = 0.7$ (red line), $\alpha = 1.2$ (blue line), $\alpha = 2$ (purple line).

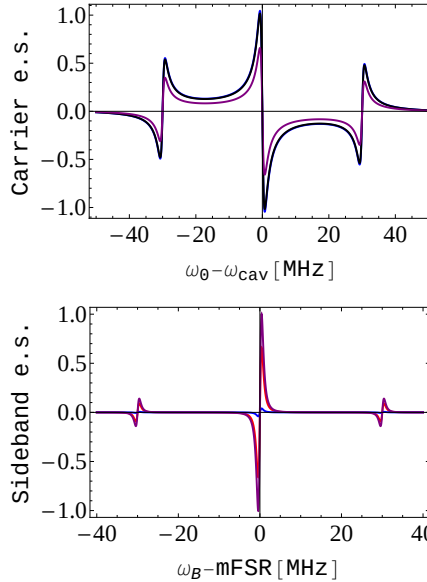


Fig. 3.10: Simulated error signals plots for different values of modulation indices β . Modulation parameters: $\omega_A = 30$ MHz, $\omega_B = 226$ MHz, $\alpha = 0.7$. Cavity features: $r_{in} = r_{out} = 0.99$, FSR = 113 MHz. Above (carrier error signal): $\beta = 1$ (black line), $\beta = 0.7$ (blue line), $\beta = 1.2$ (purple line). Blue and black line are superimposed. Below (sideband e.s.): $\beta = 1$ (red line), $\beta = 0.2$ (blue line), $\beta = 1.4$ (purple line).

CHAPTER 4

Experimental apparatuses and results

In this chapter I describe the apparatuses used and the achieved experimental results. In section 4.1, the technical description of GP2 ring laser gyroscope and the characterization of its ring cavity are reported. In section 4.2, the developed injection-locked laser reference source for the interferometric diagnostics is characterized. In section 4.3, I present the tabletop experiment we made to verify the measurement technique worked out for the length measurement and control. Finally, in section 4.4 I describe the GP2 diagonal resonators, the electronic setup used to implement the length measurement technique and obtained results.

4.1 GP2 ring laser gyroscope

4.1.1 Motivation and goal

GP2 is the seed device for the next generation heterolithic active-stabilized RLs. It exhibits an innovative design, in order to gain a long term stability and accuracy of the scale factor k_S , via a precise control of the cavity geometry and the active medium parameters. The goal is to test the control strategies and, in particular, to implement the length stabilization of its diagonal cavities using optical interferometric techniques. Figure 4.1 shows a picture of GP2 at INFN-Pisa laboratories (Pisa, Italy), where it has been fully setup in June 2015 (see figure 4.1).

In this section, I report a description of the GP2 design and the characterization of its optical cavity and piezo-nanopositioning system.

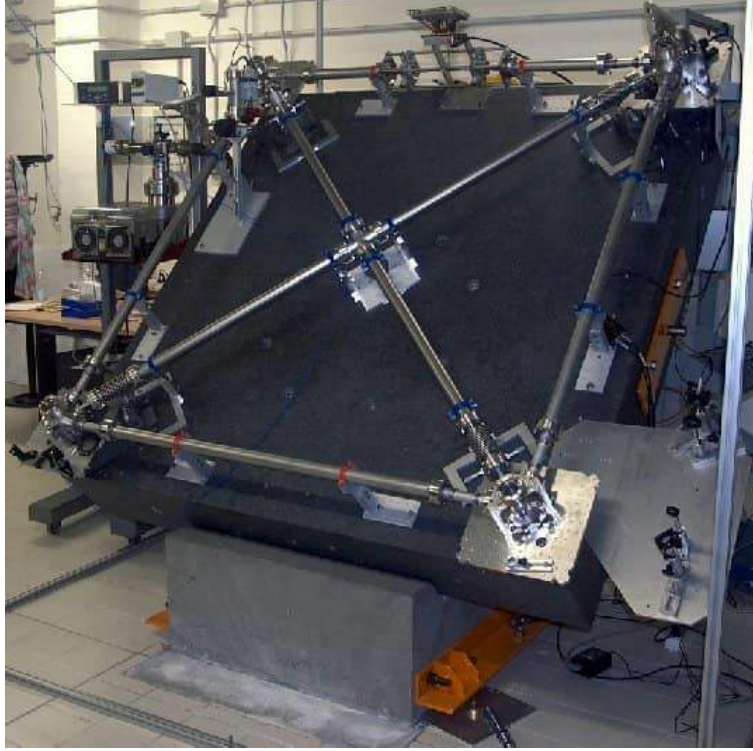


Fig. 4.1: GP2 ring laser installed at INFN-Pisa laboratory.

4.1.2 Optical ring cavity characterization

A drawing of GP2 is shown in figure 4.2. As our previous devices, it has a square optical cavity placed on a granite support. This is fixed on a concrete base attached to the ground. The granite slab is oriented along the local latitude ($43^{\circ}42'42''48$ N), in order to maximize the Sagnac signal and minimize the orientation errors on scale factor due to local tilts. The optical setup consists in four supermirrors, each one contained in a steel vacuum tight box placed at the corners of a square 1.60 m length in side. Steel pipes, connected by the mirror holders, define the vacuum chamber. As a novelty respect to the standard square cavity RLs, the GP2 chamber encloses also the optical paths between the opposite mirrors of the cavity. The whole vacuum chamber, connected to an external reservoir through a junction, is filled with a mixture of He-Ne. To keep the gas mixture clean from contaminants, especially hydrogen produced by the outgassing of the steel chamber, a chemical getter pump (SAES) is located in a small chamber communicating with the gyro. The

capacitive discharge for laser excitation, consisting in two cylindrical electrodes placed around a pyrex capillary with a diameter of 4 mm, is located in the middle of a side of the cavity. The laser operates in the single mode TEM_{00} .

The resonator mirrors (ATFilms) have a curvature radius of $R = 2$ m. The substrate is made of $1'' \times 10$ mm fused silica, and the coating, produced by Ion Beam Sputtering, has a thickness of $6 \mu\text{m}$. The mirrors have been theoretical designed in order to have, at 45deg angle of incidence, a reflectivity $r = 99.999\%$ and a transmission $t = 0.3$ ppm. The RL output power in the single mode regime is of few nW. The mirrors are accessible through big optical transparent windows installed parallel to them on the holders. The window allows the circulating beams to exit the cavity and to be monitored. Figure 4.3 shows the laser spot transmitted through one of the corner mirrors and acquired by a webcam. An external optical setup detects the beat frequency. The optical windows allow also to an external He-Ne laser source to be injected in the diagonal resonators.

To check the quality factor Q of the ring cavity, we made a ring-down time τ measurement of the laser by short-circuiting the discharge capacitor. Figure 4.4 shows the laser intensity decay trace acquired by the oscilloscope. Fitting to data the exponential function

$$I = I_0 + Ce^{-t/\tau}, \quad (4.1)$$

where I_0 is the initial intensity and C a numerical constant, we have obtained a measure for the ring-down time $\tau = 154.4 \pm 0.5 \mu\text{s}$. This ring down time corresponds to a quality cavity factor $Q = 2\pi c\tau/\lambda = 4.6 \times 10^{11}$ [122], and thus to a finesse $\mathcal{F} = \lambda Q/p = 4.6 \times 10^4$, being p is the perimeter length. The actual mirror reflectivity (assumed equal for all four mirrors) can be estimated from the ring cavity measurement by using the relation $\tau = 1/(4(1-r)FSR)$, being $4(1-r)$ the total ring cavity losses, and results to be $r = 0.99997$.

In table 4.1 the features of GP2 are summarized. A preliminary Sagnac frequency beat is shown in figure 4.5 [66].

Table 4.1: GP2 ring cavity features.

Parameter	Value
R	2 m
L	$\simeq 1.600$ m
FSR	$\simeq 46.88$ MHz
r^2	0.99997
$\mathcal{F}$	4.6×10^4
Q	4.6×10^{11}
P_{out}	< 5 nW

4.1.3 Mechanical features and piezo nano-positioning system

The optical cavity of GP2 is fixed on a slab of Black Afrika granite, a rock widely used for precision metrology applications because of the following properties: dimensional stability (being the material free of internal tension), thermal stability (the linear expansion is much lower than for steel or cast iron, being the thermal expansion coefficient of $6.5 \times 10^{-6} \text{ K}^{-1}$), hardness (being comparable to tempered steel), high flatness accuracy (its surface can be machined in order to obtain a flatness better than that obtained with traditional materials), high bending strength and insensitivity to mechanical overloading. To guarantee a preliminary well positioning of the corner mirrors, the slab has been machined with a precision better than $10 \text{ }\mu\text{m}$. The use of a lightened granite block, realized gluing together layers of granite, allows to reduce the weight of the construction (the weight of the granite slab is of ~ 1.2 ton), and to improve the long term thermal and mechanical stability.

To get a regular square cavity, the mirror holders are equipped with a piezo nano-positioning system. Since each mirror displacement can be described as a variation in the space of the position of its curvature center, the geometry of the optical path along the square loop is completely determined by the 12 centres coordinates (12 degree of freedom). As showed in chapter 2, in reference [68] we have defined the eigenvectors basis of the cavity deformations, identified the

rigid body motion of the cavity, and then classified the residual 6 optical cavity deformations once the diagonal lengths are stabilized. For this reason, in GP2 a total of 6 piezo-electric transducers (PZT) is used. One holder is provided with a 3-axial PZT, while the other three with a 1-axial PZT moving the mirror along the diagonal. The geometry of the ring cavity will be optimized driving the piezo transducers according to a control procedure. Any non rigid-body deformation can be corrected by a linear combination of piezo movements and rigid-body deformations. This idea has been in part expressed in reference [68] and reference [123], and is to date the object of further dedicated works.

Figure 4.6 shows the piezo nano-positioning system at the mirrors holders, and the electronics for driving the actuators. Each piezo stage has a dynamic range of 80 μm for an applied voltage of 10 V. The 1-axial transducer has been characterized in terms of frequency response [124]. To this aim, the actuator has been excited with a random noise, and the mirror displacement has been acquired by means of a laser based proximity sensor having a resolution of ~ 100 nm. The data, plotted in figure 4.7, show that the frequency response is flat up to 80 Hz, where a resonance appears. This sets the limit of the control bandwidth to about few tens of Hz.

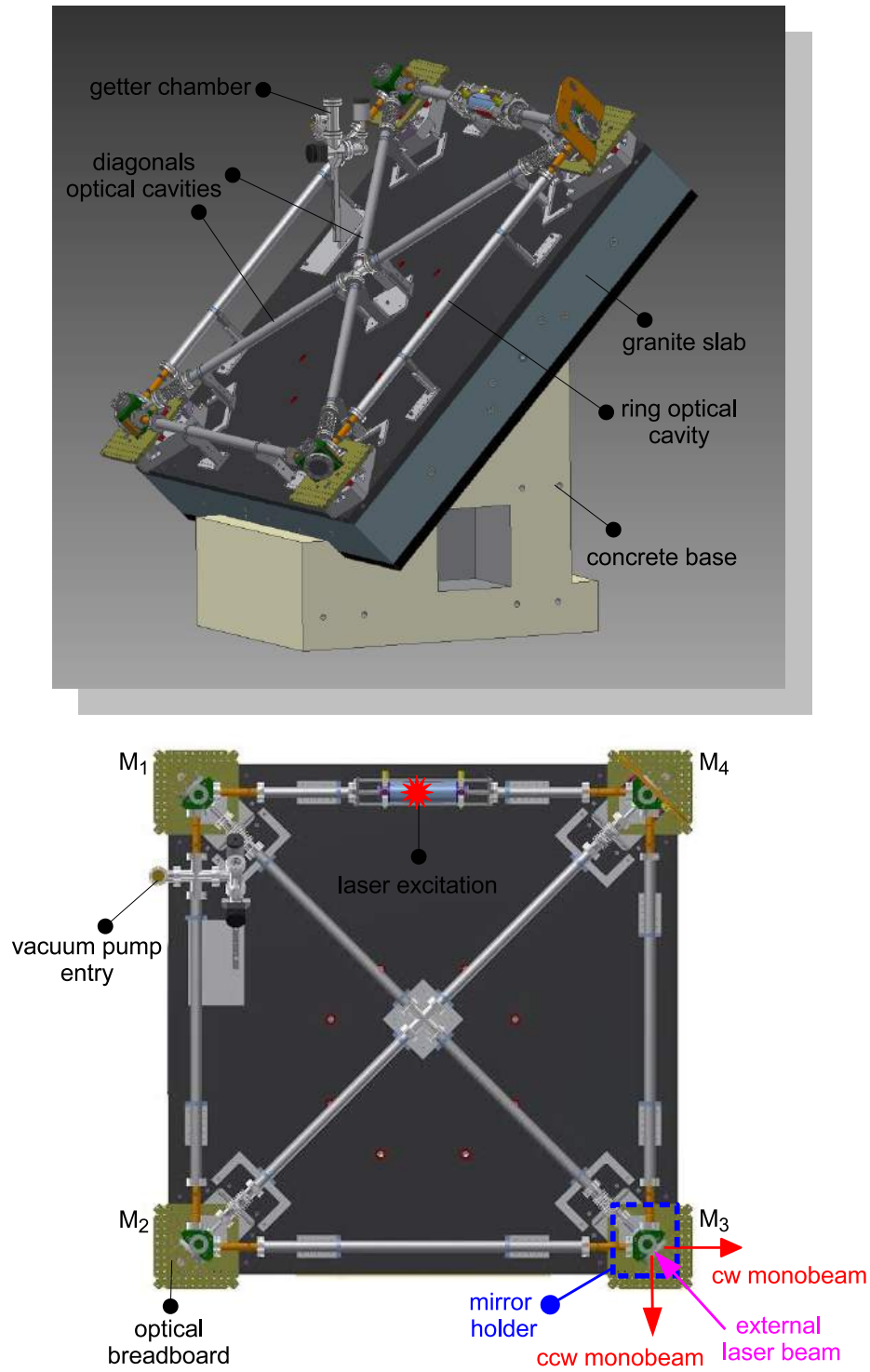


Fig. 4.2: Drawing of GP2 ring laser.

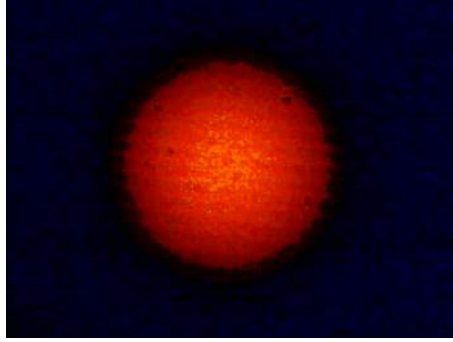


Fig. 4.3: Laser spot transmitted through a cavity mirror of GP2, acquired by a webcam. The spot is circular (ellipticity ~ 1.05), with a calculated size on cavity mirrors of ~ 2.3 mm (intended as 4 times the $1/e^2$ beam intensity radius).

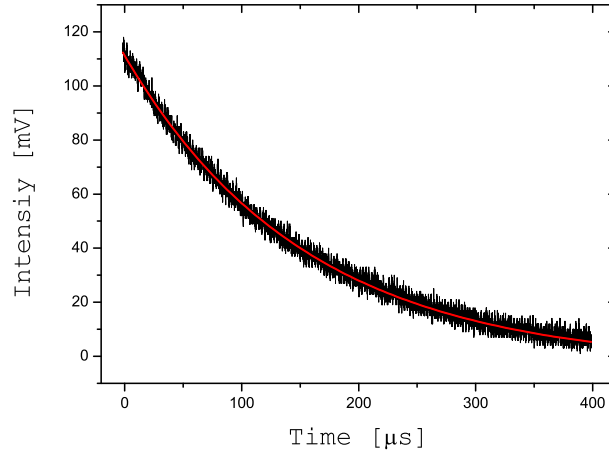


Fig. 4.4: Ring-down time measurement of GP2 ring cavity. Black line: trace of laser intensity decay acquired by the oscilloscope. Red line: exponential fit of data points. Fitted exponential function: $I = I_0 + Ce^{-t/\tau}$. Fit results: $\tau = 154.4 \pm 0.5$ μs ; $I_0 = -3.35 \pm 0.10$ mV; $C = 114.63 \pm 0.14$ mV.

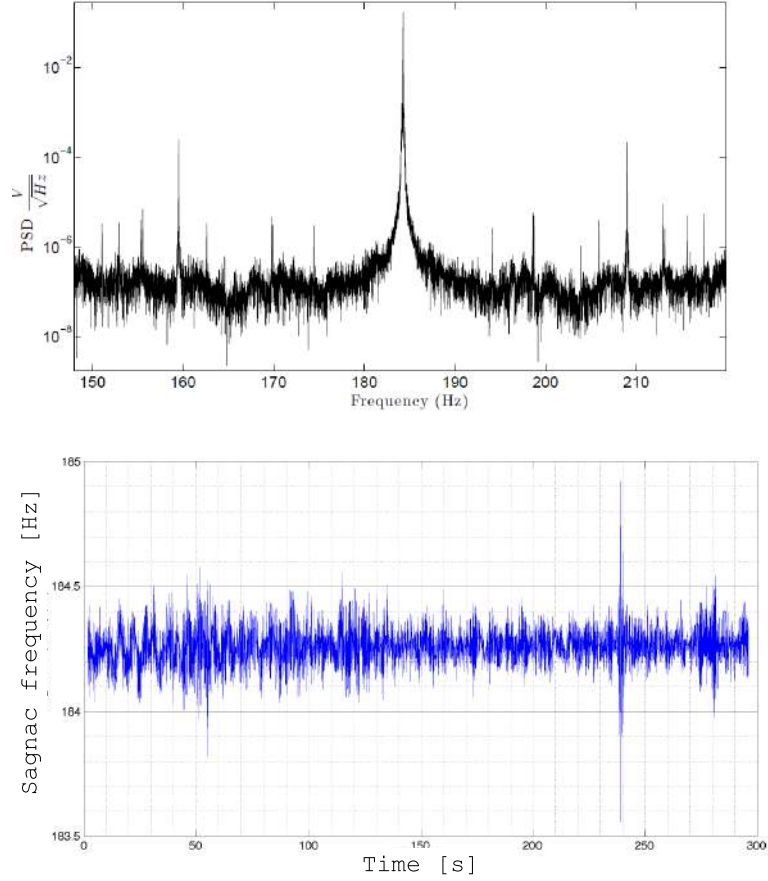


Fig. 4.5: Earth's rotation measurement from GP2. Power spectral density (above) and instantaneous Sagnac frequency (below) of GP2 data [66]. when it was installed in INFN laboratories in San Piero a Grado (Pisa). A Sagnac response peak at 184 Hz is observed, in concordance with the size of the cavity and the latitude of INFN laboratories in San Piero a Grado (Pisa), where it operated for a short period before to be moved to Pisa.

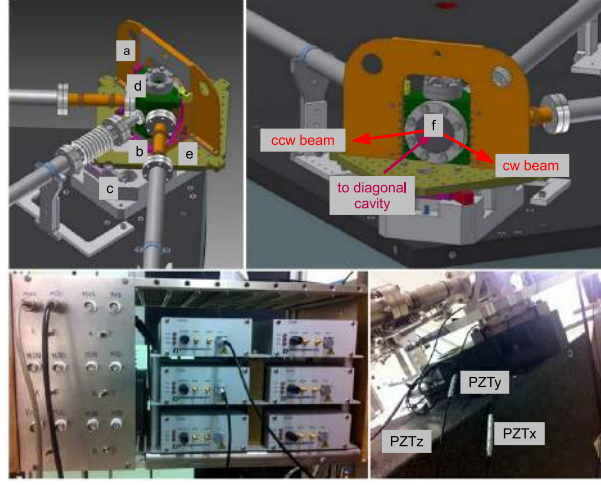


Fig. 4.6: Detail of ring cavity mirrors holders and of piezo nano-positioning system. Above: piezo system installed at one of the cavity mirror holder (on the left), and detail of mirror chamber (on the right). a: additional component for piezo-stage moving mirror along the vertical direction; b: piezo stage; c: micrometric screws; d: mirror holder; e: optical bench; f: optical window. Below: piezo drivers control units (on the left), and HV connections (on the right).

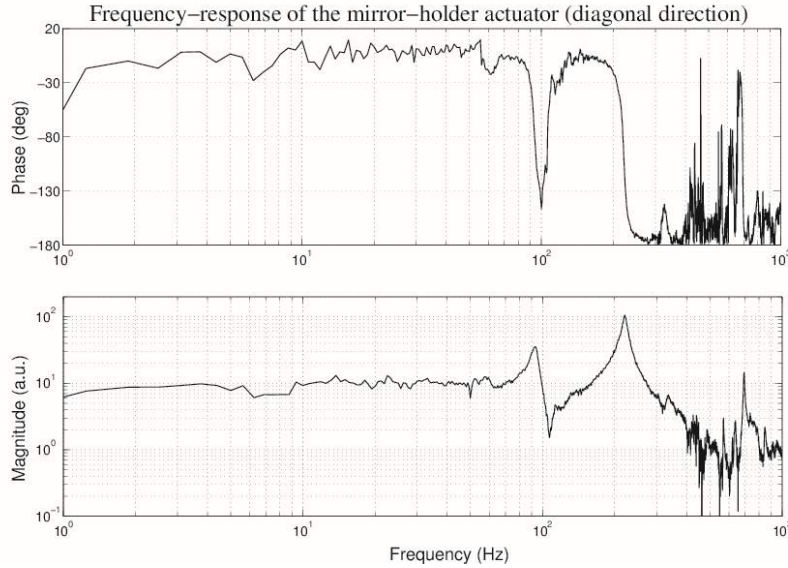


Fig. 4.7: Measurement of the 1-axial PZT frequency response. The piezo transducers has been excited with a random noise, and the mirror displacement has been acquired by means of a laser based proximity sensor (resolution ~ 100 nm).

4.2 Development of the light source

4.2.1 Motivation and goal

In order to provide a frequency reference suitable for the ring laser geometry stabilization, a stable light source at 633 nm with a good linewidth (~ 100 kHz) and an high output power ($\simeq 10$ mW) is needed. The wavelength of the light source is dictated by the fact that we use an iodine-stabilized He-Ne laser as reference frequency. Since the output power of the He-Ne/ I_2 laser is only of few hundreds μ W, we developed a system based on a 15 mW diode laser, frequency stabilized to the reference laser via optical injection-locking technique [125]. This setup allows us to increase the output power without degrading the iodine spectral purity. A photo of the injection-locking setup is shown in figure 4.8.

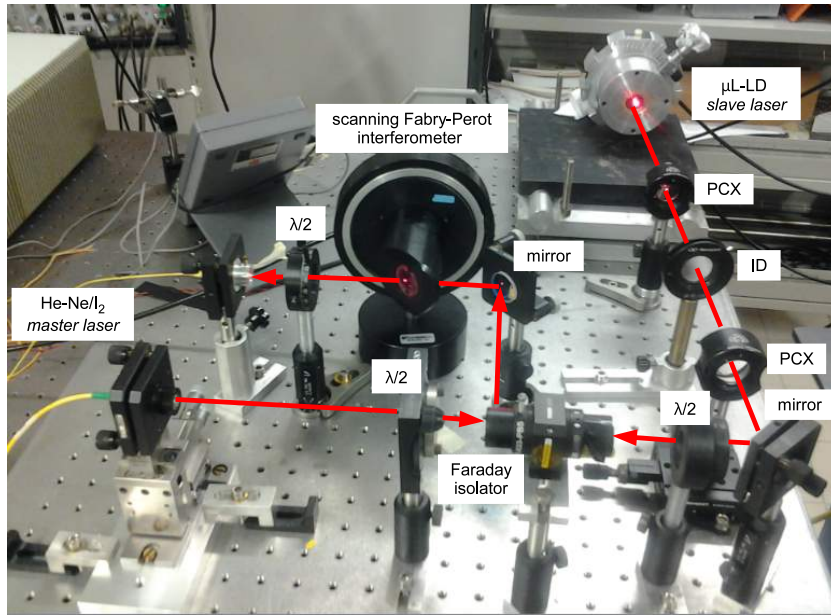


Fig. 4.8: Injection locking setup. The light from the reference laser is coupled into a single mode optical fiber, and sent into the master laser cavity by an air path, after passing through a Faraday isolator. Half wave plates ($\lambda/2$) and plano convex lenses (PCX) are used for polarization and laser spot optimization, respectively. One of the two lenses is mounted on the optical table through a 2-axis platform. ID: iris diaphragm.

In the following, before to present the results of the characterization of developed light source, I will describe in details the two laser sources acting as *reference laser* and *probe slave* in the injection-locking scheme.

4.2.2 Reference laser

Molecular iodine I_2 holds a unique position among optical frequency standards, because it offers a wide spectrum of optical transitions suitable for metrological calibration. It is one of the primary frequency standards recommended for the "mise en pratique du mètre" [126]. The He-Ne at 633 nm, frequency stabilized with the Lamb-dip technique on the hyperfine structure components of iodine, is one of the better quality standards based on I_2 [127]. The system we used, shown in figure 4.9, has been provided by the Institute of Laser Physics of Novosibirsk (Russia). It consists in a cw master-slave laser system, where a He-Ne slave laser is phase-locked to a He-Ne master laser. The latter is stabilized to the frequency of a selectable optical transition of a sample of $^{127}I_2$, contained in a fused silica cell placed inside the master laser resonant cavity. The output power of our system is of about 300 μ W. A collimated laser beam with a measured waist radius of $\simeq 0.40$ mm and a linewidth (FWHM) of $\simeq 100$ kHz is provided.

A block-scheme of the laser system is shown in figure 4.10. The resonators of master and slave lasers have a length of 0.275 m and 0.175 m, respectively. The two cavities are made of spherical mirrors with radius of curvature $R = 0.5$ m, and the transmissivity is of about 1.7% for the output couplers and 0.3% on the other side. The power density has been optimized with regard to the SNR of the detected iodine hyperfine peaks. A Peltier element, attached to the side arm of the cell containing the molecular sample, keeps the iodine at a temperature of $17^\circ C$. Furthermore, the whole laser box is stabilized to a value nearly above the room temperature ($> 26^\circ C$) through an external circuit.

The master laser has two piezoelectric transducers: a fast PZT is used for applying a 5 kHz modulation to the cavity length and also for providing fast corrections to the cavity length; a slow PZT, with larger dynamic range,

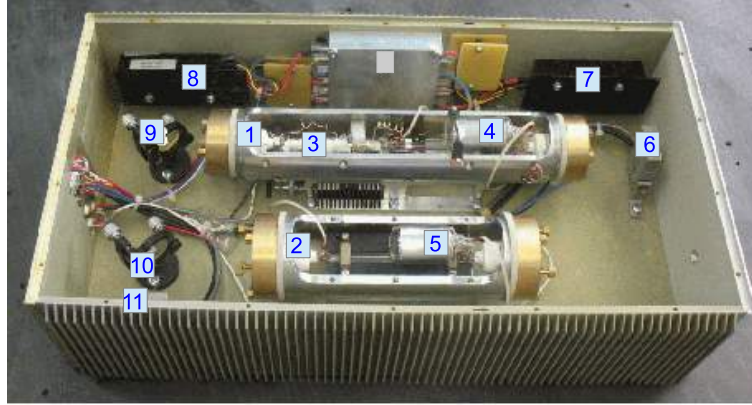


Fig. 4.9: Iodine-stabilized laser system. 1: master laser. 2: slave laser. 3: iodine cell. 4,5: getter. 6,11: photodiodes (FD-263, *Evita^R* Electronic Components) 7,8: high voltage power supply (*LASOS^R*). 9,10: plane mirrors.

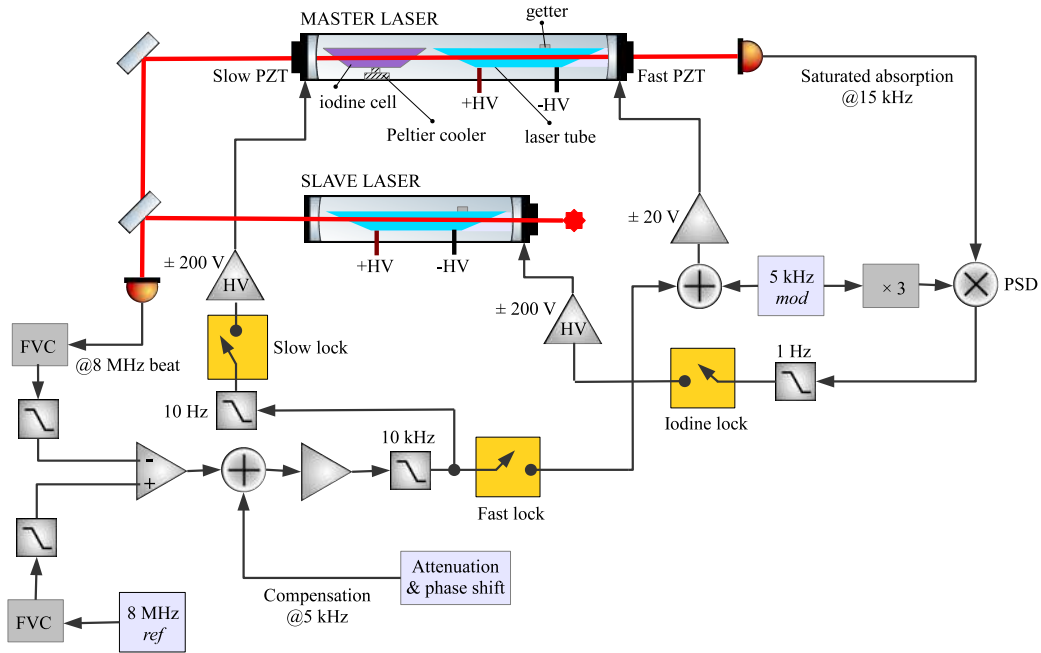


Fig. 4.10: Block scheme of the iodine stabilized He-Ne laser system. The laser tubes and the iodine cell contained in the master laser has both lateral windows wedged at the Brewster angle, to avoid parasitic interference effects and obtain a high polarization purity. FVC: Frequency to Voltage Converter; PSD: Phase Sensitive Detector; HV: High Voltage.

compensates thermal and mechanical frequency drifts.

The stabilization of the laser system consists in two steps:

1. in the first, the frequency of the master is offset-locked to the slave laser;
2. in the second, the frequency of the master is locked to the molecular line by tuning the slave laser cavity. In this situation, the reference laser output is frequency stabilized with respect to the molecular reference, with an offset of 8 MHz.

The control system is based on the third derivative detection of the saturated absorption signal. The plasma tube, filled with isotopic pure ^{20}Ne , provides laser gain over a frequency region of about 1 GHz, and the iodine cell introduces a loss due to a Doppler-limited absorption coincident with the neon transition. When the laser is tuned to the center of an iodine hyperfine transition, a sub-Doppler reduction in absorption occurs, due to transition saturation, and leads to a slight increase in the laser output power. This can be detected by modulating the laser at a frequency $f = 5$ kHz, and detecting the output power change at a frequency $3f$.

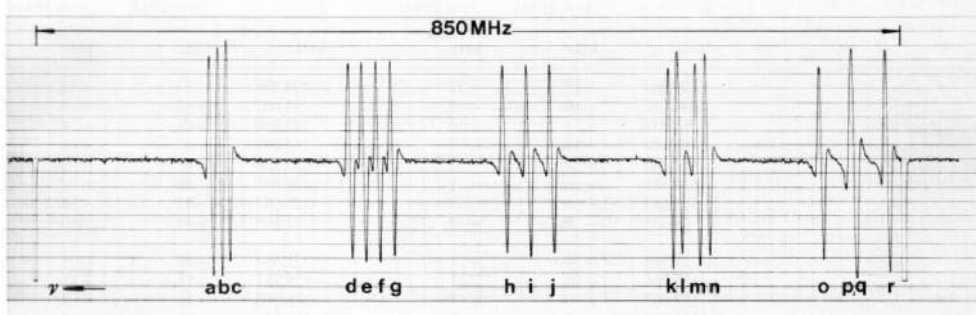


Fig. 4.11: Spectrum of third harmonic of the absorbing molecule $^{127}\text{I}_2$ in the R127 (11-5) transition [128].

The principle of the stabilization consists in keeping the laser wavelength resonant with one of the hyperfine components of the R(127) 11-5 transition, by adjusting the length of the cavity. The spectrum of the absorption lines of iodine, observed by means of third-harmonic detection, is reported in figure 4.11, while the frequency intervals between the hyperfine components are listed in table 4.2. The lock is manual: the hyperfine components are observed on oscilloscope and the iodine peak is locked on real time. The component we adopted is the i component. The corresponding frequency stability is of $\sim 10^{-12}$ at 100 s and few parts in 10^{11} at 1 hour (figure 4.12).

Table 4.2: Frequency intervals between hyperfine components of absorption lines of iodine. First column: notations of hyperfine components as utilized in quoted bibliography; the systematic notation proposed in [126, 129] has also been indicated. Second column: frequency intervals between a_x component ($x = 4, 5, \dots, 21$) and a_{13} component, taken as reference. Third column: estimated standard deviation σ . Unit: MHz [130].

$\lambda = 633 \text{ nm}, {}^{127}\text{I}_2 \text{ R}(127) \text{ 11-5}$					
Reference: component i (a_{13}), $f = 473612214.8 \text{ MHz}$					
Component	$f(a_x) - f(a_{13})$	σ	Component	$f(a_x) - f(a_{13})$	σ
a_4 r	-320.6	0.1	a_{13} i	0	-
a_5 q	-292.7	0.5	a_{14} h	21.939	0.005
a_6 p	-290.3	0.5	a_{15} g	125.694	0.005
a_7 o	-263.0	0.1	a_{16} f	138.892	0.005
a_8 n	-162.814	0.005	a_{17} e	152.255	0.005
a_9 m	-153.801	0.005	a_{18} d	165.116	0.005
a_{10} l	-137.994	0.005	a_{19} c	283.006	0.005
a_{11} k	-129.959	0.005	a_{20} b	291.100	0.005
a_{12} j	-21.565	0.005	a_{21} a	299.931	0.005

4.2.3 Probe laser

In the development of a frequency standard, diode lasers are very attractive. Compared to the most laser types, they exhibit a low cost and high compact design, and provides high optical power, high efficiency level, low intrinsic intensity noise and wavelength tunability. However, the output beam is highly divergent, has often a multi-mode spectrum and the rectangular shape of the diode junction leads to an astigmatic light spot. The lenses required for beam correction must be properly selected and mounted, in order to avoid optical feedback that can degrade the laser performance through loss in power or reduction in life. Visible-red diode lasers ((630 – 680) nm) suffer for additional problems. Owing to the fundamental limit of the material system (AlGaInP/GaAs) at this wavelength range, they are more sensitive to temperature fluctuation, to optical mirror damage due to the higher photon energy, and to the efficiency decreasing due to the gradual aging over time [131]. This

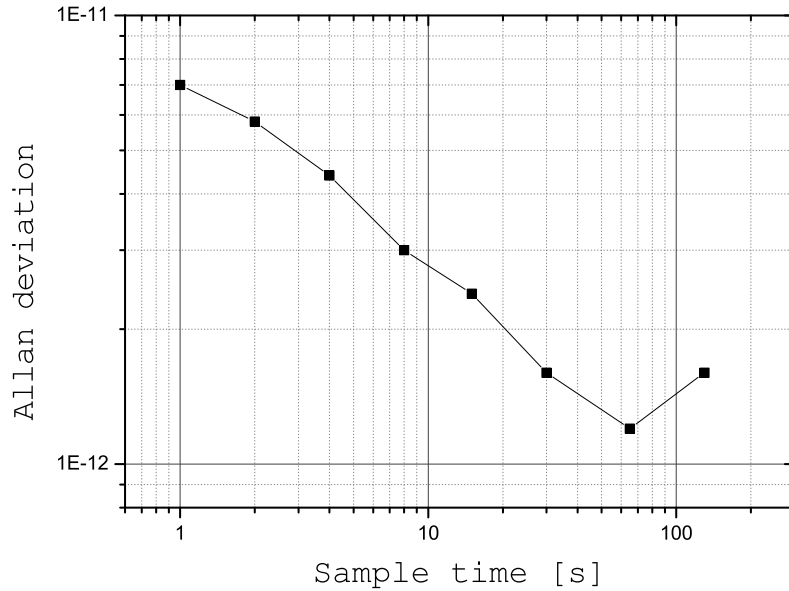


Fig. 4.12: Allan deviation of the iodine-stabilized He-Ne laser [courtesy of D. Yu. Primakov - Institute of Laser Physics of Novosibirsk (Russia)].

is particularly true at the shortest wavelengths.

In order to achieve a suitable red diode light source acting as slave laser in the injection locking setup, an appropriate chip must be selected. As in [131–133], the light source we used is a commercial microlens-coupled cw diode laser (Model VPSL-0635-015-x-9-B, Blue Sky Research) operating in the wavelength range (630 – 640) nm. A schematic drawing of the microlens laser is shown in figure 4.13.

The laser package is equipped with a virtual point source microlens that provides a 15 mW circular, diffraction-limited output beam and, reflecting a small fraction of the beam back to the laser, forces it to oscillate in a single longitudinal mode. The measured beam diameter is of 1 mm. The chip is mounted inside a cylindrical aluminium assembly (figure 4.14). Wavelength tuning is obtained by changing the drive current and the junction temperature. In order to avoid mode hops instability, the junction temperature is stabilized within 1 mK by using a Peltier element and a thermistor. A star-shaped aluminium wrapping is used to allow the heat dissipation. To reach the operation at the wavelength of 633 nm and the maximum output power, the diode laser

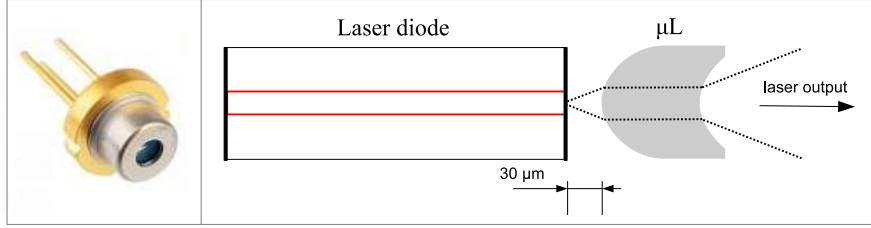


Fig. 4.13: Micro-lens-coupled diode laser. The virtual point source micro-lens is glued at a distance of $30\ \mu\text{m}$ from the chip output mirror. The laser cavity has an optical length of 3 mm. An output power of 15 mW with a linewidth of 1.8 MHz is provided.

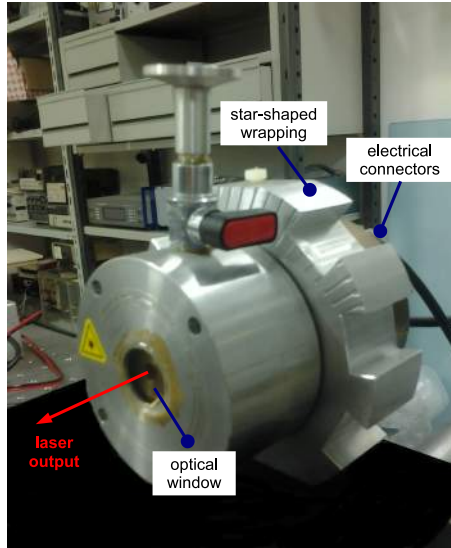


Fig. 4.14: Diode laser assembly.

has to be cooled at about 3.6°C . The laser package is placed inside a sealed aluminium chamber in order to avoid water vapour condensation.

4.2.4 Experimental results

The optical injection of the reference laser into the diode laser can be represented as the non linear coupling between two oscillators, that is the two laser sources: a master laser (He-Ne/ I_2) and a slave laser (diode laser). Relevant parameters for locking to occur are the frequency detuning between the two optical fields, and the ratio of the free running laser power to the injection power [134]. If the frequency detuning between the slave and master

lasers is made small enough, or the injected signal power large enough, the slave laser is forced to oscillate at the injected signal frequency and is locked to its phase.

The frequency range within which the diode laser can be finely tuned close locked to the master laser frequency is called *locking range* $\Delta\nu_L$. In the semiclassical approximation for the Hamiltonian of the radiation field produced in the slave laser cavity coupled with the radiation injected from the outside [134], the locking range depends on the laser sources features according to

$$\Delta\nu_L = k_T \frac{FSR}{2\pi} \sqrt{\frac{P_{inj}}{P}}, \quad (4.2)$$

where FSR and P are the free spectral range and the power of the free-running diode laser, respectively, while P_{inj} is the injected power entering the diode laser. The injection coupling efficiency is described by the coefficient k_T . It takes into account the percentage losses of the two laser beams for its non ideal spatial mode matching and of the slave cavity for the transmission from the output mirror.

The aim of the injection locking is to transfer the spectral features and the stability of a low power oscillator to a high power one. This can be obtained by accurately tuning the slave emission frequency to the master emission frequency. The coupling between the two lasers can be implemented by using a Faraday optical isolator: the input port of the isolator is used as input for the diode laser, while the light from the master is sent into the rejection port of the output polarizer. The resulting injection-locked laser beam is transmitted through the output polarizer. Combining the Faraday isolator with half-wave plates, almost all the slave power is available in the output beam (only few parts of 10^3 of the diode is transmitted).

In our experiment, the light of the reference laser is fiber coupled (P1-630A-FC-1, Thorlabs) and injected into the slave laser through the rejection port of an optical isolator (IO-5-633-PBS, Thorlabs). The slave beam enters into the optical isolator by an air path. The use of the fiber is of key importance to get an efficient coupling between the oscillators, since it allows to match the two laser spatial modes and polarizations. Before to be coupled into the fiber, the reference laser pass through a Faraday optical isolator, that minimize

the optical feedback to laser, and a couple of cylindrical lenses for the spot optimization.

The tuning of the free-running diode laser frequency is obtained by adjusting the junction temperature (coarse) and current (fine). The usable working points, i.e. the measured injection locking states, as a function of the tuning parameters, are plotted in figure 4.15. In the above graph, the slave laser current is reported as a function of the slave laser temperature. At each value of temperature, several working points occur tuning the slave laser current. The graph below reports the output power available at each injection locking state. At diode laser working point $(T, I_{diode}) = (3.6\text{ }^{\circ}\text{C}, 67.9\text{ mA})$ a maximum output power of $\simeq 14\text{ mW}$ is achieved. As shown in figure 4.16, this injection locking regime can be obtained with an injected power as small as $2\text{ }\mu\text{W}$, which means that the injection gain, or optical amplification factor, i.e. the ratio of the output power to the injected power, is $\simeq 7000$ and is limited by the diode laser maximum power.

The measured locking range data as a function of the injection ratio P_{inj}/P is shown in figure 4.17. In the typical working condition, when all the reference laser beam coupled with the PM fiber is sent to the diode laser, a locking-range of 2.8 GHz is achieved. Fitting the best line to the experimental data provided a coupling efficiency $k_T = 0.72$. Details are in the caption.

Figure 4.18 shows the evolution of the spectrum of the slave laser as its current is tuned to optimally locked regime. When the master and the slave frequencies are mismatched, broad frequency peaks on the oscilloscope move as the current is adjusted to the slave (*uninjected regime*). However, when the detuning is smaller than $\Delta\nu_L$ part of the diode laser power locks to the master (*partially injected regime*). Once injection locked, the frequency peak appears sharp, and follows the master laser even as the laser is scanned over the locking range (*full injected regime*). A comparison between the spectrum of the free running (black line) and phase controlled (wine line) signal is shown in figure 4.19. The features of the developed injection-locked laser source are summarized in table 4.3.

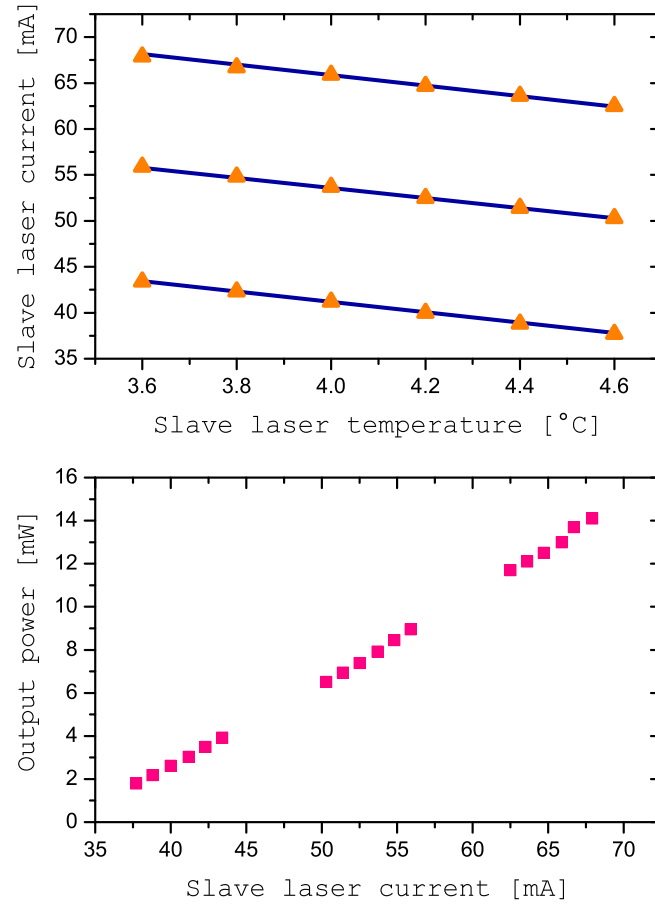


Fig. 4.15: Injection locking working points. Above: the blue lines represents the isofrequency curves, along which the slave laser beam has the same frequency of the master one. The measured injection locking states are indicated by orange triangles. Below: output power (pink squares) measured as a function of the slave laser current in the locking regime .

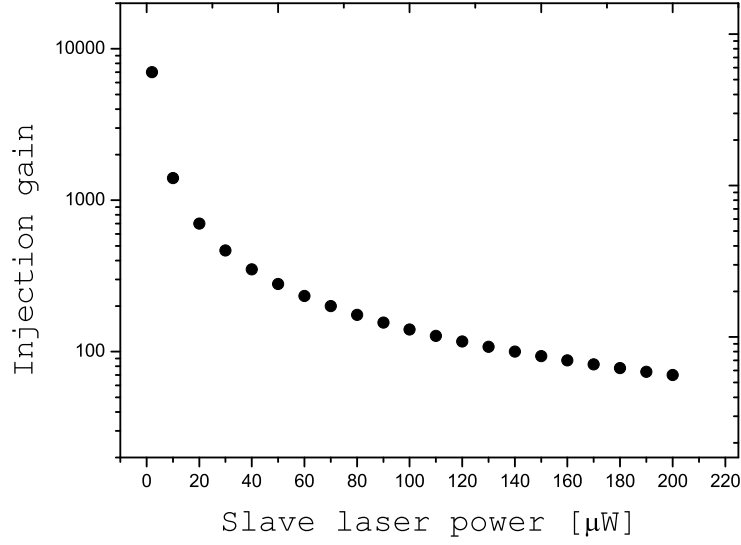


Fig. 4.16: Injection locking gain measured as a function of the slave laser power in the locking regime. The maximum measured optical amplification factor is of ~ 7000 , corresponding to a full injection locking at the maximum output power of 14.1 mW of the diode laser with an injected power of 2 μW .

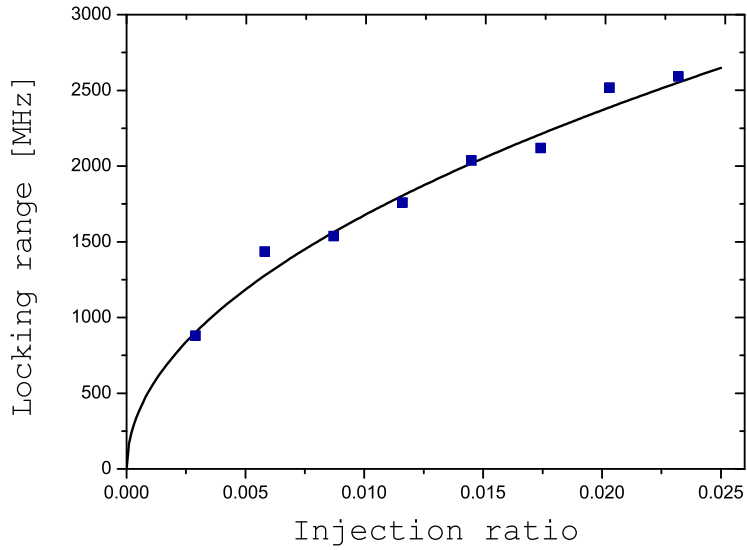


Fig. 4.17: Measured injection-locking range as a function of the injection power ratio. Solid blue squares: locking range measured by tuning the master laser frequency. Solid black line: best fit to the experimental data.

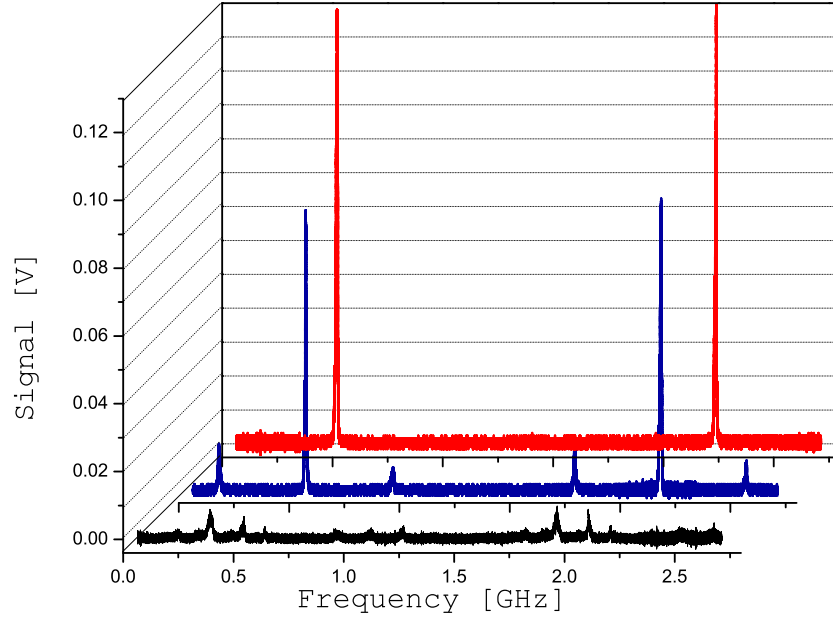


Fig. 4.18: Evolution of the slave laser spectrum as its current is tuned to optimally locked state. The Fabry-Perot scan in the three conditions are shown. Red line: fully injected; blue line: partially injected; black line: uninjected.

Table 4.3: Optical-injected laser source features.

Parameter	Value
Wavelength	632.8 nm
Output power	14.1 mW
Allan deviation	10^{-11} at 100 s
Locking range	2.8 GHz
Injection gain	~ 7000
Diode laser temperature	3.6 °C
Diode laser current	67.9 mA

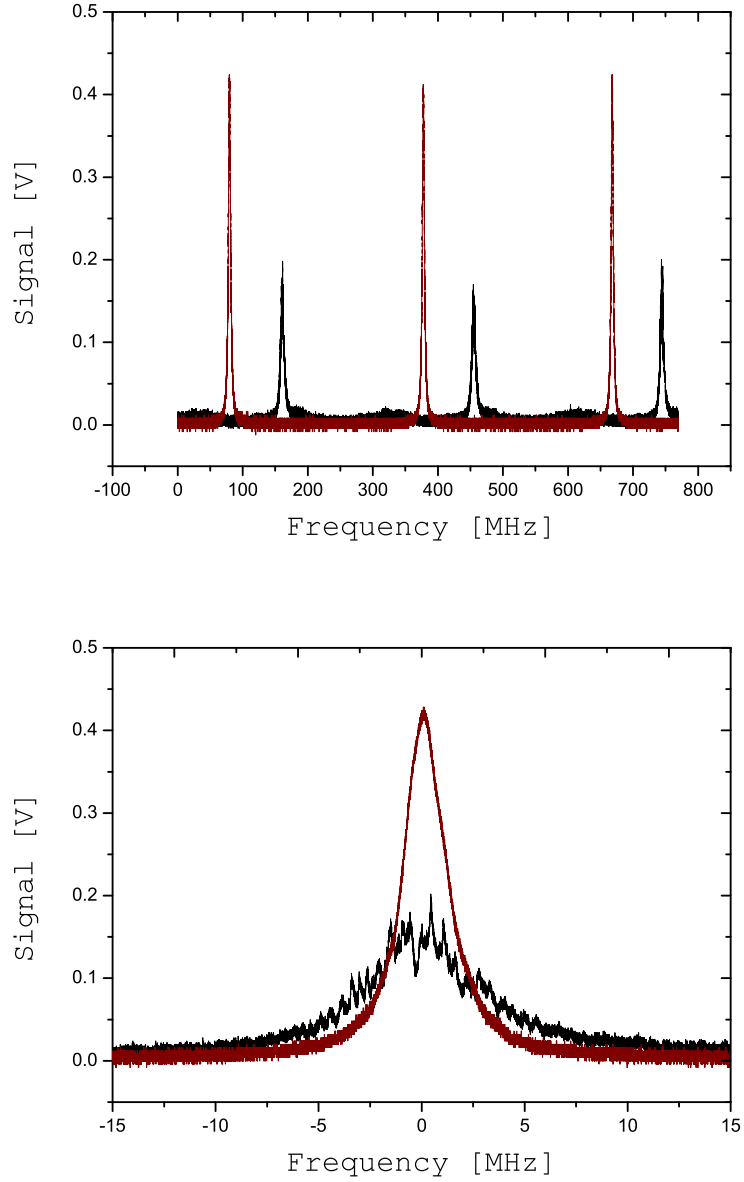


Fig. 4.19: Injection locking output spectral properties. The spectrum of the Fabry-Perot scan has been acquired in the injection locking working conditions, closing off (black line) or turning on (wine line) the reference laser beam. Above: two free spectral range of the Fabry-Perot interferometer. Below: comparison between free running and injected laser peak.

4.3 Tabletop experiment

4.3.1 Motivation and goal

The measurement technique presented in chapter 3 has the purpose to set equal the lengths of two linear resonators. Before to use it as length metrology tool for the ring laser, we made a tabletop experiment to verify the method. The used setup consists of two Fabry-Perot resonators simulating the diagonals of a He-Ne ring cavity on an optical bench. A block scheme of the developed apparatus is shown in figure 4.20. In the following I describe the laser optics and the test cavities. Both parts have been specifically designed and optimized in order to simulate the ring cavity diagonals on an optical bench. Details about the readout electronics are also given. Finally, the experimental results that demonstrate the capability of the technique, mainly presented in reference [135], are reported.

4.3.2 Test cavities features

The two cavities used in the tabletop experiment consist of spherical concave supermirrors, produced by REO company, typically used in He-Ne ring lasers. Figure 4.3.2 shows a drawing of the mechanical design of the resonators. The mirrors are allocated in holders provided with kinematic tip/tilt mounts, for a precise adjustment of their positions, and connected by Invar spacers, to improve the stability of the mechanical assembly. A piezoelectric translator is attached to the output mirror, in order to scan the cavity and implement the feedback correction to the cavity length. The measured mechanical response of the transducer is 2.2 nm/V (full range voltage: 700 V). A picture of the mirror holders is shown in figure 4.21. The setup is mounted onto an optical table equipped with a pneumatic isolation system that isolates it from vertical vibrations, and enclosed by a lead-wood shielding for thermo-acoustic insulation.

The parameters of the resonators are summarized in table 4.4. They have an identical mechanical design but, while the output mirrors have equal nominal specifications, the reflection coefficients of the input mirrors are different,

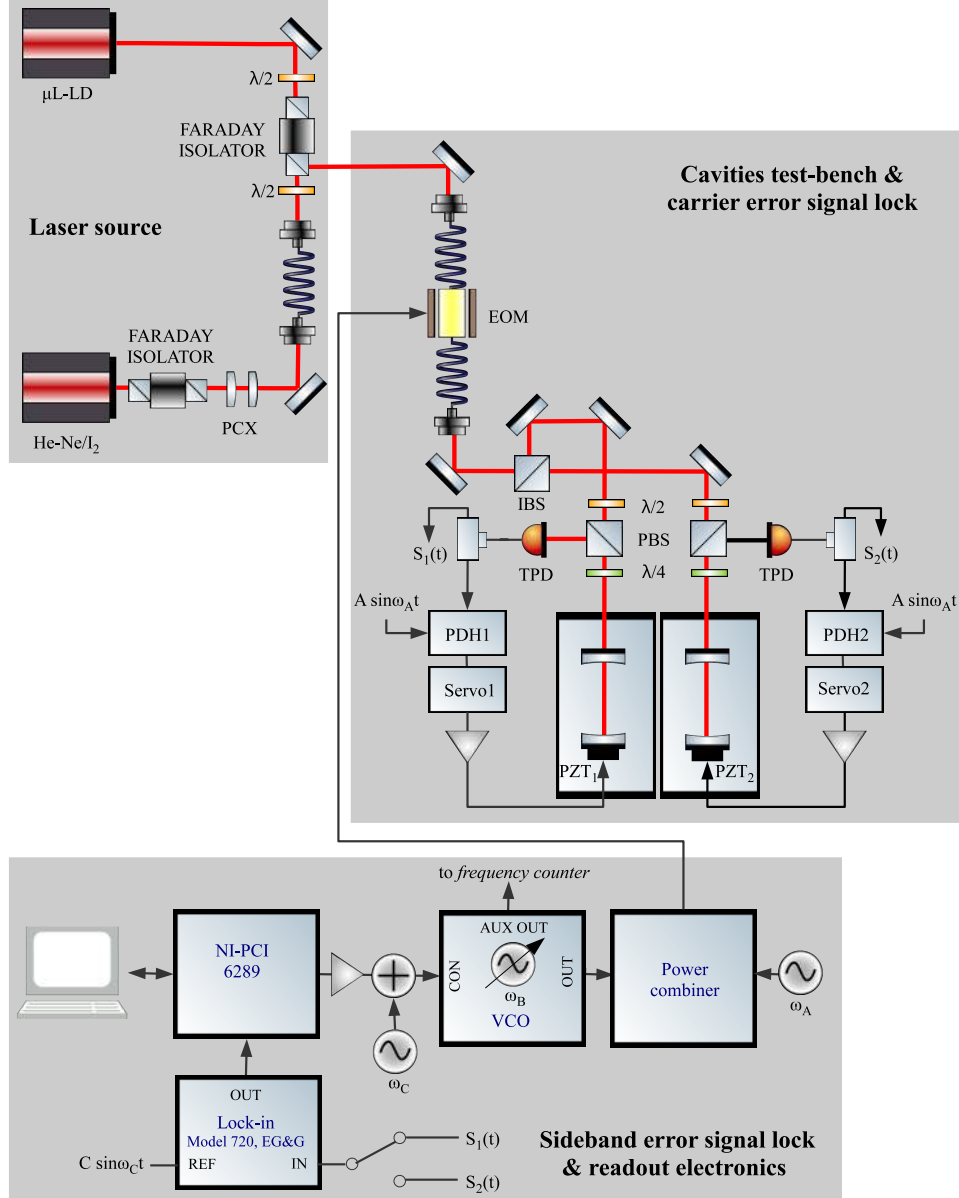


Fig. 4.20: Block scheme of the tabletop experimental apparatus. The upper part represents the optical breadboard with the injection locking setup. $\mu\text{L-LD}$ – LD: micro-Lens Laser Diode, M: Mirror, $\lambda/2$: Half Wave Plate, $\lambda/4$: Quarter Wave Plate, PBS: Polarizing Beam Splitter, IBS: Intensity Beam Splitter, OI: Optical Isolator, VCO: Voltage Controlled Oscillator, TPD: Transimpedance Photodiode, PDH: Pound Drever Hall detection stage.

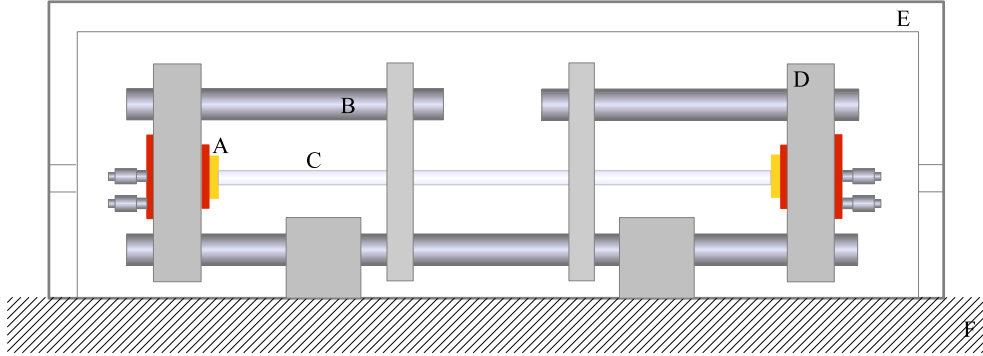


Fig. 4.21: Mechanical assembly of the Fabry-Perot cavities used in the experiment. A: concave spherical mirror; B: invar bars; C: capillary for laser beam alignment; D: aluminium tip-tilt mirrors holder; E: thermoacoustic insulation; F: damped optical table.

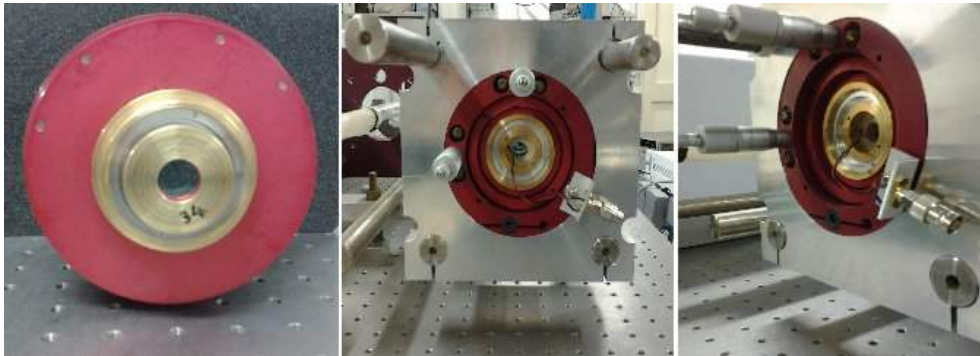


Fig. 4.22: Fabry-Perot cavities mirror assembly. The mirror, positioned in a brass container, is allocated in an aluminum holder provided with kinematic tip/tilt mounts. The photo on the right shows the piezoelectric translator attached to the output mirror too. The piezoelectric transducer is a hollow cylinder with a length and diameter of 20.3 mm, and a thickness of 1.5 mm.

leading to different cavity quality factors. The transmission spectra from the two cavities, interrogating with the external laser source, are shown in figure 4.3.2. Being typical ring laser mirrors, the optical coating is optimized in order to have a reflectivity at the level of 99.999% at 45 deg angle of incidence. Nevertheless, they have a good reflectivity at normal incidence too. The transmission curve of a ring laser supermirror when operated at normal incidence is reported in figure 4.25.

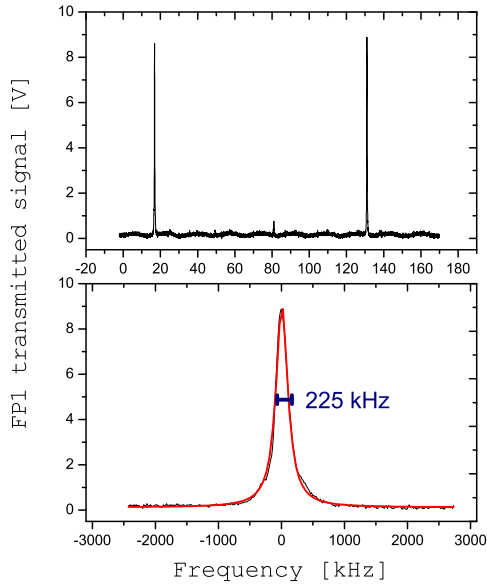


Fig. 4.23: Free spectral range spectrum and transmission linewidth of Fabry-Perot cavity 1. From the gaussian fit (red line; $\chi_R^2 = 0.99$) a linewidth of 225 kHz is estimated.

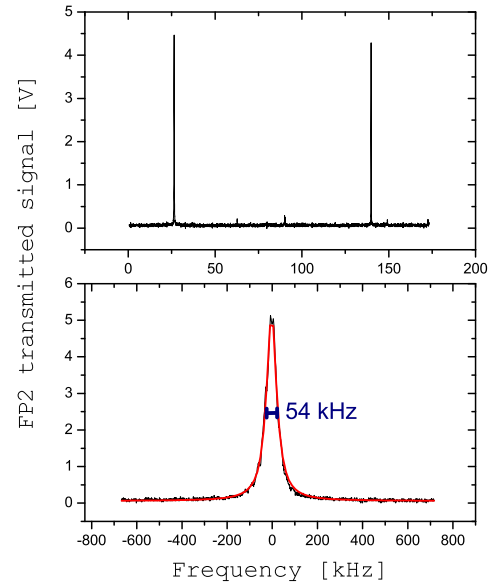


Fig. 4.24: Free spectral range spectrum and transmission linewidth of Fabry-Perot cavity 2. From the gaussian fit (red line; $\chi_R^2 = 0.99$) a linewidth of 54 kHz is estimated.

Table 4.4: Fabry-Perot cavities features.

Parameter	Cavity 1	Cavity 2
R	4 m	4 m
D	$\simeq 1.32$ m	$\simeq 1.32$ m
FSR	$\simeq 113$ MHz	$\simeq 113$ MHz
r_{in}^2	0.988	0.997
r_{out}^2	0.999	0.999
$\mathcal{F}$	504	2094
$FWHM$	225 kHz	54 kHz

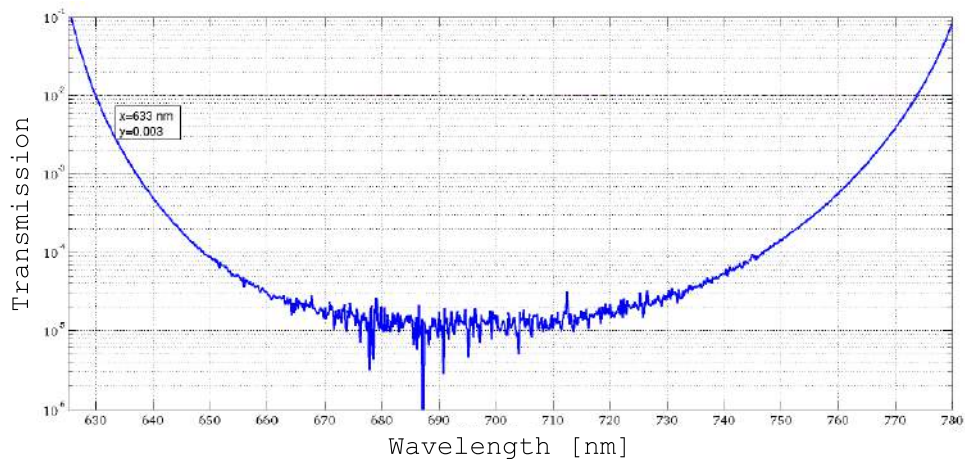


Fig. 4.25: Spectrophotometer measurement of the transmission spectrum of a ring laser supermirror when operated at normal incidence.

4.3.3 Readout and control electronics

The injection-locked diode laser beam is triple modulated at ω_A , ω_B and ω_C , as reported in equation 3.9. This is done by means of a single fiber-coupled integrated electro-optic phase modulator (EOM, Model PM635, Jenoptik). The modulation parameters used in the experiment are reported in table 4.5.

Table 4.5: Modulation parameters.

Modulation frequencies	Value
ω_A	30 MHz
ω_B	680 MHz ($\simeq 6\text{FSR}$)
ω_C	33 kHz
Modulation amplitude	Value
Δ	200 kHz
Modulation indices	Value
α	0.7
β	1.0

The modulated laser beam is then splitted with a 50:50 intensity beam splitter. One half of the laser intensity is reflected by the surface and sent to the cavity 1, while the other half of the intensity is sent to the cavity 2 after being reflected by a plane mirror (see figure 4.20). Before entering into the cavity, each beam passes through the optical elements needed to implement a PDH demodulation of the reflected light: an half-wave plate ($\lambda/2$), a polarizing beam-splitter (PBS) and a quarter-wave plate $\lambda/4$. The light reflected from each cavity, after passing through the quarter-wave plate and being reflected by the PBS, is detected with a silicon avalanche photodiode (APD), whose block diagram is shown in figure 4.26. The APD module output is amplified by an rf amplifier (Model W50ATC (0.01-50 MHz, 50dB), Trontech) and then is split using a 2-way equal-split Wilkinson power divider. One output of the power divider is demodulated at ω_A , according to the standard PDH scheme, in order to obtain the error signal $\epsilon_0(\omega)$ for the carrier lock. The second output of the divider is phase-detected by a digital lock-in amplifier (Model

7260, EG&G) referred to the modulation frequency ω_C . The amplitude of the reflected signal in quadrature with the modulation is used as error signal $\epsilon_S(\omega)$ for the sideband lock.

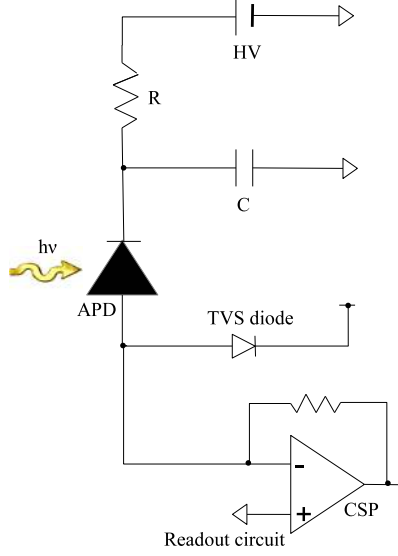


Fig. 4.26: Block diagram of the Si APD (Avalanche Photodiode) peripheral circuit. APD module consist of an APD, a bias power supply and a low-noise preamplifier circuit. HV: bias power supply (250 V); R: current limiting resistance (10 k Ω to 100 k Ω); C: capacitor 0.1 μ F or more (as close to APD as possible); TVS diode: Transient-Voltage-Suppression diode for excess voltage protection; CSP: high-speed Charge Sensitive Preamplifier.

The carrier error signals ϵ_0 are used to apply corrections to the cavity PZTs, in order to control the cavity mirror displacements. The feedback loops are two analog PI circuits with a bandwidth of about 1 kHz. The loop filter circuit is shown in figure 4.27. The sideband error signals ϵ_S are used to apply corrections to a VCO (Model ZX95-1300+, Mini-Circuits) generating the sidebands at $\omega_B = mFSR$. Once the two cavity resonance frequencies are locked to the same laser carrier frequency, the error signals $\epsilon_{S(1,2)}$ are processed by a LabView program running on a PC equipped with a DAQ board (Model NI-PCI 6289, National Instruments). It locks the VCO frequency alternatively to chosen harmonic of FSR_1 or FSR_2 , until the two interference orders $n_{1,2}$ are determined with the required precision. A microwave frequency counter (Model 5343A, Agilent/HP), connected to the auxiliary output of the VCO,

counts the $FSR_{1,2}$ data with a gate-time of 1 s. These are acquired by the PC via GPIB interface, using a LabView program.

4.3.4 Experimental results

The measured carrier and sideband error signals are shown in figure 4.28 and figure 4.29 respectively. The simulated error signals and the residuals of the measured ones are also shown. The carrier error signals are used to apply corrections to the cavity PZTs, in order to implement a first feedback loop that locks the two cavities resonances frequencies to the same optical frequency. The feedback signals controlling the two cavity lengths are reported in figure 4.30. In the same graph I reported the room temperature variation data, measured using a PT100 RTD sensor and acquired by a LabView program via a MCC USB-2404 Series board. The comparison between the traces shows that the main effect is a thermal expansion, in addition to a drift of the cavity mechanics. The residual noise in the error signal expressed as equivalent displacement noise for the two stabilized cavities is shown in figure 4.31.

The sideband error signal is used to implement the second feedback loop, that locks the VCO frequency to a multiple of the FSR. A gaussian fit on the distribution of the frequency counts provides the frequency mean value of the sample f_C and its standard deviation σ . To each cavity length L corresponds a distribution of the FSR data centered around $f = mFSR$. This is clearly shown in figure 4.32. Here, I changed the length of one cavity manually, moving the position of its output mirror. The FSR data distributions acquired before (FSR A) and after (FSR B) are shown.

Since the optical frequency is stabilized at the level of 1 part in 10^{11} , the uncertainty on the mode number difference δn_D is given by the FSR uncertainty σ_i/f_{ci} . For a given value of the linewidth of the laser source, the FSR uncertainty decreases as cavity finesse increases. We have studied the dependence of σ_i on the harmonic m too, and our experimental results suggest an improvement in FSR determination as m^{-1} . Figure 4.33 shows the comparison between the frequency data distribution obtained, for a same cavity, modulating the laser source at $\omega_B = 2FSR$ and that obtained modulating at $\omega_B = 6FSR$.

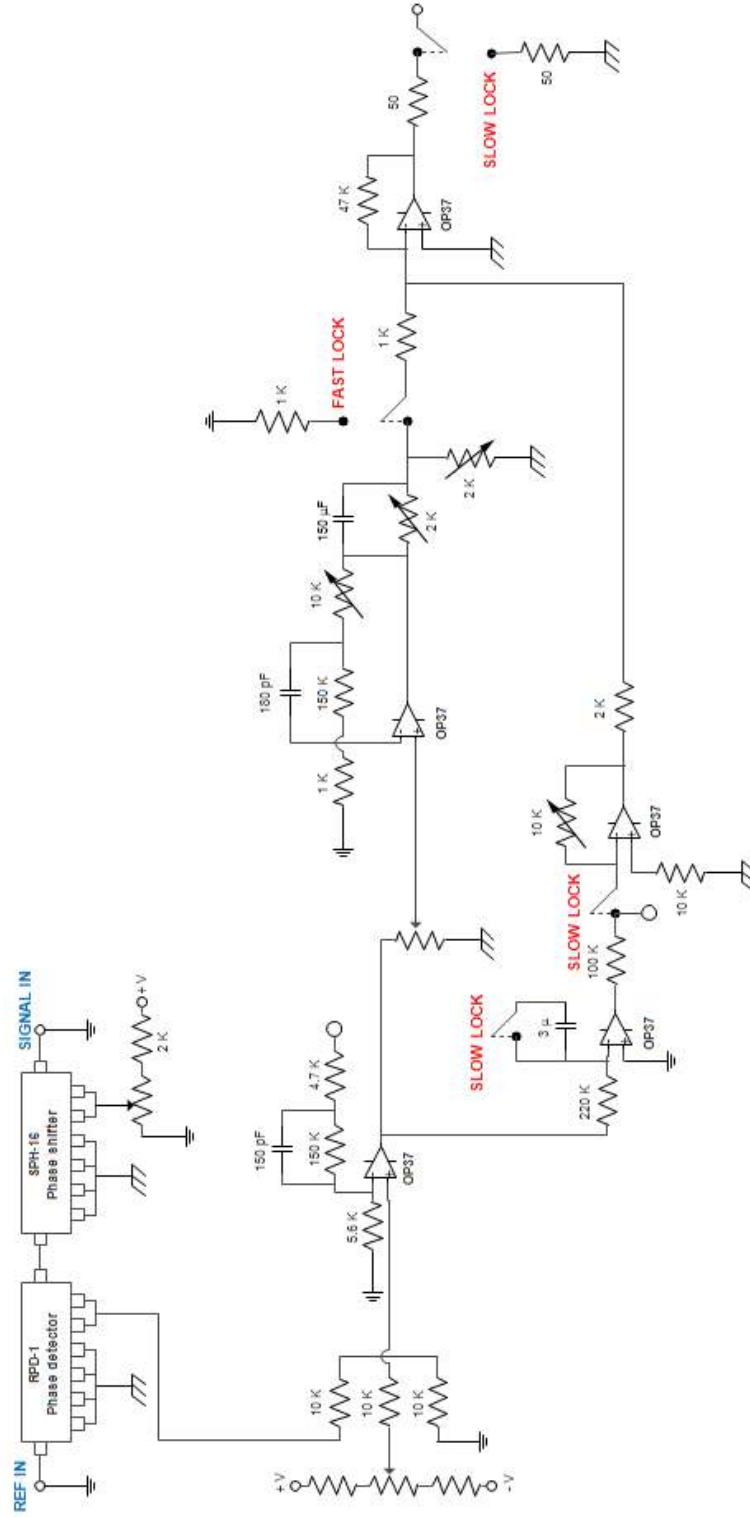


Fig. 4.27: Feedback loop filter PI circuit.

An example of the FSR frequency estimation is shown in figure 4.34 and figure 4.35 respectively for cavity 1 and cavity 2. The two FSRs are determined to few parts in 10^7 .

The mode number difference n_D distribution calculated from the measurements of figure 4.34 and figure 4.35 is reported in figure 4.36. It has been determined with an uncertainty of 1.6, corresponding to a length difference of 500 nm. The gaussian fit results on the distributions of the frequency counts and mode number difference are summarized in table 4.6.

Table 4.6: Gaussian fit results.

Parameter	Cavity 1	Cavity 2
Central frequency f_{ci}	681211560 Hz	680000798 Hz
Standard deviation σ_i	230 Hz	140 Hz
FSR uncertainty σ_i/f_{ci}	$\sim 3 \times 10^{-7}$	$\sim 2 \times 10^{-7}$
Reduced Chi-square χ_R^2	0.997	0.998

Parameter	Mode number difference
Central value n_D	7427
Standard deviation σ_{n_D}	1.6
Reduced Chi-square χ_R^2	0.997

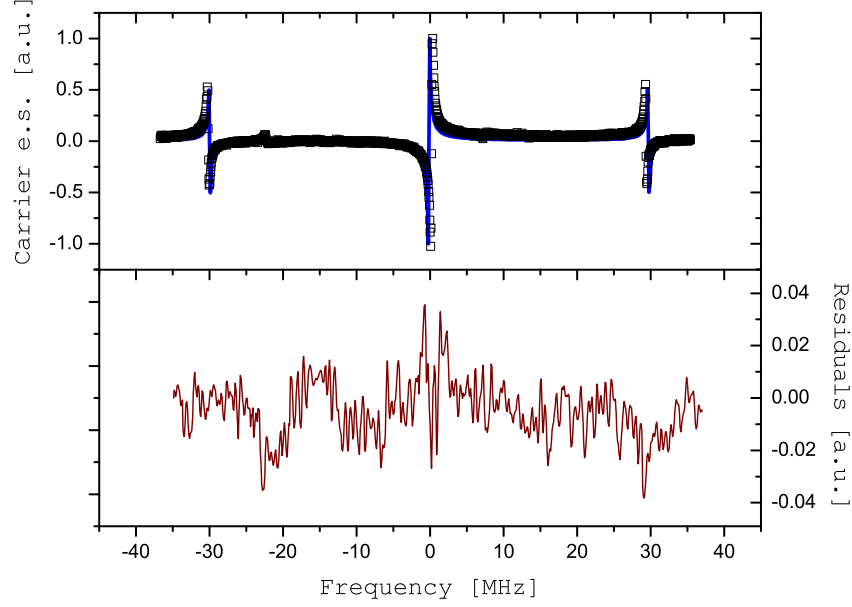


Fig. 4.28: Carrier error signal. Above: normalized simulated (continuous blue line) and measured (black squares) carrier error signal versus ω_0 , when $\omega_B \simeq 6FSR$. Sweep time: 100 ms. Slope: 6.85 V/MHz. Symmetry: 0.98. Below: residuals of the measured signal from the simulated one.

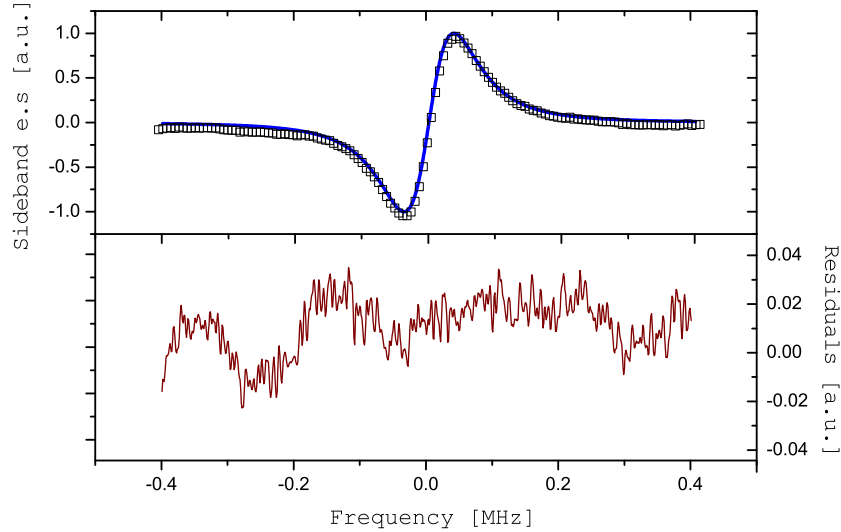


Fig. 4.29: Sideband error signal. Above: normalized simulated (continuous blue line) and measured (black squares) carrier error signal versus ω_B . Lock-in sensitivity: 10 V/1 mV, lock-in time constant: 640 μ s, sweep time: 5 s. Slope: 136.31 V/MHz. Symmetry: 0.96. Below: residuals of the measured signal from the simulated one.

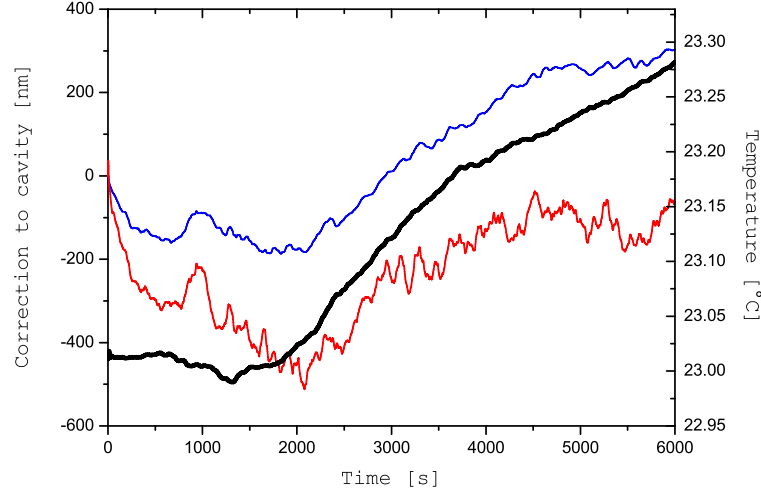


Fig. 4.30: Closed-loop corrections to Fabry-Perot cavities. Thin traces: closed-loop corrections to the cavities output mirror positions (blue line: cavity 1; red line: cavity 2). Thick trace: room temperature.

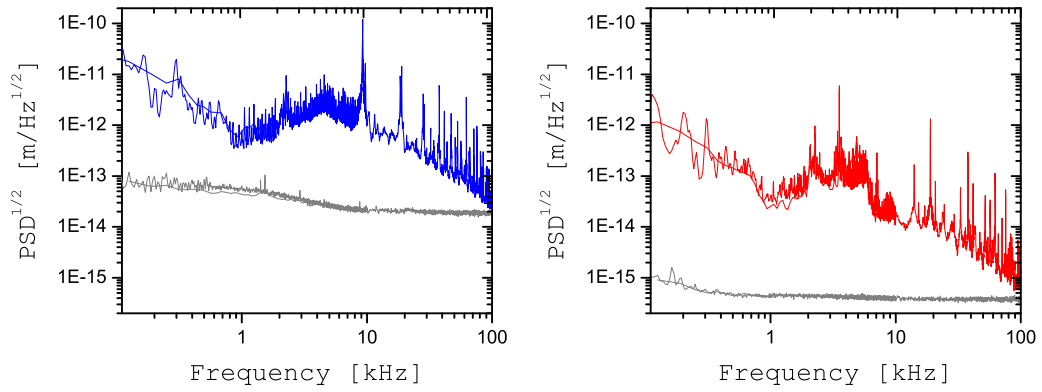


Fig. 4.31: Power spectral density of the residual displacement noise for the Fabry-Perot cavities. On the left: cavity 1; on the right: cavity 2. Gray lines are the electronic noise levels measured out of resonance.

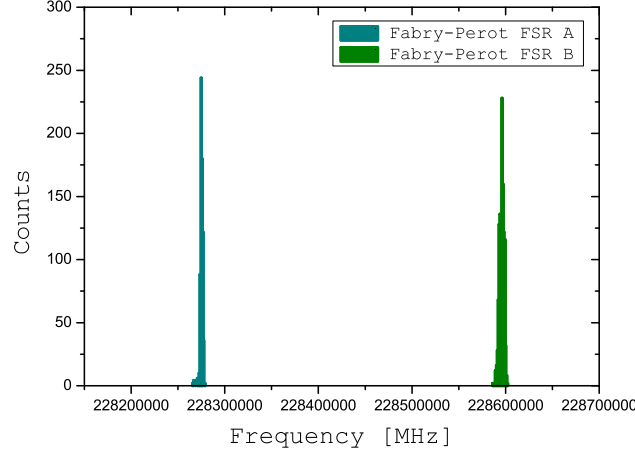


Fig. 4.32: FSR distribution displacement test. Distribution of $2FSR$ frequency counts corresponding to two different lengths of the same Fabry-Perot cavity, obtained by manually moving the input mirror holder. Total measurement time: 25 minutes. From the gaussian fits on the two distributions we obtain an estimate for the central frequency and the standard deviation σ . Cavity A (left): $f_{cA} = 228275122$ Hz, $\sigma_A = 1100$ Hz, $\chi_R^2 = 0.996$. Cavity B (right): $f_{cB} = 228595040$ Hz, $\sigma_B = 2000$ Hz, $\chi_R^2 = 0.997$. The calculated length displacement is $(1.8393 \pm 0.00010) \times 10^{-3}$ m.

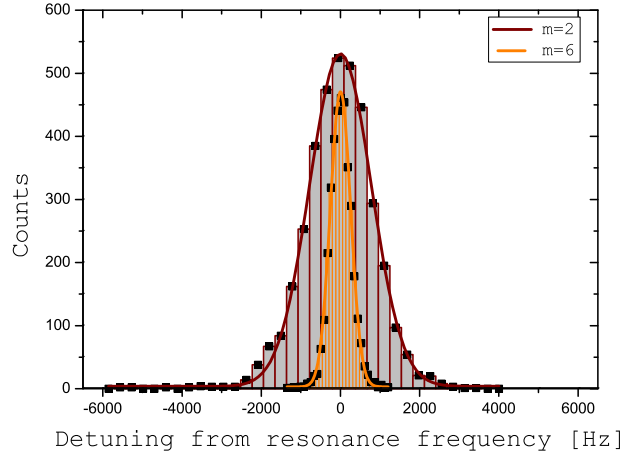


Fig. 4.33: FSR distribution for two different value of modulation harmonic m . Orange line: modulation at $\omega_B = 2FSR$; $\sigma_i = 730$ Hz. Brown line: modulation at $\omega_B = 6FSR$; $\sigma_i = 250$ Hz. Total measurement time: 140 min. Frequency counting-time: 1 s.

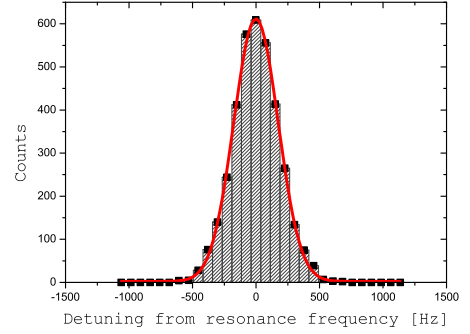
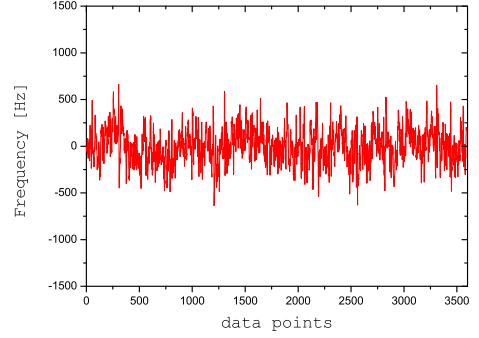
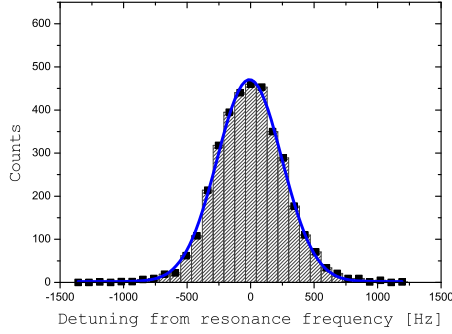
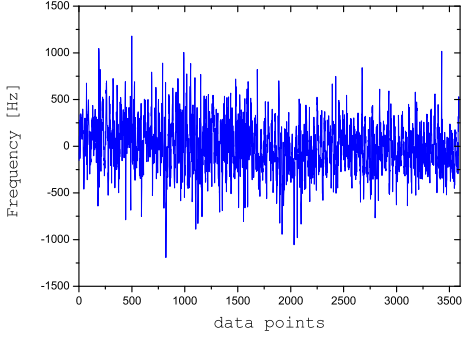


Fig. 4.34: FSR frequency estimation for the cavity 1. Frequency counting stability (above) and distribution (below) for the measurement of $6FSR$ for the cavity 1. Total measurement time: 140 min. Frequency counting gate-time: 1 s.

Fig. 4.35: FSR frequency estimation for the cavity 2. Frequency counting stability (above) and distribution (below) for the measurement of $6FSR$ for the cavity 2. Total measurement time: 140 min. Frequency counting gate-time: 1 s.

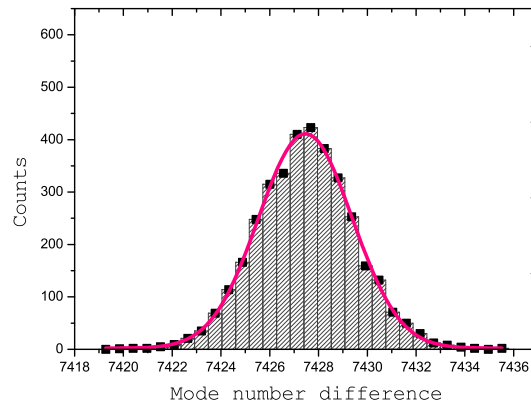


Fig. 4.36: Distribution of the mode number difference n_D obtained from the FSR measurements of figure 4.34 and figure 4.35.

4.4 Stabilization of GP2 diagonal resonators

4.4.1 Motivation and goal

The results achieved in the tabletop experiments motivated us to apply the length metrology technique to the diagonals of GP2. In this work I have studied mainly the optical signals related to the cavity geometrical parameters: 1) the laser gyro optical emission frequency, and 2) the diagonal cavities optical frequency resonances. The effects of the stabilization of the geometry on the Sagnac signal will be considered as a final goal and it is not included in this thesis. The compatibility of the worked out length metrology with the ring laser sensor is presented here as final result.

4.4.2 Diagonals in ring cavity features

The diagonals of a square RL form naturally two Fabry-Perot cavities, since the square geometry guarantees that opposite mirrors are parallel. In GP2, the vacuum chamber that encloses each diagonal consists in two steel pipes that are connected through a 4-way cross central piece. To guarantee a more mechanical stability, each pipe of the diagonal chamber is supported, as for the ring chamber, by aluminum mounts attached to granite. The diagonal vacuum chamber has been assembled in June 2015. In the same month we have started to implement the length measurement and control technique described in chapter 3, already tested in the tabletop experiment.

In figure 4.37 the FSR spectrum and the transmission linewidth of one diagonal cavity, interrogated with the reference laser, is reported. The mirror coating provides a high finesse of the diagonal cavities too. The measured cavity finesse is of ~ 920 , corresponding to a mirror reflectivity at normal incidence of about 99.66 %. The parameters of the diagonal cavities are summarized in table 4.7.

Let's notice that the stability of the diagonal resonators is guaranteed by that of the ring cavity. In fact, for a linear cavity made of two equal spherical mirrors having radius of curvature R and separated by a length l , the condition of stability is $0 \leq l/R \leq 2$ [83]. For the diagonal resonators, this condition

applies clearly with $l \equiv D = L\sqrt{2}$ and coincides with that of ring cavity in the sagittal plane.

The alignment of the diagonal resonator is made by using external optics only, as shown in the next section, without acting on the cavity mirror position. This guarantees that both diagonals and ring cavities resonate. A fine adjustment of the ring cavity alignment doesn't affect those of the diagonal cavities.

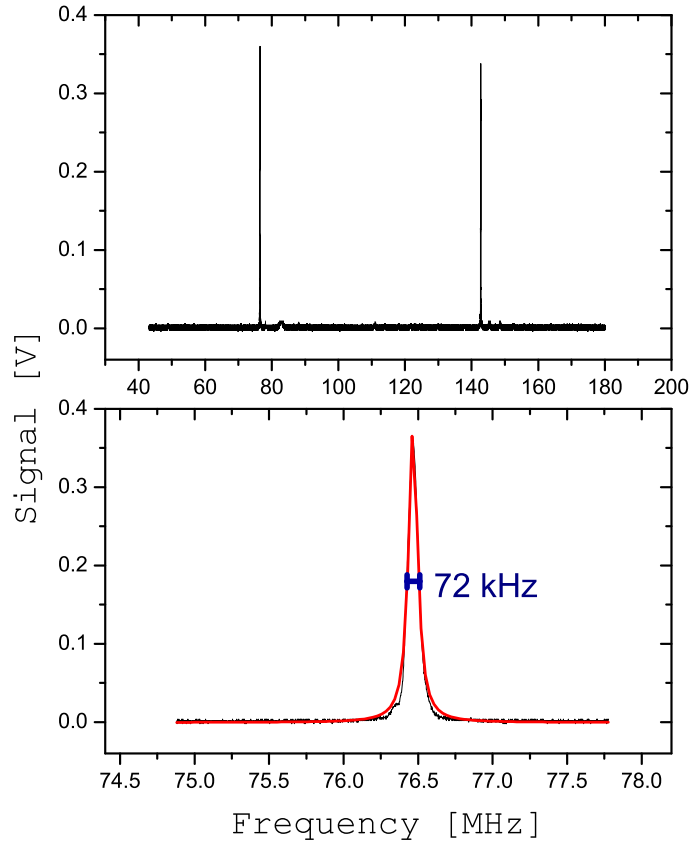


Fig. 4.37: Free spectral range and transmission linewidth of the diagonal resonators. From the gaussian fit (red line; $\chi_R^2 = 0.99$) a linewidth of 72 kHz is estimated.

Table 4.7: GP2 diagonal cavities features.

Parameter	Value
R	2 m
D	$\simeq 2.26$ m
FSR	$\simeq 66$ MHz
r^2	0.997
$\mathcal{F}$	920
$FWHM$	72 kHz

4.4.3 Readout and control electronics

To ensure a long-term run of the RL with the diagonal stabilization constraint, we divided the stabilized laser beam with a 50:50 bifurcated fiber (PMC630-50B-APC, Thorlabs) and we modulated the resulting two beams by using two identical fiber-coupled integrated electro-optic phase modulators (EOM, Model PM635, Jenoptik) 4.38 driven by a modulation signal $M_{i=1,2}(t)$

$$M_{i=1,2}(t) = \alpha \sin(\omega_A t) + \beta \sin(\omega_{Bi} + \Delta \sin(\omega_C t))t, \quad (4.3)$$

where, as in section 3.4, ω_A is the frequency modulation providing the Pound-Drever-Hall signal for carrier lock; ω_{Bi} is the frequency modulation for the detection of the i -th dynamical cavity resonance; ω_C is a frequency dithering applied for shifting the FSR detection in the kHz frequency range. The injection locking setup is the same we used in the tabletop experiment (see section 4.3). However, we made some adjustments, replacing the fiber into which the reference laser is coupled with a polarization maintaining one, and removing the PCX lenses used in the previous setup to optimize the diode laser spot (being no more necessary). In addition, the whole source laser has been closed in a wood box covered by a plexiglass cover that screens from acoustic disturbances and air currents. This makes the light source operation more stable.

The modulated beams are then coupled into two patch cables 10 m long (P1-630PM-FC-10, Thorlabs), in order to cover the distance between the op-

tical table and the ring laser (figure 4.39). The different fibers are joined by using polarization-maintaining mating sleeves (ADAFCPM2, Thorlabs). Due to fiber insertion losses, one twelfth of the reference laser output power can exit each fiber branch; a maximum power of ~ 1 mW is achieved at each end.

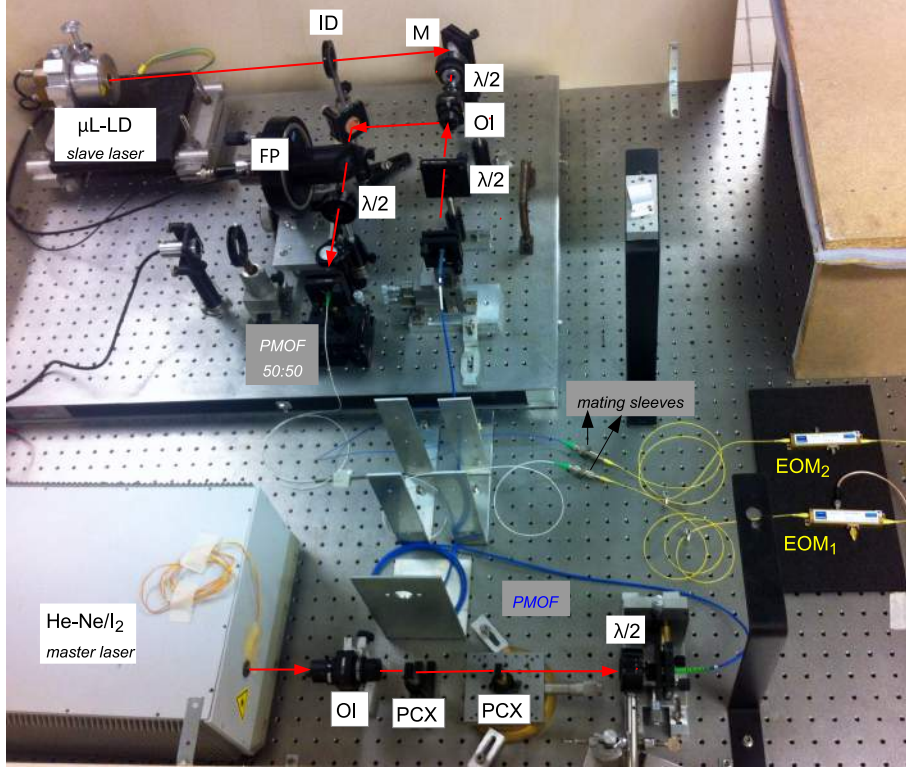


Fig. 4.38: Injection locking setup for GP2 diagonals interrogation. ID: iris diaphragm. M: mirror. $\lambda/2$: half-wave plate. OI: Faraday optical isolator. PCX: cylindrical convex lenses. PMOF: Polarization Maintaining Optical Fiber. EOM: Electro-Optic Modulator. FP: Fabry-Perot scanning interferometer.

To get an efficient coupling between the interrogating laser beam and the diagonal cavity, the two spatial modes must be matched. To match the laser beam mode to that of the diagonal resonator the fiber is terminated with an adjustable aspheric collimator (CFC-8X-A, Thorlabs). The focal of the collimator is changed by rotating its outer barrel. Once the desired position is reached, the adjustment is locked into place. The diagonal has to be interrogate with a laser beam having the cavity waist in the middle of the cavity. To this aim, first I prepared the beam on the optical table to obtain the desired waist at a distance $l > D/2$. Then, I fixed the fiber on the optical breadboard,

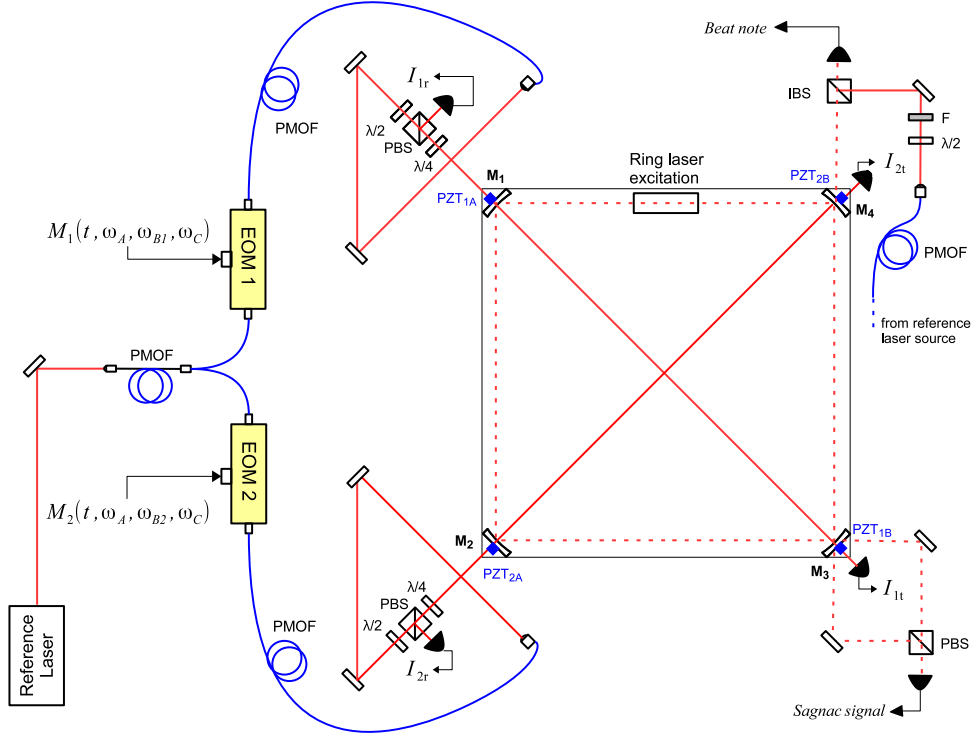


Fig. 4.39: Block scheme of GP2 experimental apparatus. The laser source block represents the injection locking setup. EOM: Electro-Optic Modulator. PBS: Polarizing Beam Splitter. IBS: Intensity Beam Splitter. PMOF: Polarization Maintaining Optical Fiber. $\lambda/2$: Half Wave Plate. $\lambda/4$: Quarter Wave Plate. PZT: Piezoelectric Transducer. F: neutral filter.

through a 2-axis platform, so that, by using two mirrors, it covers the needed distance to match the cavity. The matching is finally obtained by tilting the two mirrors and finely adjusting the focal of the collimator.

Before entering into the diagonal cavity, the laser beam passes through the optical elements needed to implement a PDH demodulation of the reflected light: an half-wave plate ($\lambda/2$), a polarizing beam-splitter (PBS) and a quarter-wave plate $\lambda/4$. The light reflected from each cavity, after passing through the quarter-wave plate and being reflected by the PBS, is detected by a Si PIN photodiode (S1722, Hamamatsu). Both the optical elements and the photodiode are mounted on the optical breadboard installed close to the cavity input mirror. The light transmitted from the diagonal cavity is detected by a

photodiode and amplified using a current amplifier (bandwidth: 4 kHz; gain: 1 GV/A; FEMTO, Laser Components). As for the ring cavity, the two laser spots transmitted from each diagonal are observed by using a webcam. This monitor is of great help in the cavity pre-alignment procedure too.

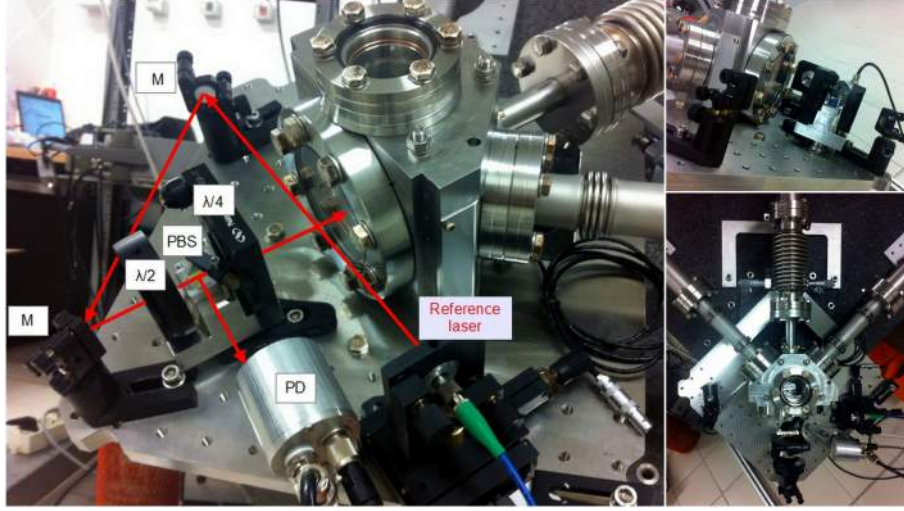


Fig. 4.40: Optical setup to implement the Pound-Drever-Hall locking of GP2 diagonals lengths. The optical elements are mounted on the optical breadboard installed close to the cavity input mirror. $\lambda/4$: Quarter Wave Plate; $\lambda/2$: Half Wave Plate; PBS: Polarizing Beam Splitter; M: Mirror.

In GP2 we use a digital PDH feedback system for the stabilization of the absolute lengths of its diagonals. Digital demodulation, filtering and feedback are carried out by using a multi-frequency digital lock-in amplifier (UHF2, Zürich Instruments). The block scheme of the signal processing and length control is shown in figure 4.4.3. The dual lock-in amplifier receives in input the two beams intensities I_{1r} and I_{2r} reflected from the two cavities, and synthesizes the phase modulations. For each diagonal resonator, it provides two phase error signals, ϕ_{Ai} at the modulation frequency ω_A and ϕ_{Ci} at the modulation frequencies ω_C , and generates two digital PIDs. One PID drives the PZTs of both mirrors forming each diagonal, while the other PID drives the VCO that synthesizes the modulations at ω_{Bi} .

A digital PID controller is more reliable respect to an analog one, since it is immune to electric noise, has a faster response and the controller parameters can be programmed and optimized easily.

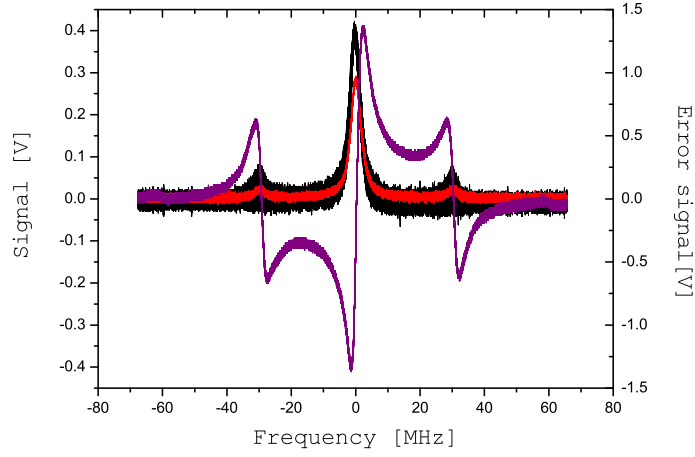
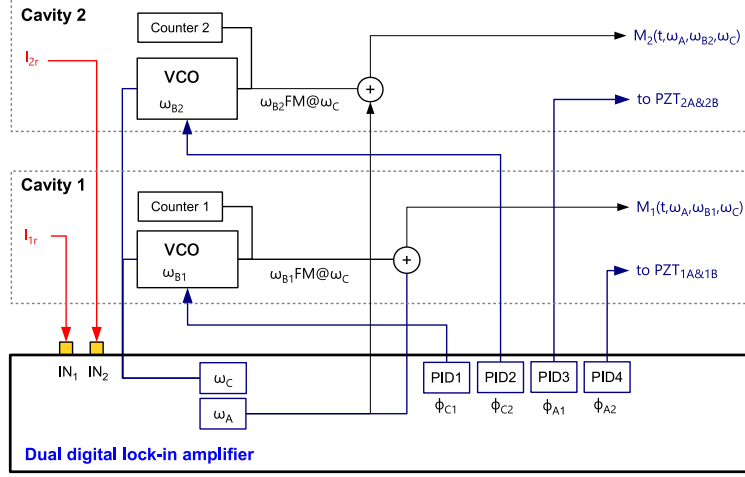


Fig. 4.41: Block scheme of the digital Pound-Drever-Hall feedback system. The dual digital lock-in amplifier (UHF2, Zürich Instruments) receives as input the beam intensities I_{1r} and I_{2r} reflected from the two cavities. It provides as output the phase error signals ϕ_{A1} , ϕ_{A2} , ϕ_{C1} , ϕ_{C2} that are processed by 4 digital PIDs. These lock the cavity lengths and the radio-frequencies ω_{B1} and ω_{B2} . In the above of the figure, the typical digital PDH signals, obtained by using a test Fabry-Perot scanning interferometer ($FSR = 1.56$ GHz), are shown. Red line: reflected laser spectrum; black line: transmitted laser spectrum; magenta line: PDH carrier signal. Modulation frequencies: $\omega_A = 30$ MHz, $\omega_C = 15$ kHz.

4.4.4 Experimental results

Optical frequency measurement

The measurement of the RL optical emission frequency is at the basis of the ring cavity perimeter control, and gives precise information about the average motion of the cavity mirrors. In GP2 the optical frequency measurement is obtained by beating the single beam emission of the ring against our reference laser. The setup we have used is shown in figure 4.39. A picture of the setup is shown in figure 4.42.

The two beams are superimposed with an intensity beam splitter on a fast

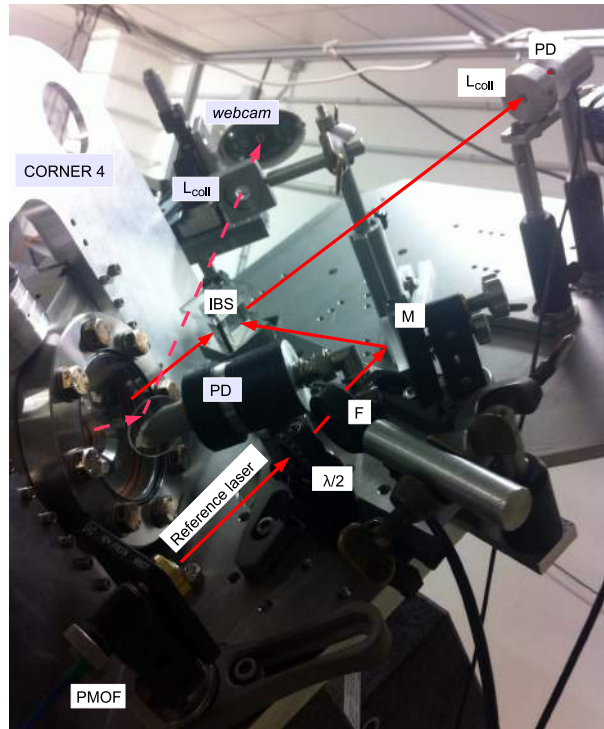


Fig. 4.42: Experimental setup for the measurement of GP2 optical frequency. The clockwise beam, exiting mirror 4, and the reference laser are superimposed with an intensity beam splitter on a photodetector. The setup used to monitor via webcam the light transmitted by diagonal cavity 2 is also shown. PMOF: polarization maintaining optical fiber. HWP: half-wave plate. F: neutral filter. M: mirror. IBS: intensity beam splitter. PD: photodetector. L_{coll} : collimating lens.

photodetector, and the signal is acquired using a real-time spectrum analyzer (RSA 6114A, Tektronix). As the two laser beams interrogating the diagonal cavities, the reference laser arrives at the optical bench through a patch cable 10 m long (P1-630PM-FC-10, Thorlabs); to this aim, one end of the 50:50 bifurcated fiber is used (see 4.4.3). The polarization of the reference laser is matched to that of the ring by a $\lambda/2$ plate. To observe the beat signal, the intensities of the two beams must be balanced. Clearly the intensity of the beat note increases with the square root of the product of the two intensities, but if one beam is much more intense than the other, the beat note is lost. Since the GP2 output power in the single mode regime is < 5 nW, while the reference beam power coupled into the fiber is ~ 1 mW, the intensity adjustment is made inserting a neutral filter before the $\lambda/2$ plate.

The beat note has been observed when the reference laser wavelength is kept resonant with different components of the iodine spectrum. In figure 4.43 an example of the acquired beat signal is reported (beat note around 135 MHz; f component of iodine spectrum). Both the spectrum and the spectrogram of the beat signal are shown. A drift of the central frequency is visible; as a consequence, two mode jumps are observables.

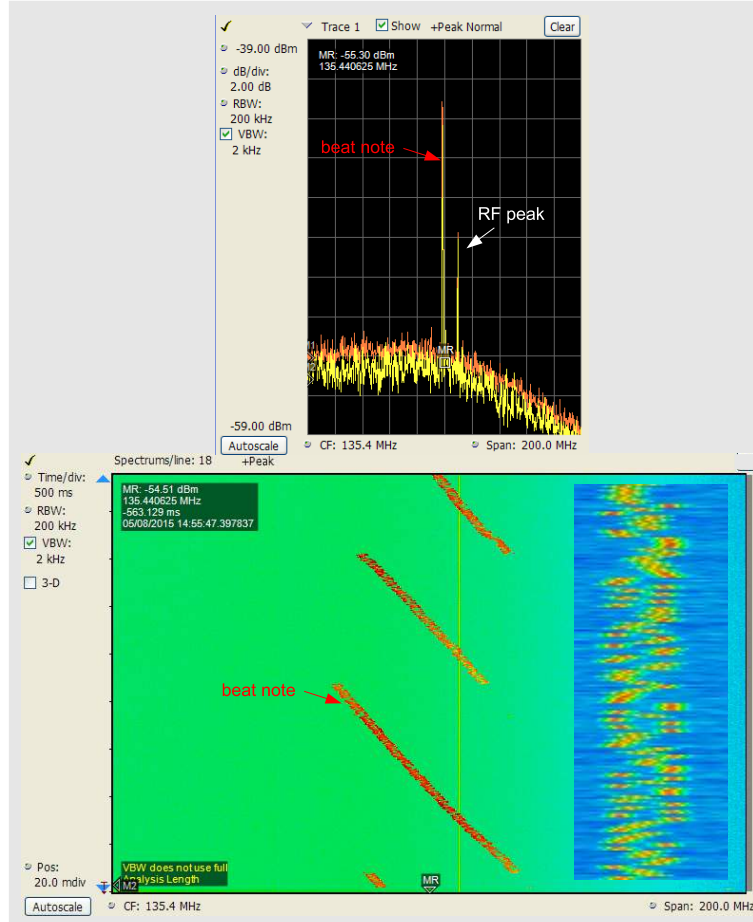


Fig. 4.43: Beat signal between the ring laser emission and the reference laser source. The spectrum (above) and the spectrogram (below) of the beat signal are shown. In addition to the beat note peak at around 135 MHz (f component of iodine spectrum, SNR:13 dB, RBW: 200 kHz), a peak at 145 MHz, due to the radiation emitted by the plasma excitation circuit, is observed. The spectrogram graph shows the long-term drift of the central frequency. Two mode jumps are clearly observed. Acquisition time: ~ 20 minutes.

Diagonals lengths locking performance

For each input channel, that receives in input the intensity reflected from each cavity, a demodulator is programmed activating an oscillator at frequency ω_A and selecting the phase of the detected quadrature, its filter order and bandwidth. The measured carrier error signal, acquired by a digital oscilloscope, is shown in figure 4.44.

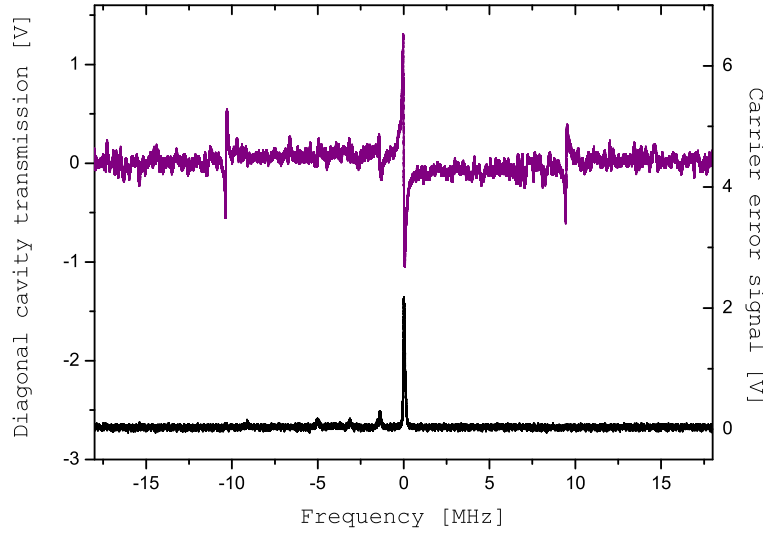


Fig. 4.44: Carrier error signal extracted from one of the GP2 diagonal cavities. From top to bottom: carrier error signal (magenta line), cavity transmission (black line). Modulation frequency: $\omega_A = 10$ MHz. Lock-in parameters: filter order 12 dB/Oct, bandwidth 5 kHz.

A digital PID receives in input the demodulator output and calculates the voltage corrections that are applied to the cavity mirrors, in order to implement the feedback loop that locks the two cavities resonance frequencies to the same optical frequency. The corrections are acquired by a PC equipped with a DAQ board (NI-PCI 6289, National Instruments) using a LabView program. The transmitted intensities and the error signals are acquired too. Figure 4.45 shows the acquisition of the zero-mean error signal of one of the two cavities when the feedback loop is enabled. Due to the limited bandwidth of the actuators, the residual frequency error oscillates around the zero set

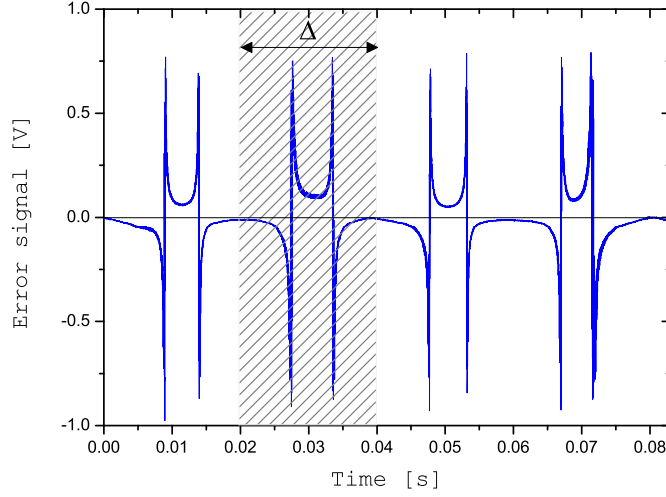


Fig. 4.45: Closed-loop acquisition of carrier error signal extracting from one of the two GP2 diagonal cavities. The asymmetry in the periodic repetition of the signal is due to the PZT hysteresis, enhanced by the non horizontal configuration of the RL. $\Delta = 6$ MHz.

point with a peak-to-peak amplitude of few MHz, corresponding to few nm. A sub-nanometer length stabilization of the diagonals cavities is hence obtained with a signal integration time of few hundred seconds.

The feedback signals controlling the two diagonal cavity lengths are reported in figure 4.46. Both signals follow the daily temperature fluctuation in the laboratory, that to date is not provided by a temperature stabilization system. The switch off of the air-conditioning/heating, indicated with a dashed blue line in the graphs, clearly affect the trend of the two signals. The difference between the two signal corrections, plotted in the same graph, accounts for time-dependent temperature gradients in the laboratory, differential mechanical relaxations, and also slightly different mechanical responses of the piezoelectric translator stages. The interruption of the measurement is typically due to the unlock of the reference laser from the hyperfine structure components of iodine, caused by the temperature drift.

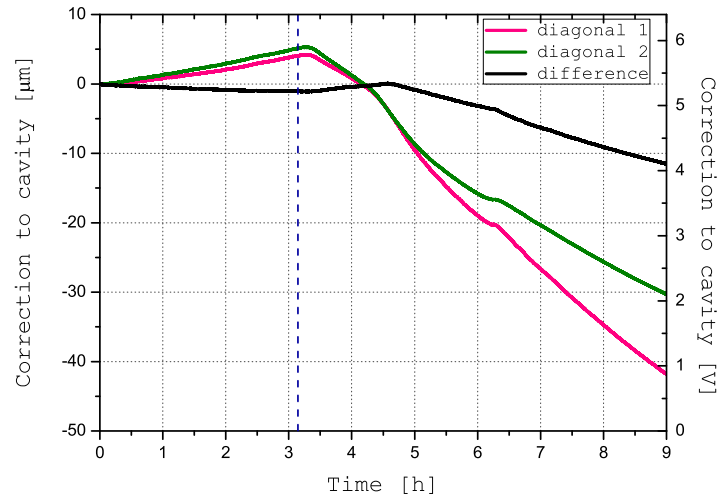


Fig. 4.46: Closed-loop corrections to both mirrors forming the GP2 diagonal cavities. Left axis: calculated corrections to cavity lengths versus time; right axis: voltage feedback signals versus time. Pink line: diagonal 1; green line: diagonal 2; black line: difference between the two signal corrections. The acquisition time is of 9 hours, starting from 4.52 pm. The dashed blue line indicates the switch off time of the air-conditioning in the laboratory at 8:00 pm.

CHAPTER 5

Conclusions and Future work

This chapter concludes the thesis with a discussion of the studies made, the setup developed and the achieved results. Some topics are also proposed in continuation of this work.

Results

This study has been conducted to investigate the use of a length metrology technique as a tool to improve the sensitivity of ring laser gyroscopes with a square shaped optical cavity. The aim is to reduce the fluctuation associated with the deformation of the optical cavity, stabilizing with a sub-nanometer accuracy the variation of the mirror inter-distances. The subject is an active research area in this field, since the deformation of the cavity geometry strongly affects the sensitivity of the present most stable ring lasers, and could be the ultimate limitation to the performance of the next-generation large frame devices. The sub-nanometer accuracy target is a strict requirement for the measurement of very tiny effects on rotation, as the Lense-Thirring effect, that are of the order of 1 part in 10^9 of the Earth rotation. This thesis reports the first experiment specifically worked out to solve this problem, and proposes a proved technique.

The approach used in the past relies into lock the ring laser emission frequency to an optical reference, in order to control the ring cavity perimeter. Since in this case the variation of the mirror inter-distances remains uncontrolled, we have chosen the lengths of the diagonal cavities formed by the opposite mirrors in square ring lasers as additional observables to constraint the geometry.

A beam steering simulation of the light propagation along a square ring

cavity, whose mirrors configuration is close to the regular shape, has demonstrated that:

- when the lengths of the two diagonals are locked to the same value, the perturbations to the mirror positions affect only quadratically the ring laser perimeter and scale factor;
- with this constraint, the regular square geometry corresponds to a saddle-point of the perimeter; this means that it's possible to implement an experimental procedure to approach the square configuration, by monitoring the perimeter length;
- the stabilization of both diagonals lengths better than 1 part on 10^{11} provides a high rejection from the ring laser scale factor of the mirror perturbations, with a residual fluctuation at level of 1 part in 10^{10} , even if their lengths differ up to 1 μm .

To stabilize the absolute length of the two diagonal cavities, an original interferometric technique has been worked out. Each diagonal is interrogated by an external laser beam, phase-modulated with a combination of three frequencies, allowing to lock the two cavities to the same laser carrier frequency and, at the same time, to determine an integer multiple of the two free spectral ranges (FSR). A suitable reference laser source, based on a single diode laser injection locked to a frequency stabilized He-Ne/Iodine frequency standard, has been developed. The technique has been verified in a tabletop experiment, using two Fabry-Perot cavities that simulate, on an optical bench, the diagonals of a He-Ne ring laser. We demonstrated:

- a robust operation of the reference laser source; in typical operational conditions, the laser system has an output power of ~ 14 mW, a locking range of 2.8 GHz, and a power amplification of the order of 7000;
- a stable locking of the resonance frequencies of the two cavities to the same reference laser, providing a length stabilization at the level of 1 part in 10^{11} at 100 s;
- the determination of the two FSRs to few parts in 10^7 , resulting in an error in the measurement of the cavity length difference of 500 nm.

The achieved results have encouraged us to apply the developed length metrology technique to GP2 ring laser. As a diagnostic of the perimeter length variation, the beat note of the gyro optical emission frequency with the reference laser beam has been detected. We conclude that:

- the carrier error signals have been successfully extracted, using the Pound-Drever-Hall technique, and the feedback correction to the cavities lengths has been implemented by a digital PID circuit; the measurement of the FSR of the two diagonals has been not implemented yet;
- the piezo actuators have been successfully tested;
- due to limited bandwidth of piezo transducers, the residual frequency error oscillates with an amplitude of few MHz around the zero set point; a sub-nanometer length stabilization is obtained with a signal integration time of few hundred seconds.

A length metrology technique has been provided, and its compatibility with a He-Ne square ring laser gyroscope has been demonstrated. To date, the main limit to the use of this tool are the uncontrolled environmental conditions in the laboratory, and the sources of noise connected to mechanical features of the device or to the light source. The primarily required upgrade is to equip the laboratory with an environmental control, in order to achieve a stable air temperature of $20^{\circ}\text{C} \pm 0.1^{\circ}\text{C}$, as defined by ISO 1.

Future work

We may consider the following subjects as future research strategies in continuation of this work:

- to stabilize both the diagonals and the perimeter lengths, so as to observe on this last the effect of residual degrees of freedom;
- to fully apply the length metrology technique to the ring laser, by measuring the FSR of the diagonal resonators. We have to take into account

that an unavoidable uncertainty on the measurement of the instantaneous length of the cavity is the acoustic. Since in a RL the mirror position actuators have a limited bandwidth (< 100 Hz) due to mirror holders inertia, we are investigating two possible solutions: to tightly lock the laser to the diagonal cavities using an acousto-optic modulator, that compensates this noise by shifting the laser frequency; to enlarge the bandwidth up to ~ 5 kHz, by using in vacuum small piezo transducers that directly act on mirrors;

- to extend the tunability of the developed laser source, in order to be able to test the passive operation of GP2 (clockwise and counter-clockwise resonators probed by an external light source).

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